

## Evaluating the contribution of slow recombination in localized subgap states to the excess quasiparticle population in ordered superconductors

Eva Gurra<sup>1,2,\*</sup>, Douglas A. Bennett<sup>1</sup>, Shannon M. Duff<sup>1</sup>, Michael R. Vissers<sup>1</sup>, and Joel N. Ullom<sup>1,2,†</sup>

<sup>1</sup>*National Institute of Standards and Technology, Boulder, Colorado 80305, USA*

<sup>2</sup>*Department of Physics, University of Colorado, Boulder, Colorado 80309, USA*



(Received 7 July 2025; accepted 13 January 2026; published 5 March 2026)

An excess density of quasiparticles is widely observed in superconducting films. This excess causes performance degradation in a variety of superconducting devices, including decoherence in qubits. In this Letter, we evaluate the hypothesis of Beshpalov *et al.* [Phys. Rev. Lett. **117**, 117002 (2016)] that the quasiparticle excess is caused by anomalously slow recombination at low quasiparticle densities due to localization in subgap states. We probe the density of states in aluminum and niobium films using current-voltage measurements of tunnel junctions and extract upper bounds on the energy scales of the subgap states and gap smearing. With these parameters, we evaluate the density-dependent recombination times calculated by Beshpalov *et al.* and find that slow recombination is not predicted to occur at observed quasiparticle densities in aluminum- and niobium-based superconducting devices. These results suggest that the quasiparticle excess in ordered superconductors is primarily due to nonthermal sources of quasiparticle generation and not slow recombination.

DOI: [10.1103/v181-yd74](https://doi.org/10.1103/v181-yd74)

**Introduction.** The performance of superconducting quantum circuits depends on a pristine environment to ensure high coherence and reliability. One bottleneck is the presence of broken Cooper pairs known as quasiparticles. It is widely observed that the quasiparticle density in superconducting devices is higher than what is expected from the equilibrium Bardeen-Cooper-Schrieffer (BCS) theory, which predicts an exponential decrease in the density at low temperatures. Excess quasiparticles are a source of decoherence in Josephson-junction-based superconducting qubits [1–4], loss in superconducting resonators [5–7], and noise in kinetic inductance detectors (KIDs) [8–10].

The steady-state quasiparticle population is reached when generation mechanisms are in balance with loss processes. BCS theory predicts the thermal quasiparticle density at finite temperature to be

$$n_{\text{qp}}(T) = \nu_0 \sqrt{2\pi k_B T \Delta} \exp[-\Delta/(k_B T)], \quad (1)$$

where  $\nu_0$  is the two-spin density of states (DOS),  $k_B$  is the Boltzmann constant, and  $\Delta$  is the superconducting gap energy. Consider an aluminum film with  $\nu_0 = 2.32 \times 10^{10} \text{ eV}^{-1} \mu\text{m}^{-3}$  and  $\Delta = 200 \mu\text{eV}$ . At experimentally achievable temperatures of 10 (50) mK, the expected thermal quasiparticle density is  $\sim 10^{-95} \mu\text{m}^{-3}$  ( $\sim 10^{-14} \mu\text{m}^{-3}$ ). In contrast, measured quasiparticle densities in aluminum, using a

variety of techniques, are typically  $1\text{--}10 \mu\text{m}^{-3}$  [5,9,11,12] and not lower than  $\sim 10^{-2} \mu\text{m}^{-3}$  [2,4,13,14].

One possible explanation for the excess quasiparticles is pair breaking by photons, phonons, or particles with energy larger than  $2\Delta$  that are not in thermal equilibrium with the superconductor. Examples include photons from warmer stages of the experimental apparatus [15–19] and phonons produced in the underlying substrate by background radiation [4,20–23]. However, observations of the quasiparticle excess persist despite a range of mitigation efforts such as careful filtering of electrical connections to the environment, and known pair-breaking mechanisms such as background radiation have not been shown to be sufficient to explain the excess. It is therefore interesting to consider other explanations for excess quasiparticles.

An alternative explanation is that quasiparticle loss rates are slower than the predictions of BCS theory. The dominant loss process, recombination, occurs when two quasiparticles form a Cooper pair with the simultaneous emission of a phonon. Beshpalov *et al.* [24] predict that recombination can be exponentially suppressed at low quasiparticle densities when quasiparticles are trapped in subgap states, which arise due to impurities and disorder in real superconducting films [25–28]. Reference [24] further hypothesizes that this slowdown in recombination, along with the nonequilibrium generation of quasiparticles, for example, by background radiation, is sufficient to explain the quasiparticle excess.

In this Letter, we extract the key parameters needed to evaluate the theory of Ref. [24] and assess whether its predictions for slow recombination explain the quasiparticle excess in two commonly used ordered superconductors: elemental aluminum and niobium. We analyze current-voltage curves of superconducting tunnel junctions and extract the gap smearing, as described by Abrikosov-Gor'kov (AG) theory [29], and an upper bound for the Lifshitz tail of localized states [25]

\*Contact author: [eva.gurra@colorado.edu](mailto:eva.gurra@colorado.edu)

†Contact author: [joel.ullom@nist.gov](mailto:joel.ullom@nist.gov)

which, to the best of our knowledge, have not been characterized in aluminum and niobium. Using these parameters, we calculate the range of quasiparticle densities where slow-recombination theory holds and find that the onset of slow recombination is predicted to occur at densities lower than those observed in various aluminum and niobium devices.

*Theory of slow recombination.* In BCS theory, we have a gapped superconductor where quasiparticle excitations cannot exist inside the gap. Quasiparticles generated by a pair-breaking event might initially have an energy well above the gap, but they subsequently relax to delocalized states near the gap edge via phonon emission. The steady-state quasiparticle density is achieved when the pair-breaking and recombination rates are in balance. In BCS recombination,  $\dot{n}_{qp} = -\bar{\Gamma} n_{qp}^2$ , where  $\bar{\Gamma}$  is a material-specific recombination constant [30]. Thus, the BCS recombination rate  $\Gamma_{BCS}$  is

$$\Gamma_{BCS}(n_{qp}) = \bar{\Gamma} n_{qp}. \quad (2)$$

The theory of slow recombination presented in Ref. [24] considers spatial gap fluctuations caused by imperfections and disorder present in real materials that can generate a tail of states below the gap whose density decays exponentially with energy relative to the gap edge. Now, quasiparticles that relax to the gap edge via phonon emission still have available states below the gap to settle into. When a quasiparticle populates a subgap state, it is spatially localized, and its decay terminates at an energy  $\varepsilon_c$  below the gap edge [31]. This energy depends on the gap smearing and characteristic energy decay of the Lifshitz tail, which we extract from density of states fits. From  $\varepsilon_c$ , a localization radius  $r_c$  is defined that sets the range of quasiparticle densities where slow-recombination theory holds (see Appendixes A and B for details).

