

Disordered yet directed: The emergence of polar flocks with disordered interactions

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Flocking is a prime example of how robust collective behavior can emerge from simple interaction rules. The flocking transition has been studied extensively since the inception of the original Vicsek model. Here, we introduce a self-propelled particle model with quenched disorder in the pairwise alignment interaction couplings akin to a spin-glass model. We find that increasing the variance of the coupling distribution can promote (rather than destroy) the emergence of global polar order. In particular, we show that our model can display a flocking phase even when the majority of the interaction couplings are antialigning. Activity is the key ingredient to reduce frustration in the system as it allows local particle clustering combined with self-organization of the particles to favor neighborhoods with strong cooperative interactions.

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How can simple interaction rules lead to robust and unexpected collective behavior? In statistical mechanics, paradigmatic examples are abound, from simple equilibrium spin systems to out-of-equilibrium systems of interacting self-propelled particles (SPP) [1–4]. In models of self-propelled particle systems, local consumption of energy can be used to self-organize into ordered states through local alignment interactions alone, leading to striking nonequilibrium behaviors, such as flocking [5–9]. Following earlier attempts to model flocking in animal groups [10], Vicsek *et al.* introduced a model of identical noisy self-propelled particles interacting solely through a local polar alignment rule akin to spins in the XY model [5]; this model displays an order-disorder phase transition [5,11]. Interestingly, despite only having local alignment interaction rules, this model exhibits long-range order in the ordered phase in two dimensions [6,7,9,12–19], as activity allows the system to escape the Mermin-Wagner-Hohenberg theorem [20,21]. Flocking remains under intense scrutiny both theoretically [16,22–24] and experimentally including in animal groups [8,25,26], bacterial colonies [27], cytoskeletal filaments driven by molecular motors [28], or synthetic colloidal systems [29–31].

Flocking has been shown to be both more robust in some respects and more fragile in others, than originally thought. Indeed, it was recently shown to arise in the absence of explicit alignment interactions and even when these interactions should in fact prevent it [32–39]. Populations of particles with set inter- and intrapopulation interaction rules can dis-

play novel phases not possible in the original Vicsek model [40–45]. Conversely, although the exact details of the metric alignment rule and implementation of noise in the model do not change the asymptotic properties of the Vicsek model [46–49], the fragility of the ordered state to heterogeneity in SPP models is evident through the influence of obstacles [50–52], spatial anisotropy [53], or even dissenters [54,55]. Discrete- and continuous-symmetry flocks with rotational anisotropy have also recently been shown to be metastable [52,56].

Previous works have considered extensions of the continuous-time formulation of the Vicsek model or “flying XY” model and have studied the influence of density dependent velocities [57], chirality [40,58], and excluded volume interactions [59,60]. Recent studies have focused on the impact of quenched and annealed random field disorder on flocks with homogeneous interaction couplings. Paradoxically, these nonequilibrium systems have been found to be more robust to quenched disorder than the corresponding equilibrium systems [61–67].

In equilibrium systems, the role of quenched disorder on phase transitions has been the subject of intense study in the context of models of disordered spins [68–71]. Disordered spin systems generically display unresolved frustration arising from the impossibility of satisfying all particle interactions at the same time. This notably leads to the so-called spin-glass phase, characterized by islands of locally aligned spins but no long-range order [71,72]. While recent papers have explored geometrical frustration in high-density active systems [73] and nonreciprocal interactions in XY spin systems [74], our understanding of the impact of disordered alignment interactions on the emergence of flocking is still crucially lacking.

Here, we introduce the *active disordered XY* (ADXY) model in which disorder is quenched in the couplings, which are drawn from a Gaussian distribution, as commonly done in equilibrium spin glasses. Quenched disorder is thus

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introduced at the level of the couplings, a pairwise interaction property rather than a property intrinsic to the particle or the environment. Upon increasing the variance of these spin-spin couplings, we observe nonzero global polar order in systems in which there would otherwise be no long-range order [75]. Despite the fact that flocking is disrupted through other forms of coupling heterogeneity, in this case, the variance in the couplings promotes, rather than destroys, flocking; activity leads to a spatial self-organization of the particles that helps decrease frustration in the system.

