

Noise fingerprints of magnon scattering channels

Yuri Nishiwaki¹, Shoki Nezu¹, and Koji Sekiguchi^{2,3,4,*}¹Graduate School of Engineering Science, *Yokohama National University*, Tokiwadai 79-5, Yokohama 240-8501, Japan²Institute of Advanced Science, *Yokohama National University*, Tokiwadai 79-5, Yokohama 240-8501, Japan³Institute of Multidisciplinary Sciences, *Yokohama National University*, Tokiwadai 79-5, Yokohama 240-8501, Japan⁴Faculty of Engineering, *Yokohama National University*, Tokiwadai 79-5, Yokohama 240-8501, Japan

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We show that low-frequency magnonic noise provides statistical fingerprints of nonlinear scattering-network configurations in a yttrium iron garnet waveguide. By tuning the magnetic field, we control whether three-magnon splitting and confluence are kinematically allowed and change the active scattering pathways. When three-magnon processes are allowed, the noise exhibits abrupt anomalies and distinct fluctuation classes, including random-telegraph and $1/f$ -type spectra; otherwise, it approaches a near-white-noise form and evolves smoothly. Simulations support multistep magnon redistribution through competing nonlinear scattering pathways. Fluctuation statistics thus provide an experimental probe of dynamical-state selection in nonlinear magnon scattering networks.

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I. INTRODUCTION

Magnonics explores information transport and signal processing based on spin waves and their quanta, magnons, offering a wave-based and potentially low-dissipation complement to charge electronics. Low-damping magnetic insulators support phase-coherent magnon propagation and frequency-selective microwave functionality [1–5], forming a central materials platform within spintronics and wave-based computing concepts [6–9]. Many magnonic devices operate under finite-amplitude microwave excitation, where linear-response descriptions no longer capture the dynamics. In this regime, magnon-magnon interactions open multiple energy-transfer pathways in frequency–wave-vector (f , k) space through three- and four-magnon processes, and the system forms a nonequilibrium mode network whose dominant pathways are determined by the dispersion and magnetic field [10–22]. Identifying which scattering channels are active and how pathway dominance switches is therefore important for both nonlinear magnon transport and device-level spectral stability.

Low-frequency noise is widely used in semiconductor systems as a probe of internal channel dynamics and stochastic switching. Noise spectroscopy has provided microscopic insight into channel structure and fluctuation mechanisms in nonlinear and defect-mediated conduction. In magnonic systems, previous studies have established that low-frequency noise is sensitive to nonlinear magnon dynamics, including the onset of nonlinear scattering and auto-oscillation, and have reported Lorentzian and $1/f^\alpha$ -type noise under microwave

excitation [23–28]. However, whether the statistical form of the noise can distinguish different dynamical states of a nonlinear scattering network remains largely unexplored.

In this paper, we address this question by controlling the kinematic availability of three-magnon splitting and its inverse process, confluence, through the magnetic field. We show that the resulting changes in the nonlinear scattering network produce qualitatively distinct noise responses and fluctuation statistics. Importantly, distinct noise states also emerge within the same three-magnon-accessible condition, demonstrating that the noise contains information beyond the mere presence or absence of a nonlinear scattering process. These results establish low-frequency noise as a sensitive probe of dynamical-state selection in nonlinear magnon scattering networks.

II. EXPERIMENTAL METHODS

Figure 1(a) shows a schematic of the measurement setup. A continuous-wave microwave signal excites spin waves through a 50- Ω copper microstrip line, and the reflected signal is detected through a circulator using a precision spectrum analyzer. An external magnetic field $\mu_0 H_{\text{ext}}$ is applied along the x axis. The sample is a 10- μm -thick yttrium iron garnet (YIG) film grown by liquid-phase epitaxy on gadolinium gallium garnet and patterned into a 20 mm $\times$ 2 mm waveguide. Figure 1(b) shows the calculated dispersion relation versus wave vector along the guide. The kinematic accessibility of three-magnon splitting and confluence linking f_{exc} and $f_{\text{exc}}/2$ is determined by $\mu_0 H_{\text{ext}}$.

