

Relaxation and decay of collective modes in open quantum systems

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In a quantum many-body system, such as atoms, molecules, and atomic nuclei, interparticle interactions generate a complex quantum level scheme that can be analyzed progressively: from the simplest picture of independent quasiparticles moving in a self-consistent mean field to more complete descriptions that include residual interactions and the opening of decay channels above some excitation thresholds. Residual interactions lead to two seemingly contrasting phenomena: the emergence of collective modes (such as rotational and vibrational excitations) associated with specific symmetries, and internal thermalization, which gives rise to features of quantum chaos and causes these collective modes to damp. The picture becomes more intricate once decay channels become energetically accessible, allowing the system to decay irreversibly by means such as particle emission and thus transition to an open quantum regime. In this work, we investigate the competition between internal many-body dynamics and explicit, irreversible decay. These processes can occur on comparable timescales, leading to an initial stage of rapid dynamics followed by equilibration. We show that the decay of the faster components drives the system into an exponentially decaying regime, characterized by a noisy decay current. For this study, we adopt a general, albeit simplified, model of a collective mode embedded in a chaotic background and coupled to open decay channels. The interplay of timescales revealed in this framework appears to reflect universal features of open quantum systems.

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I. INTRODUCTION

In a realistic many-body quantum system, the effects of interactions become increasingly important as the excitation energy and level density rise [1–4]. In the basis of simple mean-field states, and after introducing the canonical transformation to quasiparticles in the case of superconducting pairing, the stationary states of the full Hamiltonian emerge as complex superpositions involving many components. In fact, with the growth of excitation energy, the measure of non-chaotic or nonthermalized states rapidly approaches zero in a macroscopic system, even in the presence of very weak two-body interactions [5]. In this regime, a statistical approach becomes justified, based on the concepts of quantum chaos and the use of random matrix ensembles. These ensembles provide a limiting case of the universal description of complex many-body spectra, constrained only by exact conservation laws [6,7]. Self-averaging and the eigenstate thermalization hypothesis are central to this picture, emphasizing that the relevant physical properties of a single typical many-body eigenstate in a given system are essentially the same as those found in nearby eigenstates, or in averages over an ensemble

of Hamiltonians. As noted in Ref. [8], this property was already recognized by Landau and Lifshitz, who formulated it explicitly in their *Statistical Physics* textbook.

Numerous studies of specific many-body quantum systems, such as atoms, molecules, nuclei, interacting spins, atomic traps, artificial solid-state devices, and prototypes of quantum computers, support the current understanding of thermalization, which emerges from intrinsic interactions within the system, even in the absence of an external heat bath [3,5,8–10]. Yet, local deviations from a fully chaotic spectral pattern, often associated with coherent collective dynamics, remain of particular interest. Robust collective modes can persist through the friction-like process of chaotization, resulting in the spectral regions where the corresponding collective strength is concentrated being now spread over many complicated microscopic eigenstates. In classical physics, such regions are referred to as scars, remnants of ordered dynamics. Their quantum counterparts were discussed in Ref. [11].

Giant resonances (GRs) [12,13], found nearly universally in nuclei, represent a prominent example of such regions. In the idealized picture, the original collective excitations correspond to stationary vibrational modes with well-defined frequencies or excitation energies. In practice, however, they are not truly stationary, as they acquire finite widths due to relaxation processes. These processes can be broadly divided into two categories: (1) damping caused by effective friction, resulting from interactions (enhanced at high-level density) with numerous noncollective degrees of freedom absorbing the collective energy and (2) decay into the continuum (such

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as the particle emission) that becomes possible once the excitation energy exceeds a certain threshold and the system becomes open.

The internal damping processes, type (a), give rise to a region of stationary states that approach the mathematical limit of chaotic ensembles, although residual concentration of the original collective strength remains locally enhanced. For example, the ubiquitous giant dipole resonance appears, as a function of excitation energy, as a broad peak in the strength of the electromagnetic dipole radiation or absorption. The external processes of irreversible decay, type (b), can in turn be classified into equilibrium and pre-equilibrium components [14]. These distinctions reflect the quantum time evolution of the initial nonstationary state, created by a specific excitation or measurement, and its interaction with the decay process, which may involve multiple decay channels.

In this work, we address these questions from a time-dependent perspective. This framework has long been recognized as offering a significant natural advantage in the study of nonstationary dynamics [15]. Its relevance has grown further in recent years due to its exceptional ability to describe decay processes, starting from the simple tunneling up to complex situations such as multiparticle decay and correlated emission [16,17]. It also highlights subtle features of nonexponential decay [18] that, due to the internal dynamics, may even become oscillatory.

Building on our previous work [19], where relaxation in stable chaotic nuclear systems was analyzed, we now extend this study to open quantum systems. We explore both the fully chaotic regime and the regime dominated by a strong collective mode. This setting captures two competing mechanisms: internal relaxation driven by complexity that reflects the interplay between collectivity, chaos, and external irreversible decay shaped by the location of the decay threshold and associated near-threshold physics, including possible discontinuities.

Rather than focusing on a particular physical system, our aim is broader. We seek to develop a general framework for describing decay in the presence of complex internal dynamics within an open many-body quantum system and to contribute to a deeper understanding of the underlying phenomena. We also embrace a philosophy, similar to that of our previous studies [19], when the chosen models incorporate a physically accurate description of collectivity and decay (openness), while the complexity and randomness of the many-body interactions are represented by a single randomly selected realization that captures all generic properties of interest, including universal equilibration as well as pre-exponential and exponential decay.

II. FESHBACH PROJECTION FORMALISM

To describe the interplay between discrete nuclear states and the continuum, we employ the Feshbach projection formalism [20,21], which provides a formally exact method for describing the evolution of a quantum system within a subspace by means of an effective, energy-dependent Hamiltonian that is generally non-Hermitian because the restricted subspace remains coupled to the rest of the Hilbert space. We summarize here only the essential aspects and terminology

of the Feshbach formalism relevant to our approach; detailed discussions can be found in the original papers [20,21] and recent reviews [22,23].

The total Hilbert space is decomposed into two orthogonal subspaces using the projection operators $\mathcal{P}$ and $\mathcal{Q}$, which satisfy $\mathcal{P} + \mathcal{Q} = 1$, $\mathcal{P}^2 = \mathcal{P}$, $\mathcal{Q}^2 = \mathcal{Q}$, and $\mathcal{P}\mathcal{Q} = 0$. While the orthogonality condition can be relaxed in more general treatments, for simplicity we restrict ourselves here to orthogonal projectors. Although this subdivision is arbitrary, here we separate states by their asymptotic behavior, distinguishing bound from unbound. The subspace $\mathcal{Q}$ that we refer to as the *internal* or *intrinsic* subspace contains a discrete bound set of many-body states $\mathcal{Q} = \sum_1 |1\rangle\langle 1|$. In contrast, the subspace $\mathcal{P}$ that we refer to as *external* and to be excluded represents the scattering continuum and contains configurations in which one or more particles belong to unbound, asymptotic scattering states. This external subspace is composed of reaction states $|cE\rangle$, which are labeled by a continuous energy E and a discrete set of quantum numbers c . The set of quantum numbers, referred to as the *channel* c , includes all discrete quantum numbers needed to specify the asymptotic configuration—such as the discrete state of the residual nucleus and all angular momenta describing the final motion. For practicality, we focus here on particle-emission channels relevant on nuclear timescales and driven by the strong interaction; other channels such as γ (photon) decay or β decay (via the weak interaction) can be treated within the same formalism, but their much longer timescales make them irrelevant for the physics of interest here. For simplicity, here we do not consider more complex multiparticle scattering states that would require several continuous variables, although such extensions are straightforward. With proper normalization, the projector onto the continuum subspace takes the form

$$\mathcal{P} = \sum_c \int_{\epsilon_{tr}^{(c)}}^{\infty} dE |cE\rangle\langle cE|, \quad (1)$$

where $\epsilon_{tr}^{(c)}$ denotes the threshold energy for channel c .

Starting from the full Schrödinger equation $H|\Psi\rangle = E|\Psi\rangle$, we decompose the wave function as $|\Psi\rangle = |\Psi_Q\rangle + |\Psi_P\rangle$, where $|\Psi_Q\rangle = \mathcal{Q}|\Psi\rangle$ and $|\Psi_P\rangle = \mathcal{P}|\Psi\rangle$. Projecting onto the subspaces $\mathcal{Q}$ and $\mathcal{P}$ and eliminating $|\Psi_P\rangle$, one obtains an effective Schrödinger equation in the internal subspace:

$$\mathcal{H}_{\text{eff}}(E)|\Psi_Q\rangle = E|\Psi_Q\rangle, \quad (2)$$

where the energy-dependent non-Hermitian effective Hamiltonian is

$$\mathcal{H}_{\text{eff}}(E) = H_{QQ} + \Sigma(E), \quad (3)$$

with the complex self-energy term

$$\Sigma(E) = H_{QP} \frac{1}{E - H_{PP}} H_{PQ} = \Delta(E) - \frac{i}{2}W(E). \quad (4)$$

Here, $H_{AB} \equiv AHB$ for $A, B \in \{\mathcal{P}, \mathcal{Q}\}$, and Δ and W represent Hermitian and anti-Hermitian parts of self-energy. The Hermitian part $\Delta(E)$ accounts for virtual excursions into continuum, and the non-Hermitian part $W(E)$ describes the irreversible loss of probability from the internal space through the open decay channels.