I. ACTIVE DISORDERED XY MODEL

We study the dynamics of N pointlike particles interacting solely via metric alignment interactions akin to an off-lattice extension of a two-dimensional (2D) disordered XY model. The particles' dynamics are governed by the following overdamped Langevin equations:

$$\dot{\mathbf{r}}_i = v_0 \hat{\mathbf{p}}_i, \quad (1a)$$

$$\dot{\theta}_i = \frac{1}{n_i} \sum_{j \in \mathcal{N}_i} K_{ij} \sin(\theta_j - \theta_i) + \eta \xi_i, \quad (1b)$$

where $\{\mathbf{r}_i, \theta_i\}$ are, respectively, the position and orientation of particle $i \in \{1, \dots, N\}$ at time t , n_i is the number of particles in the set of neighbors $\mathcal{N}_i = \{j : |\mathbf{r}_i - \mathbf{r}_j| \leq \sigma_I, i \neq j\}$, η is a constant noise strength, and ξ_i is a Gaussian white noise with zero mean and unit variance. Particles thus move at constant speed v_0 along a self-propulsion direction $\hat{\mathbf{p}}_i = (\cos \theta_i, \sin \theta_i)$ that is subject to rotational diffusion with persistence time $\tau_p \propto \eta^{-2}$ and a polar alignment force with couplings K_{ij} from neighboring particles $j \in \mathcal{N}_i$. Our implementation of the polar forces means that the alignment is nonadditive, which is a more realistic continuous-time version of the original Vicsek model [76].

The coupling strength between each particle pair is given by K_{ij} , where $K_{ij} > 0$ gives ferromagnetic alignment and $K_{ij} < 0$ antiferromagnetic alignment. The couplings are independent and identically distributed quenched random variables drawn from a Gaussian distribution $K_{ij} \sim \mathcal{N}(\bar{K}, \sigma_K^2)$, with $K_{ij} = K_{ji}$ as is found in many spin-glass models such as the Sherrington-Kirkpatrick (SK) model [69,77]. In fact, in the limit of $\sigma_I \rightarrow \infty$, we recover a Langevin formulation akin to the SK model, as the spin interactions are infinite ranged and the particle positions in Eq. (1a) are irrelevant.

In what follows, we numerically solve Eq. (1) in a square box of size L using an Euler-Maruyama method with time step Δt . Unless stated otherwise, we use periodic boundary conditions, fix $\Delta t = 0.01$, $v_0 = 1$, and $\sigma_I = 1$, and start from initial configurations with random positions and orientations at average density $\rho = N/L^2$. We note that Vicsek-like models are subject to strong finite-size effects; here, we have taken all necessary precautions to ensure that our results are robust, including large numerical simulations of up to order 10^5 particles.

II. ORDER-DISORDER TRANSITION

The degree of global polar order in a flocking system is measured through the time-averaged global polar order pa-

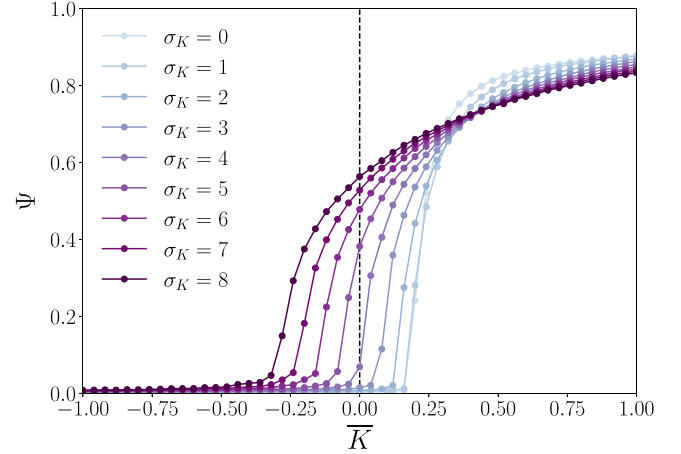


FIG. 1. Polar order parameter Ψ vs $\bar{K}$ for increasing values of σ_K . Simulations were performed with $N = 9 \times 10^4$, $\rho = 1$, $\eta = 0.4$. The black dashed line denotes $\bar{K} = 0$.

