

Anomalous wave-packet dynamics in one-dimensional non-Hermitian lattices

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Non-Hermitian (NH) systems have attracted great attention due to their exotic phenomena beyond Hermitian domains. Here, we systematically study the wave-packet dynamics in general one-dimensional NH lattices and uncover several unexpected phenomena. The group velocity of a wavepacket during the time evolution in such NH lattices is governed not only by the real part of the band structure but also by its imaginary part. The momentum also evolves due to the imaginary part of the band structure, which can lead to a *self-induced Bloch oscillation* in the absence of external fields. Furthermore, we discover that the wave-packet dynamics can exhibit disorder-free NH jumps even when the energy spectra are entirely real, which we call the *NH wave-packet jump*. Finally, we show that the NH wave-packet jumps can lead to both positive and negative temporal Goos-Hänchen shifts at the edge.

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Introduction. Wave dynamics lies at the heart of a variety of fundamental phenomena in lattice systems, such as Bloch oscillations [1,2], dynamic localization [3,4], and Anderson localization [5–8]. Although these phenomena were originally discovered for electronic wave functions, similar phenomena turn out to exist also in classical light. Beyond lattice physics, wave dynamics also exhibits various unique phenomena for light. A prominent example is the propagation of a light beam at an interface between media with different refractive indices, where the reflected beam undergoes a spatial displacement, known as the Goos-Hänchen shift [9–17], leading to an important application in sensing [18–20]. Recently, the counterpart of this effect in the time domain, i.e., temporal Goos-Hänchen shift (TGHS), was proposed in a dispersive linear medium with a time boundary [21] and subsequently demonstrated in Hermitian synthetic discrete-time heterolattices [22].

On the other hand, considerable attention has been devoted to wave dynamics in non-Hermitian (NH) systems with dissipation, which are described by NH Hamiltonians. Recent studies have shown that when disorder originates from dissipation rather than on-site potentials, conventional Anderson localization is replaced by a quantized jump behavior, termed the NH jump [23–36]. This phenomenon stems from complex energy spectra and localized eigenstates, which exhibits a unique feature of NH systems. Another intriguing example is the NH skin effect [37–53], where eigenstates become localized at the boundary under open boundary conditions (OBCs) in NH lattices with asymmetric couplings. This effect has

been shown to originate from a nontrivial point-gap topology defined in the complex energy plane [54–56]. It has been noted that the skin effect can give rise to unconventional wave reflections at boundaries, where the reflected velocity of a wave packet deviates from the conventional case, called the dynamic skin effect [57–59]. In addition, self-acceleration of wave packets accompanied by momentum changes has also been found in NH lattices with or without skin effects [60–65]. Despite these advances, a general theory of wave dynamics in NH lattices is still not fully developed, and a variety of unique physical phenomena remain largely unexplored.

In this work, we systematically study the wave-packet dynamics in general one-dimensional NH lattices. The group velocity of a Gaussian wave packet in such systems is governed not only by the slope of the real part of band structure but also by its imaginary part, when the wave packet evolves with time. Based on this anomalous group velocity, we find a *self-induced Bloch oscillation* without electric fields. We further discover disorder-free NH jumps in NH lattices under OBCs with entirely real spectra, as found also in Refs. [66,67]. We call this phenomenon the *NH wave-packet jump*. We can explain this phenomenon by the complex band structure under periodic boundary conditions (PBCs). We also find that when a wave packet is close to the edge, the NH wave-packet jumps can also lead to positive and negative TGHSs, accompanied by corresponding frequency blueshifts and redshifts, respectively. We also find the phase transitions among dynamic skin effects, TGHSs, and NH wave-packet jumps, thereby establishing a fundamental connection between the NH skin effect and observable wave dynamics.

Anomalous group velocity in NH lattices. We consider general one-dimensional single-band NH lattice models. (As we will point out, most of the results hold also for multiband models.) The Hamiltonian we consider in real space is

$$H = \sum_n V a_n^\dagger a_n + \sum_{m,n} (J_m^R a_{n+m}^\dagger a_n + J_m^L a_n^\dagger a_{n+m}), \quad (1)$$

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where $a_{n+m}^\dagger$ (a_{n+m}) is the creation (annihilation) operator for the $(n+m)$ th lattice site with the coupling order $m = 1, 2, 3, \dots$ ranging all the sites. $J_m^{R(L)}$ denotes the coupling strength from left to right (right to left) and can take complex values, while $V = \Delta + i\gamma$ is the on-site energy, with Δ (γ) being the real (imaginary) potential. The corresponding band structure calculated from the Bloch Hamiltonian under the PBC reads $E(k) = V + \sum_m (J_m^R e^{-imk} + J_m^L e^{imk}) = E_R(k) + iE_I(k)$, with $E_R(k)$ [$E_I(k)$] being the real (imaginary) part of the band structure and k being the Bloch momentum. Since V just contributes to the overall shift of the energy in complex plane and does not affect the wave-packet dynamics, we set $V = 0$ without loss of generality in the rest of the Letter.

