

Tunable intertwining via collective excitations

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The intertwining of multiple order parameters is a widespread phenomenon in equilibrium condensed matter systems, yet its exploration is often hindered by the complexity of real materials. Here, we present a controlled study of intertwined orders in a minimal and versatile driven-dissipative quantum-engineered platform. We consider a Bose-Einstein condensate at the intersection of two optical cavities, realizing two competing copies of a $\mathbb{Z}_2$ symmetry-breaking superradiant phase transition characterized by density-wave orders. Using periodic drives that exploit dynamical symmetry reduction, we show that collective excitations can be harnessed to stabilize a variety of out-of-equilibrium intertwined orders. Going beyond the conventional phenomenology involving Landau orders, we show the emergence of a larger class of out-of-equilibrium intertwined phases, including intertwining of purely time-crystalline orders, as well as between Landau and time crystal orders. These results should be observable in state-of-the-art experimental setups.

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Introduction. Complex quantum materials have ushered in the development of new paradigms of physics, exploring a confluence of ordered states in correlated electron systems. Such materials are often characterized by rich internal symmetries, hosting multiple symmetry-breaking channels, where distinct phases can coexist, and even induce novel ordered states. Foremost among these are the concepts of intertwined phases characterized by multicomponent order parameters and fluctuation-driven vestigial phases described by composite orders [1]. Though these were first studied in the context of high-temperature cuprate superconductors [2–4], they are now actively explored in diverse solid-state material families, such as kagome systems [5,6], heavy fermions [7,8], or transition metal dichalcogenides [9,10].

However, a controlled study of such novel orders in the solid state remains challenging. Recently, both classical and quantum light-matter interactions have been proposed as a tool to manipulate competing orders in solid-state systems. Notable examples include photoinduced superconductivity [11–13], Floquet engineered quantum matter [14], chiral superconductors [15], and light-matter intertwined states [16–21]. An interesting example arises in the attractive Hubbard model, where via competition between multiple order channels, a tailored drive dynamically stabilizes the elusive η -paired superconducting phase, where the pairs condense at the Brillouin-zone corner momenta [22,23]. Spatially modulated

density-wave orders naturally lend themselves to intertwining, as seen, for example, in the context of pair-density-wave superconductors [24]. A realm where matter-wave orders are rather prevalent is cavity-coupled cold atomic gases. Prominent examples include fermionic and bosonic superradiant density-wave orders [25–27] and supersolidity [28,29]. As these platforms are amenable to tailoring symmetries, they present an opportunity to explore competing orders in a controlled manner. Intertwining was demonstrated in Bose-Einstein condensate (BEC)-cavity setups [30,31] via the engineering of direct interactions that couple different order parameters.

In this work, we combine both aspects of coupling to quantum light and driving to show that periodic driving can be harnessed as a tool to realize a variety of nonequilibrium intertwined phases: intertwined Landau orders, intertwined time crystal (TC) orders, and intertwining of Landau and TC orders. We explore a minimal model hosting competing order parameters, realized experimentally by placing a BEC at the intersection of two crossed cavity modes, each facilitating a $\mathbb{Z}_2$ symmetry-breaking superradiant phase (SP) transition characterized by density-wave ordering [29,32]. Threading together the disparate notions of parametric resonance, discrete time crystals [33,34], and dynamical symmetry reduction leading to massive Goldstone (GS) modes, we show that periodic driving of collective excitations offers a promising route to realizing a variety of novel dynamical phases.

We consider a BEC trapped at the intersection of two crossed cavity modes and illuminated by a transverse pump, as depicted in Fig. 1(a). Photons can scatter from the BEC into either cavity through Raman processes, which leads to the recoil of the atoms into a finite momentum state and simultaneous population of the cavity field, generating a lattice potential for the atoms. Using a mode expansion that retains the three lowest atomic states, one obtains the following

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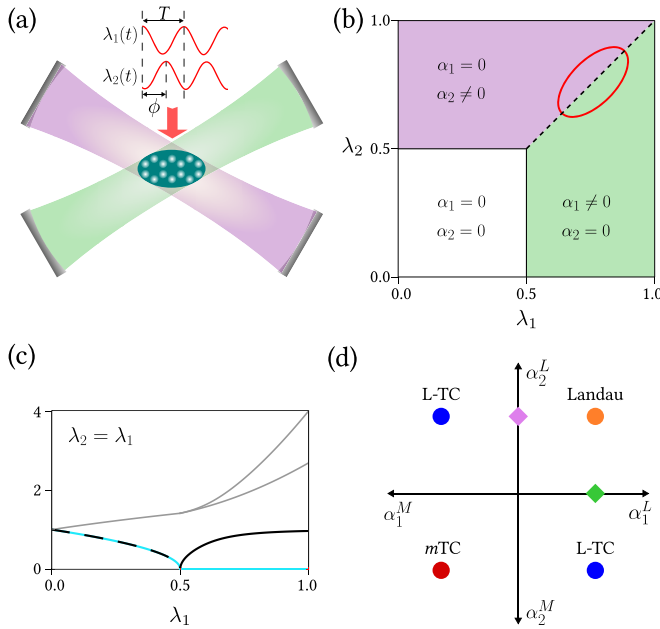


