

Static impurity in a mesoscopic system of $SU(N)$ fermionic matter waves

Juan Polo^{1,*}, Wayne J. Chetcuti^{1,2}, Anna Minguzzi², Andreas Osterloh¹, and Luigi Amico^{1,3,4}

¹Quantum Research Center, *Technology Innovation Institute*, Abu Dhabi 9639, United Arab Emirates

²Université Grenoble Alpes, CNRS, LPMMC, 38000 Grenoble, France

³Dipartimento di Fisica e Astronomia “Ettore Majorana,” *University of Catania*, Via S. Sofia 64, 95123 Catania, Italy

⁴INFN-Sezione di Catania, Via S. Sofia 64, 95123 Catania, Italy



(Received 28 November 2024; revised 9 June 2025; accepted 23 February 2026; published 15 April 2026)

We investigate the effects of a static impurity, modeled by a localized barrier, in a one-dimensional mesoscopic system comprised of strongly correlated repulsive $SU(N)$ -symmetric fermions. For a mesoscopic sized ring under the effect of an artificial gauge field, we analyze the energy spectrum, the particle density, and the current flowing through the impurity at varying interaction strengths, barrier heights, and number of components. We find that the physics of the system is governed by the competition between effective single-particle process and the formation of a high-stiffness spin-correlated state associated with the phenomenon of fractionalization of the flux quantum characterizing the N -component fermionic system. Our findings provide a route to probe the response of $SU(N)$ fermions to effective magnetic fields; at the same time, they hold significance for fundamental understanding of localized impurity problems.

DOI: [10.1103/tcmp-nf52](https://doi.org/10.1103/tcmp-nf52)

Introduction. The interplay between localized impurities and correlations in quantum many-body systems is an important topic for both basic and applied physical science, ranging from mesoscopic physics [1] and nanoelectronics [2] to high- T_c superconductivity [3] and spin liquids [4]. In this regard, Kane and Fisher carried out a groundbreaking work where they investigated an interacting electronic system at low energy and confined in an infinitely long wire interrupted by a single localized barrier [5,6]. Focusing on repulsively interacting particles, even arbitrary small barriers were found to tend to infinity (under renormalization group flow). Such remarkable results triggered a series of studies that shed light on different aspects of strongly correlated matter [4,7–10].

The emergence of quantum technology has ushered in a stage of addressing impurity problems, marked by the ability to manipulate systems on a fundamental level using newly engineered physical platforms such as cold atoms and superconducting networks. This unprecedented control enables the exploration of such systems in the presence of impurities with remarkable flexibility and precision over relevant parameters, including the characteristics of barriers and the nature of particle correlations [11–16]. Concurrently, applications have arisen, leveraging the interplay between impurity and correlations to craft quantum devices with enhanced performances, spanning from Josephson junction-based devices [17–23] to rotation sensors [24,25], and interferometers wherein the static impurity can serve as a matter wave beam splitter [26–29].

Our study focuses on a localized impurity in a one-dimensional mesoscopic system of N -component fermions, which are particles with N internal degrees of freedom that can be treated as an effective spin. On increasing the number of components, the Pauli exclusion principle relaxes allowing N particles to occupy the same quantum state, with interactions effects expected to be enhanced, leading to interesting physics [30]. We note that as $N \rightarrow \infty$, keeping the number of particles $N_p < N$ provides a “bosonic” limit [31].

Specifically, we consider $SU(N)$ -symmetric fermions with component-independent repulsive interactions [32–34]. Besides other approaches [5–10], the problem can be investigated through persistent currents flowing in annular matter wave circuits pierced by an effective magnetic field. The latter has been pursued in circuits of spinless bosons [21,35,36]. However, the spin degrees of freedom provide a significant complexity in impurity physics. When confined in mesoscopic ring-shaped potentials, at strong interactions $SU(N)$ fermions sustain persistent currents with fractional flux quanta ϕ_0/N_p , with ϕ_0 being the bare flux quantum of the free fermion case [37–39]. This phenomenon of fractionalization can be visualized as the formation of a collective state in which N_p particles are arranged on the ring with a high stiffness conferred by spin correlations. We refer to the latter as “ring droplet.” This behavior, combined with the knowledge that spin correlations and symmetries are known to play a crucial role in impurity problems [40], provides a compelling motivation for our study. We note that $SU(N)$ fermions are experimentally realizable with alkaline earthlike cold atoms [30,41–45]. A recent study has proposed a protocol to experimentally realize $SU(N)$ -symmetric systems through shielded ultracold molecules, which would circumvent the limitations of lack of tunable interactions through Feshbach resonances and different particle statistics [46,47].

*Contact author: juan.polo@tii.ae

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International](https://creativecommons.org/licenses/by/4.0/) license. Further distribution of this work must maintain attribution to the author(s) and the published article’s title, journal citation, and DOI.

In this Letter, we show that the underlying physics of $SU(N)$ matter waves flowing through a barrier is characterized by the interplay between its strength and the particle's interaction, with unique features arising from the N -component fermionic particles. Specifically, (1) we find a distinctive response of the system to the presence of the impurity with an $SU(N)$ -dependent energy gap formation mechanism, (2) we demonstrate that impurity screening results from the competition between an effective single-particle behavior (combined effect of interaction and impurity) and the unavoidable collective phenomenon implied by the formation of the aforementioned ring droplet, and (3) the impurity screening displays a marked dependence on the effective magnetic field through the particle density. To support the above statements, we analyze the energy spectrum of the system, particle's density, and persistent current that we study through a combination of numerical methods (exact diagonalization) and analytical techniques (Bethe ansatz) when applicable. Discussions on the physical implications of our findings are drawn in the conclusions.