The recombination of these localized quasiparticles depends on the spatial overlap of their wave functions. Considering the average distance between quasiparticles,  $r$ , there are two limits: (1) The low quasiparticle density limit where  $r \gg r_c$ , so quasiparticles are further away from each other and recombination is exponentially suppressed and (2) the high quasiparticle density limit where  $r \ll r_c$ , so the quasiparticles are spatially close to each other, and we recover the BCS result.

The slow-recombination rate as a function of quasiparticle density,  $n_{qp}$  can be expressed as

$$\Gamma_B(n_{qp}) \approx D \bar{\Gamma} r_c^{-3} \left( 29 \frac{n_{qp,c}}{n_{qp}} \right)^{\beta/3} e^{-\left( 29 \frac{n_{qp,c}}{n_{qp}} \right)^{1/3}}, \quad (3)$$

where  $D \approx 0.06$  and  $\beta = 0.41$  [24]. The critical quasiparticle density,  $n_{qp,c} = 5 \times 10^{-3} r_c^{-3}$ , sets the density range for which slow recombination holds. Therefore, recombination is exponentially suppressed when  $n_{qp} < n_{qp,c}$ , while for  $n_{qp} > n_{qp,c}$ , the BCS result is recovered.

*Density of states fitting.* To evaluate the recombination rate of Eq. (3), we carefully characterize deviations from the BCS density of states. As we can see in Fig. 1, in real materials there is both a rounding of the BCS singularity and subgap states, captured by the energy scales  $\varepsilon_g$  and  $\varepsilon_t$ , respectively. Here, we outline our fitting routine that combines Abrikosov-Gor'kov theory [29] to capture the rounding of the BCS singularity with a Lifshitz tail [25] to capture the subgap

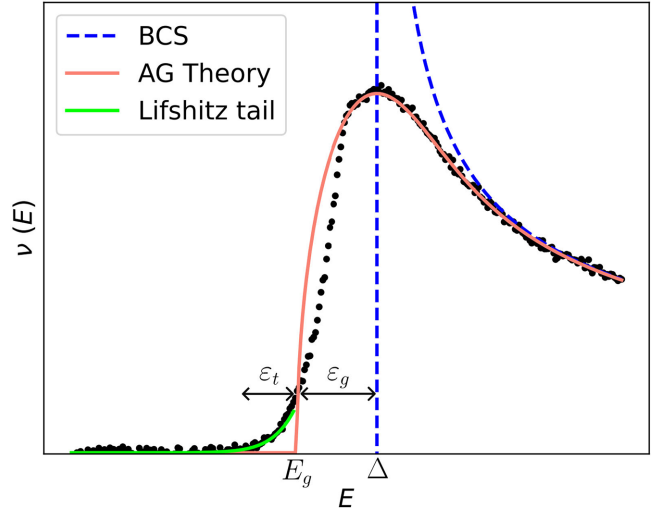


FIG. 1. Black points represent a typical density of states measurement. Dashed blue curve is the calculated BCS density of states with a hard gap at  $E = \Delta$ . Nonidealities in the DOS can be expressed as a broadening of the BCS singularity and subgap states below the shifted hard gap at  $E = E_g$ , which is quantified by Abrikosov and Gor'kov [29] theory (salmon curve) with energy  $\varepsilon_g$  and Lifshitz tail [25] (green curve) with a characteristic energy decay  $\varepsilon_t$ , respectively.

states. Nonidealities in the BCS DOS have been theoretically studied and observed. AG described the effect of magnetic impurities by a pair-breaking parameter  $\eta$  [29]. For small values of  $\eta$ , the DOS retains a hard gap but the gap edge shifts to  $E_g$  due to the rounding of the BCS singularity as can be observed in Fig. 1. The scale of the rounding is  $\varepsilon_g = \Delta - E_g$ . Details of the AG DOS formulation are given in Appendix A.

Larkin and Ovchinnikov [25] later showed that the problem of magnetic impurities is the same as considering fluctuations in the pairing potential. At the mean-field level, the AG result is recovered. Beyond mean field, rare optimal fluctuations (for example, in the presence of disorder) result in subgap states described by a Lifshitz tail. The characteristic decay energy of the tail is  $\varepsilon_t$  as shown in Fig. 1. The terminal quasiparticle energy  $\varepsilon_c$  is related to  $\varepsilon_t$ . Here, we make a step toward identifying the effects of these fluctuations in our data.

To extract the characteristic decay of the Lifshitz tail, we use a stretched exponential of the form

$$\nu_L(E) \propto \exp \left[ - \left( \frac{E_g - E}{\varepsilon_t} \right)^\xi \right], \quad (4)$$

where  $\xi$  is the characteristic exponent. Larkin and Ovchinnikov [25] derived an exponent  $\xi_{LO}$  by studying optimal fluctuations of  $\Delta$  in the Usadel equation close to the mean-field gap edge. Details on the functional form of the Lifshitz tail and the resulting  $\varepsilon_c$  and  $r_c$  parameters for both the original formulation in Ref. [24] with  $\xi_{LO}$  and the consequences of an alternative formulation of the Lifshitz tail with  $\xi_{MS}$  given in Refs. [32,33] are described in Appendix B.

Thus, we have a DOS which we can express as

$$\nu(E) = \begin{cases} \nu_{AG}(E, \Delta, \eta), & E \geq E_g, \\ \nu_L(E, E_g, \varepsilon_t), & E < E_g. \end{cases} \quad (5)$$