FIG. 1. (a) Schematic setup of the cross cavity. The BEC (turquoise patch) is illuminated by a transverse laser pump, from which photons scatter into cavities modes 1 and 2 (green and purple). The light-matter couplings $\lambda_{1,2}$ are modulated with time period T and a phase difference ϕ . (b) Static phase diagram of the model in Eq. (1) and schematic asynchronous driving scheme (red ellipse). White, green, and purple regions correspond to normal phase, and superradiant phase for cavity 1 and 2, respectively. Solid (dashed) lines indicate second (first)-order phase transitions. The line $\lambda_1 = \lambda_2$ is endowed with U(1) symmetry. (c) Polariton energies for $\lambda_1 = \lambda_2$, with Goldstone (Higgs) branches shown in cyan (black) and the higher-energy branches in gray. (d) Schematic depiction of intertwining, with $\alpha_i^{L,M}$ indicating Landau (L) and TC (M) order with respect to cavity i . Green (purple) diamonds indicate single-mode superradiance in cavity 1 (2) in the static system.

effective low-energy Hamiltonian describing the coupling between the two cavity modes and the atoms $H = H_1 + H_2$, with [32]

$$H_i = \omega_a a_i^\dagger a_i + \omega_b b_i^\dagger b_i + \frac{\lambda_i}{\sqrt{N}} (a_i^\dagger + a_i)(b_i^\dagger b_0 + b_0^\dagger b_i), \quad (1)$$

where ω_a and ω_b are, respectively, the detuning of the transverse pump relative to the cavity and atomic energies, λ_i are the light-matter couplings, N is the number of atoms, and a_i, b_i denote the annihilation operators for the cavity and the atomic state i , respectively. The atomic number satisfies the constraint $b_1^\dagger b_1 + b_2^\dagger b_2 + b_0^\dagger b_0 = N$.

Each $i = 1, 2$ on its own describes a simple $\mathbb{Z}_2$ -symmetric Dicke model, invariant under the parity operation $P_i(a_i, b_i) = (-a_i, -b_i)$ leading to a $\mathbb{Z}_2 \times \mathbb{Z}_2$ symmetry for the combined system. For $\lambda_{1,2} < \lambda_c = \sqrt{\omega_a \omega_b}/2$, the system is in the normal phase (NP), characterized by $\langle a_{1,2} \rangle = 0$ and $\langle b_{1,2} \rangle = 0$. When one of the couplings λ_i crosses the quantum critical point λ_c , the corresponding parity symmetry is spontaneously broken, and the system enters a superradiant phase (SP) characterized by finite $\langle a_i \rangle$ and $\langle b_i \rangle$. The two superradiant phases, associated with distinct cavities, are separated by a first-order phase transition. In standard parlance, α_1 and α_2 constitute

competing order parameters. We display the phase diagram of the static system in Fig. 1(b).

For $\lambda_1 = \lambda_2 = \lambda_0$, the Hamiltonian is endowed with a U(1) symmetry [29], whereby (a_1, b_1) and (a_2, b_2) can be rotated into each other continuously. Along the U(1) line, the excitation spectrum of the NP for $\lambda_0 < \lambda_c$ features two degenerate polariton branches, one of which fully softens at $\lambda_0 = \lambda_c$ where the U(1) symmetry breaks spontaneously. This leads to a gapless Goldstone mode and a gapped Higgs mode in the SP, as shown in Fig. 1(c), where the associated Landau potential in the α_1, α_2 space assumes the Mexican hat shape, where $\alpha_{1,2} = \text{Re}\langle a_{1,2} \rangle$. When the U(1) symmetry is spontaneously broken, both order parameters become nonzero, resulting in a supersolid phase of the bosons, which was observed experimentally [29]. Tuning slightly away from the U(1) line, however, lifts the polariton degeneracy in the NP and renders the Goldstone mode massive.

Periodic driving. In the simpler Dicke model, a $\mathbb{Z}_2$ -symmetry-preserving drive protocol was shown to result in new dynamical phases that break time translation symmetry [35,36], whenever the drive was parametrically resonant with a polaritonic mode. Here, we consider an asynchronous driving protocol in which $\lambda_{1,2}(t) = \lambda_0 + \epsilon \cos(\omega t \mp \phi/2)$, with ϵ and ω the driving amplitude and frequency, respectively, and a phase lag ϕ between the driven cavities. When $\phi = 0$, we have a fully U(1)-symmetry-preserving drive. Then, for $\lambda_0 - \epsilon > \lambda_c$, the Goldstone mode in the SP remains pinned at zero energy and cannot be resonantly coupled to. The other polaritonic and Higgs modes can be excited via straightforward parametric resonance akin to the Dicke model.