We discuss the wave-packet dynamics under this lattice obeying the Schrödinger-type equation [57,62,64]

$$i \frac{d}{dt} \vec{\psi}(t) = H \vec{\psi}(t), \quad (2)$$

where $\vec{\psi}(t)$ is the wave function describing the amplitude of the wave at each site at time t . We assume a Gaussian wave packet at $t = 0$, whose amplitude at site n is $\psi_n(0) = \frac{1}{(2\pi\sigma^2)^{1/4}} e^{-\frac{(n-n_0)^2}{4\sigma^2}} e^{ik_0(n-n_0)}$. Here, k_0 , n_0 , and σ are the initial momentum, central position, and width of the wave packet, respectively.

We define the momentum-space wave packet by the Fourier transformation of $\psi_n(t)$ as $\psi_k(t) \propto \sum_n \psi_n(t) e^{-ikn}$. Assuming infinite-size lattice, we can analytically solve Eq. (2) and find $\psi_k(t) = C e^{-ikn_0} e^{-iE_R(k)t} e^{-\sigma^2(k-k_0)^2 + E_I(k)t}$, where C is a constant. We call the momentum at which $\psi_k(t)$ is maximum $k_{\max}(t)$. This $k_{\max}(t)$ will evolve in time t as [54,60,68,69] (see also the Supplemental Material [70])

$$k_{\max}(t) = k_0 + \frac{t}{2\sigma^2} \left. \frac{dE_I(k)}{dk} \right|_{k=k_{\max}}. \quad (3)$$

Thus, $k_{\max}$ will change with time if the imaginary part of the band is nonzero [$E_I(k) \neq 0$], and it will increase (decrease) for $dE_I(k)/dk > (<)0$, and eventually approaches k^* with the maximum $E_I(k)$.

Since the change of $k_{\max}$, the center of mass of the wave packet $\bar{n}$ will evolve with time as [60,62–64,70]

$$\bar{n}(t) = \frac{\sum_n n |\psi_n(t)|^2}{\sum_n |\psi_n(t)|^2} = n_0 + \bar{v}_g(t)t, \quad (4)$$

where $\bar{v}_g(t) \equiv \langle v_g \rangle$, with $v_g \equiv dE_R(k)/dk$, and the momentum-space average of a function $f(k)$ is defined by $\langle f(k) \rangle \equiv \int_{-\pi}^{\pi} dk |\psi_k(t)|^2 f(k) / \int_{-\pi}^{\pi} dk |\psi_k(t)|^2$. Note that $\bar{v}_g(t)$ is nothing but the ordinary group velocity for Hermitian systems, which is given by the average of $dE(k)/dk$. We, however, find that the corresponding group velocity of this non-Hermitian system is

$$V_g(t) \equiv \frac{d\bar{n}(t)}{dt} = \bar{v}_g(t) + \frac{d\bar{v}_g(t)}{dt}t, \quad (5)$$

which is different from the Hermitian system by the second term. This second term can be rewritten as $\frac{d\bar{v}_g(t)}{dt}t = 2t[\langle E_I v_g \rangle - \langle E_I \rangle \bar{v}_g]$ [70], which we call the *anomalous group velocity*. Note that the existence of anomalous group velocity