rameter $\Psi = \langle \psi(t) \rangle_t$, where

$$\psi(t) = \frac{1}{N} \left| \sum_{i=1}^N \hat{\mathbf{p}}_i(t) \right|, \quad (2)$$

where Ψ takes values from $\psi = 0$ (disorder) to $\psi = 1$ (order). Averages are taken over time (after the system reaches steady state) as well as realizations of the random couplings K_{ij} (usually 20–50 realizations).

For homogeneous couplings— $\sigma_K = 0$ and $K_{ij} = \bar{K}$ for all pairs (i, j) —a disorder-to-order phase transition is observed as the value of $\bar{K}$ is increased [11,57,59,78,79]. We confirm this result here. For $\sigma_K > 0$, we also observe such a transition (see Fig. 1). However, as σ_K is increased, the onset of flocking strikingly shifts to negative values of $\bar{K}$. In the region $\bar{K} \leq 0$, order is promoted as σ_K increases, and we are able to recover a significant degree of global order, despite the fact that the majority of interactions are *antialigning*. In the case $\sigma_K = 0$, recent studies argue that Vicsek-type models with metric-free interactions behave as do their metric counterpart [48,49]. Here, we confirm that our observations are robust and extend to systems with non-Gaussian distributions of random couplings and systems with topological (rather than metric) interactions (see details in Supplemental Material [80]).

To understand the emergence of this mechanism, we compare snapshots from simulations with small and large σ_K values in Fig. 2 (see Supplemental Material [80] for movies). To start, we consider systems with $\bar{K} = 0$, in which there are an equal number of aligning and antialigning interactions. At $\bar{K} = 0$ and $\sigma_K = 1$, the polar order parameter is $\Psi = 0.01$: The system is disordered and particles point in random directions [Fig. 2(a)]. Upon increasing the coupling dispersion to $\sigma_K = 8$, the steady-state polar order parameter increases to $\Psi = 0.57$. This increase in the polar order parameter corresponds to the appearance of clusters of locally aligned particles [Fig. 2(b)]. Indeed, as σ_K is increased, the distribution of number of neighbors n_i within the interaction radius becomes more right skewed and particles are more likely to have a greater number of neighbors, as shown in Fig. 2(c).