In contrast, for $\phi \neq 0$ [red trajectory in Fig. 1(b)], even though the time-averaged drive preserves the U(1) symmetry, the instantaneous symmetry is $\mathbb{Z}_2 \times \mathbb{Z}_2$. This implies that all polaritonic excitation modes, including the original massless Goldstone mode, acquire a fluctuating gap. This is similar in vein to the drive used to induce η -paired superconducting states in Ref. [22]. There, the time-averaged drive preserves the full SO(4) symmetry of the system, while the instantaneous Hamiltonian typically preserves only the lower SU(2) symmetry. In our work, a measure of the U(1) symmetry breaking can be obtained by $\Delta(\phi) = |\overline{\lambda_1(t) - \lambda_2(t)}| \sim |\sin(\phi/2)|$, where overline denotes time average and $-\pi \leq \phi \leq \pi$. Note, however, that discrete time translation invariance $H(t + nT) = H(t)$ generated by $\mathcal{T}_T^n$ for integer n and $T = \frac{2\pi}{\omega}$ now endows the Hamiltonian with an additional symmetry $\mathbb{Z}^T$. We use the superscript T to differentiate it from the time-independent symmetries. For $\phi \neq 0$, the system is not amenable to a simple analysis of coupled Mathieu oscillators as in the Dicke model and necessitates other approaches. *High-frequency expansion.* To shed light on the effects of asynchronous driving, we employ a high-frequency or Magnus expansion. To this end, we rewrite the Hamiltonian as

$$H = H_0 + \epsilon [O_1 \cos(\omega t - \phi/2) + O_2 \cos(\omega t + \phi/2)], \quad (2)$$

where $H_0 = H|_{\epsilon=0}$ contains the time-independent part and $O_i = (a_i + a_i^\dagger)(b_i^\dagger b_0 + b_0^\dagger b_i)/\sqrt{N}$. A high-frequency expansion approximates the Floquet Hamiltonian as $H_F = \sum_{n=0}^{\infty} T^n \Omega_n$, where $T = 2\pi/\omega$ is the period of the drive. Though this is technically valid only for large driving

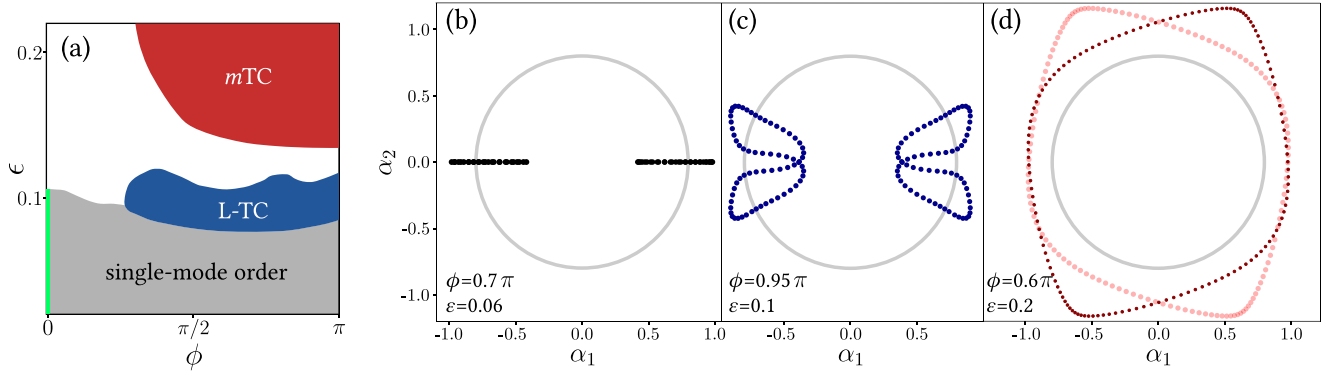


FIG. 2. (a) Phase diagram of the Higgs mode in the (ϕ, ϵ) plane, with $\kappa = 0.1$, $\lambda_0 = 0.85$, and $\omega = 0.92$, satisfying parametric resonance with the Higgs polariton branch. All frequencies and units are in terms of ω_a , which has been set equal to 1. Here, $\phi = 0$ is the U(1)-symmetry-broken phase, shown in green. While the white region is devoid of a coherent steady state, gray, blue, and red regions indicate periodic symmetry-broken phases (see text). Representative trajectories in the (α_1, α_2) plane are shown in panels (b), (c), and (d), respectively, where gray circle indicates the static potential minimum. In panel (d), the two steady-state trajectories are shown in dark and pale markers as a guide for the eye. For a complete set of steady states, see Supplemental Material [38].