$SU(N)$ fermionic matter wave currents. Consider N_p strongly interacting N -component fermions of mass m residing on a mesoscopic one-dimensional ring-shaped optical lattice composed of N_s sites threaded by an effective magnetic flux ϕ . The ring contains an impurity in the form of a localized potential barrier, breaking the discrete translational invariance. Such a scenario can be modeled through the multicomponent Fermi-Hubbard Hamiltonian [33,34]

$$\mathcal{H} = \sum_{j=1}^{N_s} \left[-t \sum_{\alpha}^N (e^{i\Omega} c_{j,\alpha}^{\dagger} c_{j+1,\alpha} + e^{-i\Omega} c_{j+1,\alpha}^{\dagger} c_{j,\alpha}) + \sum_{\alpha \neq \beta}^N U_{\alpha\beta} n_{j,\alpha} n_{j,\beta} + \sum_{\alpha}^N \lambda_{j,\alpha} n_{j,\alpha} \right], \quad (1)$$

where $c_{j,\alpha}^{\dagger}$ ($c_{j,\alpha}$) creates (destroys) a fermion with color α on site j and $n_{j,\alpha} = c_{j,\alpha}^{\dagger} c_{j,\alpha}$ is the local number operator. The parameters t and U correspond to the hopping and on-site interaction energies, respectively. Unless explicitly stated, we adopt isotropic interactions $U_{\alpha\beta} = U$, turning Eq. (1) into the $SU(N)$ Hubbard model. In this case, (1) it is a model that well represents experimentally realizable lattices of alkaline earthlike cold atoms [33,43] and (2) the system is, for $\lambda_{\alpha} = 0$, Bethe ansatz integrable in the continuous limit of vanishing lattice spacing as it tends to the Gaudin-Yang-Sutherland model [48,49]. The barrier of strength λ is equal for all colors localized at site j_0 such that $\lambda_{j,\alpha} = \lambda \delta_{j,j_0}$, taken to be positive. All energies will be given in units of t and we consider an equal number of particles per component.

The artificial gauge field $\Omega = (2\pi\phi)/(N_s\phi_0)$ is introduced through the Peierls substitution $t \rightarrow te^{i\Omega}$ [50]. For neutral cold atoms, the synthetic field can be introduced through various means [51], resulting in a Hamiltonian of the same form as that in Eq. (1) albeit with a different parametric expression for the elementary flux quantum ϕ_0 , which encodes the physical nature of the specific implementation. One instance is that of inducing rotation through stirring [52,53], which is particularly suitable in the presence of a barrier, where $\phi_0 = \hbar/(mR^2)$, with R being the radius of the ring (taken

to be equal to 1 in our case). In the presence of a flux, the many-body spectrum in the free particle regime is piecewise parabolic, caused by energy level crossings between parabolas of well-defined angular momentum per particle, denoted by ℓ , to counteract the increase in flux piercing the system. Consequently, the energy spectrum is periodic in ϕ with a period fixed by ϕ_0 akin to that of particles in a periodic potential. Therefore, following Leggett, $E(\phi)$ define the ‘‘Bloch bands’’ of the problem in which the magnetic flux plays the role of the momentum [54].

In order to characterize the flow of $SU(N)$ fermionic particles through a localized barrier, we utilize the spatial density $\langle n_j \rangle$ and the persistent current $I(\phi)$, which is the system's response to the applied field ϕ . At zero temperature, the persistent current is obtained from the ground-state energy E_0 : by the relation $I(\phi) = -\partial E_0/\partial \phi$. We employ the Hellmann-Feynman theorem to calculate the species-wise current $I_{\alpha}(\phi)$, which for lattice systems reads $I_{\alpha}(\phi) = -(2it\pi)/(\phi_0 N_s) \sum_{j=1}^{N_s} \langle e^{i\Omega} c_{j,\alpha}^{\dagger} c_{j+1,\alpha} - \text{H.c.} \rangle_{\text{GS}}$, with $\langle \cdot \rangle_{\text{GS}}$ being the ground-state expectation value. The resulting persistent current profile is a sawtooth shape with the jumps corresponding to changes in the system's angular momentum due to level crossings. In cold atom implementations, the sawtooth character of the current gives rise to the discrete steps in the angular momentum per particle as observed in experiments [53].

Fermionic matter wave currents, without an impurity, in the strongly interacting regime exhibit a reduced periodicity depending on the nature of interactions the particles are subjected to. For repulsive interactions, the phenomenon arises from energy crossings between the ground-state and higher-energy excitations characterized by different spin quantum numbers. Such a feature leads to an energy landscape consisting of N_p piecewise parabolic segments per flux quantum [37] (see Ref. [49] for attractive case). The reduced periodicity of the current reflects a fractional flux quantum ϕ_0/N_p mediated by energy level crossings at $n\phi_0/(2N_p)$ for odd integer n . The phenomenon of fractionalization is specific to a mesoscopic system with periodic boundary conditions reflecting the formation of a collective state, dubbed ring droplet, in which the specific spin correlations give rise to a stiff particle arrangement [37].