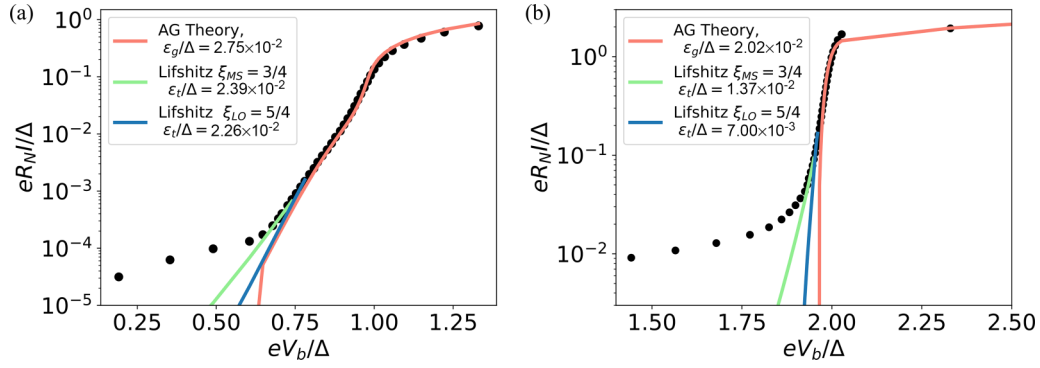


FIG. 2. Measured current-voltage curves for superconducting tunnel junctions (black points) and fits using AG theory (salmon curve) to extract  $\varepsilon_g/\Delta$  and two Lifshitz tails (formulation is given in Appendix B) to extract  $\varepsilon_t/\Delta$  for exponents  $\xi_{MS}$  (green curve) and  $\xi_{LO}$  (blue curve). (a) Data and fits for an Al-AlOx-AlMn NIS junction at 89 mK. Salmon curve includes a voltage-dependent temperature calculated using a model of NIS junction self-cooling [34]. (b) Data and fits for an Nb-AlOx-Nb SIS junction at 67 mK.

In this model, there are three free parameters,  $\Delta$ ,  $\eta$ , and  $\varepsilon_t$ . Parameter  $\varepsilon_g$  is constrained by  $\Delta$  and  $\eta$  (see Appendix A).

**Results.** To probe the DOS of aluminum and niobium, we take current-voltage measurements of tunnel junctions: (1) AlMn-AlOx-Al normal metal-insulator-superconductor (NIS) junctions and (2) Nb-AlOx-Nb superconductor-insulator-superconductor (SIS) junctions. The current through the junction as a function of applied voltage can be written as

$$I(V_b) = \frac{1}{eR_N} \int_{-\infty}^{\infty} dE v_l(E) v_r(E - eV_b) \times (f_r(E - eV_b) - f_l(E)), \quad (6)$$

where  $e$  is the electron charge,  $R_N$  is the normal resistance of the junction, and  $v_l$ ,  $v_r$  are the DOS of the left and right junction electrodes, respectively. We assume  $f(E) = (e^{E/k_B T} + 1)^{-1}$  to be the Fermi function and find minimal changes to our results from considering other occupation functions. A more detailed discussion on possible occupation functions, details of the measurements, and a summary of all the measured devices are included in the Supplemental Material [35].

Representative fits to the current-voltage data using the DOS in Eq. (5) are shown in Fig. 2. We observe that AG theory captures our data near  $\Delta$  but diverges from the data in the subgap. Including the Lifshitz tail improves the fit of the subgap data but does not capture points in the deep subgap where a linear leakage current persists. This leakage current is commonly observed in tunnel junctions at low voltages in both measurements of aluminum and niobium NIS junctions [13,36–39] and niobium tri-layer SIS junctions [40,41]. By choice, our DOS model does not include a separate term to capture this deep subgap leakage to ensure that  $\varepsilon_t$  is an upper bound on this quantity.

A variety of phenomena can cause subgap leakage including junction barrier pinholes and flux trapping. In addition, photon-assisted tunneling causes both subgap leakage and gap smearing [37]. The Dynes parameter  $\Gamma$  is commonly used to describe both leakage current and gap smearing [42]. Fits of our measured current-voltage curves using the Dynes model are shown in the Supplemental Material [35], where we find Dynes parameters similar to many literature values.

Because of the various contributions to subgap leakage and gap smearing that are unrelated to the true density of states of the superconductors, our fits for  $\varepsilon_g$  and  $\varepsilon_t$ , represent upper bounds on these energies.

We can make the upper bounds on  $\varepsilon_g$  and  $\varepsilon_t$  more realistic by considering measurements in the literature that achieved lower Dynes parameters than we found in our devices. These lower Dynes parameters can be attributed to better shielding, filtering, and junction quality, and it can be argued that they provide a more accurate representation of the true density of states. In particular, for aluminum NIS junctions, Saira *et al.* [13], and for niobium NIS junctions, Häfner *et al.* [36], achieved record-low Dynes parameters. We predict the associated  $\varepsilon_g$  and  $\varepsilon_t$  values for these devices using their published Dynes parameters and techniques given in Appendix C. A summary of the fit parameters  $\varepsilon_g$  and  $\varepsilon_{t,LO}$ , and calculated  $r_c$  and  $n_{qp,c}$  values are given in Table I for our aluminum and niobium measurements and for the more ideal aluminum and niobium measurements in the literature.

**Discussion.** The parameters extracted from fitting the current-voltage curves give an upper bound on gap smearing  $\varepsilon_g$  and Lifshitz tail decay  $\varepsilon_t$ . Using these to calculate localization radii  $r_c$ , we can predict an upper bound on the critical quasiparticle density  $n_{qp,c}$  that marks the onset of the exponential slowdown in recombination.

We plot the predictions of the slow-recombination theory of Ref. [24] and BCS theory in Fig. 3. The red and green curves are calculated using  $\varepsilon_g$  and  $\varepsilon_t$  from the more ideal junction measurements in the literature [13,36], and from our own tunnel junction measurements, respectively. The  $n_{qp,c}$  values predicted from these localization radii give a realistic upper bound on the range of cut-off quasiparticle densities for the onset of slow recombination. We assess the validity of the slow-recombination theory by comparing these  $n_{qp,c}$  values to both the lowest measured densities in the literature and to more typical measured values.

For aluminum, we can see in Fig. 3(a) that the highest  $n_{qp,c} = 2.9 \times 10^{-2} \mu\text{m}^{-3}$ , calculated for our measured devices, is smaller than typical densities measured in qubits, which fall between  $\sim 0.04$  and  $0.1 \mu\text{m}^{-3}$  [2,4,12], and  $\sim 3$  orders of magnitude smaller than the lowest densities observed in aluminum resonators, which are about  $\sim 10 \mu\text{m}^{-3}$  [5,9,11].