frequencies (beyond the range of resonant frequencies we are interested in), it is nonetheless illustrative of how ϕ generates terms that lead to intertwining. Here, $\Omega_0 = H_0$, whereas the first nontrivial term reads

$$2\pi i\hbar\Omega_1 = \epsilon \sin\left(\frac{\phi}{2}\right)[H_0, O_1 - O_2] + \frac{\epsilon^2}{2} \sin\phi[O_1, O_2], \quad (3)$$

where the leading-order term in ϵ is indeed proportional to $\Delta(\phi)$. Looking at this term, we can separate $H_0 = h + \lambda_0(O_1 + O_2)$, where h is the noninteracting part of H_0 , and note that it only involves the commutators $[h, O_i]$ and $[O_1, O_2]$. For nonzero phase lag ϕ , the former yields the terms $(a_i^\dagger - a_i)(b_i^\dagger b_0 + b_0^\dagger b_i)$ and $(a_i^\dagger + a_i)(b_i^\dagger b_0 - b_0^\dagger b_i)$, which couple to both quadratures of a given cavity mode, akin to the interpolating Dicke-Tavis Cummings model. The latter generates a direct coupling between the two cavities $(a_1^\dagger + a_1)(a_2^\dagger + a_2)(b_1^\dagger b_2 - b_2^\dagger b_1)$ mediated by the atoms. This is reminiscent of the $a_1^\dagger a_2 + \text{H.c.}$ term that couples to another atomic excitation mode engineered in Ref. [31] to induce intertwining. Already to leading order in ϵ , clearly $\phi \neq 0$ breaks the U(1) symmetry of the Floquet Hamiltonian H_F , while preserving $\mathbb{Z}_2 \times \mathbb{Z}_2$, reminiscent of the $\text{SO}(4) \rightarrow \text{SU}(2)$ symmetry reduction in the problem of η pairing [22]. This consequently endows the original Goldstone mode with an effective mass without recourse to complex Floquet protocols as done in Ref. [37].

While we here only show the leading-order results of the Magnus expansion in T , higher orders involve nested commutators of h , O_1 , and O_2 (see Supplemental Material [38]), highlighting the potential for intertwined orders. Caution needs to be exercised regarding the convergence of such high-frequency expansions when one approaches parametric instabilities. In what follows, using a mean-field approach, we will show that the notion of intertwining generalizes in the driven problem to involve intertwining of the usual Landau and discrete time-crystalline orders.

Numerical results. We explore the stationary states in the presence of cavity dissipation by numerically integrating the

equations of motion for the Lindbladian time evolution of the mean-field expectation values $\langle O \rangle$, where $O = a_i, b_i^\dagger b_i, b_i^\dagger b_0, b_0^\dagger b_0, b_i^\dagger b_2, b_2^\dagger b_1$, with $i = 1, 2$. The mean-field equations are presented in the Supplemental Material [38] and are solved numerically for various values of ϵ , ω , and ϕ . For simplicity, we set $\omega_a = \omega_b = 1$ measured in units of the atomic recoil energy E_r , which in the limit of zero dissipation yields $\lambda_c = 1/2$. This recoil energy that depends on the atomic mass and the wavelength of the pump laser sets the scale in experiments, where typically $E_r \sim 100$ kHz. The experimentally relevant case of $\omega_b \ll \omega_a$ is discussed in the Supplemental Material [38]. Both cavities are dissipative with a rate κ , which is also measured in units of E_r , and we only consider driving in the static ordered phases in what follows.

To characterize the plethora of phases obtained, we introduce cavity Landau order parameters via averaging over n drive cycles of period T , $\alpha_i^L = \int_0^{nT} \alpha_i(t) dt$. As α_i^L is a $\mathbb{Z}_2$ order parameter, it signals two degenerate steady states connected by P_i . In addition to conventional Landau order, the driven system exhibits discrete time-crystalline order, where a mean-field observable A satisfies $\langle A(t + nT) \rangle = \langle A(t) \rangle$ for integer $n > 1$. Focusing on the period-doubling case ($n = 2$), we define $\alpha_i^M = \tilde{\alpha}_i(\Omega = \frac{\omega}{2})$, with $\tilde{\alpha}_i(\Omega)$ denoting the corresponding Fourier component. Such a TC also constitutes a $\mathbb{Z}_2$ order, with two degenerate solutions connected by $\mathcal{T}_T \alpha_i(t) = \alpha_i(t + T) = -\alpha_i(t)$. In what follows, we discuss different intertwined phases that emerge in the driven system, whose

TABLE I. The different intertwined phases for the two cavity modes in the asynchronously driven system with $\lambda_0 > \lambda_c$ and $\phi > 0$ are summarized. For the case $\phi < 0$, the same table holds but with the cavity indices permuted.