III. FIELD-DEPENDENT MAGNONIC NOISE AND NONLINEAR SCATTERING

Figures 1(c) and 1(d) show representative time traces $A(t)$ at 29 and 64 mT. From $A(t)$, we obtain the power spectral density $P(f)$ and define the magnonic noise power P by

*Contact author: sekiguchi-koji-gb@ynu.ac.jp

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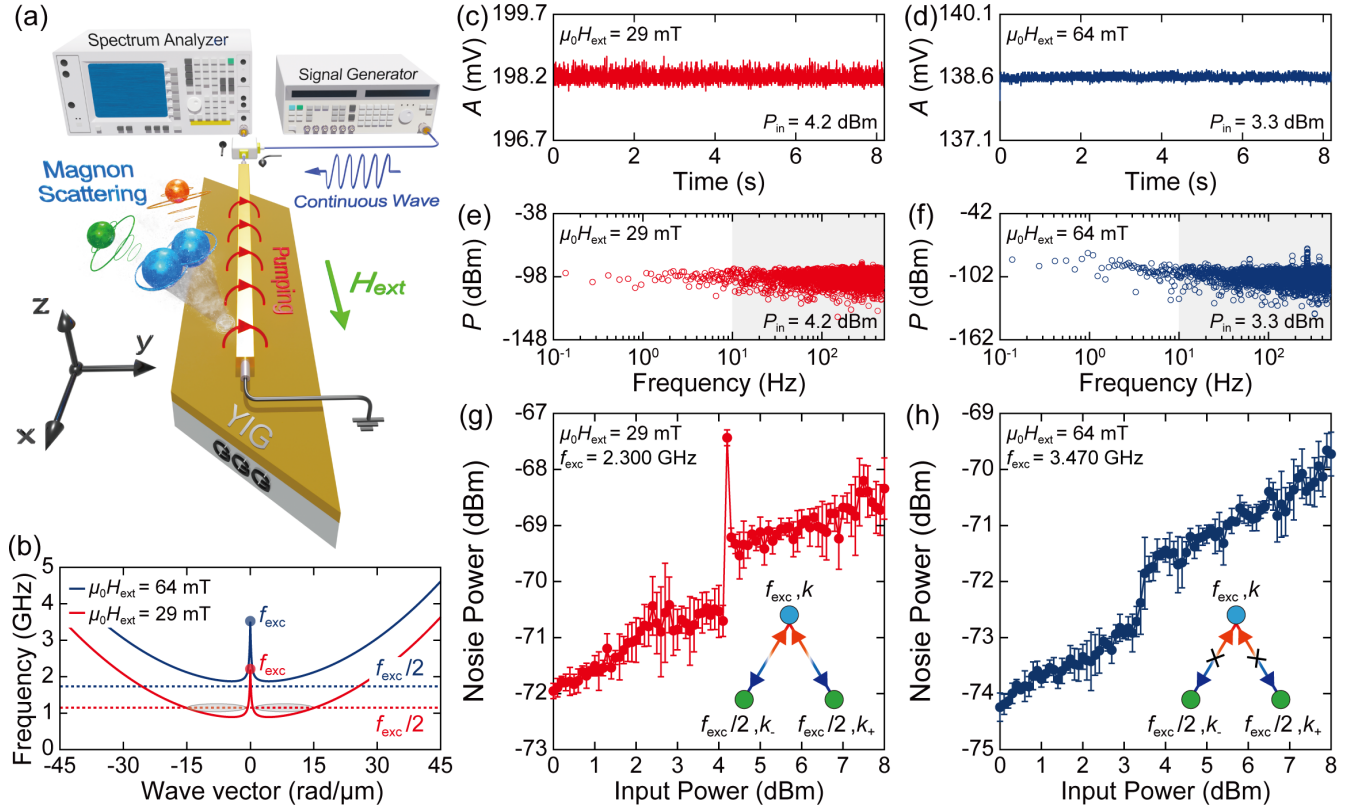