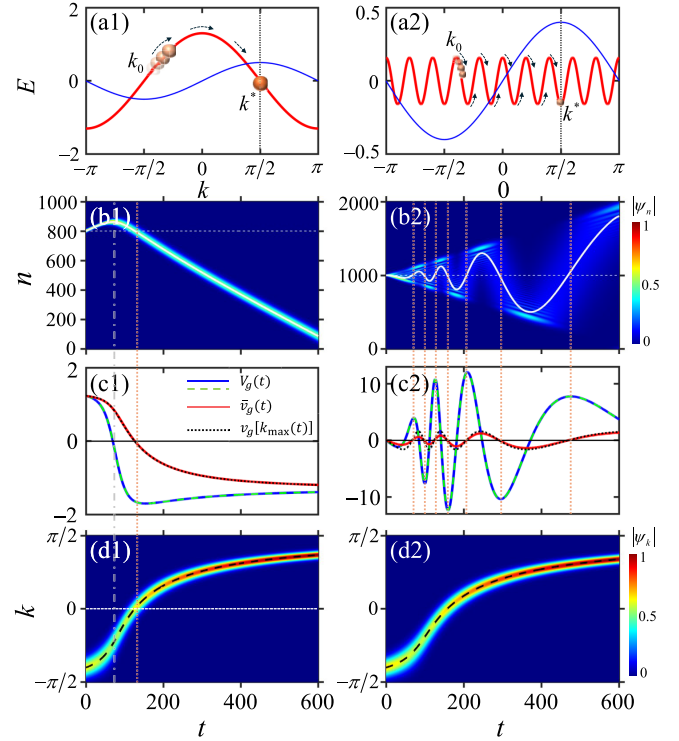


FIG. 1. (a1), (a2) Band structures, $E_R(k)$ (red lines) and $E_I(k)$ (blue lines). (b1), (b2) The evolution of a wave packet in NH lattices, where the white solid lines denote the predicted center of mass $\bar{n}$ according to Eq. (4). The horizontal white dashed lines are at n_0 . (c1), (c2) The evolution of group velocities $\bar{v}_g(t)$ (red lines), $v_g[k_{\max}(t)]$ (black dotted lines), the numerical $V_g(t)$ (blue solid lines), and the predicted $V_g(t) = v_g[k_{\max}(t)]$ according to Eq. (5). The vertical gray dash-dotted line in panels (b) and (c) denotes $V_g = 0$, while the red dotted lines highlight the agreement of zero points of $\bar{n}(t) = n_0$ with $\bar{v}_g(t) = v_g[k_{\max}(t)] = 0$. (d1), (d2) The evolution of the wave packet in momentum space with the theoretical $k_{\max}(t)$ denoted by the black dashed lines. The parameters are $J_1^L = 0.9$, $J_1^R = 0.4$, and $J_{n>1}^L = 0$ in panels (a1)–(d1), while $J_1^L = 0.2$, $J_1^R = -0.2$, and $J_{10}^L = J_{10}^R = 0.08$ in panels (a2)–(d2). $k_0 = -0.4\pi$ and $\sigma = 5$.

and the evolution of momentum $k_{\max}$ [Eq. (3)] were first found by Muschietti and Dum in 1993 [68].

We now numerically demonstrate the anomalous wave-packet dynamics considering the Hatano-Nelson model [74], which is Eq. (1) with $J_m^{R(L)} \neq 0$ only for $m = 1$. For such a model, $E_I(k)$ has the maximum value at $k^* = \pi/2$ [Fig. 1(a1)]. We simulate the real-space dynamics of a wave packet and plot the normalized wave function $|\psi_n(t)|/\sqrt{\sum_n |\psi_n(t)|^2}$ [23–26,57,62] in Fig. 1(b1), with the center of mass $\bar{n}(t)$ highlighted by a white solid line. The maximum momentum $k_{\max}(t)$ of the wave packet evolves toward k^* , consistent with the theoretical prediction of Eq. (3) (black dashed line) [Fig. 1(d1)].

In Fig. 1(c1), we plot the group velocities of the corresponding time evolution. One sees that $\bar{v}_g(t)$ (red solid line) agrees well with $v_g[k_{\max}(t)]$ (black dotted line), which means that the momentum-space average of $v_g = dE_R(k)/dk$ is well approximated by its value at $k_{\max}(t)$. However, as we noted earlier, the true group velocity of NH system should be

different from $\bar{v}_g(t)$. Indeed, in a solid blue line, we plot the true group velocity calculated from the numerically obtained values of $\bar{n}(t)$, and we see a clear deviation from $\bar{v}_g(t)$. The numerically obtained group velocity agrees perfectly with the theoretically predicted value of $V_g(t)$ as given in Eq. (5), plotted in a green dashed line. We note an intrinsic relation between $\bar{v}_g$ and V_g ; they are related by $\bar{v}_g(t) = (1/t) \int_0^t V_g(t') dt'$. This relation implies that the average velocity of the wave packet evolving to time t is exactly $\bar{v}_g(t)$ [see also Eq. (4)]. Since $\bar{v}_g(t) \approx v_g[k_{\max}(t)]$, we see that the average velocity over time t is given by the instantaneous value of $v_g = dE_R(k)/dk$ at $k = k_{\max}(t)$. We note that Eqs. (3)–(5) also hold in multiband cases (see the Supplemental Material [70]). We note that the momentum is not conserved here even though the system has a translational symmetry [70].