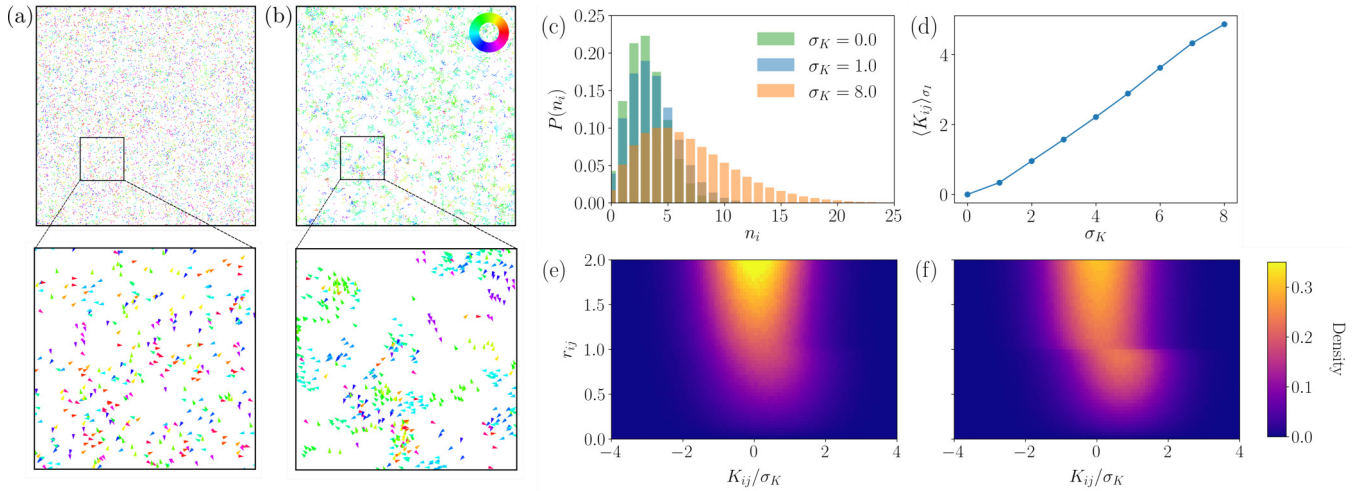


FIG. 2. Local clustering of particles. Snapshots from simulations with $\sigma_K = 1$ in panel (a) and $\sigma_K = 8$ in panel (b), with $N = 10^4$, $\rho = 1$, $\eta = 0.4$, and $\bar{K} = 0$. Close-up views of the simulation box are shown below with length $L/5 = 20$. The colored arrows give the particle orientations $\theta \in [0, 2\pi)$. See movies S1 and S2 and their descriptions in the Supplemental Material [80]. (c) Normalized histograms for the number of neighbors n_i within the radius of interaction $\sigma_l = 1$ for the systems in panels (a) and (b) with $\sigma_K = 1$ in blue and $\sigma_K = 8$ in orange, as well as $\sigma_K = 0$ (noninteracting particles) in green. (d) Mean K_{ij} over interacting pairs in steady state against σ_K for $\bar{K} = 0$. 2D histograms of distance between particle pairs $r_{ij} = |\mathbf{r}_i - \mathbf{r}_j|$ against their coupling value for $\sigma_K = 1.0$ [panel (e)] and $\sigma_K = 8.0$ [panel (f)]. Coupling values have been normalized by σ_K . Data are averaged over time (at intervals of $t = 100$ from $t = 3000$ to 3500), and realizations, with $N = 10^4$, $\rho = 1$, $\eta = 0.4$, and $\bar{K} = 0$. Note that histograms remain symmetric for values of $r_{ij} > 2$, not shown here.

To investigate this clustering further, we look at the relationship between particle pair rescaled couplings K_{ij}/σ_K and their separation distance r_{ij} in Figs. 2(e) and 2(f). Owing to the Gaussian nature of the coupling distribution, we would expect the joint probability distribution $P(K_{ij}/\sigma_K, r_{ij})$ to be peaked around $K_{ij}/\sigma_K = 0$ and fall off symmetrically if particles were clustered randomly. This is what we observe for distances $r_{ij} > \sigma_l$. However, for $r_{ij} < \sigma_l = 1$, we observe a clear shift toward positive coupling values, which becomes more pronounced as σ_K increases. In other words, an increase in σ_K promotes the creation of more cohesive clusters of highly aligning particles. Indeed, we find that the mean coupling within the interaction radius $\langle K_{ij} \rangle_{\sigma_l}$ increases monotonically with σ_K , as shown in Fig. 2(d). While the variance in the couplings introduces an inherent frustration in the particle orientations, the activity allows particles to self-organize to find other particles to satisfy their interactions, thus decreasing the global frustration in the system. However, this does not imply that all antiferromagnetic couplings ($K_{ij} < 0$) become irrelevant. Even at large σ_K , a significant proportion of interactions remain frustrated, in the sense that they contribute unfavorably to the energy (see Supplemental Material [80]). The formation of highly aligned clusters alone does not necessarily imply global polar order. We simulated an annealed variant in which K_{ij} remains fixed while particles i and j are neighbors, but is updated if they separate and later come back within each other's interaction radius. This update rule ought to favor local clusters of highly aligning particles. However, in Fig. 3, we clearly observe a loss of global cooperation when the coupling distribution is replaced with this type of annealed disorder. For $\bar{K} < 0$, increasing the coupling variance no longer leads to global polar order [Fig. 3(a)]. Comparing a single realization of the quenched and annealed model variants in Fig. 3(b), the mean K_{ij} over interacting pairs rapidly increases to a