$SU(N)$ -dependent spectral gap formation. The presence of an impurity in a single-component quantum system, including bosons with repulsive or attractive interactions, is known to split *all* the degeneracies in $E_0(\phi)$ opening a spectral gap Δ [21,35,36,53]. At these avoided crossings, a coherent superposition of different angular momentum states is created. Consequently, the persistent current's sawtooth shape starts to smoothen out at weak barriers, eventually becoming a sinusoid as λ gets large enough.

For the present case of interacting $SU(N)$ fermions with an impurity, only specific degeneracies in $E_0(\phi)$ are found to split. The mechanism behind these selective gap openings lies in the internal degrees of freedom [55]. Indeed, as the barrier in Eq. (1) commutes with the quadratic $SU(N)$ Casimir operators with eigenvalue s , which characterizes each piecewise parabola of the energy landscape (see Supplemental Material [56–58]), it is unable to couple states with different values of s [59]. In turn, the Bloch bands and the corre-

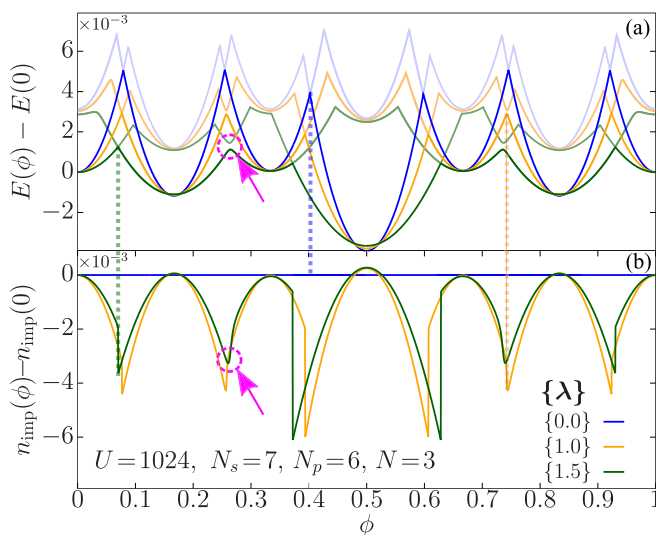


FIG. 1. Profiles of the energy $E(\phi)$ (top) and corresponding density at the impurity site n_{imp} (bottom) against the effective magnetic flux ϕ in the regime of strong interactions $U/t = 1024$ for various barrier strengths λ/t in a ring of N_s sites. Dotted lines highlight the connection between the energy peaks and density minima, while circles indicate the opening of the gaps in $E(\phi)$ and designated changes in n_{imp} . The quadratic Casimir values s for each energy parabola are $\{6, 3, 3, 0, 3, 3, 6\}$. Results obtained with exact diagonalization of the $SU(N)$ Hubbard model using the parameter set indicated in the figure.

sponding persistent current landscape are found to display a nontrivial dependence on s . Specifically, depending on N_p and N , the ground and excited states intersect each other as U increases: N_p dictates the amount of energy level crossings and N governs the quantity and ϕ dependence of the energy gaps through the $SU(N)$ Casimir. Such behavior reflects the dependence of the system's ground state on s . Consequently, only crossings characterized by the same Casimir value can be split by the barrier to generate a gap (Fig. 1).

For weak barriers, we find that Δ scales as λ/U (see Supplemental Material [56]). The $1/U$ behavior can be understood by a perturbative calculation around the Bethe ansatz results $\lambda = 0$ as $U \rightarrow \infty$ [57,60], which gives the energy contribution of the spin part scaling as t^2/U [57]. For the case of stronger barriers, we find that the gap scales nonlinearly with λ , as $\Delta(U, \lambda) \approx \frac{\lambda^\gamma}{U}$ with $\gamma > 1$ outside the linear response behavior expected at low λ .

We note that impurity and fractionalization compete with each other. In particular, the opening of the gap makes the energy landscape “protected” from fractionalization as soon interactions are strong enough to overcome the gap Δ between the ground-state and the higher excitation energy (characterized by different spin quantum numbers). Therefore, the low-lying spectrum at weak interaction in the presence of the impurity is not affected by the correlations emerging due to flux quantum fractionalization. In this sense, such regime is effectively single particle.

Screening of the local impurity. Due to the barrier's presence, there is a global minimum in the particle's density at the site where it is located, denoted by n_{imp} , meaning that particles tend to avoid residing in the barrier to minimize

$E(\phi)$. The reduced value of n_{imp} with decreasing N_p/N reflects the loosening of the Pauli principle (enabling more particles to reside at the impurity site, which enhances the barrier—see Supplemental Material [56]). The increase of n_{imp} with the effective interaction NU signals that the fluid screens the barrier. For weak U , the rate at which the barrier is screened $\partial n_{\text{imp}}/\partial U$ is larger with increasing N [61]. In contrast, $\partial n_{\text{imp}}/\partial U$ decreases with N for intermediate and large U , with n_{imp} saturating to a value, coinciding with that of hardcore bosons/spinless fermions, that is independent of N as $U \rightarrow \infty$. We note that when N_s is comparable to N_p , the density at the barrier is found to be nonmonotonic with U (see Supplemental Material [56]). Indeed, such specific screening properties arise because the presence of the impurity sets the aforementioned effectively weak interaction regime in which the spin correlations implied by the flux-quantum fractionalization are not effective. At larger interactions, instead, the flux-quantum fractionalization is relevant, and the properties of the system are dominated by the formation of the “ring droplet” of N_p particles.