Self-induced Bloch oscillations. The combined effect of momentum evolution and anomalous group velocity allows for a self-induced Bloch oscillation in the absence of an external electric field. This phenomenon happens when $E_R(k)$ oscillates faster than $E_I(k)$. To achieve such a situation, we consider Hermitian long-range couplings $J_{10}^R = J_{10}^L$ and non-Hermitian nearest-neighbor coupling $J_1^L = -J_1^R$. The band structure is plotted in Fig. 1(a2). According to Eq. (3), the wave packet moves in momentum space toward $k^* = \pi/2$, at which $E_I(k)$ becomes maximum.

Figure 1(b2) shows the wave-packet evolution in real space along with the center of mass $\bar{n}(t)$, indicating a self-induced Bloch oscillation. The envelope amplitude of the Bloch oscillation increases linearly with time, unlike the Hermitian case where it remains constant [1,2], and its slope is determined by the maximum value of $v_g = dE_R(k)/dk$ [70]. The period of oscillation is not constant, and this is because the momentum of the wave packet does not change linearly but becomes slower in time, as one can see from Fig. 1(d2).

NH wave-packet jumps with real spectra. So far, we have considered lattices that are relatively large compared to the width σ of the wave-packet. When the lattice is smaller, NH wave packet jumps occur. In Figs. 2(c1) and 2(c2), we plot the time evolution of a wave packet in a lattice of size 60 and 100, respectively. We observe that jumps of the wave function occur during the evolution, which are the NH wave-packet jumps we are going to analyze. We note that the NH jumps found here are associated with wave-packet dynamics in systems with entirely real spectra under OBCs [see black dots in Fig. 3(a)], which are in contrast to previous studies of NH jumps with disorders and complex spectra [23–36].

We consider the same wave packet $\psi_n(0)$ at $t = 0$ in real space as before. In momentum space, when the lattice is finite, the wave function is $\psi'_k(0) = \sum_n \psi_n(0) e^{-ik_m n}$, where $k_m = 2\pi m/N$, with $m = -N/2, \dots, N/2 - 1$. In Fig. 2(a1), we show the finite-size momentum-space wave function $\psi'_k(0)$ for $N = 60$ and the infinite-size counterpart $\psi_k(0)$. We see that $\psi'_k(0)$ overlaps with $\psi_k(0)$ near the initial momentum $k_0 = 0$, and deviates from it for other momenta. The evolution of the wave packet in momentum space is then $\psi'_k(t) = c_k(t) e^{-iE_R t}$, with $c_k(t) = \psi'_k(0) e^{E_I(k)t}$ being the amplitude of each momentum at time t . The momentum distribution function $c_k(t)$ with $c_{k=k^*}(t)$ highlighted by the blue dashed line is shown in Fig. 2(b1). One sees that $c_{k=k^*}(t)$ gradually increases and exceeds other $c_{k \neq k^*}(t)$ near $t_c \approx 17$, and then dominates the