The $\mathcal{H}_{\text{eff}}(E)$ along with W and Δ are all operators within the discrete internal subspace $\mathcal{Q}$ and can be represented as matrices. The imaginary and real parts of the self-energy matrix elements are expressed in terms of the coupling amplitudes $A_1^c(E)$ as

$$W_{12}(E) = 2\pi \sum_{\substack{c \\ E > \epsilon_{\text{tr}}^{(c)}}} A_1^c(E) A_2^{c*}(E), \quad (5)$$

$$\Delta_{12}(E) = \sum_c \text{PV} \int_{\epsilon_{\text{tr}}^{(c)}}^{\infty} \frac{A_1^c(E') A_2^{c*}(E')}{E - E'} dE', \quad (6)$$

where the sum in Eq. (5) runs only over *open channels*, i.e., those with $E > \epsilon_{\text{tr}}^{(c)}$. This condition is implemented by assuming $A_1^c(E) = 0$ for $E < \epsilon_{\text{tr}}^{(c)}$; here PV stands for the principal value. The coupling amplitudes, expressing matrix elements of H_{QP} in the above equations, are defined through energy-dependent channel vectors that belong to the internal space $\mathcal{Q}$,

$$|A^c(E)\rangle = H_{QP}|cE\rangle. \quad (7)$$

The corresponding amplitude for a discrete state $|1\rangle$ is then given by

$$A_1^c(E) = \langle 1|A^c(E)\rangle = \langle 1|H|cE\rangle. \quad (8)$$

The energy dependence of the channel vector $|A^c(E)\rangle$ is governed by reaction kinematics and available at given energy phase space, and thus factorizes from the structural part as

$$|A^c(E)\rangle = a^c(E)|c\rangle, \quad (9)$$

where $a^c(E)$ is a smooth scalar column function of energy that, in practical cases, is well known and can be computed from the simple potential scattering. Its low-energy behavior is analytically determined by the corresponding properties of spherical Bessel or Coulomb functions, reflecting the available phase space near threshold. A detailed discussion can be found in Refs. [15,24–26]. The constant vector $|c\rangle$, often referred to as the channel vector or doorway state, contains the structural part of the channel and represents the many-body configuration of the asymptotic system.

The limit of the weak coupling to continuum is particularly instructive. In this regime, the imaginary part of the effective Hamiltonian can be treated perturbatively, using as a basis the eigenstates $|\alpha\rangle$ of the Hermitian operator $H_{QQ} + \Delta(E)$, the last item is sometimes called Thomas-Ehrman shift [27,28]. The energy width of a state $|\alpha\rangle$ is then given as a sum over partial widths:

$$\Gamma_\alpha = \langle \alpha|W|\alpha\rangle = \sum_c \Gamma_{c\alpha}, \quad \Gamma_{c\alpha} = \gamma_c(E_\alpha) |\langle \alpha|c\rangle|^2, \quad (10)$$

where each $\Gamma_{c\alpha}$ is the product of a spectroscopic factor $|\langle \alpha|c\rangle|^2$ and a single-particle width $\gamma_c(E) = 2\pi |a^c(E)|^2$. This limit makes the structural separation and the connection of $a^c(E)$ to the phase space transparent. It also highlights that the channel vectors are generally normalized, which in practical nuclear studies is reflected through various sum rules that depend on angular momentum coupling, particle number, and the types of channels [15]. For the model discussion below, and without loss of generality, we simply assume $\langle c|c\rangle = 1$. It should be noted, however, that different channels are not orthogonal.

The distribution of partial widths $\Gamma_{c\alpha}$ across many-body states is of fundamental interest, as it reflects how the continuum coupling is shared among the available configurations. This also gives access to the channel strength distribution $|\langle \alpha|c\rangle|^2$ that provides insight into configuration mixing, collectivity, and the physical nature of the decay. The width and strength distributions arise in a variety of physical contexts involving energy dissipation, transport, and complex many-body dynamics, including nuclear and astrophysical processes. They also play a central role in statistical approaches, such as the random matrix theory, where they reflect fluctuation properties, the onset of chaos, and superradiant transitions.

In the low-energy spectral regimes of the main physical interest, the number of open channels is typically low, much smaller than the dimensionality of the internal space $\mathcal{Q}$. In many cases, there may be only a single open channel. Since the rank of the imaginary part of the effective Hamiltonian, $W(E)$ that has a factorized form (5), is equal to the number of open channels, the subspace it spans is generally of low dimension. In the limit of very strong coupling to the continuum, the imaginary term $W(E)$ becomes dominant in the effective Hamiltonian. In this regime, the mentioned earlier superradiant transition occurs [22], analogous to phenomena in quantum optics. As the coupling increases, the eigenstates of the system undergo a collective reorganization: a small number of states, equal to the number of open channels, align strongly with the decay channels and acquire large widths. These are referred to as *superradiant states*. The remaining states, by orthogonality, decouple from the continuum and become long-lived acquiring vanishingly small widths. This dynamical segregation of resonances is a generic feature of open quantum systems [22].

It is beyond the scope of this presentation, but the full formulation of the Feshbach formalism can also be applied to scattering processes that originate in the external space $\mathcal{P}$. This leads to a scattering matrix $S_{c'c}(E)$, which describes the matrix elements of the evolution operator between different channels. Importantly, the factorized form of the imaginary part W is closely related to the unitarity of the scattering matrix [15,22,24], some comments about that are also given in the Appendix.

The advantage of the Feshbach formalism is that the exact dynamics can be described entirely within the intrinsic space, represented by a discrete Hilbert space, in a manner largely identical to that of a closed system, even though the Schrödinger equation with the effective Hamiltonian is no longer linear and Hermitian. This is the strategy we adopt, and all further references to states, channels, and matrices pertain to states and operators within this discrete intrinsic space.

III. RELAXATION OF OBSERVABLES IN AN OPEN SYSTEM

We start with a model study extending the discussion of the relaxation process in Refs. [19,29]. Utilizing the Feshbach formalism discussed in Sec. II, we consider an initial state $|1\rangle$ that belongs to the internal subspace $\mathcal{Q}$ of the total Hilbert space. This is not an eigenstate of the full Hamiltonian but rather a typical initial state created by some external pro-

cess like excitation or measurement. This state undergoes the subsequent relaxation governed by the multitude of internal interactions. Due to complexity of the system, we assume the Hermitian part of the Hamiltonian being represented by an instance of the Gaussian orthogonal ensemble (GOE) [6] of $N \times N$ matrices, where the random matrix elements H_{12}^{GOE} have the normal distribution with zero mean and variance given by

$$\langle H_{12}^{\text{GOE}} H_{34}^{\text{GOE}} \rangle = \frac{\lambda^2}{N} (\delta_{13} \delta_{24} + \delta_{14} \delta_{23}). \quad (11)$$

The energy scale here is set by the parameter λ .

Irreversible decays outside the intrinsic space are captured by the evolution governed by the effective non-Hermitian Hamiltonian $\mathcal{H}$ that, in this introductory model, is taken to have the typical form [see Eqs. (5), (8), and (9)],

$$\mathcal{H} = H^{\text{GOE}} - \frac{i\gamma}{2} \mathbf{A} \mathbf{A}^\dagger. \quad (12)$$

Here, $\mathbf{A}$ is a matrix composed of N_c columns, each representing a decay amplitude into a randomly oriented normalized open-channel vector $|c\rangle$ in Eq. (8). In this simplified model, we also assume the channel vectors to be energy-independent, an approximation valid within an energy region sufficiently far from thresholds. The parameter γ controls the overall strength of coupling to the continuum. Hermitian corrections from the continuum, Δ , are assumed to be already incorporated into the Hermitian part H^{GOE} of the model Hamiltonian (12).

The survival amplitude of the initial state is given by the matrix elements

$$f_{21}(t) = \langle 2 | \exp(-i\mathcal{H}t) | 1 \rangle. \quad (13)$$

The limit of a closed system (no decay, $\gamma = 0$) described by a GOE Hamiltonian has been studied in detail in Refs. [8,29,30], and, in the context of nuclear physics, in Ref. [19].

In the time domain, where the fine details of the individual spectra are washed out, the presence of many significant components in the evolving complex wave function leads to strong self-averaging [19]. As a result, the survival probabilities of a given state or the dynamics of any complex observable for any specific Hamiltonian realization are nearly indistinguishable from the ensemble average. This self-averaging property allows the universal function to be obtained from the ensemble average. After averaging over the GOE ensemble, the survival amplitude becomes diagonal and state-independent [19]:

$$\overline{f_{21}(t)} = \delta_{21} f(t). \quad (14)$$

This results in the universal time dependence of the survival amplitude given by the Bessel function J_1 :

$$f(\tau) = \frac{1}{\tau} J_1(2\tau), \quad (15)$$

where time is measured in dimensionless units $\tau = \lambda t$, assuming $\hbar = 1$ and using the characteristic energy scale λ of the ensemble (11). In this closed-system regime, the evolution of

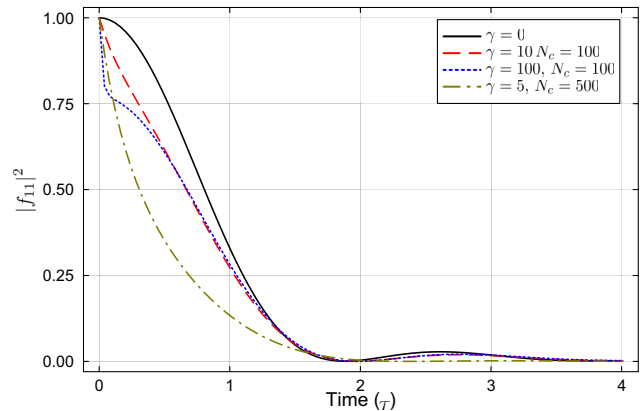


FIG. 1. Survival probability as a function of time.

any observable follows the general form

$$Q(\tau) = Q(0) |f(\tau)|^2. \quad (16)$$

While this result relies on ensemble averaging, the time-dependent properties under consideration exhibit self-averaging like many other key results that motivate the use of random matrix theory. Naturally, certain spectral details differ between realizations, but the following results are nonetheless generic despite being based on single realizations.