Intertwining	Broken	α_1^L	α_2^L	α_1^M	α_2^M
Landau	$\mathbb{Z}_2 \times \mathbb{Z}_2$	$\neq 0$	$\neq 0$	0	0
mTC	$\mathbb{Z}_2^T$	0	0	$\neq 0$	$\neq 0$
L-TC	$\mathbb{Z}_2 \times \mathbb{Z}_2^T$	$\neq 0$	0	0	$\neq 0$

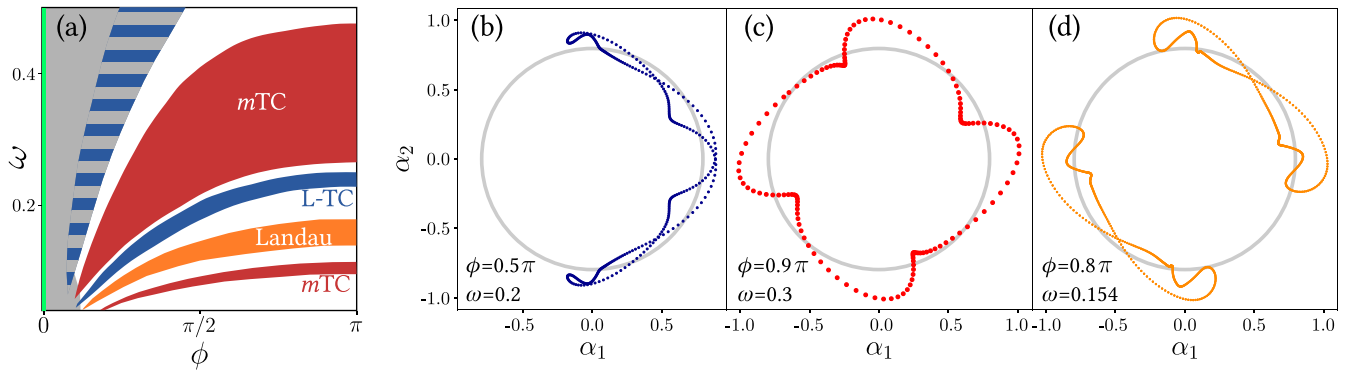


FIG. 3. (a) Phase diagram associated with the GS mode in the (ϕ, ω) plane, with $\epsilon = 0.1$, $\lambda_0 = 0.85$, and $\kappa = 0.17$. The white, green, and gray regions are the same as in Fig. 2. Blue, red, and orange regions indicate symmetry-broken phases, with representative steady-state trajectories shown in panels (b), (c), and (d), respectively (see text). Here, gray circle shows the static potential minimum. The shaded region in panel (a) indicates coexistence of gray and blue. Highly fine-tuned steady states occupying very narrow regions have been removed from the lowest-frequency regime. For a complete set of steady states, see Supplemental Material [38].

characterizing order parameters and corresponding broken symmetries are summarized in Table I.

We first discuss the case of the resonantly driven Higgs mode. We set $\lambda_0 = 0.85$ and modulate the drive with frequency $\omega = \Delta_H = 0.92$, matching the energy Δ_H of the Higgs branch in the static system [see Fig. 1(c)]. Note that this is different from the usual parametric driving schemes where one would choose $\omega = 2\Delta_H$, as here the relative gap from the Goldstone mode is targeted. The resulting phase diagram in the (ϕ, ϵ) plane for $0 \leq \phi \leq \pi$ (changing the sign of ϕ merely exchanges the role of the cavities) is depicted in Fig. 2(a). Here, the white region indicates a disordered chaotic phase. On the other hand, the gray, blue, and red regions describe time periodic steady states with distinct symmetry properties that we will discuss below. Note that these are not the simple Arnold tongues one might expect from standard parametric resonances, rather they stem from a complex interplay of the underlying symmetries.

For comparatively small $\epsilon < \kappa$, we find two T periodic steady states (gray region) with $\alpha_1^L \geq 0, \alpha_2(t) = 0$ (vice versa) for $0 \leq \phi \leq \pi$ ($-\pi \leq \phi \leq 0$), representing a single-mode order. This region has broken $\mathbb{Z}_2$ symmetry and is devoid of intertwining [see representative trajectories in the (α_1, α_2) plane in Fig. 2(b)]. When $\epsilon \approx \kappa$, for $|\phi| > \phi_0^b \approx 0.3\pi$, we see a bifurcation to the blue regime [see Fig. 2(c) for representative trajectories]. Here, for $\phi > 0$, the α_1 mode shows $\mathbb{Z}_2$ Landau order with $\alpha_1^L \neq 0$ and T periodicity, while α_2 manifests period-doubled TC order with $\alpha_2^M \neq 0$, but $\alpha_2^L = 0$. This region is characterized by broken $\mathbb{Z}_2 \otimes \mathbb{Z}_2^T$ symmetry and four degenerate steady-state solutions are connected by the operations of P_1 and $\mathcal{T}_T$ (see extended data in the Supplemental Material [38]). Note that the roles of $\alpha_{1,2}$ are switched when $-\pi < \phi < 0$ or $\pi < \phi < 2\pi$. Curiously, these states exist as metastable solutions when initial conditions are randomly sampled. The energy levels of the four solutions cross at $\phi = \pi$, where all four states are equally probable, due to the emergent symmetry where the Hamiltonian is invariant under the composite operation of $(a_1, b_1) \leftrightarrow (a_2, b_2)$ and $t \rightarrow t \pm T/2$. The blue region, therefore, realizes a novel phase of

matter, which displays dynamically intertwined Landau-time crystal (L-TC) order.