TABLE I. Summary of key fit parameters ( $\varepsilon_g/\Delta$ ,  $\varepsilon_{t,LO}/\Delta$ ) extracted from measured aluminum and niobium devices and lower Dynes parameter devices reported in the literature, calculated localization radii  $r_{c,LO}$ , and critical quasiparticle density  $n_{qp,c}$ .

| Device    | $\varepsilon_g/\Delta$ | $\varepsilon_{t,LO}/\Delta$ | $r_{c,LO}$ (nm) | $n_{qp,c}$ ( $\mu\text{m}^{-3}$ ) |
|-----------|------------------------|-----------------------------|-----------------|-----------------------------------|
| Al NIS    | $2.75 \times 10^{-2}$  | $2.26 \times 10^{-2}$       | 553             | $2.9 \times 10^{-2}$              |
| Ref. [13] | $3.56 \times 10^{-7}$  | $2.47 \times 10^{-7}$       | 2920            | $2 \times 10^{-4}$                |
| Nb SIS    | $2.02 \times 10^{-2}$  | $7 \times 10^{-3}$          | 121             | 3                                 |
| Ref. [36] | $1.28 \times 10^{-3}$  | $1.18 \times 10^{-3}$       | 188             | 0.75                              |

The very lowest measured quasiparticle density of  $\sim 5 \times 10^{-3} \mu\text{m}^{-3}$  [14] falls below the highest  $n_{qp,c}$  predicted for our measured devices but is still over an order of magnitude larger than the lower  $n_{qp,c} = 2 \times 10^{-4} \mu\text{m}^{-3}$  calculated for the more ideal literature DOS measurements. Since the lowest measured quasiparticle density is an order of magnitude higher than the most realistic upper bound for the onset of slow recombination, and since typical quasiparticle densities in qubits and resonators are larger than  $n_{qp,c}$  derived from our own devices, we conclude that slow recombination is not the cause of excess quasiparticle densities observed to date in aluminum.

Observed quasiparticle densities in niobium vary widely but consistently exceed the critical densities shown by the green and red dashed curves in Fig. 3(b). A recent result in a niobium-based qubit device [43] shows qubit lifetime saturation for Nb at  $\approx 1.1$  K, which corresponds to a quasiparticle density of  $33 \mu\text{m}^{-3}$ , higher than the  $n_{qp,c}$  for both the red and green curves in Fig. 3(b). This is consistent with the temperatures where the internal quality factor saturates in microwave kinetic inductance detectors, which serves as another probe of the excess quasiparticle density. For typical high-quality niobium films, with critical temperatures  $T_c$  ranging from 9.2–9.5 K, and high-quality factor niobium superconducting radio frequency cavities [44], the quality factor saturation occurs between  $\sim 0.1$  and  $0.2 T_c$  yielding quasiparticle densities as low as  $11\text{--}17 \mu\text{m}^{-3}$  [45,46] and as high

as  $10^3\text{--}10^4 \mu\text{m}^{-3}$  [47,48]. The lowest measured density of  $11 \mu\text{m}^{-3}$  [45] is comparable to our highest  $n_{qp,c} = 3 \mu\text{m}^{-3}$ , calculated for our measured devices, and is about 1 order of magnitude higher than the lower  $n_{qp,c} = 0.75 \mu\text{m}^{-3}$ , calculated from the most ideal DOS measurements in the literature. Therefore, we conclude that slow recombination is not the cause of the excess densities in niobium.

**Conclusion.** We have evaluated the theory of slow quasiparticle recombination [24] for ordered aluminum and niobium. The theory hypothesizes that the quasiparticle excess universally observed in superconducting films is caused by slow recombination and weak nonequilibrium generation. Quantitative evaluation of the slowdown requires detailed knowledge of the superconducting density of states. Using our own density-of-states measurements from superconducting tunnel junctions as well as measurements from the literature, we have extracted values for the AG gap smearing  $\varepsilon_g$  and the subgap Lifshitz tail  $\varepsilon_t$  for ordered films of aluminum and niobium. With these parameters, we calculate critical quasiparticle densities below which slow recombination should be present. We find that the predicted onset of slow recombination occurs at quasiparticle densities below the densities observed in superconducting films. While our results do not disprove a connection between subgap states and slow recombination, our results do show that the current theory for slow recombination [24] does not explain the quasiparticle excess in commonly used ordered super-

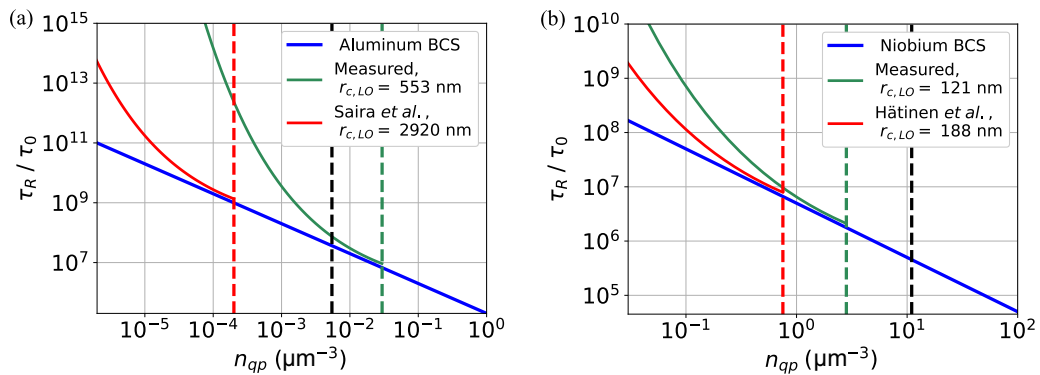


FIG. 3. Recombination time  $\tau_R$  in units of the electron-phonon coupling time  $\tau_0$  [30] plotted as a function of  $n_{qp}$  for aluminum (left) and niobium (right), where  $\tau_0 \bar{\Gamma} = \frac{4\Delta^2}{v_0(k_F T_c)^2}$ . Blue curves correspond to the BCS recombination time  $\tau_R = 1/\Gamma_{\text{BCS}}$  [Eq. (2)]. Red and green curves correspond to recombination time  $\tau_R = 1/\Gamma_B$  [Eq. (3)] from literature and our measurements, respectively. Dashed vertical red and green lines denote  $n_{qp,c}$  values from the literature and our measurements, respectively. Dashed vertical black lines denote the lowest measured  $n_{qp}$  in the literature. (a) Predicted recombination times in aluminum where  $\tau_{0,Al} \bar{\Gamma}_{Al} \approx 5 \times 10^{-6} \mu\text{m}^3$ . Our measurements yield  $n_{qp,c} = 2.9 \times 10^{-2} \mu\text{m}^{-3}$  and Saira *et al.* [13] yields a lower  $n_{qp,c} = 2 \times 10^{-4} \mu\text{m}^{-3}$ . The lowest measured  $n_{qp}$  in the literature is  $5 \times 10^{-3} \mu\text{m}^{-3}$  [14]. (b) Predicted recombination times in niobium where  $\tau_{0,Nb} \bar{\Gamma}_{Nb} \approx 2 \times 10^{-7} \mu\text{m}^3$ . Our measurements yield  $n_{qp,c} = 3 \mu\text{m}^{-3}$  and Häfner *et al.* [36] yields a lower  $n_{qp,c} = 0.75 \mu\text{m}^{-3}$ . The lowest measured  $n_{qp}$  observed in the literature is  $\sim 11 \mu\text{m}^{-3}$  [45].

conductors. This conclusion motivates both the consideration of alternative theories of slow recombination and the search for sources of nonequilibrium quasiparticle generation. Recent theoretical work in Ref. [49] reached similar conclusions to our own about slow recombination in subgap states in aluminum.