Now, we look at the relaxation dynamics in an open system. Figure 1 illustrates representative examples of relaxation dynamics for single realizations of the random Hamiltonian. The black solid line $\gamma = 0$ agrees with the result in Eq. (15), showing the characteristic oscillations in equilibration process. The other lines display the results for a different number of open channels, N_c , and corresponding strengths γ ; all in units of $\lambda = 1$. In this calculation, the total dimension of the Q space was $N = 1000$.

The key observations can be characterized in the following way. With the channels as random vectors, when the decay strength is weak and is distributed over many states, the spectrum is represented by very narrow (long-lived) resonances; thus, the internal relaxation, as a much faster process, is not significantly impacted by the openness of the system. The result would be essentially the same as for the $\gamma = 0$ limit. The deviations become noticeable when coupling to the continuum grows. The red dashed curve for $\gamma = 10$ and $N_c = 100$ channels shows that the initial behavior is significantly modified by the decay, although at remote times the characteristic oscillations, Eq. (15), are still observed. The blue dotted curve for $\gamma = 100$ and $N_c = 100$ highlights the superradiance effect [22] and a different limit of the timescales. Initially, we see a fast and sharp exponential drop that reflects the irreversible decay of those initial components of the wave function that are directly coupled to the external subspace $\mathcal{P}$ via corresponding amplitudes $\mathbf{A}$; afterwards, the relaxation proceeds similarly to the $\gamma = 0$ case, as only the trapped components remain. Finally, the last curve (shown by the dash-dotted line) corresponds to the situation when the number of open channels is large while the coupling to the continuum is sufficiently strong placing the system in a fully overlapping resonance regime. In the latter limit, we observe a decay curve closely resembling a typical irreversible exponential decay.

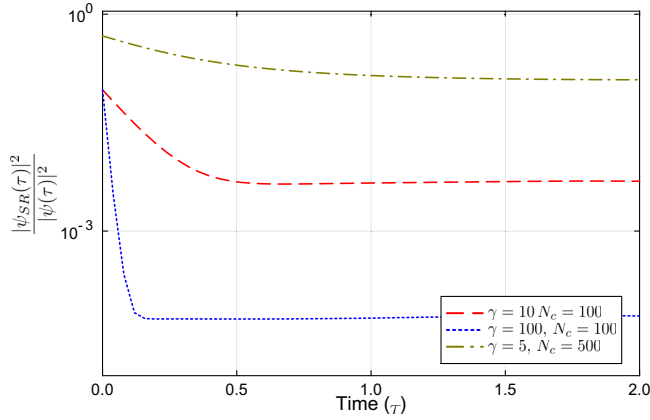


FIG. 2. Weight of the wave function component in the channel subspace is shown as a function of time.

In order to further illustrate the relaxation mechanism in an open system with a chaotic internal structure, in Fig. 2, we show, as a function of time, the weight of the part of the wave function that belongs to the superradiant or doorway subspace spanned by the channel vectors included in $\mathbf{A}$. Within the intrinsic space $\mathcal{Q}$, the subspace spanned by the channel vectors A^c is often called the *doorway* or *superradiant* subspace, which we denote below as $\mathcal{Q}_{\text{SR}}$. In mathematical terms, this subspace coincides with the non-null sector (range) of the non-Hermitian part of the Hamiltonian W in Eq. (5). States with support in $\mathcal{Q}_{\text{SR}}$ couple directly to the excluded space $\mathcal{P}$ and therefore decay rapidly. Superradiant states are those lying fully within this subspace, whereas states orthogonal to it, i.e., with zero overlap with all A^c , do not couple directly to the continuum and thus carry zero direct width in Eq. (10). Because the channel vectors are not generally orthonormal, the projector onto the channel subspace is formally given by

$$\mathcal{Q}_{\text{SR}} = \mathbf{A}(\mathbf{A}^\dagger \mathbf{A})^{-1} \mathbf{A}^\dagger. \quad (17)$$

The component of the wave function that lies within this superradiant subspace is defined as

$$\psi_{\text{SR}}(t) = \mathcal{Q}_{\text{SR}} \psi(t). \quad (18)$$

This part of the state is directly aligned with the decay channels and undergoes the immediate irreversible decay. In contrast, the complementary components, orthogonal to the superradiant subspace, can only decay after internal mixing transfers part of their amplitude into the decay-aligned doorway sector.

In Fig. 2, the weight of the superradiant component of the wave function $|\psi_{\text{SR}}(t)|^2$ is shown relative to the norm $|\psi(t)|^2$. Given complete randomness and assuming the equivalent channels, at the initial moment, the weight of superradiant component is roughly given by the ratio of the dimensionalities N_c versus N . As seen in this figure, the initial dynamics are dominated by the irreversible decay until the equilibration is reached. Then, the fraction of the wave function coupled to continuum remains approximately unchanged because the intrinsic relaxation is a uniform process in the internal space, which does not favor doorway subspace spanned by the chan-

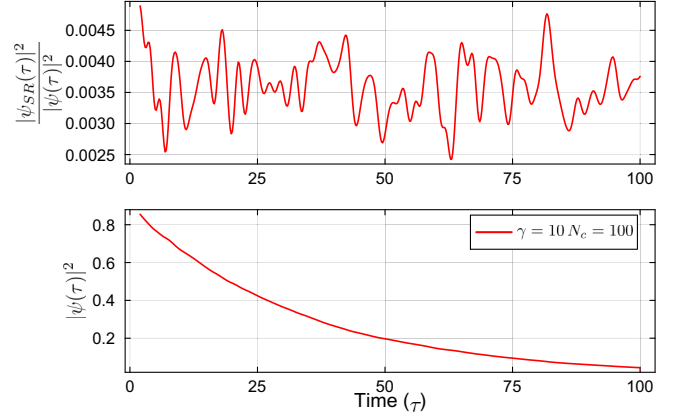


FIG. 3. Numerical results showing the long-time dynamics for a representative case with $\gamma = 10$ and $N_c = 100$. The upper panel shows the weight of the wave function projected onto the channel subspace (superradiant, doorway subspace) magnified to reveal fine-scale fluctuations that appear as a flat line in Fig. 2. These fluctuations reflect chaotic internal mixing and quantum noise in the decaying subspace. The lower panel shows the corresponding decay of the total norm that follows an average exponential trend.

nel vectors. The timescale of the initial equilibration process is determined by the strength γ of the continuum coupling.

The long-time dynamics are illustrated in Fig. 3 using a representative case with $\gamma = 10$ and $N_c = 100$. The upper plot shows the same weight of the wave function that belongs to the channel subspace but it is significantly magnified (this region appears as a straight line in Fig. 2). This magnification allows one to see chaotic characteristics specific to a particular realization of the ensemble. All results shown previously in Figs. 1 and 2, as well as the lower plot of this figure, display randomly selected individual GOE realizations, yet they are universal and indistinguishable from the average or from the results of another realization.

The observed chaotic dynamics reflect the complexity of internal mixing that keeps the components in the decaying superradiant subspace random. At the same time, the internal mixing introduces new components that subsequently decay irreversibly, thus reducing the overall normalization of this wave function. On average, this process produces the exponential decay, as seen in the lower panel. However, on a finer scale, the fluctuations observed in the decay signal reflect the underlying random quantum dynamics of internal mixing within the system. This extends the concept of oscillatory features in the decay curve, originally discussed in the context of a minimal two-level model [18], to systems with complex internal mixing. Related questions about modulated decay curves have also arisen in connection with the so-called GSI time-modulated decay observations, although the origin and interpretation of these effects remain under discussion [31,32]. The fluctuations observed in the decay can also be viewed through the lens of quantum noise, a concept widely studied in condensed matter and material physics, including the shot noise in strongly correlated systems [33], Johnson noise in precision thermometry [34], and the long ago established fundamental connection between noise and dissipation in irreversible processes [35].

IV. GIANT RESONANCE IN THE CONTINUUM

A. Qualitative model of a giant resonance

We now turn to the case of a nuclear GR, a collective excitation of the system involving coherent contributions from many simple mean-field configurations (e.g., of particle-hole character or pair deformation in a Fermi system with pairing) that share the same global quantum numbers characterizing the symmetry of the excitation. As discussed in Sec. I, this represents a particularly interesting situation where coherent dynamics persist against the background of quantum many-body chaos.

We carry out our discussion starting from the stationary-state formalism for closed systems and then, with the aid of the Feshbach projection formalism, provide a non-Hermitian Hamiltonian-based description for decay. However, our ultimate goal is to elucidate the time-dependent dynamics of a collective oscillatory-like motion. As already emphasized before, this perspective is naturally appropriate for open quantum systems, is technically well suited, and provides the best insight into the physical connection with experimental observables.