Besides capturing the screening effect, the density profile can also provide information about the current when a barrier is present. In particular, Fig. 1(b) shows the flux dependence of the density reflecting both the energy landscape and the current in the system. As particles increase their flow, the barrier counteracts this motion by reducing the density at the impurity increasing its effectiveness. When the flow starts to reduce, i.e., when the ground-state energy decreases, the density at the impurity increases again reaching a maximum at zero current. We also note that smoothened gaps and energy crossings correspond to smooth and discontinuous jumps of $n_{\text{imp}}(\phi)$, which is also reflected in the density-density correlations (see Supplemental Material [56]).

Persistent current through a localized barrier. The interplay between the impurity strength and interaction manifests itself in the persistent current profile. Focusing on the current's maximum amplitude I_{max} , we find that it exhibits a nonmonotonic behavior as a function of λ and U (Fig. 2). Such behavior arises because of the aforementioned competition between the effective single-particle current, leading to the screening of the impurity and thus to a current increase, and the formation of the ring droplet, resulting in the suppression of the current (the ring droplet leads to an effective increase of the mass of the particles). In the following, we analyze the above competition as a function of interaction U and number of fermion components N . For weak interactions, and fixed λ , the current amplitude increases with U at a rate that is more pronounced with increasing N —A similar behavior occurs in the density n_{imp} (see Supplemental Material [56]). We note that $N_p/N = 1$ leads to a vanishing Pauli principle constraint. In this case, and at weak interactions, the bosonic limit of persistent currents is recovered [35]. For intermediate interactions such that $U/\lambda > 1$, where the phenomenon of fractionalization becomes more pronounced, the I_{max} reflects the impurity screening with a clear dependence on U . In this regime, the current undergoes a “hybridization” becoming a smoothed yet fractionalized one with cusps, exhibiting a reduced periodicity (see Supplemental Material [56]). Subsequently, I_{max} emerges from the cusped parabolas. We point out that in this regime,

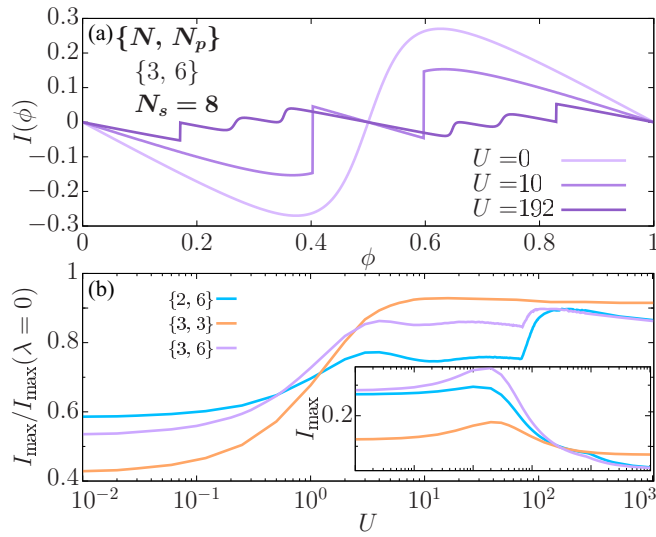


FIG. 2. (a) Current $I(\phi)$ vs flux ϕ showcasing the interplay between the characteristic fractionalized sawtooth and a smoothed profile given by the presence of the impurity. (b) Maximum persistent current amplitude $I_{\max}$ as a function of interactions U/t in the presence of a barrier with strength $\lambda/t = 1$ for different number of particles N_p and components N . In the main panel, the current is normalized by the maximum current computed at $\lambda = 0$ at all values of interactions, while inset shows the bare value of $I_{\max}$ vs U . The competition between screening of the barrier and fractionalization results in $I_{\max} U^\nu$, with $\nu \approx 0.7$ for $U/t \ll 1$ and $\nu \approx -0.8$ for $U/t \gg 1$. Results obtained with exact diagonalization of the $SU(N)$ Hubbard model.

even for $N_p/N = 1$, our persistent current is markedly different from that of single-component bosons—This feature is natural since in the latter system, no fractionalization occurs [53]. Such difference is clearly showcased in Fig. 2(b) in which the persistent current displays additional maxima and specific cusps that are a direct consequence of the fractionalization phenomenon (occurring because of specific level crossings implied by fractionalization).

In the limit of strong interactions, as $U/\lambda \gg 1$, the impact of the impurity is drastically reduced. Nonetheless, we find that the current is strongly suppressed. Such an effect primarily originates from the fractionalization whose rate is enhanced with N [37]. Consistently, the persistent current displays a perfect sawtooth shape with a reduced periodicity of $1/N_p$ exhibiting negligible signs of smearing.

Discussions and conclusions. We considered a single localized impurity embedded in a one-dimensional mesoscopic system of repulsive $SU(N)$ fermions subjected to an artificial gauge field with flux ϕ . The physics is characterized by the interplay between the impurity strength and interaction through the phenomenon of flux-quantum fractionalization that is a genuine feature of multicomponent systems [37]. Density-density correlations corroborate the two different regimes (see Supplemental Material [56]): For weak interactions, the correlations change gradually, signaling the effective single-particle behavior, while in the strongly interacting regime, a bunching effect occurs, meaning that correlations in the ring

droplet change discontinuously only for persistent currents with different winding numbers.