FIG. 1. Measurement scheme and field-dependent magnonic-noise response. (a) Setup for reflection detection of spin-wave signals in a YIG waveguide using a spectrum analyzer in the zero-span mode. (b) Calculated dispersion showing that the kinematic feasibility of three-magnon splitting and confluence linking f_{exc} and $f_{\text{exc}}/2$ depends on $\mu_0 H_{\text{ext}}$. (c), (d) Representative $A(t)$ traces at f_{exc} for $\mu_0 H_{\text{ext}} = 29$ mT and 64 mT. (e), (f) Corresponding low-frequency power spectra $P(f)$ obtained from fast Fourier transform (FFT) of $A(t)$. (g), (h) Input-power dependence of the magnonic noise power (integrated P over $10 \text{ Hz} \leq f \leq 500 \text{ Hz}$).

integrating $P(f)$ over $10 \text{ Hz} \leq f \leq 500 \text{ Hz}$. The corresponding spectra $P(f)$ are shown in Figs. 1(e), 1(f), 1(g), and 1(h) show the input-power dependence of the noise power. Each point represents the average of five repeated measurements under identical conditions. In both field conditions, the noise increases with input power, but the magnitude and functional form differ strongly. At 29 mT, the noise reaches a much larger overall level and shows a clearly nonmonotonic dependence, with a suppression near 2.8 dBm and a sharp peak near 4.1 dBm. At 64 mT, the variation is smoother and smaller in amplitude, without comparable peak-dip structure. From the dispersion relation in Fig. 1(b), three-magnon splitting is kinematically allowed at 29 mT but not at 64 mT. Three-magnon splitting of a strongly driven mode can be understood in the framework of the first-order Suhl instability, corresponding to the parametric decay of the driven mode into a pair of lower-frequency magnons. The much stronger noise response at 29 mT is associated with the opening of this additional nonlinear scattering channel, consistent with the onset of three-magnon processes allowed by the dispersion.

To relate the noise features to changes in the nonlinear scattering network, we examine frequency-swept spectra around the carrier $f_c = f_{\text{exc}}$ (Fig. 2). The spectra at 29 and 64 mT are shown in Figs. 2(a) and 2(b), respectively. Symmetric sidebands satisfying $2f_c = f_+ + f_-$ identify four-magnon scattering products. Although three-magnon processes are also active under the 29 mT condition, these symmetric

sideband pairs provide a direct measure of the four-magnon contribution. We therefore use the lower-sideband peak amplitude V_{four} as a measure of four-magnon scattering strength and compare it with the noise power shown in Figs. 2(c) and 2(d).

At 29 mT, V_{four} shows stepwise increases with input power, while the magnonic noise power is suppressed near 2.8 dBm and exhibits a sharp peak near 4.1 dBm [Figs. 2(a) and 2(c)]. At 64 mT, both the spectral evolution [Fig. 2(b)] and the noise response [Fig. 2(d)] vary more smoothly. The spectra show that the sideband peak amplitude decreases at high power, while the inset indicates spectral broadening with nearly conserved integrated intensity. At high power, the sideband peak amplitude decreases owing to spectral broadening, while the integrated sideband intensity remains nearly conserved, indicating that the peak amplitude alone no longer provides a direct measure of the total four-magnon population.

IV. MICROMAGNETIC ANALYSIS OF NONLINEAR SCATTERING DYNAMICS

To examine the redistribution of magnons in (f, k) space, we perform micromagnetic simulations (MuMax3) under the 29 mT condition, where three-magnon splitting is kinematically allowed by the spin-wave dispersion. Under the high-power excitation used here, both three- and four-magnon scattering processes can contribute to the magnon dynamics.