Future work in this area should also look at recombination in disordered superconductors, where slow recombination is predicted to occur at higher quasiparticle densities and thus may be more easily visible. Measurements of recombination in various disordered materials have been inconclusive [6,50,51], but recent work in Ref. [52] shows a link between quasiparticle localization and quasiparticle relaxation in disordered  $\beta$ -Ta resonators. Future work should also evaluate whether quasiparticle localization effects are suppressed by excitation by thermal phonons and readout photons.

**Acknowledgments.** We thank NIST ERB reviewers Jordan Wheeler and John Teufel for valuable feedback on the manuscript. We thank John Zasadzinski for helpful input on Dynes parameter measurements in niobium. We thank the U.S. Army Research Office for financial support through the Quantum Computing in the Solid State with Spin and Superconducting Systems (QC-S<sup>5</sup>) program. E.G. acknowledges financial assistance Award No. 70NANB23H027 from the U.S. Department of Commerce, National Institute of Standards and Technology.

**Data availability.** The data that support the findings of this Letter are not publicly available. The data are available from the authors upon reasonable request.

**Appendix A: Abrikosov-Gor'kov theory DOS formulation.** Here, we provide the functional form of the AG [29] density of states used to fit data in Figs. 2 and 5. To model the rounding of the BCS singularity, we start with the mean-field treatment of the Usadel equation [25,27],

$$iE \sin \theta_{AG} + \bar{\Delta} \cos \theta_{AG} - \bar{\Delta} \eta \sin \theta_{AG} \cos \theta_{AG} = 0, \quad (\text{A1})$$

where  $\eta$  is an effective depairing parameter,  $\theta_{AG}$  is the pairing angle, and  $\bar{\Delta}$  is the average value of the superconducting gap. Using this equation to solve for  $\theta_{AG}$ , the gapped quasiparticle density of states can be expressed as

$$\nu_{AG}(E) = \text{Re}(\cos \theta_{AG}). \quad (\text{A2})$$

In this case, the BCS coherence peak is smeared and the hard gap moves to  $E_g$  [32,53],

$$E_g = \bar{\Delta}(1 - \eta^{2/3})^{3/2}. \quad (\text{A3})$$

From this, we obtain  $\varepsilon_g = \bar{\Delta} - E_g$ . In the limit of  $\eta \rightarrow 0$ , we recover the BCS density of states.

**Appendix B: Lifshitz tail DOS formulation.** Larkin and Ovchinnikov [25] obtained an exponent  $\xi_{LO} = (8 - d)/4$  by looking at optimal fluctuations of  $\Delta$  in the Usadel equation close to the mean-field gap edge. The problem was revisited by Meyer and Simon [32,33], where further studies of the nonlinearity of the Usadel equation were done, and an exponent  $\xi_{MS} = (6 - d)/4$  was derived. This description is applicable for energies further from the gap edge [53]. Reference [24] derives the localization radius for the Lifshitz tail with exponent  $\xi_{LO} = 5/4$ , and here we also consider an

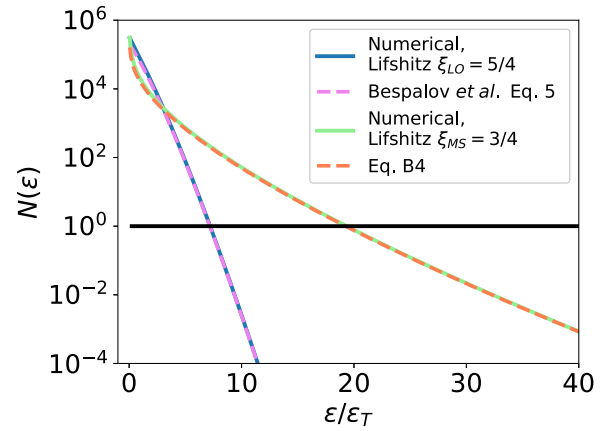


FIG. 4. Comparison of the numerical solution of the number of localized states that overlap with a given state,  $N(\varepsilon)$ , to the approximate solutions. For  $\xi_{LO} = 5/4$ , the numerical solution (blue) is compared to the approximate solution given in Eq. (5) of Beshpalov *et al.* [24] or Eq. (B2) in this Letter (magenta). For  $\xi_{MS} = 3/4$ , the numerical solution (green) is compared to the approximate solution given in Eq. (B4) (orange). The black line marks  $N(\varepsilon) = 1$  and the energy at which the curves intersect with this line define  $\varepsilon_c$ .

extension of those results for the Lifshitz tail with exponent  $\xi_{MS} = 3/4$ . These two localization radii give a range of critical densities  $n_{qp,c}$  that mark the onset of slow-recombination theory.

Starting with the formulation used in Ref. [24], the Lifshitz tail for a three-dimensional (3D) film has an exponent  $\xi_{LO} = 5/4$  and can be expressed as

$$\nu_{LO}(\varepsilon) = A \nu_0 (\varepsilon/\varepsilon_t)^{9/8} \exp[-(\varepsilon/\varepsilon_t)^{5/4}], \quad (\text{B1})$$

where  $A = 0.79 \sqrt{\varepsilon_t/\Delta}$  and  $\varepsilon = E_g - E$  is the distance from the gap edge. To compute the number of localized states that overlap with a given state, we integrate over the Lifshitz tail:

$$N(\varepsilon) = L^3(\varepsilon) \int_{\varepsilon}^{\infty} d\varepsilon' \nu_L(\varepsilon') \approx N_T (\varepsilon/\varepsilon_t)^{1/8} \exp[-(\varepsilon/\varepsilon_t)^{5/4}], \quad (\text{B2})$$