Below, we summarize our results for the energy spectrum of the system, density at the impurity site n_{imp} , and persistent current $I(\phi)$.

The impurity opens energy gaps $\Delta \sim 1/U$ selectively, following an $SU(N)$ dependent mechanism [see Fig. 1(a)]. At weak interactions, the flux-quantum fractionalization has a small influence on the impurity physics as long as the gap to the higher energy excitations is smaller than Δ . In such a regime, the system is characterized by an effective single-particle spectrum landscape (renormalized by the interaction). For increasing interactions, the fractionalization of the flux quantum becomes relevant and signals the formation of a ring droplet.

By looking at the density at the impurity site n_{imp} as a function of U , we can establish that initially the barrier is screened with a rate dependence that becomes larger with N : As more particles can reside at the impurity site, the impurity is screened faster since the effective repulsion is enhanced as NU in $SU(N)$ systems. As $U \rightarrow \infty$, n_{imp} is found to saturate to a value that is independent of N because of the formation of the ring droplet with a stiff particle arrangement [38]. Besides capturing the screening effect, n_{imp} displays a marked dependence on ϕ , reflecting the periodicity of the $SU(N)$ fermions flux quantum fractionalization [see Fig. 1(b)]. In a cold atom system, n_{imp} would be accessible, for example, through *in situ* density measurements (see Ref. [62] for a recent work on a one-dimensional fermionic gas). Such a feature can be exploited to access to the phenomenon of fractionalization in cold atoms experiments beyond the interference dynamics [63]. We note that the latter, being related to the momenta of particles and therefore to the current, is also found to be affected by the impurity.

As a result of the aforementioned competition between effective single-particle behavior and ring droplet formation, the current amplitude $I_{\max}$ exhibits a *non monotonic behavior* as a function of the interaction and parametrized by the impurity strength λ . While at weak U , $I_{\max}$ increases due to the suppression of the impurity as observed in the density n_{imp} , this is not the case on going to stronger interactions. In this regime, the current decreases as the result of the spin correlations characterizing the ring droplet. Such distinctive features of $SU(N)$ matter wave emerge in Fig. 2.

The study of $SU(N)$ fermions confined in rings poses an interesting challenge from both an analytical and a numerical standpoint. Analytical techniques have been found at strong interactions [57], while numerically one faces known problems coming from periodic boundary conditions [as, e.g., in density matrix renormalization group (DMRG)] combined with a high degeneracy in the ground state. To address these limits, further developments are being explored [64,65]. On the analytical side, we remark that our study is in a very different regime in which the standard one-dimensional Luttinger theory applies, as the Fermi sphere in our system is hardly filled [9] nonetheless, we expect to obtain similar features as the ones discussed in this letter. As shown in our results and as predicted by Bethe ansatz [37], the fractionalization is a direct consequence of the coupling between the spin and “charge” rapidities. However, Luttinger theory

implements spin-charge separation, with the caveat that the two sectors in the Luttinger Hamiltonian (in the mesoscopic regime) are still coupled through certain constraints on spin and charge quantum numbers [66]; as a result, the charge and spin current operators J_c and J_s , respectively, are functionally related. Because of the latter feature, the factorization shows up in terms of energy crossings in the energy response to Aharonov-Bohm flux ϕ $E_c(J_c; \phi) \propto (J_c - \phi/\phi_0)^2$ (see also Ref. [67]). Moreover, the selective gap openings we discussed in our Letter hold true also in the Luttinger liquid picture. In this context, a full renormalization group description [68] including the barrier effects would provide an interesting step forward.

We highlight that static impurities are of high relevance for quantum technology. We note that the experimental realization based on alkali-earth atoms that was proposed recently for rings with a spin impurity [69] can be pursued for the present case with nearly the same ingredients. The setup considered here can also provide the basis for current-based simulators and interferometers [29,36,51,70–72] utilizing N -component matter waves.

Acknowledgment. We thank Enrico C. Domanti and Sam Carr for useful discussions.

Data availability. The data supporting the findings of this study can be generated using standard methods and procedures described within the article.