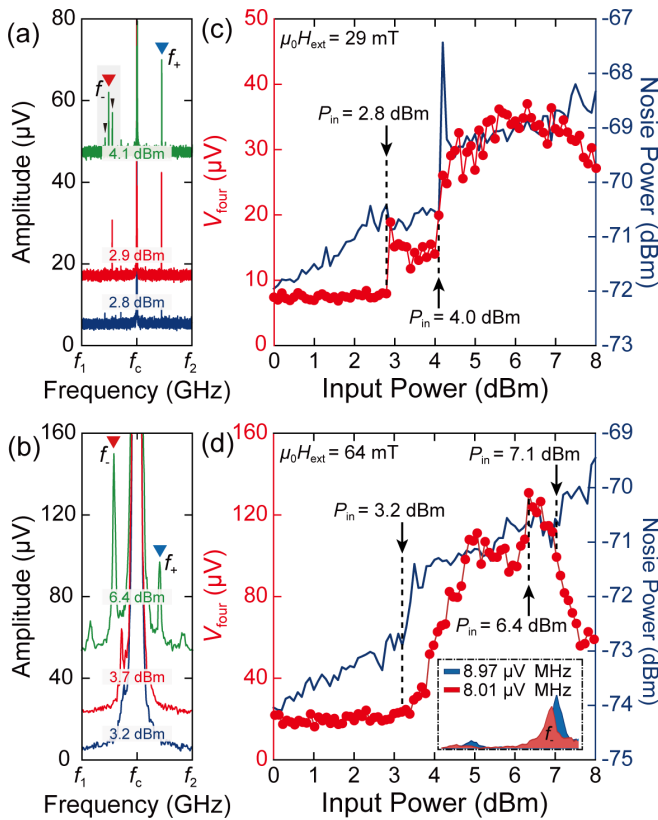


FIG. 2. Four-magnon sidebands and their correlation with magnonic noise. Spectra around the carrier ($f_c = f_{\text{exc}}$) at $\mu_0 H_{\text{ext}} = 29$ mT [panel (a)] and 64 mT [panel (b)]. Input-power dependence of the lower-sideband amplitude V_{four} (defined as the maximum amplitude in the band below f_c), compared with the magnonic noise power, showing stepwise changes and emergence of multipeak structure at 29 mT [panel (c)] and a smooth evolution at 64 mT [panel (d)]. The inset shows the corresponding spectrum as a function of frequency.

The simulations use the same static-field and excitation-frequency conditions as in the experiment. A sinusoidal microwave magnetic field is applied locally in the region corresponding to the microstrip line, with the rf field oriented perpendicular to the static bias field. Details of the simulations are provided in the Appendix.

Spatiotemporal Fourier analysis yields the frequency-wave-vector intensity map shown in Fig. 3(a). In addition to the directly excited component at $f_{\text{exc}} = 2.30$ GHz, a pronounced population appears around $f_{\text{exc}}/2 = 1.15$ GHz over a finite wave-vector range, consistent with three-magnon splitting. Finite-wave-vector spectral weight is also observed around f_{exc} , indicating redistribution toward higher-wave-vector states through nonlinear scattering. To enhance the visibility of the finite-wave-vector components, the spatially averaged magnetization was subtracted at each time step before applying the spatial Hanning window and Fourier transform. Consequently, the uniform $k = 0$ component is suppressed in Fig. 3(a). The intensity appearing near $k = 0$ at frequencies slightly below the calculated f_{FMR} originates predominantly from spectral leakage of finite- k components into the $k \approx 0$ region caused by the spatial windowing. Thus, the apparent downward frequency shift is an analysis-induced

artifact arising from this spectral leakage, rather than a physical shift of the uniform ferromagnetic resonance (FMR) mode. The indicated wave-vector windows define the regions used for the time-resolved analysis in Fig. 3(b).