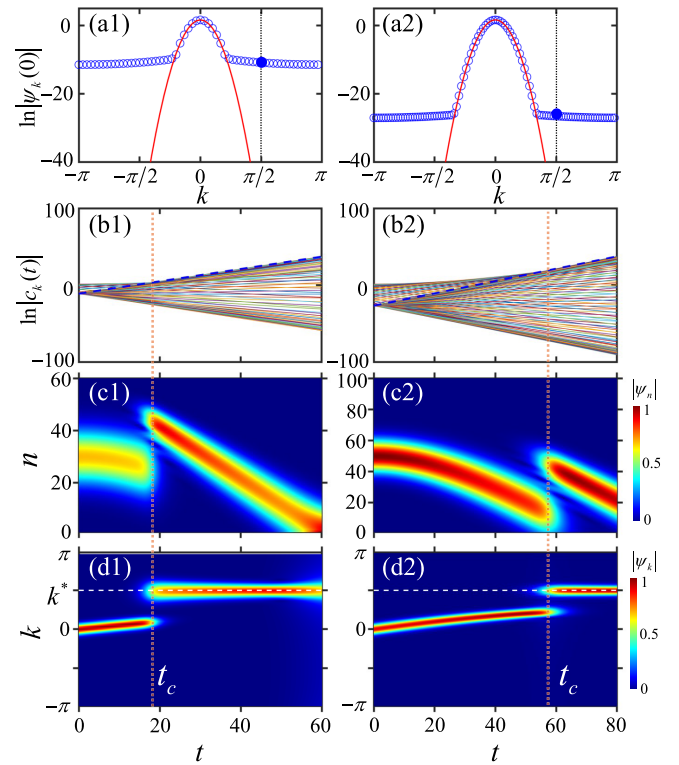


FIG. 2. (a1), (a2) Initial wave packet in momentum space with a finite (blue circles) and an infinite size (red lines). The blue dots indicate the values for $k = k^* = \pi/2$. (b1), (b2) The evolution of $c_k(t)$, where $c_{k=k^*}(t)$ is highlighted by the blue dashed lines. The evolution of the wave packet in (c1), (c2) real and (d1), (d2) momentum spaces under OBCs, where the horizontal white dashed lines denote $k = k^* = \pi/2$. The vertical red dotted lines in panels (b), (c), and (d) highlight the time t_c when the NH wave-packet jump occurs. The sizes in panels (a1)–(d1) and (a2)–(d2) are $N = 60$ and $N = 100$, respectively. Other parameters are $J_1^L = 0.9$, $J_1^R = 0.1$, $J_{m>1}^{L,R} = 0$, $k_0 = 0$, and $\sigma = 5$.

evolution. Consequently, the wave packet jumps to the state with $k = k^* = \pi/2$, as shown in the real and momentum-space evolution in Figs. 2(c1) and 2(d1), respectively.

Since the initial momentum distribution $\psi'_k(0)$ is related to the size of the lattice, the jump time t_c depends on the size N . This is demonstrated in Figs. 2(a2)–2(d2), where we show another case with $N = 100$ and keep other parameters the same. In addition to the jump time, jump position can also be identified [70]. It should be noted that the wave dynamics remains the same under both OBCs and PBCs, as the wave packet propagates in the bulk. The NH wave-packet jumps disappear when the lattice size is larger than $2\pi\sigma^2$ [70]. We also find that such NH wave-packet jumps occur in a wide range of NH lattices, not restricted to the ones with asymmetric couplings [70]. For multiband lattices, NH wave-packet jumps can occur between different bands (see the Supplemental Material [70]).

NH wave-packet jumps close to the edge. We now discuss that when a wave packet approaches the edge of a system exhibiting the skin effect and has a purely real spectrum under the OBC, NH wave-packet jumps can also occur, but with a different mechanism.