Giant resonances, can exhibit a variety of symmetry properties, such as monopole, quadrupole, and octupole modes, as well as spin and spin-isospin excitations like the Gamow-Teller and spin-dipole resonances. GR is a collective excitation and can be built or superimposed upon essentially any initial state, which can itself represent a collective excitation. Mathematically, a GR can be expressed as the state obtained by applying the corresponding excitation operator to a given initial state. In many physical situations, the underlying simplicity of nuclear motion or geometry causes the resulting state to exhibit a high degree of collectivity, with the strength concentrated within a relatively narrow region of the excitation spectrum; when this occurs, the excitation is identified as a giant resonance.

The most prominent nuclear example of this kind is the giant dipole resonance (GDR), which, in the simplest picture, can be viewed as a harmonic oscillation of protons relative to neutrons of dipole character, while the center of mass remains fixed; the operator thus carries the quantum numbers of angular momentum and parity $J^\pi = 1^-$. Owing to its general and simple character, the GDR is present in nearly all nuclei. Because of the strength of the residual dipole-dipole interaction, the GDR is significantly shifted upward relative to the corresponding simple particle-hole excitations of dipole symmetry, often placing it above the continuum threshold (see illustration in Fig. 4).

In what follows, we adopt the simplest picture of a single collective mode, which can, for example, represent the common case of a spherical nucleus in its ground state with a giant dipole resonance built upon it. For brevity and to avoid the repeated use of general terms, we adopt the notation d and will often refer to the collective excitation, its strength, and the corresponding operator as the dipole, and to the giant resonance as the GDR, with the understanding that, without loss of generality, the discussion applies to any collective excitation.

Even within a nondecaying (bound) nuclear system, a giant resonance is not a pure stationary state of the nucleus.

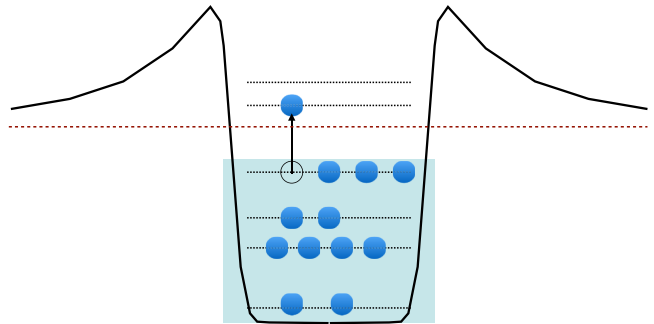


FIG. 4. Illustration showing the rough microscopic picture of the nuclear giant resonance. The ground state of the nuclear Fermi system in the mean-field picture corresponds to the lower single-particle energy levels being fully occupied. A coherent superposition of particle-hole excitations with the same spin and parity forms a collective state associated with the giant resonance. The nuclear mean-field potential is finite, as illustrated, featuring a threshold and a potential barrier. Above the threshold, nucleons can irreversibly decay. The dynamics under discussion involve a combination of intrinsic collective motion with natural damping and decay. The decay is sensitive to both the energy and the position of the threshold, for example, when tunneling under the barrier is involved, as illustrated.

The residual interactions mix this pure oscillation mode with close in energy more complicated states of the same global symmetry. As a function of excitation energy, the observed giant resonance appears as a broad peak in the spectrum, concentrating the entire transition strength within a characteristic energy width. The width of the giant resonance is inversely related to the lifetime of the coherent oscillation, reflecting the effects of damping, mixing, and redistribution of collectivity. In fact, the width can be substantial, such that the lifetime of the GDR as a collective mode corresponds to only a few oscillation periods. A significant portion of this width arises from the internal damping mechanism: The original simple collective mode mixes with nearby noncollective states of the same symmetry but more complex structure: a microscopic origin of friction. The resulting picture of the giant resonance state $|d\rangle$ is that of a collective oscillation. In a simple basis $|k\rangle$, such as particle-hole excitations illustrated in Fig. 4, we can represent it as a vector with components d_k ,

$$|d\rangle = \sum_k d_k |k\rangle. \quad (19)$$

It is worth recalling that the quadratic dipole-dipole interaction term $H_{kk'} \propto d_k d_{k'}$ in the Hamiltonian is responsible for the collectivity, being analogous to the restoring force in an oscillator that drives the collective oscillation. The collectivity is manifested in the correlated distribution of the amplitudes $\langle d|\alpha\rangle$ for the exact eigenstates $|\alpha\rangle$ of the system, with energies E_α around the central point E_0 found in the original approximation. For strongly collective cases, such as the GDR, these amplitudes $\langle d|\alpha\rangle$ can often be approximated (see Ref. [23], Sec. 25.2) by a Breit-Wigner form characterized by the spreading width γ ,

$$|\langle d|\alpha\rangle|^2 = \frac{s}{2\pi} \frac{\gamma}{(E_\alpha - E_0)^2 + \gamma^2/4}, \quad (20)$$

where s is the characteristic spacing between levels of the same symmetry in the giant-resonance energy region. The damping, and the resulting width γ , arise from other incoherent interaction terms of average strength v that mix the collective mode with nearby noncollective configurations, producing a frictionlike effect. This leads to

$$\gamma = 2\pi \frac{v^2}{s}. \quad (21)$$

B. Giant resonance and continuum effects

The GR, or part of it, may lie in the nuclear continuum, as is commonly the case for the GDR (see illustration in Fig. 4). In such situations, the observed damping width is increased by the explicit and irreversible decay into the continuum of reaction states. In this section, we formulate the problem and discuss analytical limits that can be treated explicitly. In particular, we review GR within the stationary-state formalism, extended to open systems described by an energy-dependent non-Hermitian Hamiltonian. This analytic perspective is valuable because it reveals the underlying physics more transparently than numerical simulations alone and provides a natural bridge to the numerical time-dependent studies presented in the following section.

To account for both continuum effects and internal mixing in the giant resonance problem, we consider a schematic effective Hamiltonian matrix restricted to a sector with the fixed global symmetry of interest. This matrix includes the original shell-model contributions, denoted by H^0 , a factorized collective interaction term H^d of the relevant symmetry, and the coupling to continuum channels represented by $\mathcal{H}^c$,

$$\mathcal{H}_{kk'} = H_{kk'}^0 + H_{kk'}^d + \mathcal{H}_{kk'}^c. \quad (22)$$

The shell-model part H^0 can be taken in the diagonalized form,

$$H_{kk'}^0 = \epsilon_k \delta_{kk'}. \quad (23)$$

The collective interaction part H^d in these bases contains the amplitudes d_k of the collective mode and their coupling to states k (corresponding to the complicated configurations of the same symmetry) through amplitudes f_k ,

$$H_{kk'}^d = \kappa d_k d_{k'} + f_k f_{k'}; \quad (24)$$

here, κ is the strength of the multipole interaction. The amplitudes f_k are responsible for friction-like decoherence of the collective mode. The coupling to the continuum is described by the non-Hermitian term $\mathcal{H}^c$ of the same form as in Eq. (12), which, for simplicity of this analytic consideration, is assumed to include just a single collective decay channel

$$\mathcal{H}_{kk'}^c = -\frac{i}{2} A_k A_{k'}. \quad (25)$$

The natural next step is to diagonalize the non-Hermitian Hamiltonian $\mathcal{H}$ in order to obtain its complex eigenfunctions,

$$|\alpha\rangle = \sum_k \alpha_k |k\rangle, \quad (26)$$

with generally complex eigenvalues $\mathcal{E}$ satisfying

$$\mathcal{H}|\alpha\rangle = \mathcal{E}|\alpha\rangle. \quad (27)$$

For notational convenience, we introduce the following abbreviation for typical sums that depend on the complex energy eigenvalue $\mathcal{E}$:

$$\langle xy \rangle = \sum_k \frac{x_k y_k}{\mathcal{E} - \epsilon_k}. \quad (28)$$

The factorization of the model interactions leads to the set of coupled algebraic equations,

$$D(1 - \kappa \langle d^2 \rangle) - F \langle df \rangle + \frac{i}{2} A \langle dA \rangle = 0, \quad (29)$$

$$F(1 - \langle f^2 \rangle) - \kappa D \langle df \rangle + \frac{i}{2} A \langle fA \rangle = 0, \quad (30)$$

$$A \left(1 + \frac{i}{2} \langle A^2 \rangle \right) - \kappa D \langle Ad \rangle - F \langle Af \rangle = 0. \quad (31)$$

The unknown sums contain amplitudes of the elements of the effective Hamiltonian:

$$D = \sum_k \alpha_k d_k, \quad F = \sum_k \alpha_k f_k, \quad A = \sum_k \alpha_k A_k. \quad (32)$$

If all shell-model states $|k\rangle$ are below the continuum threshold, $\epsilon_k < \epsilon_{\text{th}}$ and $A_k = 0$, the resulting secular equation for the energy contains only Hermitian parts,

$$[1 - \kappa \langle d^2 \rangle][1 - \langle f^2 \rangle] - \kappa [\langle df \rangle]^2 = 0. \quad (33)$$

This equation (its roots here are shifted from the original energies ϵ_k of the particle-hole excitations $|k\rangle$) describes the shape of the giant resonance still unperturbed by the continuum effects but with the damping being fully included through couplings f_k . Without the damping terms, the secular equation reduces to

$$1 - \kappa \langle d^2 \rangle = 0. \quad (34)$$

If $\bar{\epsilon}$ is the average energy of the collective excitations, defining the centroid of the shell-model dipole states, the GR energy for $\kappa > 0$ can be estimated as

$$\mathcal{E}_0 = \bar{\epsilon} + \kappa \sum_k d_k^2. \quad (35)$$

With $\kappa > 0$, the dipole strength is concentrated much higher in energy than typical shell-model individual dipole transitions, $\mathcal{E}_0 \gg \bar{\epsilon}$.