To resolve the temporal evolution of these populations, Fig. 3(b) shows the integrated FFT intensity in selected regions of (f, k) space: the low- and high- k components at f_{exc} , the finite- k component at $f_{\text{exc}}/2$, and the low- and high- k components around f_{FMR} . Here, f_{FMR} specifies the frequency window centered at the calculated uniform FMR frequency and does not exclusively represent the $k = 0$ uniform mode. Immediately after the microwave excitation is applied, the low- k component at f_{exc} increases, followed by pronounced growth of the finite- k population at $f_{\text{exc}}/2$. At later times, the high- k component at f_{exc} increases, accompanied by a redistribution of spectral weight around f_{FMR} from the low- to the high- k region. After the excitation is turned off, the directly excited low- k component at f_{exc} rapidly decreases, whereas finite- k populations around $f_{\text{exc}}/2$, f_{FMR} , and f_{exc} persist.

This temporal sequence indicates that the high- k population at f_{exc} is not generated solely by a direct transfer from the half-frequency band, but instead reflects redistribution through multiple nonlinear scattering channels and intermediate magnon states. Representative pathways are illustrated on the calculated spin-wave dispersions in Fig. 3(c). Process 1 represents three-magnon splitting, in which an excitation-frequency magnon decays into two magnons near $f_{\text{exc}}/2$. Process 2 represents four-magnon scattering that redistributes the excitation-frequency population toward finite wave vectors. Process 3 represents the confluence of split magnons, in which two magnons near $f_{\text{exc}}/2$, with wave vectors k_1 and k_2 , combine to populate a state near f_{exc} with wave vector $k_1 + k_2$. The coexistence and temporal overlap of these pathways are consistent with a multistep redistribution of magnons through the nonlinear scattering network.

V. NOISE FINGERPRINTS OF DISTINCT DYNAMICAL STATES

At high input power, multiple nonlinear pathways are active simultaneously, making individual channel contributions difficult to separate quantitatively. We therefore examine the low-power regime, where four-magnon sidebands are not resolved and the number of active scattering channels is reduced. Figure 4(a) shows the low-power noise behavior at 29 and 64 mT. At 29 mT, the noise power increases strongly with input power and separates into distinct fluctuation states, whereas at 64 mT it varies smoothly and remains smaller in magnitude. In Fig. 4(a), the 29 mT data points are color coded by fluctuation class, with random telegraph noise (RTN) points shown in red and burstlike points in blue. As the input power is varied, these two classes appear in an alternating sequence across neighboring data points, making the RTN-burst pattern directly visible in the plot.

At 29 mT, the time traces $A(t)$ exhibit distinct fluctuation states depending on the input power, including two-level RTN and burstlike fluctuations [Figs. 4(b) and 4(d)]. As the input power is varied, these states appear in an alternating sequence, as shown in Fig. 4(a). This alternating behavior