similar level in both cases, signifying highly aligned clusters. However, while the global order of the quenched disorder simulation steadily increases over time to a steady state of $\psi_\infty \approx 0.6$, the simulation with the annealed disorder variant remains globally disordered. This suggests that quenched disorder generates an effective interaction network capable of sustaining alignment beyond individual clusters. Recently, a nonreciprocal variant of this model was also observed to show strong clustering, yet no global polar order [81]. Taken together, these results suggest that the key ingredients for global ordering in the ADXY model with $\bar{K} < 0$ and $\sigma_K > 0$ are activity and reciprocal couplings with quenched disorder.

III. SELF ORGANIZATION AND CLUSTERING PROMOTE POLAR ORDER

To systematically characterize the phases of our model, we first measure the polar order parameter Ψ for a fixed noise strength $\eta = 0.4$ while varying ρ , $\bar{K}$, and σ_K [Figs. 4(a) and

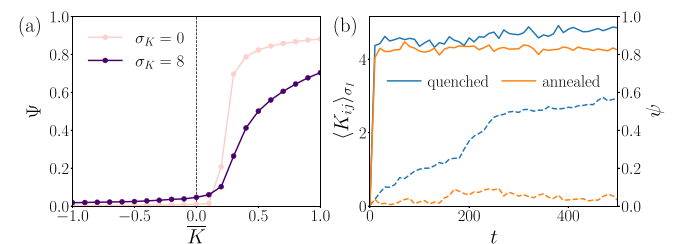


FIG. 3. (a) Polar order phase transition against $\bar{K}$ for the annealed variant of coupling disorder. Simulation parameters were $N = 10^4$, $\rho = 1$, and $\eta = 0.4$. (b) Comparison of quenched and annealed coupling disorder for one realization with $\bar{K} = 0$, $\sigma_K = 8$. Mean K_{ij} over interacting pairs over time (solid lines) and polar order ψ over time (dashed lines).