- [1] B. L. Altshuler, P. A. Lee, and W. R. Webb, *Mesoscopic Phenomena in Solids* (Elsevier, Amsterdam, 2012).
- [2] G. L. Timp and R. E. Howard, Quantum mechanical aspects of transport in nanoelectronics, *Proc. IEEE* **79**, 1188 (1991).
- [3] H. Alloul, J. Bobroff, M. Gabay, and P. J. Hirschfeld, Defects in correlated metals and superconductors, *Rev. Mod. Phys.* **81**, 45 (2009).
- [4] A. Kolezhuk, S. Sachdev, R. R. Biswas, and P. Chen, Theory of quantum impurities in spin liquids, *Phys. Rev. B* **74**, 165114 (2006).
- [5] C. L. Kane and M. P. A. Fisher, Transport in a one-channel Luttinger liquid, *Phys. Rev. Lett.* **68**, 1220 (1992).
- [6] C. L. Kane and M. P. A. Fisher, Transmission through barriers and resonant tunneling in an interacting one-dimensional electron gas, *Phys. Rev. B* **46**, 15233 (1992).
- [7] J. von Delft and H. Schoeller, Bosonization for beginners—refermionization for experts, *Ann. Phys.* **510**, 225 (1998).
- [8] H. Saleur, Lectures on non perturbative field theory and quantum impurity problems, in *Topological Aspects of Low Dimensional Systems: Session LXIX* (Springer, Berlin, 2002), p. 473.
- [9] T. Giamarchi, *Quantum Physics in One Dimension* (Clarendon Press, Oxford, 2003), Vol. 121.
- [10] C. Rylands and N. Andrei, Quantum impurity in a Luttinger liquid: Exact solution of the Kane-Fisher model, *Phys. Rev. B* **94**, 115142 (2016).
- [11] P. Manju, K. S. Hardman, M. A. Sooriyabandara, P. B. Wigley, J. D. Close, N. P. Robins, M. R. Hush, and S. S. Szigeti, Quantum tunneling dynamics of an interacting Bose-Einstein condensate through a Gaussian barrier, *Phys. Rev. A* **98**, 053629 (2018).
- [12] S. Léger, J. Puertas-Martínez, K. Bharadwaj, R. Dassonneville, J. Delaforce, F. Foroughi, V. Milchakov, L. Planat, O. Buisson, C. Naud, *et al.*, Observation of quantum many-body effects due to zero point fluctuations in superconducting circuits, *Nat. Commun.* **10**, 5259 (2019).
- [13] J. Puertas Martínez, S. Léger, N. Gheeraert, R. Dassonneville, L. Planat, F. Foroughi, Y. Krupko, O. Buisson, C. Naud, W. Hasch-Guichard, S. Florense, I. Snyman, and N. Roch, A tunable Josephson platform to explore many-body quantum optics in circuit-qed, *npj Quantum Inf.* **5**, 19 (2019).
- [14] R. Kuzmin, N. Grabon, N. Mehta, A. Burshtein, M. Goldstein, M. Houzet, L. I. Glazman, and V. E. Manucharyan, Inelastic scattering of a photon by a quantum phase slip, *Phys. Rev. Lett.* **126**, 197701 (2021).
- [15] S. Mistakidis and A. Volosniev, *Physics of Impurities in Quantum Gases* (MDPI, Basel, 2022).
- [16] S. Léger, T. Sépulcre, D. Fraudet, O. Buisson, C. Naud, W. Hasch-Guichard, S. Florense, I. Snyman, D. M. Basko, and N. Roch, Revealing the finite-frequency response of a bosonic quantum impurity, *SciPost Phys.* **14**, 130 (2023).
- [17] M. Nadeem, M. S. Fuhrer, and X. Wang, The superconducting diode effect, *Nat. Rev. Phys.* **5**, 558 (2023).
- [18] A. M. Hrisu and Yu. V. Nazarov, Coulomb blockade due to quantum phase slips illustrated with devices, *Phys. Rev. B* **83**, 174511 (2011).
- [19] M. Trahms, L. Melischek, J. F. Steiner, B. Mahendru, I. Tamir, N. Bogdanoff, O. Peters, G. Reecht, C. B. Winkelmann, F. von Oppen, *et al.*, Diode effect in Josephson junctions with a single magnetic atom, *Nature (London)* **615**, 628 (2023).
- [20] C. Ryu, P. W. Blackburn, A. A. Blinova, and M. G. Boshier, Experimental realization of Josephson junctions for an atom SQUID, *Phys. Rev. Lett.* **111**, 205301 (2013).
- [21] D. Aghamalyan, M. Cominotti, M. Rizzi, D. Rossini, F. Hekking, A. Minguzzi, L.-C. Kwek, and L. Amico, Coherent superposition of current flows in an atomtronic quantum interference device, *New J. Phys.* **17**, 045023 (2015).
- [22] G. Valtolina, A. Burchianti, A. Amico, E. Neri, K. Khani, J. A. Seman, A. Trombettoni, A. Smerzi, M. Zaccanti, M. Inguscio, and G. Roati, Josephson effect in fermionic superfluids across the BEC-BCS crossover, *Science* **350**, 1505 (2015).
- [23] V. P. Singh, J. Polo, L. Mathey, and L. Amico, Shapiro steps in driven atomic Josephson junctions, *Phys. Rev. Lett.* **133**, 093401 (2024).
- [24] C. Ryu, E. C. Samson, and M. G. Boshier, Quantum interference of currents in an atomtronic SQUID, *Nat. Commun.* **11**, 3338 (2020).
- [25] O. Adeniji, C. Henry, S. Thomas, R. C. Sapp, A. Goyal, C. W. Clark, and M. Edwards, Double-target BEC atomtronic rotation sensor, [arXiv:2411.06585](https://arxiv.org/abs/2411.06585).
- [26] R. M. Godun, M. B. d’Arcy, G. S. Summy, and K. Burnett, Prospects for atom interferometry, *Contemp. Phys.* **42**, 77 (2001).
- [27] S. A. Haine, Quantum noise in bright soliton matterwave interferometry, *New J. Phys.* **20**, 033009 (2018).