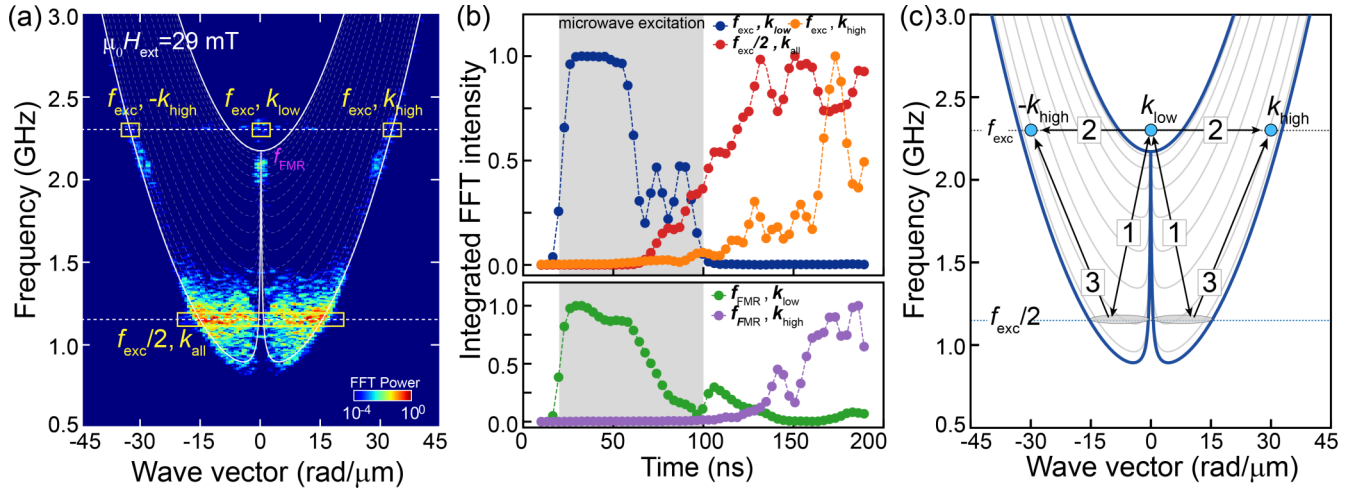


FIG. 3. Micromagnetic simulation of scattering-induced redistribution. (a) Frequency–wave-vector intensity map obtained from spatiotemporal Fourier analysis at $\mu_0 H_{\text{ext}} = 29$ mT. The horizontal dashed lines indicate the excitation frequency f_{exc} , half the excitation frequency $f_{\text{exc}}/2$. The indicated low- and high- k regions define the wave-vector windows used for the time-resolved analysis. (b) Time evolution of the integrated FFT intensity in the selected wave-vector windows: the low- and high- k components at f_{exc} , the finite- k component at $f_{\text{exc}}/2$, and the low- and high- k components around f_{FMR} . The shaded region indicates the microwave-excitation period. (c) Calculated spin-wave dispersion relations for different propagation angles between the wave vector and the equilibrium magnetization, together with representative nonlinear scattering pathways in the magnon network. (1) Three-magnon splitting: $(f_{\text{exc}}, 0) \rightarrow (f_{\text{exc}}/2 + k) + (f_{\text{exc}}/2 - k)$. (2) Four-magnon scattering: $(f_{\text{exc}}, 0) + (f_{\text{exc}}, 0) \rightarrow (f_{\text{exc}}, +k) + (f_{\text{exc}}, -k)$. (3) Three-magnon confluence of split magnons: $(f_{\text{exc}}/2k_1) + (f_{\text{exc}}/2k_2) \rightarrow (f_{\text{exc}}, k_1 + k_2)$.

is reproducibly observed across measurements and parameter scans at this field. The corresponding low-frequency spectra $P(f)$ show a Lorentzian component during the RTN intervals [Fig. 4(c)] and a spectrum dominated by a $1/f^\alpha$ background during the burstlike intervals [Fig. 4(e)], with the burstlike intervals generally exhibiting larger fluctuation amplitude.

At 64 mT, in contrast, the time trace shows neither RTN nor burstlike behavior [Fig. 4(f)], and the corresponding spectrum is close to white noise over the measured band [Fig. 4(g)], indicating the absence of dominant slow fluctuation processes. The measurements therefore resolve three distinct spectral classes—Lorentzian, $1/f^\alpha$, and white—

under different magnetic-field and pathway conditions, with the multiclass and alternating behavior appearing only when additional nonlinear scattering pathways are available.

These spectra reflect the effective number and spread of fluctuation timescales: A Lorentzian component is consistent with a dominant two-level fluctuation process, a $1/f^\alpha$ -type background with a broader distribution of fluctuation timescales, and a near-white-noise spectrum with the absence of dominant slow fluctuation processes.