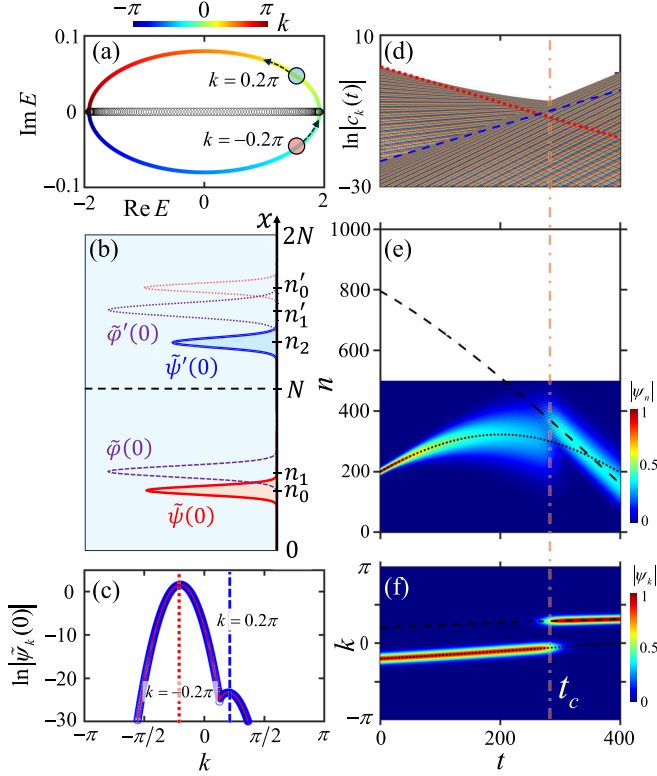


FIG. 3. (a) Energy spectra under the PBC (colored line) and OBC (black dots) for a finite lattice of size $N = 500$. The red (blue) circle denotes the initial state of the initial (auxiliary) wave packet, which moves along the spectra denoted by the black dashed arrow. (b) Relative position of different wave functions. (c) The initial wave packet $\tilde{\psi}(0) + \tilde{\psi}'(0)$ in momentum space, where the blue dots (red line) denote the result under the finite lattice of size $2N$ (infinite case). (d) The evolution of $c_k(t)$, where the red dotted (blue dashed) line denotes the case for $k_0 = -0.2\pi$ ($-k_0 = 0.2\pi$). (e) The evolution of the wave packet in real space. (f) The evolution of the wave packet in momentum space. The black dotted (dashed) lines in panels (e) and (f) are theoretical center of mass $\bar{n}$ and $k_{\max}(t)$ for the initial (auxiliary) wave packet using Eqs. (4) and (3), respectively. The parameters are $J_L = 1$, $J_R = 0.92$, $k_0 = -0.2\pi$, and $\sigma = 5$.

We consider, as an example, the Hatano-Nelson model (i.e., $J_m^L = J_m^R = 0$ for $m \neq 1$) with $J_R = J_1^R = 0.92$ and $J_L = J_1^L = 1$. The energy spectra under the OBC (black dots) and PBC (colored line) are plotted in Fig. 3(a), where we see that the OBC spectra are real and different from the PBC spectra with a counterclockwise loop, indicating the nontrivial point-gap topology and existence of the skin effect at the left boundary under the OBC [54,55]. In Figs. 3(e) and 3(f), we plot the time evolution of a wave packet with the initial central momentum $k_0 = -0.2\pi$ in real and momentum spaces under the OBC, respectively, for a wave packet closer to and directed toward the edge. We see that an NH wave-packet jump occurs at around $t_c \approx 280$. Expanding the wave function right after the NH wave-packet jump in eigenstates under PBC, which is allowed even under OBC because PBC eigenstates form a complete biorthogonal basis, we see that the wave function is jumped to a state not with the maximum $E_l(k)$, but to a momentum opposite to the initial momentum [Fig. 3(f)]. Since

the NH wave-packet jump in Fig. 2 is to a state with the largest $E_l(k)$, a different mechanism is behind the NH wave-packet jump in Fig. 3.

As we now discuss, the NH wave-packet jump in Fig. 3 can be explained by the *reflection* at the edge. To understand the NH wave-packet jump in terms of reflection, we first perform a similarity transformation to convert the NH Hamiltonian into a Hermitian form. For the Hermitian Hamiltonian, we can use knowledge of ordinary reflection of a wave packet at the edges. Let us assume that the edge is placed at $n = 500$. A reflection at the edge can be understood by considering a hypothetically larger lattice as in the upper half $n > 500$ part of Fig. 3(e). Reflected wave packet can be identified with an *auxiliary* wave packet created at $n > 500$ moving toward $n < 500$ region. In Fig. 3(b), we plot as $\tilde{\psi}'(0)$ the initial wave packet moving up the lattice, and as $\tilde{\varphi}(0)$ the initial wave packet after the similarity transformation to a Hermitian Hamiltonian. Note that the center of the wave packet changes after the similarity transformation. The wave packet $\tilde{\varphi}'(0)$ is an auxiliary one obtained by inverting $\tilde{\varphi}(0)$ at the original edge at $n = 500$. The wave packet $\tilde{\psi}'(0)$ is the auxiliary wave packet after inverse-similarity transformation, with the opposite central momentum $-k_0$ and shifted position, which we denote by n_2 , which is different from the mirror-symmetric point of $\tilde{\psi}(t)$, which we denote by n'_0 . The reflection, in this language, is nothing but the wave packet $\tilde{\psi}(t)$ moving to $n > 500$ region while the auxiliary wave packet $\tilde{\psi}'(t)$ moving into $n < 500$ region. In Fig. 3(c), we plot the momentum-space distribution of $\tilde{\psi}(0) + \tilde{\psi}'(0)$. Since the initial (auxiliary) wave packet with the central momentum k_0 ($-k_0$) experiences loss (gain) [see the red and blue circles in Fig. 3(a)], the amplitude $|\tilde{\psi}(t)|$ ($|\tilde{\psi}'(t)|$) will decrease (increase). When the amplitude of the auxiliary wave packet $\tilde{\psi}'(t)$ exceeds that of the initial wave packet $\tilde{\psi}(t)$, as described by the momentum distribution function $c_k(t)$ in Fig. 3(d), the NH wave-packet jump occurs and the auxiliary wave packet starts to dominate the evolution. More quantitative argument is given in the Supplemental Material [70]. The jump time t_c depends on the ratio J_R/J_L , the width of the wave packet σ , and the size N (see the Supplemental Material [70]).