The matrix elements f_k describe the coupling between the collective excitation and nearby states $|k\rangle$ with the same global quantum numbers that lie close in energy but have a more complicated internal structure. This coupling accounts for the spreading of the initially sharp dipole strength over neighboring states, representing the process of thermalization. Assuming that the matrix elements f_k are nearly random and uncorrelated with the corresponding energies (in the vicinity of the unperturbed giant resonance), the average value of the term $\langle df \rangle$ vanishes in Eq. (33). The term $\langle f^2 \rangle$ accounts for the nearly random mixing among the complex eigenstates $|k\rangle$. A typical consequence of such mixing is the broadening of the giant-resonance peak that often follows a Breit-Wigner distribution as described by Eq. (20). The distribution is characterized by the density of admixed states $\rho(\epsilon) \sim 1/s$ and the average interaction strength of the admixed configurations f_k defines damping.

We now turn to the case where the resonance curve, or at least its upper portion above the threshold energy $\epsilon_{tr} = 0$, lies already within the continuum. In this regime, the coupling to continuum states must be explicitly addressed. Let us take the ϵ_k as the eigenvalues of the Hermitian problem. Considering the effective non-Hermitian Hamiltonian $\mathcal{H}_{kk'} = \epsilon_k \delta_{kk'} - \frac{i}{2} A_k A_{k'}$, we arrive at the secular equation for the complex energies $\mathcal{E}$:

$$1 = -\frac{i}{2} \sum_k \frac{A_k^2}{\mathcal{E} - \epsilon_k}. \quad (36)$$

Separating real and imaginary parts, $\mathcal{E} = E - i\Gamma/2$, the relations between the resonance position E and its width Γ are given by

$$\sum_k \frac{A_k^2}{(E - \epsilon_k)^2 + \Gamma^2/4} = \frac{4}{\Gamma}, \quad (37)$$

$$E = \frac{\Gamma}{4} \sum_k \frac{\epsilon_k A_k^2}{(E - \epsilon_k)^2 + \Gamma^2/4}. \quad (38)$$

Let us consider the limiting cases of strong and weak decay, distinguished by comparing the typical width Γ with the unperturbed level spacing between neighboring ϵ_k . To analyze the weak decay limit (such as the case of electromagnetic radiation), we isolate the dominant term in Eq. (37):

$$\frac{\Gamma}{4} \left\{ \frac{A_k^2}{(E - \epsilon_k)^2 + \Gamma^2/4} + \sum_{l \neq k} \frac{A_l^2}{(E - \epsilon_l)^2 + \Gamma^2/4} \right\} = 1. \quad (39)$$

In this limit, the width Γ is small compared to the level spacing, and the resonance energy lies near $E \approx \epsilon_k$. Under these conditions, the first term dominates, while the other contributions to the sum can be neglected. This yields the perturbative limit in Eq. (10), where $\Gamma \approx A_k^2$. In this nonoverlapping resonance limit, we can readily associate each unperturbed level k with its corresponding width Γ_k .

Addressing corrections to this result, we note that the energy shift of the level due to the open continuum channel is expected to be of higher order than the decay width itself. Therefore, the first term in Eq. (39) reduces to A_k^2/Γ . With this estimate, and by setting $A_l^2 = \Gamma_l$ for levels $l \neq k$ in the second term of that equation, we obtain

$$\Gamma_k = A_k^2 \left\{ 1 + \frac{A_k^2}{4} \sum_{l \neq k} \frac{A_l^2}{(\epsilon_k - \epsilon_l)^2} \right\} = A_k^2 + \delta\Gamma_k. \quad (40)$$

The meaning of this result is clear: The width of a resonance increases due to its coupling to other decaying states, assuming that the resulting width remains small, as for example, in the case of isolated neutron resonances near threshold, so that the resonances do not overlap. In the same approximations, the shift of the resonance energy due to the direct continuum coupling and the mixing with other levels through the continuum is given by

$$E_k = \epsilon_k \left(1 - \frac{\delta\Gamma_k}{A_k^2} \right) + \frac{A_k^2}{4} \sum_{l \neq k} \frac{\epsilon_l A_l^2}{(\epsilon_k - \epsilon_l)^2} = \epsilon_k + \delta\epsilon_k. \quad (41)$$

The opposite limit of strong decay, which is not typical in nuclear physics, corresponds to the phenomenon of superradiance. In this regime, the decay (radiation) width Γ exceeds the natural width of the original giant resonance. For such a state, the centroid of the corresponding wave function is given by

$$E = \frac{\sum_k \epsilon_k A_k^2}{\sum_k A_k^2}, \quad (42)$$

while the total width of the superradiant state accumulates the entire continuum coupling strength,

$$\Gamma = \sum_k A_k^2. \quad (43)$$

The energy dependence of the amplitudes can play a significant role in the overlapping regime and requires separate consideration. In nuclear physics, a variety of intermediate cases lie between the two extremes, including narrow low-energy neutron resonances [6], typical giant resonances broadened by continuum decay, and, in the strong coupling regime, superradiant and clustering states with large widths [36].

In the case where the main part of the giant resonance lies in the continuum region, and the complex spectrum in the region of interest is described by the level density $\rho(\epsilon)$, the main secular equation (36) for the non-Hermitian eigenvalue problem takes the form

$$1 = -\frac{i}{2} \int_{\epsilon_{tr}}^{\infty} d\epsilon \rho(\epsilon) \frac{A^2(\epsilon)}{\mathcal{E} - \epsilon}. \quad (44)$$

Here, the decay amplitudes $A(\epsilon)$ begin at the continuum threshold $\epsilon = \epsilon_{tr}$ and typically follow a power-law energy dependence determined by the type of the decay process. Close to threshold, this energy dependence is governed by the density of states in the continuum, in accordance with the Fermi Golden Rule that relates the decay width $\Gamma(\epsilon) = A^2(\epsilon)$ to the density of final states.

In the weak continuum coupling limit, by considering the giant resonance maximum on the real axis at E and evaluating the contribution from a pole in the lower part of the complex plane, close to the real axis, in Eq. (44), using the standard decomposition of a Cauchy-type integral near a simple pole, we obtain

$$\rho(E) A^2(E) = \frac{1}{\pi}. \quad (45)$$

This shows that the resonance curve is distorted by the energy threshold where the decay amplitude vanishes. The low-energy tail is simply cut off at the threshold. The behavior at slightly higher energies is typically smoothed by the random (chaotic) fluctuations of ρ resulting from the mixing of the resonance with local chaotic states that share the same global symmetry.

The general form of Eqs. (37) and (38), valid for arbitrary width Γ , defines the resonance centroid parameters $\mathcal{E} = E - (i/2)\Gamma$ in the limit of a continuous density of states through the following integrals:

$$\frac{4}{\Gamma} = \int d\epsilon \rho(\epsilon) \frac{A^2(\epsilon)}{(E - \epsilon)^2 + \Gamma^2/4}, \quad (46)$$

$$E = \frac{\Gamma}{4} \int d\epsilon \rho(\epsilon) \epsilon \frac{A^2(\epsilon)}{(E - \epsilon)^2 + \Gamma^2/4}. \quad (47)$$

Explicit expressions with a clear physical meaning can be derived in the case of a broad resonance, where the width is primarily determined by the original chaotic mixing, modulated by the energy dependence of the decay amplitudes. In this case,

$$\Gamma \approx \int d\epsilon \rho(\epsilon) A^2(\epsilon), \quad (48)$$

while the centroid of the broad resonance is given by

$$E = \int d\epsilon \rho(\epsilon) \epsilon \frac{A^2(\epsilon)}{\Gamma}, \quad (49)$$

which corresponds to the original centroid weighted by the local contribution of the decay width across the spectrum.

A similar analysis applies to the double dipole resonance, the two-phonon collective vibrational state, which has been observed in several nuclei [37]. In this case, the strong dipole transitions proceed through two one-phonon steps. The strength of each transition is primarily determined by the collective dipole degree of freedom. However, the strength distribution may differ slightly because the realistic shape of each resonance, described by the function $\rho(\epsilon)$, is influenced by the surrounding chaotic environment different for the first and second resonances. Assuming that the structures of the first and second excitations are practically identical, the overall shape of the double resonance can change from a Breit-Wigner profile to a Gaussian form. The Gaussian distribution appears to be closer to what is observed experimentally. Additional complexity arises from the increase in decay probability with energy, governed by the energy-dependent amplitudes $A(\epsilon)$, even in the absence of other competing channels.

C. Numerical studies

Having outlined the general physical principles of giant resonances, the structure of the effective Hamiltonian and the main effects involved, we now turn to a detailed numerical investigation, with the ultimate goal of using the time-dependent perspective to capture the interplay between collective dynamics, mixing with the chaotic environment, and irreversible decay. For the numerical studies reported below, we use the form of the Hamiltonian given in Eq. (22), with specific choices outlined in the following.

We consider an N -dimensional Hilbert space where the vector d is taken as a unit vector with a single nonzero component numbered $k = 1$. This choice is purely a matter of basis selection and does not affect the generality of the discussion. In the remaining $(N - 1)$ -dimensional subspace orthogonal to d , the basis is chosen such that the Hermitian Hamiltonian H is diagonal with $H_{kk} = \epsilon_k$. In the matrix $H_{kk'}$, this choice of basis leaves only the diagonal elements and the first row and first column different from zero, with $H_{1k} = H_{k1} = v_k$. The position of the giant resonance is roughly determined by the diagonal matrix element at $k = 1$, given by $H_{11} = \epsilon_1$. We use this as a reference point and measure all energies relative to it by setting $\epsilon_1 = 0$. To study the giant resonance state as

a collective mode embedded into the complex spectrum, we naturally place ϵ_1 in the middle relative to the other ϵ_k .