A further bifurcation to the red region emerges at $\epsilon > \kappa$ and $|\phi| > \phi_0^r$ (ϵ dependent) [see representative trajectories in Fig. 2(d)], where both cavity modes manifest period doubling, while maintaining $\alpha_{1,2}^L = 0$. Here, the $\mathbb{Z}_2^T$ symmetry is broken and we obtain four steady states corresponding to a dynamically intertwined multicomponent TC (mTC) order parameter (α_1^M, α_2^M) . The four solutions are related by the action of the shift operator $\mathcal{T}_T \alpha_i(t) = -\alpha_i(t)$ for each cavity mode (see Supplemental Material [38]). Coincidentally, this is equivalent to acting with P_i ; nevertheless, the underlying symmetry breaking is fully in the time domain. To summarize, driving the Higgs mode can lead to dynamically intertwined TC phases where one or both cavity modes break discrete time translation symmetry.

We now explore the regime dominated by the Goldstone mode with a drive-induced dynamical gap. A measure of this gap is given by $\Delta(\phi) \propto |\sin(\phi/2)|$, which characterizes the departure from U(1) symmetry. Driving these modes, we unearth a rich representative steady-state phase diagram in the (ϕ, ω) plane [see Fig. 3(a)]. As in the Higgs case, we here highlight representative steady-state trajectories for each intertwined phase and show a full set of stationary states in the Supplemental Material [38]. While for small ϕ we find the same phase shown in gray as in Fig. 2, for $\phi > \phi_0^{\text{GS}} \sim 0.1\pi$ we find a hierarchy of phases with shapes similar to the curve $\Delta(\phi)$. The threshold ϕ_0^{GS} increases for larger κ , reminiscent of the shifts in Arnold lobes in the presence of dissipation [39]. Note that the associated trajectories in the (α_1, α_2) plane remain in the vicinity of the static potential minimum, highlighting the low-energy nature of the underlying GS mode.

The blue and red regions [representative trajectories shown in Figs. 3(b) and 3(c), respectively] are, respectively, counterparts to the dynamically intertwined L-TC and mTC orders for resonant Higgs driving, featuring four steady states via $\mathbb{Z}_2 \otimes \mathbb{Z}_2^T$ and $\mathbb{Z}_2^T$ symmetry breaking. As before, the $\phi > 0$ blue region is characterized by finite α_1^L and α_2^M and features

a second set of degenerate metastable states, with the roles of $\alpha_{1,2}$ switched along with a level crossing at $\phi = \pi$.

Surprisingly, a new regime appears in the orange arc in Fig. 3(a) with distinct symmetry properties, driven by $\text{Im}\langle a_{1,2} \rangle$ and $\text{Re}\langle b_1^\dagger b_2 \rangle$. It features four degenerate steady states with $\alpha_{1,2}^L \neq 0$ and $\alpha_{1,2}^M = 0$, related by parity $P_{1,2}$ for any viable value of ϕ [two trajectories shown in Fig. 3(d)]. This indicates the spontaneous breaking of the $\mathbb{Z}_2 \times \mathbb{Z}_2$ symmetry while preserving the full $\mathbb{Z}^T$ time translation symmetry. This shows that driving can stabilize intertwined Landau orders. Unlike the dynamically intertwined phases, which manifest even when $\lambda_0^{(1)} \neq \lambda_0^{(2)}$, this intertwining of Landau orders requires $\lambda_0^{(1)} = \lambda_0^{(2)}$. It has its origin in the dynamical symmetry reduction breaking U(1) symmetry instantaneously, while leaving $\mathbb{Z}_2 \times \mathbb{Z}_2$ intact, reminiscent of the $\text{SO}(4) \rightarrow \text{SU}(2)$ reduction necessary for stabilizing the unconventional η pairing.

Discussion and outlook. Our results can be verified in the well-studied cross-cavity experimental setup, where controlled driving via probe fields on the cavities was already used to identify the collective Higgs and Goldstone modes [40]. We remark that these novel intertwined phases exist in the experimentally relevant parameter regime $\omega_b \ll \omega_a$ (see Supplemental Material [38]). The asynchronous driving can be implemented using a two-color pump, facilitating the independent modulation of the cavity modes. Alternatively,

we expect that the low-energy Goldstone mode is accessible via low-frequency modulation of the cavity geometries using piezoelectric devices. The different intertwined phases should be detectable via emitted cavity photons [41].

To conclude, we showed that an interplay between drive and symmetry can be used to generate novel out-of-equilibrium intertwined ordered phases in a minimal setup featuring two $\mathbb{Z}_2$ density-wave orders in a tunable cavity platform. We see that resonant driving of both the Goldstone and massive polaritonic excitations leads to the intertwining of Landau and/or time crystal order parameters. It would be exciting to explore whether this latter feature involving engineered drives exploring symmetry-enhanced parameter regimes and soft modes provides a general pathway to realizing intertwined and vestigial phases in systems with competing orders. We leave this for future work.