where  $L(\varepsilon) = (2/3)^{1/4} \xi (\varepsilon/\Delta)^{-1/4}$  and  $N_T = (4/5) A \nu_0 \varepsilon_t L(\varepsilon_T)^3$ . The coherence length  $\xi$  is calculated in the dirty limit as  $\xi \approx (\xi_{BCS} t)^{1/2}$  for a film of thickness  $t$ .  $N_T$  represents the number of localized states at  $\varepsilon \simeq \varepsilon_t$  and can be approximated as  $N_T \simeq N(\varepsilon_t) \sim g \frac{(\varepsilon_t/\Delta)^{3/4}}{\varepsilon_g/\Delta}$ , where  $g = \pi \nu_0 \Delta \xi^3$ . Localized quasiparticles end their relaxation at the terminal energy,  $\varepsilon_c$ , defined when  $N(\varepsilon_c) \simeq 1$ , which is approximately given by

$$\varepsilon_{c,LO} \approx \varepsilon_{t,LO} (\ln N_T)^{4/5}.$$

The localization radius is defined as  $r_{c,LO} = L(\varepsilon_{c,LO})/2$ . In the main text, we give the values of these parameters in Table I and plot the predicted slow-recombination time as a function of quasiparticle density in Fig. 3 for these parameters.

An alternative formulation based on Refs. [32,33] yields a Lifshitz tail for a 3D film with an exponent  $\xi_{MS} = 3/4$  and can be expressed as

$$\nu_L(\varepsilon) = A \nu_0 \exp[-(\varepsilon/\varepsilon_t)^{3/4}]. \quad (\text{B3})$$

TABLE II. Summary of key fit parameters ( $\varepsilon_g/\Delta$ ,  $\varepsilon_{t,MS}/\Delta$ ), extracted from measured aluminum and niobium devices and lower Dynes parameter devices reported in the literature, calculated localization radii  $r_{c,MS}$ , and critical quasiparticle density  $n_{qp,c}$ . This table replicates Table I except for the use of Eq. (B3) to describe the Lifshitz tail.

| Device    | $\varepsilon_g/\Delta$ | $\varepsilon_{t,MS}/\Delta$ | $r_{c,MS}$ (nm) | $n_{qp,c}$ ( $\mu\text{m}^{-3}$ ) |
|-----------|------------------------|-----------------------------|-----------------|-----------------------------------|
| Al NIS    | $2.75 \times 10^{-2}$  | $2.39 \times 10^{-2}$       | 409             | $7.3 \times 10^{-2}$              |
| Ref. [13] | $3.56 \times 10^{-7}$  | $3.31 \times 10^{-7}$       | 2510            | $3.2 \times 10^{-4}$              |
| Nb SIS    | $2.02 \times 10^{-2}$  | $1.37 \times 10^{-2}$       | 79              | 10                                |
| Ref. [36] | $1.28 \times 10^{-3}$  | $1.22 \times 10^{-3}$       | 150             | 1.5                               |

Following the same procedure as done with  $\xi_{LO}$ , we numerically compute the number of localized states that overlap with a given state, but integrating over the Lifshitz tail in Eq. (B3) this time.

Using Eq. (B3), we can approximate the number of states as

$$N(\varepsilon) = L^3(\varepsilon) \int_{\varepsilon}^{\infty} d\varepsilon' v_L(\varepsilon') \approx N_T(\varepsilon/\varepsilon_t)^{-1/2} \exp[-(\varepsilon/\varepsilon_t)^{3/4}], \quad (\text{B4})$$

where  $N_T = (4/3)A v_0 \varepsilon_t L(\varepsilon_T)^3$ . A comparison of the numerical results to the approximate results for both  $\xi_{MS}$  and  $\xi_{LO}$  cases is shown in Fig. 4.

For this formulation,  $\varepsilon_c$  is given by the equation

$$(\varepsilon_{c,MS}/\varepsilon_{t,MS})^{3/4} = \ln N_T - (1/2) \ln(\varepsilon_{c,MS}/\varepsilon_{t,MS}),$$

and we can define localization radius  $r_{c,MS} = L(\varepsilon_{c,MS})/2$ .

Table II is a summary of the fit parameters using the formulation with  $\xi_{MS}$  and the corresponding  $r_{c,MS}$  and  $n_{qp,c}$ . We can see that the  $r_{c,MS}$  values are smaller than  $r_{c,LO}$  and therefore yield a higher  $n_{qp,c}$ . These differences can be attributed to the fact that the Lifshitz tail with exponent  $\xi_{MS}$  is a better fit to the data deeper in the subgap and, therefore,  $\varepsilon_{t,MS}$  is likely inflated by contributions from subgap leakage. However, the relative ordering of the  $n_{qp,c}$  values in Table II and the observed quasiparticle densities summarized in the main text is largely unchanged from the  $\varepsilon_{t,LO}$  case of Table I. Thus, we can still conclude that the predicted onset of slow recombination is inconsistent with observed densities in aluminum and niobium devices.

*Appendix C: DOS fits based on literature data.* We look at junction measurements in the literature of NIS junctions, which have yielded even lower Dynes parameters in aluminum and niobium than what we have measured. These lower Dynes parameter values can be attributed to better filtering, shielding, and junction quality. For the aluminum measurement, capacitive shunting was employed to suppress subgap leakage due to photon-assisted tunneling, therefore yielding a very low Dynes parameter [13]. For the niobium junction, the fabrication was carefully tuned to yield junctions with low subgap leakage [36]. Therefore, we can view these measurements as getting closer to representing the true density of states for aluminum and niobium.

Based on the Dynes parameters  $\Gamma$  reported in Refs. [13], [36], we generated simulated DOS curves using the Dynes model, which was demonstrated to be a good fit to their data. The Dynes DOS is given by [42]

$$v(E) = \text{Re} \left( \frac{E + i\Gamma}{\sqrt{(E + i\Gamma)^2 - \Delta^2}} \right). \quad (\text{C1})$$

Using this model, we generate simulated data and fit the DOS using AG theory and the Lifshitz tails, as described in Appendixes A and B, respectively, to estimate  $\varepsilon_g$  and  $\varepsilon_t$ . The fits to the simulated Dynes curves are shown in Fig. 5. These values yield an improved upper bound on  $n_{qp,c}$  that better represents ordered aluminum and niobium devices.