Given a complex and chaotic nature of the nuclear Hamiltonian, we select energies ϵ_k distributed uniformly in some interval with their level spacing and all other properties as those of the GOE. In practice, this set is obtained by unfolding an instance of semicircular GOE spectrum and selecting its central part containing N values of ϵ_k . Following standard arguments of the random matrix theory and of the central limit theorem, we select the matrix elements v_k from a normal distribution centered at zero with the variance value v^2 . Thus, GR is considered as embedded in a single realization of a complex spectrum. The spectrum itself has statistical properties, characterized by the average level spacing s and the typical off-diagonal element v , which govern the energy spread of the GR state,

$$\lambda_1^2 = (H^2)_{11} - H_{11}^2 = \sum_{k \neq 1} |v_k|^2 \approx N v^2, \quad (50)$$

and thereby determine the timescale of GR decay [19]. The energy interval $\epsilon_k \in [-E_{\max}, E_{\max}]$ determines the level spacing $s \approx 2E_{\max}/N$. The qualitative model discussed above leads to the strength distribution (20), where

$$\gamma = 2\pi \frac{v^2}{s} = \pi \frac{\lambda_1^2}{E_{\max}}. \quad (51)$$

For the continuum, assuming one decay channel, we have

$$\mathcal{H}_{kk'}^c = -\frac{i}{2} \gamma(\epsilon) A_k A_{k'}, \quad (52)$$

where the threshold energy dependence is assumed to be

$$\gamma(\epsilon) = \begin{cases} \gamma_0 (\epsilon - \epsilon_{\text{tr}})^{(\ell+1/2)}, & \epsilon \geq \epsilon_{\text{tr}}, \\ 0, & \epsilon < \epsilon_{\text{tr}}, \end{cases} \quad (53)$$

which would correspond to a neutral particle decay in the partial wave ℓ . The parameter γ_0 defines the decay strength; in actual calculations, to avoid numerical issues with divergences, we also scale this energy dependence at high energies with the exponential suppression $\propto \exp(-\epsilon/\Lambda)$, where the parameter Λ is very large, giving some energy cutoff.

The vector A_k is generated randomly in orthonormal basis k . Its overlap with the dipole vector d is fixed,

$$\xi = \sum_k A_k d_k = A_1, \quad (54)$$

which determines the A_1 component, while the remaining components are chosen as independent Gaussian random variables with zero mean and a Lorentzian-shaped standard deviation centered at $\bar{\epsilon}_A$ and width γ_A :

$$|A_k|^2 \sim \frac{1}{(\bar{\epsilon}_A - \epsilon_k)^2 + \gamma_A^2}. \quad (55)$$

The randomly generated components are then scaled so that the full vector satisfies the normalization condition $\sum_k |A_k|^2 = 1$. In this way, the channel strength also has a nonuniform distribution that is closer to a typical physical reality of, for example, a single-particle decay strength distribution. The meaning of ξ^2 can be expressed as

$$\xi^2 = \langle d | \mathcal{Q}_{\text{SR}} | d \rangle = |\langle d | A \rangle|^2, \quad (56)$$

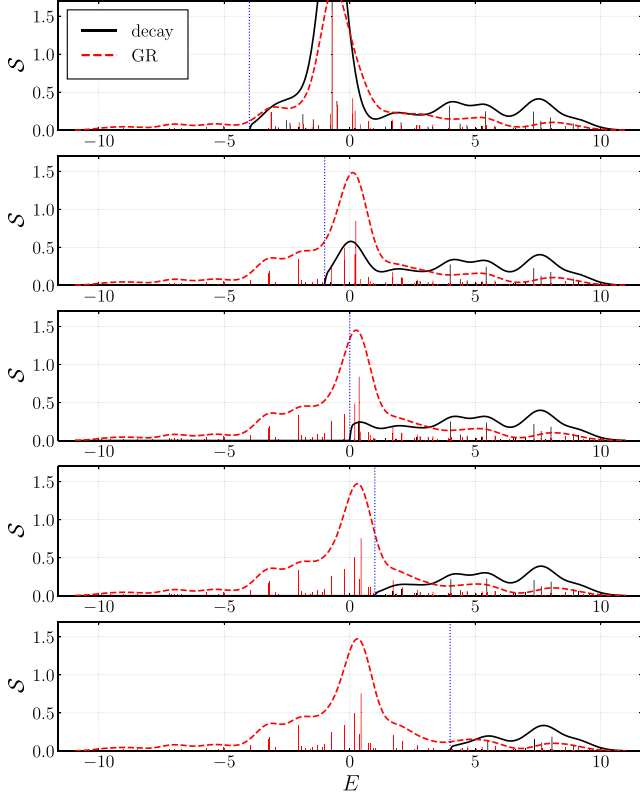


FIG. 5. Energy-dependent strength distributions for different threshold values $\epsilon_{tr} = -4, -1, 0, 1, 4$ (top to bottom). Each panel shows the decay widths $\Gamma_\alpha = -2 \text{Im } \mathcal{E}_\alpha$ (black bars) and the GR strength $|\langle d|\alpha \rangle|^2$ plotted vs E_α . The smoothed strength distributions for the giant resonance (red dashed line) and decay (solid black line) are overlaid to better enhance the representation. The vertical dashed blue line marks the threshold ϵ_{tr} in each panel.

which represents the fraction of the dipole excitation contained in the superradiant subspace and effectively a mixing parameter or an angle between vectors for channel A and for the GR state d .

In Fig. 5, we present a study of the model with $N = 100$ states within an energy region of $E_{\max} = 10$. This results in the average level spacing of $s = 0.2$. We take $v^2 = 0.1$, so that the natural spreading width of the GR is $\gamma = \pi \approx 3.14$, placing the resonance well within the $E_{\max} = 10$ range of the spectrum. The parameter $\gamma_0 = 4$ is chosen to parametrize the energy dependence of the width for the irreversible decay. We assume $\ell = 0$ and thus near the threshold the width grows as the square root of energy, Eq. (53). Finally, we select a strong mixing parameter of $\xi = 0.7$; the distribution of decay strength given by Eq. (55) is centered at $\bar{\epsilon}_A = 2$ and has the width $\gamma_A = 4$.

As already noted, GR is a collective state whose components are spread over many eigenstates within a certain energy region, approximately following the Breit-Wigner form of Eq. (20). Importantly, a fraction of these components lying above the decay threshold appear as resonances, which represents the situation we are addressing here. The strength distribution for various threshold positions is shown in Fig. 5. The poles in the complex energy scattering plane

TABLE I. For different values of ϵ_{tr} shown in the first column, the table presents the cumulative decay width of all states, the total part of the GR strength above threshold ϵ_{tr} , and the decay width Γ_{GR} . The last three columns contain data on the resonance pole α_d identified as having the largest magnitude of overlap with d . This includes the energy E_{α_d} , the decay width Γ_{α_d} , and the squared value of the highest overlap, $|\langle \alpha_d | d \rangle|^2$.

ϵ_{tr}	$\sum_\alpha \Gamma_\alpha$	$\sum_{E_\alpha > \epsilon_{tr}} \langle \alpha d \rangle ^2$	Γ_{GR}	E_{α_d}	Γ_{α_d}	$ \langle \alpha_d d \rangle ^2$
-4.0	7.3	0.99	0.97	-0.78	3.0	0.30
-1.0	3.2	0.70	0.11	0.26	0.39	0.17
0.0	2.5	0.56	0.056	0.37	0.23	0.16
1.0	2.1	0.23	0.014	0.44	0.00	0.15
4.0	1.2	0.10	0.0049	0.44	0.00	0.15

are found as the eigenvalues of the energy-dependent non-Hermitian Hamiltonian, $\mathcal{H}(\mathcal{E})|\alpha\rangle = \mathcal{E}_\alpha|\alpha\rangle$, that is determined self-consistently through iterative convergence. For each of these N poles, the real energy is plotted on the x axis, while the corresponding width is shown on the y axis. Simultaneously, the strength distribution $|\langle \alpha | d \rangle|^2$ is represented on the y axis using red bars. Formally, the GR strength distribution as a function of resonance energy, in the continuous limit, is defined as

$$S_d(E) = \sum_\alpha |\langle \alpha | d \rangle|^2 \delta(E - E_\alpha). \quad (57)$$

In the discrete case considered here, this distribution is shown as an impulse (stick) plot (with a different normalization) together with a smoothed curve obtained by replacing the delta functions with Gaussians.

The same figure also displays the widths of the resonances, $\mathcal{E}_\alpha = E_\alpha - i\Gamma_\alpha/2$, both as a stick plot of Γ_α versus E_α and as the corresponding distribution

$$S_\Gamma(E) = \sum_\alpha \Gamma_\alpha \delta(E - E_\alpha). \quad (58)$$

Although widths and strengths are fundamentally different, our choice of scales allows them to be presented on the same plot, providing a direct visual comparison of the spectral strength for both decay and dipole collectivity. In both cases, the impulse lines for individual states are shown, while the solid curves represent the smoothed distributions obtained with the Gaussian peak smoothing.