- [28] O. J. Wales, A. Rakonjac, T. P. Billam, J. L. Helm, S. A. Gardiner, and S. L. Cornish, Splitting and recombination of bright-solitary-matter waves, *Commun. Phys.* **3**, 51 (2020).
- [29] P. Naldesi, J. Polo, P. D. Drummond, V. Dunjko, L. Amico, A. Minguzzi, and M. Olshanii, Massive particle interferometry with lattice solitons, *SciPost Phys.* **15**, 187 (2023).
- [30] L. Sonderhouse, C. Sanner, R. B. Hutson, A. Goban, T. Bilitewski, L. Yan, W. R. Milner, A. M. Rey, and J. Ye, Thermodynamics of a deeply degenerate $SU(N)$ -symmetric Fermi gas, *Nat. Phys.* **16**, 1216 (2020).
- [31] H. Frahm and A. Schadschneider, On the Bethe ansatz soluble degenerate Hubbard model, in *The Hubbard Model: Its Physics and Mathematical Physics*, edited by D. Baeriswyl, D. K. Campbell, J. M. P. Carmelo, F. Guinea, and E. Louis (Springer US, Boston, MA, 1995), p. 21.
- [32] A. V. Gorshkov, M. Hermele, V. Gurarie, C. Xu, P. S. Julienne, J. Ye, P. Zoller, E. Demler, M. D. Lukin, and A. M. Rey, Two-orbital $SU(N)$ magnetism with ultracold alkaline-earth atoms, *Nat. Phys.* **6**, 289 (2010).
- [33] M. A. Cazalilla and A. M. Rey, Ultracold Fermi gases with emergent $SU(N)$ symmetry, *Rep. Prog. Phys.* **77**, 124401 (2014).
- [34] S. Capponi, P. Lecheminant, and K. Totsuka, Phases of one-dimensional $SU(N)$ cold atomic Fermi gases—From molecular Luttinger liquids to topological phases, *Ann. Phys.* **367**, 50 (2016).
- [35] M. Cominotti, D. Rossini, M. Rizzi, F. Hekking, and A. Minguzzi, Optimal persistent currents for interacting bosons on a ring with a gauge field, *Phys. Rev. Lett.* **113**, 025301 (2014).
- [36] J. Polo, P. Naldesi, A. Minguzzi, and L. Amico, The quantum solitons atomtronic interference device, *Quantum Sci. Technol.* **7**, 015015 (2021).
- [37] W. J. Chetcuti, T. Haug, L.-C. Kwek, and L. Amico, Persistent current of $SU(N)$ fermions, *SciPost Phys.* **12**, 33 (2022).
- [38] W. J. Chetcuti, Persistent currents in atomtronic circuits of $SU(N)$ fermions, Ph.D. thesis, University of Catania, 2023.
- [39] O. I. Pătu and D. V. Averin, Temperature-dependent periodicity of the persistent current in strongly interacting systems, *Phys. Rev. Lett.* **128**, 096801 (2022).
- [40] N. Andrei, K. Furuya, and J. H. Lowenstein, Solution of the Kondo problem, *Rev. Mod. Phys.* **55**, 331 (1983).
- [41] S. Taie, R. Yamazaki, S. Sugawa, and Y. Takahashi, An $SU(6)$ Mott insulator of an atomic Fermi gas realized by large-spin Pomeranchuk cooling, *Nat. Phys.* **8**, 825 (2012).
- [42] G. Pagano, M. Mancini, G. Cappellini, P. Lombardi, F. Schäfer, H. Hu, X.-J. Liu, J. Catani, C. Sias, M. Inguscio, and L. Fallani, A one-dimensional liquid of fermions with tunable spin, *Nat. Phys.* **10**, 198 (2014).
- [43] F. Scazza, C. Hofrichter, M. Höfer, P. C. De Groot, I. Bloch, and S. Fölling, Observation of two-orbital spin-exchange interactions with ultracold $SU(N)$ -symmetric fermions, *Nat. Phys.* **10**, 779 (2014).
- [44] C. Hofrichter, L. Riegger, F. Scazza, M. Höfer, D. R. Fernandes, I. Bloch, and S. Fölling, Direct probing of the Mott crossover in the $SU(N)$ Fermi-Hubbard model, *Phys. Rev. X* **6**, 021030 (2016).
- [45] S. Taie, E. Ibarra-García-Padilla, N. Nishizawa, Y. Takasu, Y. Kuno, H.-T. Wei, R. T. Scalettar, K. R. A. Hazzard, and Y. Takahashi, Observation of antiferromagnetic correlations in an ultracold $SU(N)$ Hubbard model, *Nat. Phys.* **18**, 1356 (2022).
- [46] B. Mukherjee, J. M. Hutson, and K. R. A. Hazzard, $SU(N)$ magnetism with ultracold molecules, *New J. Phys.* **27**, 013013 (2025).
- [47] B. Mukherjee and J. M. Hutson, $SU(N)$ symmetry with ultracold alkali dimers: Weak dependence of scattering properties on hyperfine state, *Phys. Rev. Res.* **7**, 013099 (2025).
- [48] B. Sutherland, Further results for the many-body problem in one dimension, *Phys. Rev. Lett.* **20**, 98 (1968).
- [49] W. J. Chetcuti, J. Polo, A. Osterloh, P. Castorina, and L. Amico, Probe for bound states of $SU(3)$ fermions and colour deconfinement, *Commun. Phys.* **6**, 128 (2023).
- [50] R. Peierls, Zur Theorie des Diamagnetismus von Leitungselektronen, *Z. Phys.* **80**, 763 (1933).
- [51] L. Amico, M. Boshier, G. Birkel, A. Minguzzi, C. Miniatura, L.-C. Kwek, D. Aghamalyan, V. Ahufinger, D. Anderson, N. Andrei, *et al.*, Roadmap on atomtronics: State of the art and perspective, *AVS Quantum Sci.* **3**, 039201 (2021).
- [52] K. C. Wright, R. B. Blakestad, C. J. Lobb, W. D. Phillips, and G. K. Campbell, Driving phase slips in a superfluid atom circuit with a rotating weak link, *Phys. Rev. Lett.* **110**, 025302 (2013).
- [53] J. Polo, W. J. Chetcuti, T. Haug, A. Minguzzi, K. Wright, and L. Amico, Persistent currents in ultracold gases, *Phys. Rep.* **1137**, 1 (2025).
- [54] A. J. Leggett, Dephasing and non-dephasing collisions in nanostructures, in *Granular Nanoelectronics*, edited by D. K. Ferry, J. R. Barker, and C. Jacoboni (Springer US, Boston, MA, 1991), p. 297.
- [55] On account of this, such behavior is different from that of bosons, even the ones with attractive interactions where fractionalization is present.
- [56] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/tcmp-nf52> for additional derivations and data, which includes Refs. [73–76].
- [57] A. Osterloh, J. Polo, W. J. Chetcuti, and L. Amico, Exact one-particle density matrix for $SU(N)$ fermionic matter waves in the strong repulsive limit, *SciPost Phys.* **15**, 006 (2023).
- [58] G. Pecci, G. Aupetit-Diallo, M. Albert, P. Vignolo, and A. Minguzzi, Persistent currents in a strongly interacting multi-component Bose gas on a ring, *C. R. Phys.* **24**, 87 (2023).
- [59] Note that the $SU(N)$ representation can be different, which would generally be indicated by a different s . However, for some N the different representations can share the same s .
- [60] M. Ogata and H. Shiba, Bethe-ansatz wave function, momentum distribution, and spin correlation in the one-dimensional strongly correlated Hubbard model, *Phys. Rev. B* **41**, 2326 (1990).
- [61] The barrier is the least effective versus interactions at $N_p = N$. In this case, the density at the impurity site coincides with that of a bosonic system of the same N_p . Such a behavior reflects the lack of a meaningful Pauli exclusion principle in the system.
- [62] A. Litvinov, P. Bataille, E. Maréchal, P. Pedri, O. Gorceix, M. Robert-De-Saint-Vincent, and B. Laburthe-Tolra, Measuring densities of cold atomic clouds smaller than the resolution limit, *Phys. Rev. A* **104**, 033309 (2021).
- [63] W. J. Chetcuti, A. Osterloh, L. Amico, and J. Polo, Interference dynamics of matter-waves of $SU(N)$ fermions, *SciPost Phys.* **15**, 181 (2023).