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Data availability. The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

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- [1] B. V. Svistunov, E. S. Babaev, and N. V. Prokof'ev, *Superfluid States of Matter* (CRC Press, Boca Raton, FL, 2015).
 - [2] E. Fradkin, S. A. Kivelson, and J. M. Tranquada, *Colloquium: Theory of intertwined orders in high temperature superconductors*, *Rev. Mod. Phys.* **87**, 457 (2015).
 - [3] R. M. Fernandes, P. P. Orth, and J. Schmalian, Intertwined vestigial order in quantum materials: Nematicity and beyond, *Annu. Rev. Condens. Matter Phys.* **10**, 133 (2019).
 - [4] E. Berg, E. Fradkin, S. A. Kivelson, and J. M. Tranquada, Striped superconductors: How spin, charge and superconducting orders intertwine in the cuprates, *New J. Phys.* **11**, 115004 (2009).
 - [5] T. Neupert, M. M. Denner, J.-X. Yin, R. Thomale, and M. Z. Hasan, Charge order and superconductivity in kagome materials, *Nat. Phys.* **18**, 137 (2022).
 - [6] Y. Wang, H. Wu, G. T. McCandless, J. Y. Chan, and M. N. Ali, Quantum states and intertwining phases in kagome materials, *Nat. Rev. Phys.* **5**, 635 (2023).
 - [7] D. Y. Kim, S.-Z. Lin, F. Weickert, M. Kenzelmann, E. D. Bauer, F. Ronning, J. D. Thompson, and R. Movshovich, Intertwined orders in heavy-fermion superconductor CeCoIn₅, *Phys. Rev. X* **6**, 041059 (2016).
 - [8] Q. Gu, J. P. Carroll, S. Wang, S. Ran, C. Broyles, H. Siddiquee, N. P. Butch, S. R. Saha, J. Paglione, J. S. Davis, *et al.*, Detection of a pair density wave state in UTe₂, *Nature (London)* **618**, 921 (2023).
 - [9] X. Liu, Y. X. Chong, R. Sharma, and J. C. S. Davis, Discovery of a Cooper-pair density wave state in a transition-metal dichalcogenide, *Science* **372**, 1447 (2021).
 - [10] C. Candelora, M. Xu, S. Cheng, A. De Vita, D. Romanin, C. Bigi, M. B. Petersen, A. LaFleur, W. A. Castro, F. Mottl, P. Orgiani, G. Vinai, M. Calandra, J. Miwa, Y. Hwang, Z. Wang, F. Mazzola, and I. Zeljkovic, Discovery of magnetic-field-tunable density modulations and spin tilting in a layered altermagnet, *Commun. Mater.* **7**, 74 (2026).
 - [11] A. Cavalleri, Photo-induced superconductivity, *Contemp. Phys.* **59**, 31 (2018).
 - [12] M. Budden, T. Gebert, M. Buzzi, G. Jotzu, E. Wang, T. Matsuyama, G. Meier, Y. Laplace, D. Pontiroli, M. Riccò, *et al.*, Evidence for metastable photo-induced superconductivity in K₃C₆₀, *Nat. Phys.* **17**, 611 (2021).
 - [13] A. de la Torre, D. M. Kennes, M. Claassen, S. Gerber, J. W. McIver, and M. A. Sentef, *Colloquium: Nonthermal pathways to ultrafast control in quantum materials*, *Rev. Mod. Phys.* **93**, 041002 (2021).
 - [14] T. Oka and S. Kitamura, Floquet engineering of quantum materials, *Annu. Rev. Condens. Matter Phys.* **10**, 387 (2019).
 - [15] S. Kitamura and H. Aoki, Floquet topological superconductivity induced by chiral many-body interaction, *Commun. Phys.* **5**, 174 (2022).
 - [16] G. Kipp, H. M. Bretscher, B. Schulte, D. Herrmann, K. Kussyak, M. W. Day, S. Kesavan, T. Matsuyama, X. Li, S. M. Langner, *et al.*, Cavity electrodynamics of van der Waals heterostructures, *Nat. Phys.* **21**, 1926 (2025).
 - [17] M. Kiffner, J. R. Coulthard, F. Schlawin, A. Ardavan, and D. Jaksch, Manipulating quantum materials with quantum light, *Phys. Rev. B* **99**, 085116 (2019).
 - [18] I.-T. Lu, D. Shin, M. Kamper Svendsen, S. Latini, H. Hübener, M. Ruggenthaler, and A. Rubio, Cavity engineering of solid-state materials without external driving, *Adv. Opt. Photon.* **17**, 441 (2025).
 - [19] F. Schlawin, D. M. Kennes, and M. A. Sentef, Cavity quantum materials, *Appl. Phys. Rev.* **9**, 011312 (2022).