A careful examination of this figure, together with the analytical discussion in Sec. IV B, highlights the role of threshold in shaping the distribution of GR strength. In the superradiant limit, where the entire giant resonance lies within the continuum (top panel), there is a visible attraction of decay strength toward the GR at lower energies. A similar interplay between dipole and decay collectivity is discussed in Refs. [38,39]. The GR peak is also locally disturbed, and increased mixing leads to the enhanced local strength. These effects are significantly reduced when the decay threshold lies at higher energy, as shown in the lower panels.

In Table I, we summarize some of the data related to the results shown in Fig. 5. For each threshold position, we provide the total decay width of all states, $\sum_\alpha \Gamma_\alpha$, the portion of the dipole strength above the threshold, and the decay width

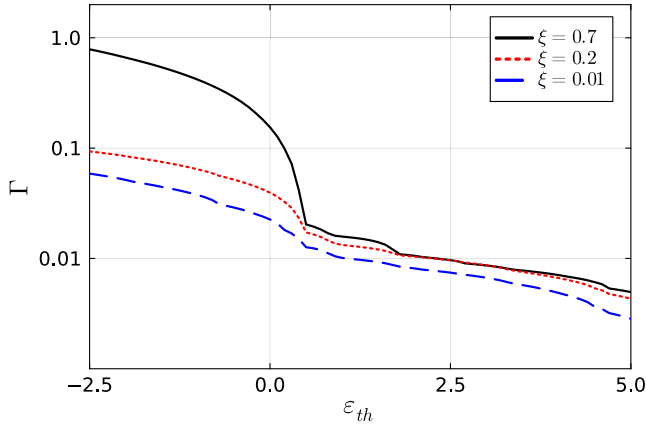


FIG. 6. The total width Γ_{GR} of the giant resonance is shown as a function of the threshold location for different values of the mixing amplitude $\xi = |\langle d|A\rangle|$.

of the GR computed as

$$\Gamma_{\text{GR}} = \sum_{\alpha} |\langle \alpha|d\rangle|^2 \Gamma_{\alpha}. \quad (59)$$

In a closed system undergoing internal relaxation only, with no irreversible decay, the decay rate of any given initial state vanishes at the initial moment $t = 0$ (see the detailed discussion and proof in Ref. [18]). This is not the case for an open system with the presence of irreversible decay. This behavior is also evident in our plots of the survival probability in Figs. 1 and 7, which show the closed-system limit where the initial time dependence of the survival probability is flat. In an open system, the decay rate is never zero; Γ_{GR} in Eq. (59) represents the irreversible decay rate for GR state d at the initial time $t = 0$. Practically, this rate can be inferred directly by measuring the initial outgoing flux of the decay products; in the GR case, a central example in this discussion, this would be the neutron flux.

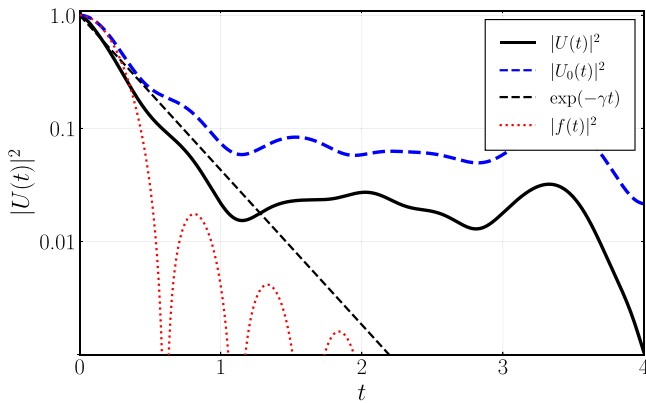


FIG. 7. Survival probability of the GR state as a function of time for the model discussed. The figure shows the case of a closed system, an open system with threshold at $\epsilon_{\text{tr}} = -1$, and the exponential that describes the natural relaxation with the rate γ [and thus Breit-Wigner strength distribution in Eq. (20)]; the behavior expected for a chaotic system is given by the Bessel function in Eq. (15).

For a resonance state α_d , whose wave function has the largest in magnitude overlap with the dipole mode d , in Table I, we report its energy and width and the value of the overlap squared.

It is evident that the impact of the decay threshold on the GR strength distribution is not significant in this model. The distortion is most pronounced when the entire GR is above threshold and the mixing is relatively strong, allowing internal and external collectivities to compete. However, since the strength of both is generally maximized at the center of the spectrum, the overall impact remains limited.

In Fig. 6, we show the dependence of Γ_{GR} on the threshold position for three different choices of the magnitude of the mixing amplitude. The impact of mixing is significant when the GR is above the threshold. This is expected because ξ^2 measures the GR component in the superradiant subspace, which promptly decays once the resonance is above threshold. In the subthreshold region, decay proceeds via the tails of the GR peak, which have a small overlap with the decay channel making the effect of the mixing overlap small.

The analysis of the time-dependent behavior and relaxation of the GR peak, including irreversible decay and the impact of chaotic mixing, is particularly insightful, as previously discussed. We turn to this topic next. An outline of the key elements of the method developed for this first-of-its-kind study involving an energy-dependent non-Hermitian intrinsic Hamiltonian is provided in the Appendix.

The time dependence of the survival probability of the initial state d is shown in Fig. 7, where we use the exact same model with the original choice $\xi = 0.7$ but the following results are obtained for a different choice of random parameters. The blue dashed curve shows the natural relaxation without coupling to the continuum, and the black curve shows the evolution for $\epsilon_{\text{tr}} = -1$. These quantities can be expressed through the matrix elements of the evolution operator

$$U_0(t) = \langle d|e^{-iHt}|d\rangle, \quad U(t) = \langle d|e^{-i\mathcal{H}t}|d\rangle, \quad (60)$$

where in the first case ($A = 0$) decay is not included and H is Hermitian, while in the second case the full propagator of an open system is considered via the Feshbach formalism, here written formally as $e^{-i\mathcal{H}t}$. The method of evaluation is discussed in the Appendix.

In this case, the continuum effect is significant and the survival probability is impacted by particle decay. The natural relaxation in the dipole model $\propto \exp(-\gamma t)$, Eq. (51), is shown with a thin dashed line.

The dynamics of the GR strength distribution for this system $\mathcal{S}_{d(t)}$, where time dependence is formally written as

$$|d(t)\rangle = e^{-i\mathcal{H}t}|d\rangle \quad (61)$$

is illustrated in Fig. 8 through a series of time snapshots. These depict the early-time evolution governed by both the intrinsic GR dynamics (friction-like interactions with complex noncollective states) and explicit particle decay. The distribution of decay widths is shown, for convenience of comparison (not to scale), by the black solid line, while the threshold energy is indicated by the dashed blue line. The distribution of decay widths is, of course, time independent. Together, these curves

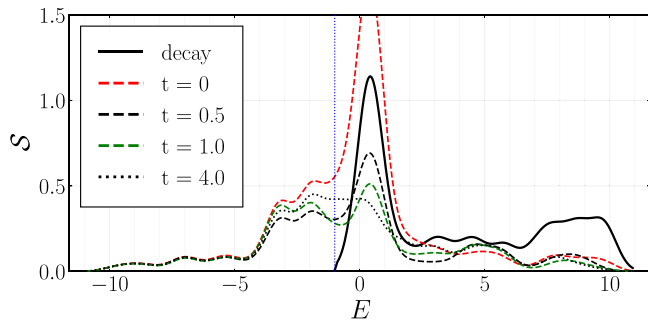


FIG. 8. Time snapshots of the GR strength distribution for an open system with threshold at $\epsilon_{tr} = -1$ are indicated by a blue vertical dotted line. The solid curve shows the decay strength distribution, while the dashed lines correspond to time snapshots at $t = 0, 0.5, 1$, and 4 . As seen in Fig. 7, the GR lifetime extends to approximately $t = 1$.

highlight the contrasting evolution of GR components located above and below the threshold.

Finally, in Fig. 9, we show the survival probability of the initial GR state relative to irreversible decay, namely, the normalization of the wave function as a function of time

$$P(t) = \sum_k |d_k(t)|^2, \quad (62)$$

which decays because the system is open. Similar to the chaotic example considered earlier in Figs. 1–3, the decay is initially fast due to a large component ξ reflecting the direct overlap of the initial state with the continuum channel (see dashed red and solid blue curves). Once this component decays, further irreversible decay proceeds through internal relaxation and mixing, which, on average, is exponential with some local noise.

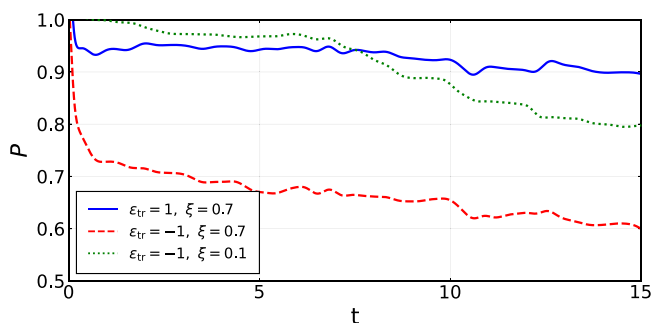


FIG. 9. Survival probability $P(t)$ of the GR state, shown relative to irreversible decay into the continuum. This reflects the total normalization of the wave function in the intrinsic space. The first two curves correspond to the same strong overlap of the initial state with the decay channel ($\xi = 0.7$), but with different threshold positions, $\epsilon_{tr} = \pm 1$. The comparison illustrates that subthreshold decay ($\epsilon_{tr} = 1$) is significantly slower, even though the initial component decays similarly. A third case with reduced overlap ($\xi = 0.1$, $\epsilon_{tr} = -1$) shows the absence of the initial rapid decay.