TGHSs induced by skin effects. The model we have just considered is the Hatano-Nelson model with $J_R/J_L = 0.92$. As we make the ratio $J_R/J_L = 0.92$ closer to 1, namely, making the system closer to the Hermitian limit, the NH wave-packet jump will turn into TGHS. In Figs. 4(b) and 4(c), we plot the reflection of a wave packet for $J_R/J_L = 0.96$ and $J_R/J_L = 0.98$. When $J_R/J_L = 0.92$, the NH wave-packet jump occurs away from the boundary, but as the ratio J_R/J_L becomes 0.96, the reflection point touches the boundary [Fig. 4(b)]. As the ratio gets even larger (such as 0.98), we observe the negative TGHS [Fig. 4(c)], namely, $\Delta t = t_2 - t_1 < 0$, with t_2 and t_1 being the time that the center of mass of $\tilde{\psi}'(t)$ and $\tilde{\psi}(t)$, respectively, touches the boundary. Note that when the ratio reaches 1, negative TGHS disappears and a simple reflection at the boundary occurs.

Now let us consider what happens when we make the ratio J_R/J_L greater than 1. In Figs. 4(e) and 4(f), we plot the cases for $J_R/J_L = 1/0.7$ and $J_R/J_L = 1/0.8$, respectively. When $J_R/J_L = 1/0.8$, we observe the positive TGHS. This is because, as described in Fig. 4(f), the initial and auxiliary

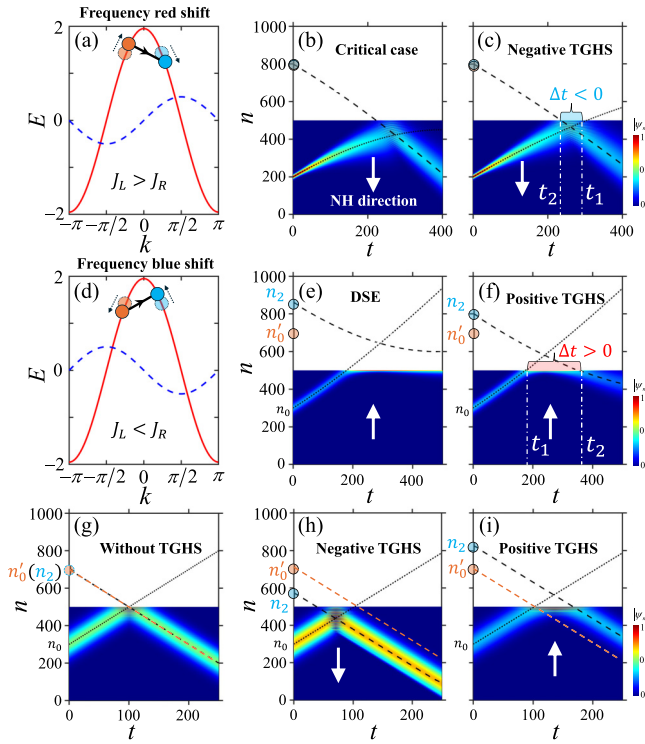


FIG. 4. Typical band structures, $E_R(k)$ (red solid lines) and $E_I(k)$ (blue dashed lines) with (a) $J_L > J_R$ and (d) $J_L < J_R$. (b), (c), (e), (f)–(i) The evolution of the wave packet in real space under the OBC. The parameters are $J_L = 1$, $J_R = 0.96$ [panel (b)], $J_L = 1$, $J_R = 0.98$ [panel (c)], $J_L = 0.7$, $J_R = 1$ [panel (e)], $J_L = 0.8$, $J_R = 1$ [panel (f)], $J_L = J_R = 1$ [panel (g)], $J_L = 1$, $J_R = 0.95$ [panel (h)], and $J_L = 0.95$, $J_R = 1$ [panel (i)]. $k_0 = -0.2\pi$, $\sigma = 5$ for panels (b) and (c), $k_0 = -0.2\pi$, $\sigma = 15$, for panels (e) and (f), and $k_0 = -0.5\pi$, $\sigma = 35$, for panels (g), (h) and (i). The red (blue) circles denote n'_0 (n_2). The red dashed lines in panels (g)–(i) denote the Hermitian cases.