In this picture, the $1/f^\alpha$ state is consistent with contributions from many competing scattering pathways with distributed characteristic timescales. The same device can

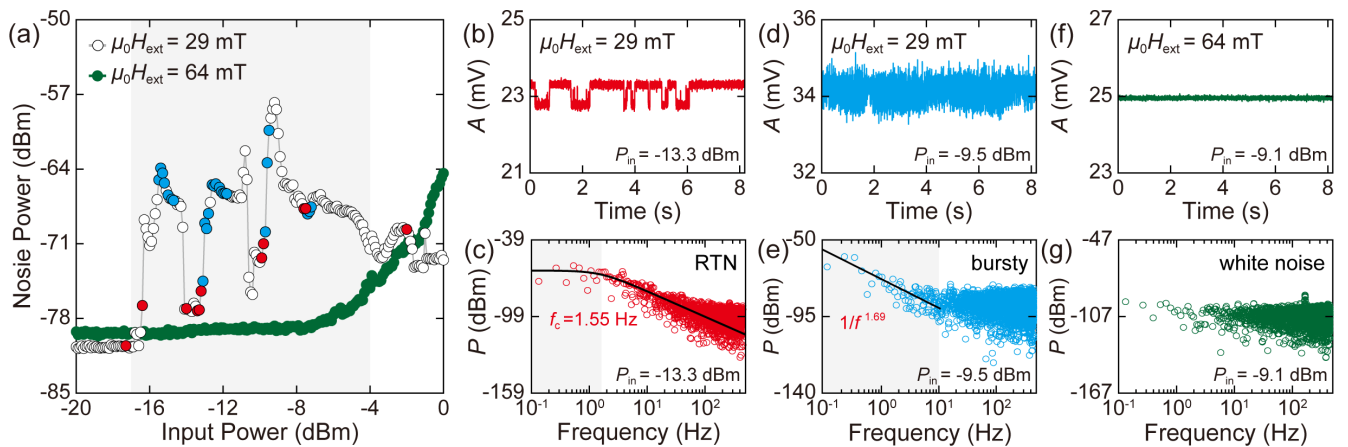


FIG. 4. Low-power magnonic-noise states. (a) Magnonic noise power vs input power P_{in} at $\mu_0 H_{\text{ext}} = 29$ mT and 64 mT. (b) Representative $A(t)$ trace at 29 mT showing (RTN). (c) Corresponding low-frequency power spectrum $P(f)$ exhibiting a Lorentzian component. (d) Representative $A(t)$ trace at 29 mT showing burstlike fluctuations. (e) Corresponding $P(f)$ dominated by a $1/f^\alpha$ background. (f) Representative $A(t)$ trace at 64 mT showing an absence of burst or RTN fluctuations. (g) Corresponding $P(f)$ at 64 mT exhibiting a white-noise spectrum. Note that the absolute spectral levels in panels (e) and (g) differ, reflecting the different fluctuation amplitudes in the corresponding time traces.

therefore occupy different fluctuation states that are resolved by low-frequency noise. Previous work reported Lorentzian and $1/f^\alpha$ magnonic noise in YIG systems [24,25]; here, we connect these spectral classes, including the white-noise state, to a field-controlled scattering mechanism. Distinct noise states appear only when three-magnon splitting and confluence are kinematically allowed, linking fluctuation class to the availability of competing nonlinear scattering pathways.

This role of noise as a state-resolving probe is analogous to its established uses in electronic devices, where low-frequency noise fingerprints reveal active transport channels and fluctuators, particularly in scaled metal-oxide-semiconductor field-effect transistor and nanoscale conductors [29]. In the present magnonic system, the noise spectrum likewise serves as a fingerprint of the dynamical configuration of the nonlinear scattering network, linking fluctuation statistics to the underlying nonlinear dynamics. These noise fingerprints thus provide a systematic way to distinguish different dynamical configurations of the nonlinear scattering network. More generally, these results show that the spectral form and temporal statistics of fluctuations can provide information on dynamical-state selection that is not captured by the average noise magnitude alone.