- [64] T. Botzung and P. Nataf, Exact diagonalization of $SU(N)$ Fermi-Hubbard models, *Phys. Rev. Lett.* **132**, 153001 (2024).
- [65] A. Weichselbaum, QSpace—An open-source tensor library for Abelian and non-Abelian symmetries, *SciPost Phys. Codebases* **40** (2024).
- [66] A. Seidel and D.-H. Lee, The Luther-Emery liquid: Spin gap and anomalous flux period, *Phys. Rev. B* **71**, 045113 (2005).
- [67] F. V. Kusmartsev, Aharonov-Bohm effect in the Luttinger liquid, *JETP Lett.* **60**, 639 (1994).
- [68] A. O. Gogolin, A. A. Nersisyan, and A. M. Tsvelik, *Bosonization and Strongly Correlated Systems* (Cambridge University Press, Cambridge, 2004).
- [69] A. Amaricci, A. Richaud, M. Capone, N. D. Oppong, and F. Scazza, Engineering the Kondo impurity problem with alkaline-earth atom arrays, *Phys. Rev. A* **112**, 043301 (2025).
- [70] L. Amico, D. Anderson, M. Boshier, J.-P. Brantut, L.-C. Kwek, A. Minguzzi, and W. von Klitzing, *Colloquium: Atomtronic circuits: From many-body physics to quantum technologies*, *Rev. Mod. Phys.* **94**, 041001 (2022).
- [71] J. Polo, W. J. Chetcuti, E. C. Domanti, P. Kitson, A. Osterloh, F. Perciavalle, V. P. Singh, and L. Amico, Perspective on new implementations of atomtronic circuits, *Quantum Sci. Technol.* **9**, 030501 (2024).
- [72] P. Naldesi, J. Polo, V. Dunjko, H. Perrin, M. Olshanii, L. Amico, and A. Minguzzi, Enhancing sensitivity to rotations with quantum solitonic currents, *SciPost Phys.* **12**, 138 (2022).
- [73] S. D. Huber, B. Theiler, E. Altman, and G. Blatter, Amplitude mode in the quantum phase model, *Phys. Rev. Lett.* **100**, 050404 (2008).
- [74] N. Yu and M. Fowler, Persistent current of a Hubbard ring threaded with a magnetic flux, *Phys. Rev. B* **45**, 11795 (1992).
- [75] M. Consiglio, W. J. Chetcuti, C. Bravo-Prieto, S. Ramos-Calderer, A. Minguzzi, J. I. Latorre, L. Amico, and T. J. G. Apollaro, Variational quantum eigensolver for $SU(N)$ fermions, *J. Phys. A* **55**, 265301 (2022).
- [76] G. Del Pace, K. Khani, A. Muzi Falconi, M. Fedrizzi, N. Grani, D. Hernandez Rajkov, M. Inguscio, F. Scazza, W. J. Kwon, and G. Roati, Imprinting persistent currents in tunable fermionic rings, *Phys. Rev. X* **12**, 041037 (2022).