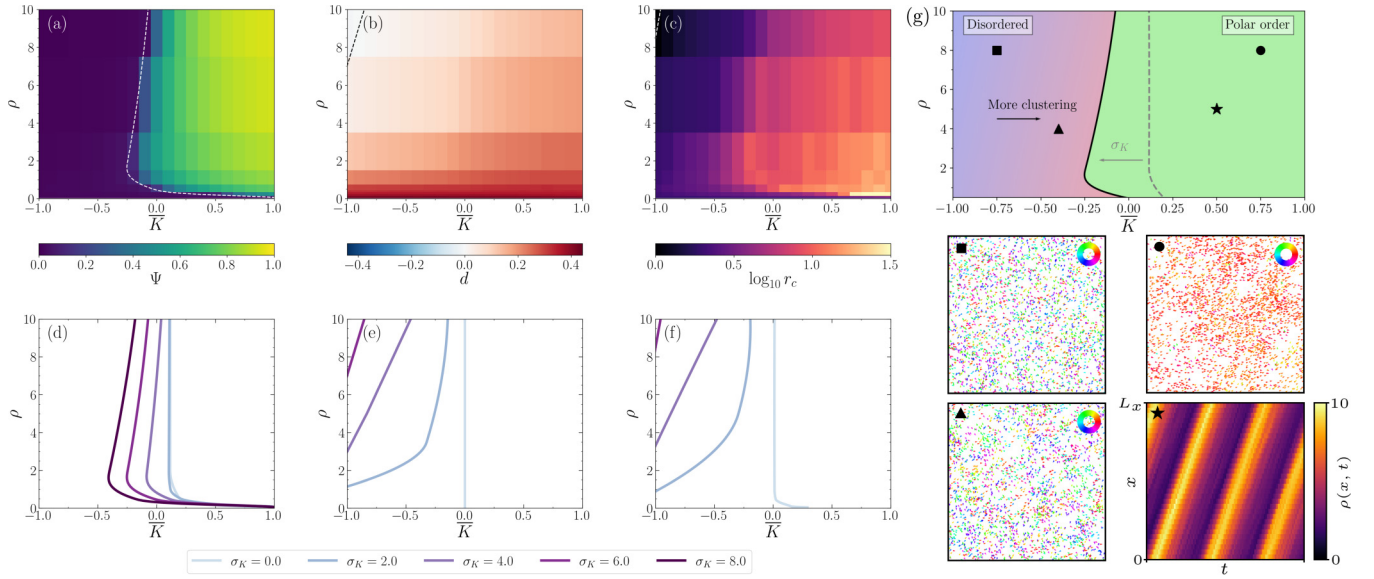


FIG. 4. Phase diagrams in the $(\bar{K}, \rho)$ plane. (a) Color map of the polar order parameter with a dashed line showing the order-disorder transition at $\Psi = 0.1$. (b) Color map of the demixing order parameter with a dashed line for the transition at $d = 0$. (c) Color map of the clustering order parameter, with a dashed line at $r_c = 1$. For panels (a)–(c), the parameters are as follows: $N = 10^4$, $\eta = 0.4$, and $\sigma_K = 6$. (d)–(f) Transition lines for various values of σ_K for the corresponding polar, demixing, and clustering order parameters shown above. Transition lines for a fixed value of $\sigma_K = 6$ are plotted on the same graph in Supplemental Material [80]. (g) Phase diagram for $\eta = 0.4$ and $\sigma_K = 6$ based on the phases determined by the three order parameters. The gray dashed line shows the polar order transition line for $\sigma_K = 0$ (i.e., no coupling disorder). This line shifts toward the left as σ_K increases. Example snapshots from simulations are shown below (zoomed in to $L/2 \times L/2$) for $\bar{K} = -0.75$, $\rho = 8$ (square); $\bar{K} = -0.4$, $\rho = 4$ (triangle); $\bar{K} = 0.75$, $\rho = 8$ (circle). Additionally, a kymograph of the density transverse to the band's direction against time is plotted for $\bar{K} = 0.5$, $\rho = 5$ (star) with $N = 10^5$, $L_x/L_y = 5$ to show the presence of banding in the polar ordered phase. See movies S3–S6 and their descriptions in the Supplemental Material [80].

4(d)]. Remarkably, for large enough values of σ_K , the system exhibits a region of parameter space with global polar order but $\bar{K} \leq 0$; this is made possible thanks to the self-sorting mechanism discussed above. Note that as the density ρ is varied, we observe reentrance in the polar order parameter not seen for homogeneous couplings ($\sigma_K = 0$); indeed, the observed critical value $\bar{K}_c$ for the onset of flocking ($\Psi > 0$) varies nonmonotonically with the density and displays a maximum around $\rho \approx 2$ for our choice of parameters.

As $\rho \rightarrow 0$, we expect a transition to disorder, as seen in the original Vicsek model [5]. At low density, the lack of interaction prevents the propagation of information. At very high densities, particles have many interacting neighbors and the self-sorting clustering mechanism needed for polar order to arise is impaired due to a greater frustration in the system; the number of interacting neighbors outweighs the number of large, positive couplings K_{ij} , thus destroying global polar order.

To further support self-organization and clustering as a mechanism to promote polar order when $\bar{K} < 0$, we define the demixing order parameter as

$$d = \left\langle \frac{1}{N} \sum_{i=1}^N \frac{\kappa_i}{n_i} - \frac{1}{2} \right\rangle, \quad (3)$$

where n_i is the total number of interacting neighbors of particle i within the radius of interaction σ_I and κ_i is the number of interacting neighbors with $K_{ij} > 0$. Their ratio therefore quantifies the proportion of ferromagnetically in-

teracting neighbors, meaning that $d = 0$ corresponds to a perfectly mixed system and $d > 0$ (resp. $d < 0$) to a system with more local ferromagnetic (resp. antiferromagnetic) interactions on average. The average $\langle \cdot \rangle$ is taken over time and realizations of K_{ij} , removing particles with no neighbors from the sum.