- [20] J. Li and M. Eckstein, Manipulating intertwined orders in solids with quantum light, *Phys. Rev. Lett.* **125**, 217402 (2020).
- [21] D. Basov, A. Asenjo-Garcia, P. J. Schuck, X. Zhu, A. Rubio, A. Cavalleri, M. Delor, M. M. Fogler, and M. Liu, Polaritonic quantum matter, *Nanophotonics* **14**, 3723 (2025).
- [22] S. Kitamura and H. Aoki, η -pairing superfluid in periodically-driven fermionic Hubbard model with strong attraction, *Phys. Rev. B* **94**, 174503 (2016).
- [23] J. Li, D. Golez, P. Werner, and M. Eckstein, η -paired superconducting hidden phase in photodoped Mott insulators, *Phys. Rev. B* **102**, 165136 (2020).
- [24] D. F. Agterberg, J. S. Davis, S. D. Edkins, E. Fradkin, D. J. Van Harlingen, S. A. Kivelson, P. A. Lee, L. Radzihovsky, J. M. Tranquada, and Y. Wang, The physics of pair-density waves: Cuprate superconductors and beyond, *Annu. Rev. Condens. Matter Phys.* **11**, 231 (2020).
- [25] K. Baumann, C. Guerlin, F. Brennecke, and T. Esslinger, Dicke quantum phase transition with a superfluid gas in an optical cavity, *Nature (London)* **464**, 1301 (2010).
- [26] K. Baumann, R. Mottl, F. Brennecke, and T. Esslinger, Exploring symmetry breaking at the Dicke quantum phase transition, *Phys. Rev. Lett.* **107**, 140402 (2011).
- [27] V. Helsen, T. Zewtler, F. Mivehvar, E. Colella, K. Roux, H. Konishi, H. Ritsch, and J.-P. Brantut, Density-wave ordering in a unitary Fermi gas with photon-mediated interactions, *Nature (London)* **618**, 716 (2023).
- [28] R. Landig, L. Hruby, N. Dogra, M. Landini, R. Mottl, T. Donner, and T. Esslinger, Quantum phases from competing short- and long-range interactions in an optical lattice, *Nature (London)* **532**, 476 (2016).
- [29] J. Léonard, A. Morales, P. Zupancic, T. Donner, and T. Esslinger, Monitoring and manipulating Higgs and Goldstone modes in a supersolid quantum gas, *Science* **358**, 1415 (2017).
- [30] S. Gopalakrishnan, Y. E. Shchadilova, and E. Demler, Intertwined and vestigial order with ultracold atoms in multiple cavity modes, *Phys. Rev. A* **96**, 063828 (2017).
- [31] A. Morales, P. Zupancic, J. Léonard, T. Esslinger, and T. Donner, Coupling two order parameters in a quantum gas, *Nat. Mater.* **17**, 686 (2018).
- [32] J. Léonard, A. Morales, P. Zupancic, T. Esslinger, and T. Donner, Supersolid formation in a quantum gas breaking a continuous translational symmetry, *Nature (London)* **543**, 87 (2017).
- [33] D. V. Else, B. Bauer, and C. Nayak, Floquet time crystals, *Phys. Rev. Lett.* **117**, 090402 (2016).
- [34] H. Keßler, P. Kongkhambut, C. Georges, L. Mathey, J. G. Cosme, and A. Hemmerich, Observation of a dissipative time crystal, *Phys. Rev. Lett.* **127**, 043602 (2021).
- [35] R. Chitra and O. Zilberberg, Dynamical many-body phases of the parametrically driven, dissipative Dicke model, *Phys. Rev. A* **92**, 023815 (2015).
- [36] P. Mognini, L. Papariello, A. U. J. Lode, and R. Chitra, Superlattice switching from parametric instabilities in a driven-dissipative Bose-Einstein condensate in a cavity, *Phys. Rev. A* **98**, 053620 (2018).
- [37] Y. Hou, Z. Fu, R. Moessner, M. Bukov, and H. Zhao, Floquet-engineered emergent massive Nambu-Goldstone modes, *Phys. Rev. B* **112**, L020305 (2025).
- [38] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/tq9z-xg7z> for the following: details on the mean-field phase diagram and fluctuation spectra of the cross-cavity system; high-frequency expansion; Heisenberg equations of motion; and extended dataset for dispersively coupled cavity.
- [39] C. Zerbe, P. Jung, and P. Hänggi, Brownian parametric oscillators, *Phys. Rev. E* **49**, 3626 (1994).
- [40] J. Léonard, A supersolid of matter and light, Ph.D. thesis, ETH Zurich, 2017.
- [41] R. Lin, C. Georges, J. Klinder, P. Mognini, M. Büttner, A. U. J. Lode, R. Chitra, A. Hemmerich, and H. Keßler, Mott transition in a cavity-boson system: A quantitative comparison between theory and experiment, *SciPost Phys.* **11**, 030 (2021).