wave packets cross at the extended region beyond the edge. As the non-Hermiticity is increased to $J_R/J_L = 1/0.7$, reflection ceases to occur [Fig. 4(e)]; this phenomenon was previously found and termed dynamic skin effect (DSE) [57,59]. Figure 5(a) is the numerically obtained phase diagram in the

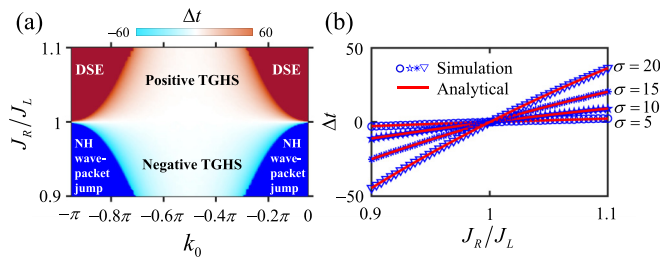


FIG. 5. (a) Phase diagram under different choices of k_0 and J_R/J_L . Other parameters are $\sigma = 5$ and $N = n_0 + 200$. (b) Relation of TGHS Δt with J_R/J_L for different σ at $k_0 = -0.5\pi$.

($k_0, J_R/J_L$) plane, which shows the phase transitions among the DSE, NH wave-packet jump, and TGHS.

For the case of $k_0 = -0.5\pi$, where $E_I(k_0)$ takes an extreme value, the momentum remains unchanged, allowing an analytical TGHS

$$\Delta t = \frac{n_2 - n'_0}{v_g(k_0)} = \frac{2\sigma^2 \ln(J_R/J_L)}{J_L + J_R}, \quad (6)$$

which shows $\Delta t > 0$ (< 0) for $J_R > (<) J_L$ and $\Delta t = 0$ for the Hermitian case with $J_R = J_L$ [see Figs. 4(g)–4(i)]. The agreement between the analytical and numerical results for different σ is demonstrated in Fig. 5(b).

We have further found that the peak momentum $k_{\max}$ exhibits unequal amplitudes during the reflection process, accompanied by a frequency (energy) shift. As we described in Figs. 4(a) and 4(d), the momenta of the incident and auxiliary states move from the red and blue dotted circles along the $+k$ ($-k$) direction to the solid circles, where the reflection occurs, indicating the frequency redshift (blueshift) for $J_L > J_R$ ($J_L < J_R$) (see the Supplemental Material [70]). Note that the above NH wave-packet jumps, DSEs, and TGHSs with phase transitions can occur in other models beyond the Hatano-Nelson model [70].

Conclusion. We have found that, beyond the static properties of point-gap topologies of the complex energy bands and eigenstate localizations of the skin effects, even in a simple setting of wave-packet propagation on a one-dimensional lattice without any external force, non-Hermitian Hamiltonian leads to phenomena quite distinct from Hermitian Hamiltonians, such as wave-packet evolution with an anomalous group velocity, self-induced Bloch oscillations, NH wave-packet jumps without disorder and with completely real spectra, and TGHSs. Our work hence bridges the gap of static nontrivial point-gap topology with skin effects and observable wave propagation phenomena. Adding external forces, temporal and spatial modulations, effects of other bands, and nonlinearity can lead to further exotic phenomena, which are the topics of future works. Extending the results to two and higher dimensions, including effects of (both real and imaginary) magnetic fields, is also an interesting perspective. Our results can be verified in a variety of experimental platforms. The most promising one might be the synthetic temporal photonic lattices, where it is possible to engineer the asymmetric couplings [46], open boundary conditions [75], and Gaussian wave packets [22]. Other platforms where our proposal can be tested include photonic ring resonators [76–78], waveguide arrays [79,80], synthetic momentum lattices [81,82], acoustic lattices [83,84], and photonic quantum walks [45,64].

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