V. CONCLUSION

In this work, we have formulated and explicitly constructed a general framework for describing collective excitations in quantum many-body systems of interacting particles in the presence of open decay channels near the excitation energy of the collective mode and relevant thresholds. To investigate these phenomena, we developed a time-dependent approach that captures the essential features of quantum many-body complexity, the relaxation of collective dynamics on this background, and the explicit processes of irreversible decay.

Our study encompasses the complex interplay of several key elements that govern the structure and dynamics of open quantum many-body systems with collective excitations. The mean field provides the essential background that defines a convenient physical basis and identifies many-body configurations. The complex interparticle interactions increase the level density and induce mixing among these configurations, giving rise to chaotic dynamics in the evolution to the regime described by the random matrix theory. At the same time, coherent components of the interaction can generate well-defined collective modes that emerge independently of the chaotic background. These collective excitations are then subject to damping, and their strength tends to concentrate in specific spectral regions leading to the formation of what is typically recognized as a giant resonance.

The opening of decay channels at specific threshold energies introduces a division between bound and unbound spectral parts of the system. Near thresholds, decay amplitudes exhibit discontinuities, which in turn modify the time evolution. Bound states can be affected by virtual coupling to the continuum, while above threshold, the real emission occurs, and the system ultimately relaxes through irreversible decay processes.

We explored these complicated mechanisms using a generic model that allows us to trace the full-time evolution of the system. The time-dependent approach naturally captures both the coherent dynamics of the initial excitation and its subsequent relaxation, driven by internal mixing and external decay.

Our results reveal a complex interplay among three distinct components of the dynamics: relaxation similar to thermalization with the formation of a chaotic background, characterized by behavior similar to that of a generic Bessel function; natural damping of the collective mode, governed by residual interactions with the background; and irreversible decay into continuum. In complex many-body systems, the irreversible decay is often slow, as it is hindered by the complexity of the evolving wave function. During the time evolution, the components of the wave function that align with the decay channel (often referred to as the doorway state) are typically small and fluctuate rapidly. The fluctuations in decay reflect a form of quantum noise, analogous to effects well known in condensed matter and material physics, such as the shot noise observed in strongly correlated systems. These noisy decay components contribute cumulatively to a slow, averaged exponential decay.

The initial stage of the dynamics discussed above is dominated by the collective excitation and its competition with fast-decaying components. At extremely short times, one may

identify a pre-exponential regime that reflects the excitation of the system into a slowly decaying collective mode. Beyond that, the early behavior is sensitive to the presence of continuum-coupled components in the initial state, as well as to the position of the decay threshold that governs how rapidly the irreversible decay can proceed. In general, components with strong coupling to the continuum decay quickly effectively removing themselves from the system and leaving behind a slower combination of natural and noise-driven relaxation of the remaining collective mode. In the long time limit, the system enters a regime of the weak residual radiation accompanied by persistent quantum fluctuations, which continue to reflect the underlying complexity of the many-body dynamics.

Although the model was deliberately schematic, it captured essential features of the full process and revealed aspects of the dynamical behavior that appear generic for many-body systems with collective excitations and decay. The approach provides a foundation for future detailed studies aimed at realistic examples and direct comparison with experimental data. Beyond nuclear physics, the results are relevant to a wide range of open quantum systems where collective dynamics, chaotic mixing, and irreversible decay interplay in nontrivial ways. This includes mesoscopic systems, where control of decoherence and dissipation is crucial, and platforms for quantum information processing, where understanding relaxation and equilibration is essential for maintaining coherence and developing fault-tolerant architectures. The emergence of thermalization through chaotic dynamics connects naturally to the eigenstate thermalization hypothesis, a central concept in modern many-body theory. These broader connections make the presented framework a promising tool for investigating fundamental aspects of nonequilibrium dynamics in both traditional and emerging quantum systems.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

APPENDIX: ENERGY-DEPENDENT AMPLITUDES IN TIME-DEPENDENT FORMALISM

Near-threshold physics is becoming a central focus in nuclear studies, as interest increasingly shifts toward nuclei far from stability and close to the drip lines. As discussed earlier, time-dependent methods offer a particularly natural framework for studying such phenomena. These approaches capture the genuine dynamical evolution of the system and inherently emphasize the most relevant physical features. One of the key advantages of time-dependent techniques is their robustness in capturing processes that span a wide range of timescales. This feature becomes especially important near decay thresholds,

where the hierarchy of dynamical timescales can be extreme and where discontinuities often arise in the traditional energy-domain description.

Although methodological aspects are sometimes considered secondary to physical insights, they often serve as the foundation for further scientific progress. Careful attention to these techniques is essential, as overlooking such details may limit the broader development and application of these tools. For this reason, we include this short Appendix to outline the technical aspects of the time-dependent method developed and employed in the present work and logic behind this strategy.

Here, we consider the energy dependence of amplitudes following the discussion in Ref. [15] for the nonunitary evolution operator

$$U(t) = \exp(-i\mathcal{H}t) = -\frac{1}{2\pi i} \int_{-\infty}^{\infty} \mathcal{G}(E) \exp(-iEt) dE, \quad (\text{A1})$$

where the propagator $\mathcal{G}(E)$ is given by the full energy-dependent non-Hermitian effective Hamiltonian,

$$\mathcal{G}(E) = \frac{1}{E - \mathcal{H}(E)}. \quad (\text{A2})$$

The factorized form of the non-Hermitian component that is closely tied to the unitarity of the scattering matrix allows us to express [15]

$$\mathcal{G}(E) = G(E) - \frac{i}{2} G(E) \mathbf{A} \left(1 + \frac{i}{2} \mathbf{A}^\dagger G(E) \mathbf{A} \right)^{-1} \mathbf{A}^\dagger G(E), \quad (\text{A3})$$

where

$$G(E) = \frac{1}{E - H} \quad (\text{A4})$$

is the Hermitian propagator unperturbed by the continuum coupling. This propagator is much easier to evaluate as a function of energy because H is an energy-independent operator and its eigenstate structure can be used. Another advantage comes from the fact that the operator $1 + \frac{i}{2} \mathbf{A}^\dagger G(E) \mathbf{A}$ works in the channel subspace being represented by the $N_c \times N_c$ matrix. The number of open channels at small excitation energy is usually relatively small.

The core challenge lies in evaluating the integral in Eq. (A1), where the propagator is generally a complex operator. In practice, this evaluation involves computing matrix elements between the initial and final states. The method based on a Chebyshev polynomial expansion, proposed in Ref. [15], is particularly useful, as it evaluates the operator through an iterative procedure similar to the Lanczos technique. This approach requires only matrix-vector multiplications, and the time resolution improves systematically as the iteration proceeds. While the method performs well in many practical applications, it becomes difficult to use in sensitive studies that require high precision. The Hamiltonian matrix must be bounded, and extending the procedure beyond several thousand iterations is generally impractical due to the significant precision loss. As a result, the method must be restricted to a limited energy region, which in turn limits the resolution at short timescales.

The strategy discussed here stems from the perspective that a good reduced-basis model can be constructed to describe the physics of interest. Practically speaking, exact numerical diagonalization is available for very large matrices, with dimensions reaching tens of thousands, which is more than sufficient for most applications. It is therefore advantageous to perform the integration in Eq. (A1) using a Fast Fourier Transform that is indeed fast, reliable, and yields an immediate evaluation of $U(t)$ at all times. However, energy discretization along the real axis, though commonly used, becomes ineffective and error-prone when the spectrum is dense. In such cases, the presence of many poles on the real axis leads to numerical instabilities.

A practical solution is to use the Berggren basis [40], which implements a contour that typically begins at the threshold and bends down into the complex plane. This approach decomposes the integral of interest into contributions from bound states, resonances, and the complex contour. In the present study, however, we are not focused on bound or resonant states but solely on understanding the time evolution. For this reason, we employ a different contour, illustrated in Fig. 10, which runs parallel to the real axis in the upper half-plane. Since there are no poles in the upper half-plane, this integration path is numerically stable. The result is largely insensitive to the offset δ , provided that δ is not too small, as a very small offset would place the contour too close to the poles on the real axis. While the contributions from the vertical edges of

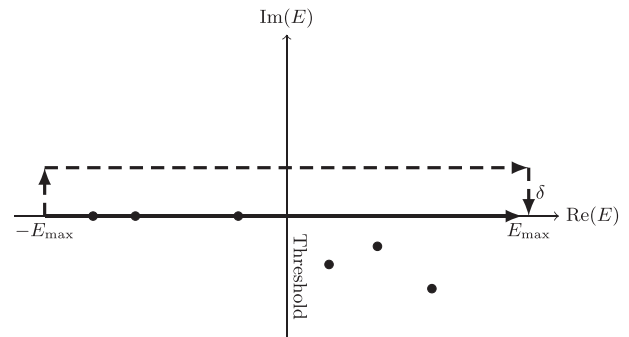


FIG. 10. Illustration of the integration contour in the complex energy plane used for evaluating the time evolution operator. The real axis corresponds to physical energies, with a threshold marked by a vertical axis line. To avoid instabilities caused by poles on the real axis, the integration is carried out along a shifted contour displaced by an offset δ into the upper half-plane (dashed horizontal path). This deformation ensures numerical stability and convergence. With proper handling of all segments of the contour, the results are largely insensitive to the specific value of δ , as long as it is not taken too small or too large.

the contour (extending upward and downward at its endpoints) are typically small due to their remoteness, including them numerically further enhances the stability of the method and its insensitivity to the offset δ .

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