In Fig. 4(b), we show the value of d in the $(\bar{K}, \rho)$ plane and draw a transition line at $d = 0$ [see Fig. 4(e) for the transition lines for several values of σ_K]. For completely mixed systems, we would expect a vertical transition line at $\bar{K} = 0$. However, we notably find regions of parameter space where $d > 0$ despite $\bar{K} < 0$, in line with the clustering mechanism described above and seen in Fig. 2(e). We also note that a heuristic mean-field calculation yields a minimal nucleation criterion for the appearance of rare cooperative subsets, confirming the presence of cooperative nuclei with positive mean coupling value when $\bar{K} < 0$ and σ_K is sufficiently large (see Supplemental Material Ref. [80] for details). The active dynamics can subsequently amplify such cooperative subsets into global order, although this lies beyond the mean-field description.

For each system, we also measure the normalized equal-time connected density correlations $C_\rho(r)$ (see Supplemental Material [80]). We define the typical cluster size r_c to be the radial distance such that $C_\rho(r_c) = 0.01$. In a homogeneous system, we would expect correlations to decay very quickly outside the radius of interaction, leading to $r_c \approx \sigma_I$. For systems with clustering, density fluctuations increase such that $r_c \gg \sigma_I$. Typical cluster sizes are reported on a log scale in Fig. 4(c) for $\sigma_K = 6$. We define

the boundary between homogeneous and clustered systems as $r_c = \sigma_l$. Note that the original Vicsek model ($\sigma_K = 0$) also displays clustering in the disordered phase close to the transition [82], though this region is much smaller in the $(\bar{K}, \rho)$ space compared to the ADXY model. Comparing Figs. 4(d)–4(f), we find that the transitions for demixing and clustering coincide with each other. Systems in this clustered disordered region display local ordering within highly aligning clusters but have not become clustered enough to propagate this information far enough to display global polar order. Similar disordered clustered states were found in another active particle model with distance dependent alignment [83], and more recently in an ADXY model with nonreciprocal alignment [81]. Finally, note that the clustering transition observed in the disordered phase is accompanied by a transition from hyperuniform to clustered states (see Supplemental Material [80]).

IV. PHASE DIAGRAM FOR THE ACTIVE DISORDERED XY MODEL

We propose a phase diagram for the ADXY model in Fig. 4(g). For $\bar{K} \ll 0$, we have the familiar disordered homogeneous phase. As $\bar{K}$ increases, clustering sets in. Once $\bar{K}$ is increased further, we reach the polar ordered flocking phase, which includes regions of parameters space where $\bar{K} \leq 0$. Note that the onset of polar order is accompanied by a sharp increase in the particles' persistence and the typical pairwise contact time (see Supplemental Material [80]). Polar order decreases monotonically with increasing noise strength η , as in the Vicsek model (see Supplemental Material [80]).

We also find a banding phase between the disordered phase and the homogeneous polar liquid phase (or Toner-Tu phase). For $N = 10^4$, this manifests as clustered systems with large density fluctuations due to finite-size effects, as shown in Fig. 2(b). However, for large enough system sizes, we recover ordered traveling bands, as reported for the original Vicsek model [23]. We found our model to be subject to even stronger finite-size effects, requiring for instance, even larger system sizes and a slab geometry to observe stable banding. A kymograph of the local density in slices transverse to the direction of travel reveals clear and stable periodic bands moving in time, with the largest density close to the front of the band, leaving a density trail in its wake (see details in Supplemental Material [80]). The presence of bands, together with a negative dip in the Binder cumulant $G = 1 - \langle \psi^4 \rangle / 3 \langle \psi^2 \rangle^2$ near the transition for small $\sigma_K > 0$ [Fig. 5(a)], supports a first-order order-disorder transition, as in the original Vicsek model [9,11]. Although for larger σ_K we could not clearly resolve this dip within the accessible system sizes, our results suggest that quenched disorder in the coupling strengths does not change the nature of the transition. We further find evidence that the polar liquid phase retains true long-range order: For $\sigma_K = 1$, the global polar order is consistent with an algebraic decay to a nonzero asymptotic value Ψ_∞ and its system-size dependence is well described by $\Psi - \Psi_\infty \sim L^{-2/3}$, consistent with the predictions of Toner and Tu [84].