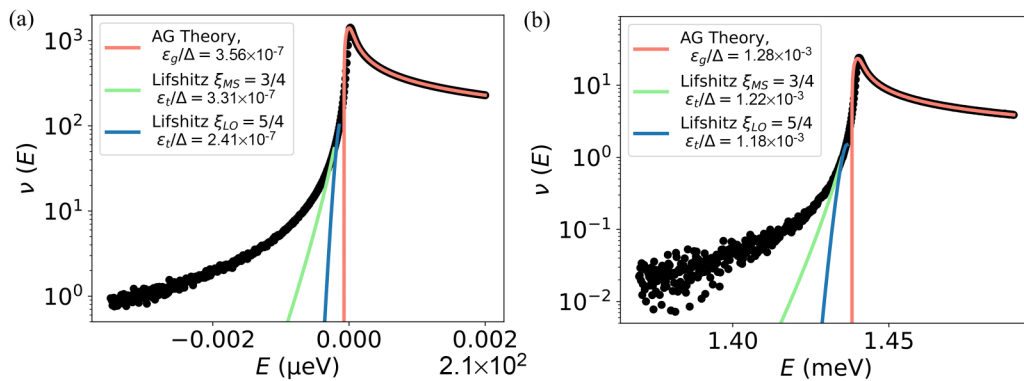


FIG. 5. Representative DOS fits to simulated data in black with AG theory (shown by the solid salmon color line) and the Lifshitz tail (shown by the solid green color line). (a) The simulated data correspond to one of the lowest Dynes parameters measured in literature for aluminum in Saira *et al.* of  $\Gamma/\Delta = 1.7 \times 10^{-7}$  [13]. The coherence length in the dirty limit is  $\xi \approx 245$  nm. (b) The simulated data correspond to a Dynes parameter of  $\Gamma/\Delta = 5.6 \times 10^{-4}$  measured in Hätinen *et al.* [36] for niobium. The coherence length in the dirty limit is  $\xi \approx 130$  nm.

- [1] L. Sun, *et al.*, Measurements of quasiparticle tunneling dynamics in a band-gap-engineered transmon qubit, *Phys. Rev. Lett.* **108**, 230509 (2012).
- [2] D. Ristè, *et al.*, Millisecond charge-parity fluctuations and induced decoherence in a superconducting transmon qubit, *Nat. Commun.* **4**, 1913 (2013).
- [3] C. D. Wilen, *et al.*, Correlated charge noise and relaxation errors in superconducting qubits, *Nature (London)* **594**, 369 (2021).
- [4] A. P. Vepsäläinen, *et al.*, Impact of ionizing radiation on superconducting qubit coherence, *Nature (London)* **584**, 551 (2020).
- [5] P. J. de Visser, D. J. Goldie, P. Diener, S. Withington, J. J. A. Baselmans, and T. M. Klapwijk, Evidence of a nonequilibrium distribution of quasiparticles in the microwave response of a superconducting aluminum resonator, *Phys. Rev. Lett.* **112**, 047004 (2014).
- [6] L. Grünhaupt, *et al.*, Loss mechanisms and quasiparticle dynamics in superconducting microwave resonators made of thin-film granular aluminum, *Phys. Rev. Lett.* **121**, 117001 (2018).
- [7] L. Cardani, *et al.*, Reducing the impact of radioactivity on quantum circuits in a deep-underground facility, *Nat. Commun.* **12**, 2733 (2021).
- [8] P. K. Day, *et al.*, A broadband superconducting detector suitable for use in large arrays, *Nature (London)* **425**, 817 (2003).
- [9] P. J. de Visser, J. J. A. Baselmans, P. Diener, S. J. C. Yates, A. Endo, and T. M. Klapwijk, Number fluctuations of sparse quasiparticles in a superconductor, *Phys. Rev. Lett.* **106**, 167004 (2011).
- [10] K. Karatsu, *et al.*, Mitigation of cosmic ray effect on microwave kinetic inductance detector arrays, *Appl. Phys. Lett.* **114**, 032601 (2019).
- [11] D. J. Temples, *et al.*, Performance of a phonon-mediated kinetic inductance detector at the NEXUS cryogenic facility, *Phys. Rev. Appl.* **22**, 044045 (2024).
- [12] K. Serniak, M. Hays, G. de Lange, S. Diamond, S. Shankar, L. D. Burkhardt, L. Frunzio, M. Houzet, and M. H. Devoret, Hot nonequilibrium quasiparticles in transmon qubits, *Phys. Rev. Lett.* **121**, 157701 (2018).
- [13] O. P. Saira, A. Kemppinen, V. F. Maisi, and J. P. Pekola, Vanishing quasiparticle density in a hybrid Al/Cu/Al single-electron transistor, *Phys. Rev. B* **85**, 012504 (2012).
- [14] T. Connolly, P. D. Kurilovich, S. Diamond, H. Nho, C. G. L. Bottcher, L. I. Glazman, V. Fatemi, and M. H. Devoret, Coexistence of nonequilibrium density and equilibrium energy distribution of quasiparticles in a superconducting qubit, *Phys. Rev. Lett.* **132**, 217001 (2024).
- [15] A. D. Córcoles, *et al.*, Protecting superconducting qubits from radiation, *Appl. Phys. Lett.* **99**, 181906 (2011).
- [16] R. Barends, *et al.*, Minimizing quasiparticle generation from stray infrared light in superconducting quantum circuits, *Appl. Phys. Lett.* **99**, 113507 (2011).
- [17] R. T. Gordon, *et al.*, Environmental radiation impact on lifetimes and quasiparticle tunneling rates of fixed-frequency transmon qubits, *Appl. Phys. Lett.* **120**, 074002 (2022).
- [18] C. H. Liu, *et al.*, Quasiparticle poisoning of superconducting qubits from resonant absorption of pair-breaking photons, *Phys. Rev. Lett.* **132**, 017001 (2024).
- [19] R. Anthony-Petersen, *et al.*, A stress-induced source of phonon bursts and quasiparticle poisoning, *Nat. Commun.* **15**, 6444 (2024).
- [20] M. McEwen, *et al.*, Resolving catastrophic error bursts from cosmic rays in large arrays of superconducting qubits, *Nat. Phys.* **18**, 107 (2022).
- [21] D. Gusenkova, *et al.*, Operating in a deep underground facility improves the locking of gradiometric fluxonium qubits at the sweet spots, *Appl. Phys. Lett.* **120**, 054001 (2022).
- [22] J. W. Fowler, *et al.*, Spectroscopic measurements and models of energy deposition in the substrate of quantum circuits by natural ionizing radiation, *PRX Quantum* **5**, 040323 (2024).
- [23] B. Loer, *et al.*, Abatement of ionizing radiation for superconducting quantum devices, *J. Instrum.* **19**, P09001 (2024).
- [24] A. Bespalov, M. Houzet, J. S. Meyer, and Y. V. Nazarov, Theoretical model to explain excess of quasiparticles in superconductors, *Phys. Rev. Lett.* **117**, 117002 (2016).
- [25] A. I. Larkin and Yu N. Ovchinnikov, Density of states in inhomogeneous superconductors, *Sov. Phys. JETP* **34**, 1144 (1972), [*Zh. Eksp. Teor. Fiz.* **61**, 2147 (1972)].
- [26] A. V. Balatsky, *et al.*, Impurity-induced states in conventional and unconventional superconductors, *Rev. Mod. Phys.* **78**, 373 (2006).
- [27] M. V. Feigel'man and M. A. Skvortsov, Universal broadening of the Bardeen-Cooper-Schrieffer coherence peak of disordered superconducting films, *Phys. Rev. Lett.* **109**, 147002 (2012).
- [28] S. E. de Graaf, *et al.*, Two-level systems in superconducting quantum devices due to trapped quasiparticles, *Sci. Adv.* **6**, eabc5055 (2020).
- [29] A. A. Abrikosov and L. P. Gor'kov, Contribution to the theory of superconducting alloys with paramagnetic impurities, *Zh. Eksp. Teor. Fiz.* **39**, 1781 (1960).
- [30] S. B. Kaplan, *et al.*, Quasiparticle and phonon lifetimes in superconductors, *Phys. Rev. B* **14**, 4854 (1976).
- [31] Reference [1] assumes zero temperature so that QPs that settle into localized states do not delocalize. At finite temperatures, it can be assumed that there are interacting populations of localized and delocalized quasiparticles as in Ref. [7]. Consequently, Ref. [1] can be interpreted as an upper bound on the effect of slow recombination in localized states.
- [32] J. S. Meyer and B. D. Simons, Gap fluctuations in inhomogeneous superconductors, *Phys. Rev. B* **64**, 134516 (2001).
- [33] J. S. Meyer, Mesoscopic phenomena driven by parallel magnetic fields, Ph.D. thesis, Universität zu Köln, 2001.
- [34] G. C. O'Neil, P. J. Lowell, J. M. Underwood, and J. N. Ullom, Measurement and modeling of a large-area normal-metal/insulator/superconductor refrigerator with improved cooling, *Phys. Rev. B* **85**, 134504 (2012).
- [35] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/v181-yd74> for further information on measured devices.
- [36] J. Härtinen, *et al.*, Efficient electronic cooling by niobium-based superconducting tunnel junctions, *Phys. Rev. Appl.* **22**, 064048 (2024).
- [37] J. P. Pekola, V. F. Maisi, S. Kafanov, N. Chekurov, A. Kemppinen, Y. A. Pashkin, O. P. Saira, M. Mottonen, and J. S. Tsai, Environment-assisted tunneling as an origin of the Dynes density of states, *Phys. Rev. Lett.* **105**, 026803 (2010).
- [38] N. R. Groll, *et al.*, Insight into bulk niobium superconducting RF cavities performances by tunneling spectroscopy.