VI. CONCLUSION

In conclusion, by changing the external magnetic field to control whether three-magnon splitting and confluence are kinematically allowed, we demonstrate that low-frequency magnonic noise serves as a probe of nonlinear scattering-network configurations in a YIG waveguide. When three-magnon processes are allowed, the noise shows abrupt high-power anomalies and distinct low-power states, reflecting changes in the active scattering pathways; when they are not, the noise and spectra evolve smoothly. Low-frequency noise therefore provides a statistical fingerprint of nonlinear magnon scattering and pathway competition. More generally, fluctuation statistics provide a sensitive experimental probe of dynamical-state selection in nonlinear scattering networks.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this

research project. The data are available from the authors upon reasonable request.

APPENDIX: MICROMAGNETIC SIMULATION AND FOURIER ANALYSIS

Micromagnetic simulations were performed using MuMax3 by numerically solving the Landau-Lifshitz-Gilbert equation. The simulated YIG film had dimensions of $9.6 \times 9.6 \times 10 \mu\text{m}^3$ and was discretized into $384 \times 384 \times 16$ cells. Periodic boundary conditions were applied in both in-plane directions. The saturation magnetization, exchange stiffness, and Gilbert damping constant were set to $M_s = 1.42 \times 10^5 \text{ A/m}$, $A_{\text{ex}} = 3.5 \times 10^{-12} \text{ J/m}$, and $\alpha = 1.0 \times 10^{-4}$, respectively, and thermal fluctuations at 300 K were included. An in-plane static magnetic field of $\mu_0 H_{\text{ext}} = 29 \text{ mT}$ was applied along the y direction. A sinusoidal microwave magnetic field at $f_{\text{exc}} = 2.300 \text{ GHz}$ was applied locally using the spatial field profile corresponding to the microstrip antenna, with the rf field perpendicular to the static bias field.

For the spatiotemporal Fourier analysis, the out-of-plane magnetization $m_z(x, y, t)$ at the upper surface of the film was recorded every 50 ps. To enhance finite-wave-vector components in the f - k_y map [Fig. 3(a)], the spatially averaged magnetization was subtracted from each snapshot. The resulting magnetization was averaged along the x direction, and a spatial Hanning window was applied along the y direction before the spatial Fourier transform. Because this procedure suppresses the uniform $k_y = 0$ component, the residual intensity near $k_y = 0$ does not represent the uniform FMR mode. In particular, spatial windowing causes spectral leakage of finite- k_y components into the $k_y \approx 0$ region, producing the apparent intensity slightly below the calculated f_{FMR} .

For the time-resolved analysis in Fig. 3(b), FFT intensities were evaluated using a 20-ns temporal window and integrated over selected frequency and wave-vector regions. The low- k_y , finite- k_y , and high- k_y regions were defined as $|k_y| < 10 \text{ rad}/\mu\text{m}$, $5 \text{ rad}/\mu\text{m} < |k_y| < 20 \text{ rad}/\mu\text{m}$, and $25 \text{ rad}/\mu\text{m} < |k_y| < 40 \text{ rad}/\mu\text{m}$, respectively. The low- and high- k_y populations were evaluated around $f_{\text{exc}} = 2.300 \text{ GHz}$ and the calculated uniform FMR frequency $f_{\text{FMR}} \simeq 2.17 \text{ GHz}$, whereas the three-magnon-splitting population around $f_{\text{exc}}/2 = 1.150 \text{ GHz}$ was evaluated in the finite- k_y region. A frequency bandwidth of 100 MHz was used for each spectral window.

The analytical spin-wave dispersions in Figs. 3(a) and 3(c) were calculated using the same magnetic parameters and film thickness as in the micromagnetic simulation for different propagation angles θ between the in-plane wave vector and the equilibrium magnetization. The $\theta = 0^\circ$ and 90° limits correspond to the backward-volume and magnetostatic-surface-wave geometries, respectively.

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