Finally, we observe that the flocks obtained for $\sigma_K > 0$ appear to be more fragile than those for $\sigma_K = 0$; indeed, the microscopic angular diffusion coefficient in the flocking

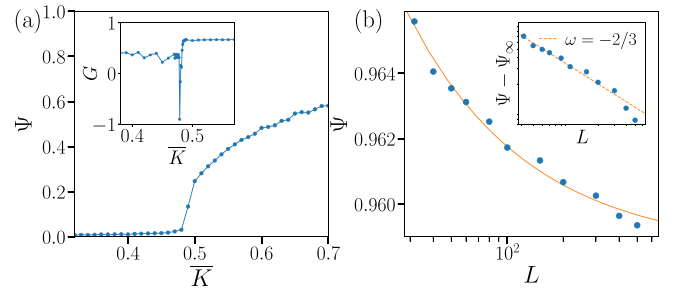


FIG. 5. Nature of the phase transition and ordered phase. (a) Polar order parameter Ψ vs $\bar{K}$ with $\sigma_K = 1$. Inset: Binder cumulant G vs $\bar{K}$ showing a negative dip around the transition point. Simulations were performed with $N = 6 \times 10^4$, $\rho = 1$, and $\eta = 0.6$. (b) Polar order parameter Ψ vs system size L with a line showing a fitted curve, which has algebraic decay to an asymptotic value of $\Psi_\infty = 0.958(5)$, fitted with a scaling exponent of $\omega = -2/3$. The inset shows the log-log fit of $\Psi - \Psi_\infty \sim L^\omega$ in a dashed line for the data (blue dots). Simulations were performed with $\rho = 1$, $\eta = 0.2$, $\bar{K} = 1$, $\sigma_K = 1$.

phase is larger when quenched disorder is present at all values of $\bar{K}$ (see Supplemental Material [80]).

V. CONCLUSION

In a swath of natural (including cellular tissues, bacterial suspensions, and bird flocks) and synthetic (including active colloids, self-propelled bots, cell colonies, and organoids) systems, agent-to-agent variability is the norm [85–89], and we expect randomly distributed alignment interactions to be the rule rather than the exception. Inspired by this, we introduced the ADXY model as an extension of existing models of self-propelled interacting particles to study the consequences of random alignment interaction for collective motion. Our main finding is that, when combined with activity, such coupling disorder can provide a general mechanism for global order, even for *negative* values of $\bar{K}$. In turn, we show that increasing the variance in the couplings can promote rather than destroy flocking, a surprising result when compared to recent studies [54,55]. Importantly, we show that this counterintuitive phenomenon is explained by a self-organization mechanism in which activity allows particles to rearrange into local neighborhoods with favorable alignment couplings, thereby reducing the local frustration present in a spatially uniform configuration. Our results further show that local clustering alone is not sufficient for global flocking, pointing instead to the crucial role of quenched reciprocal couplings in connecting local alignment to system-spanning order. In particular, we showed that the system exhibits clustering even before the onset of collective motion. Future work will attempt to explore the interplay between activity and frustration, and examine whether a spin-glass phase can exist in our model and more generally in active systems. Addressing these questions is analytically challenging and may benefit from combining analytical techniques from both active matter and spin-glass theory. While the work presented here is theoretical, our findings may offer a new perspective on the roles of heterogeneity and variability in natural swarms and cell-cell interactions.

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DATA AVAILABILITY

The data that support the findings of this article are openly available [90].

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