- [39] D. Bafia, *et al.*, Investigating the anomalous frequency variations near  $T_c$  of Nb SRF cavities, in *Proceedings of the 20th International Conference on RF Superconductivity* (JACoW Publishing, Geneva, Switzerland, 2022), pp. 885–892.
- [40] T. Noguchi, *et al.*, Subgap tunneling current at low temperature in Nb/Al-AlN/Nb SIS junctions, *IEEE Trans. Appl. Supercond.* **21**, 756 (2011).
- [41] S. K. Tolpygo, *et al.*, Properties of unshunted and resistively shunted Nb/AlOx-Al/Nb Josephson junctions with critical current densities from 0.1 to 1 mA/ $\mu\text{m}^2$ , *IEEE Trans. Appl. Supercond.* **27**, 1 (2017).
- [42] R. C. Dynes, J. P. Garno, G. B. Hertel, and T. P. Orlando, Tunneling study of superconductivity near the metal-insulator transition, *Phys. Rev. Lett.* **53**, 2437 (1984).
- [43] A. Anferov, K. H. Lee, F. Zhao, J. Simon, and D. I. Schuster, Improved coherence in optically defined niobium trilayer-junction qubits, *Phys. Rev. Appl.* **21**, 024047 (2024).
- [44] A. Romanenko, R. Pilipenko, S. Zorzetti, D. Frolov, M. Awida, S. Belomestnykh, S. Posen, and A. Grassellino, Three-dimensional superconducting resonators at  $t = 20$  mk with photon lifetimes up to  $\tau = 2$  s, *Phys. Rev. Appl.* **13**, 034032 (2020).
- [45] A. Dominjon, *et al.*, Investigation of single-crystal niobium for microwave kinetic inductance detectors, *J. Low Temp. Phys.* **194**, 404 (2019).
- [46] S. Zhu, *et al.*, High quality superconducting Nb co-planar resonators on sapphire substrate.
- [47] R. Barends, *et al.*, Niobium and tantalum high  $Q$  resonators for photon detectors, *IEEE Trans. Appl. Supercond.* **17**, 263 (2007).
- [48] T. Noguchi, *et al.*, Characteristics of very high  $Q$  Nb superconducting resonators for microwave kinetic inductance detectors, *IEEE Trans. Appl. Supercond.* **29**, 1 (2019).
- [49] P. B. Fischer and G. Catelani, Excess quasiparticles and their dynamics in the presence of subgap states, *Phys. Rev. B* **112**, L100507 (2025).
- [50] R. Barends, S. van Vliet, J. J. A. Baselmans, S. J. C. Yates, J. R. Gao, and T. M. Klapwijk, Enhancement of quasiparticle recombination in Ta and Al superconductors by implantation of magnetic and nonmagnetic atoms, *Phys. Rev. B* **79**, 020509(R) (2009).
- [51] P. C. J. J. Coumou, *et al.*, Microwave properties of superconducting atomic-layer deposited tin films, *IEEE Trans. Appl. Supercond.* **23**, 7500404 (2013).
- [52] S. A. H. de Rooij, R. Fermin, K. Kouwenhoven, T. Coppens, V. Murugesan, D. J. Thoen, J. Aarts, J. J. A. Baselmans, and P. J. de Visser, Recombination of localized quasiparticles in disordered superconductors, *Nat. Commun.* **16**, 8465 (2025).
- [53] M. A. Skvortsov and M. V. Feigel'man, Subgap states in disordered superconductors, *J. Exp. Theor. Phys.* **117**, 487 (2013).