

Spin glass dynamics on complex hardware topologies: A bond-correlated percolation approach

Viviana Gómez^{1,*}, Gabriel Téllez^{1,†}, and Fernando J. Gómez-Ruiz^{2,‡}¹Departamento de Física, Universidad de los Andes, A.A. 4976, Bogotá D.C., Colombia²Departamento de Física, Universidad Carlos III de Madrid, Avda. de la Universidad 30, 28911 Leganés, Spain

(Received 11 November 2025; accepted 24 June 2026; published 24 July 2026)

Understanding how frustration and disorder shape relaxation in complex systems is a central problem in statistical physics and quantum annealing. Spin glass models provide a natural framework to explore this connection, as their energy landscapes are governed by competing interactions and constrained topologies. We investigate the nonexponential relaxation behavior of spin glasses on network architectures relevant to quantum annealing hardware—such as finite size Chimera, Pegasus, and Zephyr graphs—where embedding constraints and finite connectivity strongly modulate the distribution of barriers and metastable states. This slow relaxation arises from the combined effects of frustration and disorder, which persist even beyond the conventional spin glass transition. Within the Fortuin-Kasteleyn-Coniglio-Klein cluster formalism, the appearance of unfrustrated cluster regions gives rise to multiple relaxation scales, as distinct domains follow different dynamical pathways across a rugged energy landscape. This framework enables a broader characterization of spin glass energy landscapes and provides physical insight into how topological constraints, frustration, and disorder jointly govern relaxation dynamics in complex networks relevant to quantum annealing architectures.

DOI: [10.1103/t6rh-7mnl](https://doi.org/10.1103/t6rh-7mnl)

I. INTRODUCTION

In the realms of quantum computing and quantum simulation, a fundamental yet highly nontrivial problem is the preparation of ground states of interacting Hamiltonians [1–4]. Quantum annealing (QA) has established itself as a theoretical and practical paradigm for addressing computationally demanding combinatorial optimization problems, with experimental implementations based on networks of coupled qubits that realize programmable interactions [5–7]. Several hardware architectures have been developed following this principle, featuring connectivity patterns such as the D-Wave Chimera, Pegasus, and Zephyr graphs, which represent successive efforts to increase qubit connectivity, embedding efficiency, and scalability [8–10]. These programmable quantum processors, exemplified by D-Wave’s annealing platforms [11–14], have enabled the exploration of nonequilibrium spin glass dynamics, where phenomena akin to the Kibble-Zurek mechanism govern defect formation and relaxation processes during the annealing trajectory [15–18].

Spin glasses, disordered magnetic systems characterized by competing ferromagnetic and antiferromagnetic interactions, offer a natural framework for formulating hard combinatorial optimization problems [19]. Their Hamiltonians, typically expressed as Ising models with quenched

disorder in the couplings, capture rugged energy landscapes with a multitude of nearly degenerate local minima separated by large barriers [20–22]. In the context of quantum adiabatic computing, such Hamiltonians serve as benchmark instances, as the adiabatic evolution seeks the ground-state configuration within these complex landscapes [23]. Owing to their intrinsic complexity, spin glass models pose a formidable challenge for classical algorithms and, at the same time, provide an ideal test bed for assessing the potential advantages of quantum approaches [24].

Spin glasses exhibit a transition to a spin glass phase at a characteristic temperature T_{SG} , below which the system freezes into a disordered but stable configuration with no conventional long-range magnetic order. This transition marks the point where the spins become trapped in a complex landscape of metastable states, leading to a dramatic slowing down of dynamics and to history-dependent phenomena such as aging [25–27]. However, in glassy and frustrated systems, it is often observed that the dynamical behavior changes well before any static transition takes place [28,29]. Specifically, spin-spin autocorrelations, which decay exponentially at high temperatures, develop a stretched or nonexponential form below a crossover scale T^* [30,31]. This dynamical crossover can arise from two main mechanisms: frustration and disorder. Percolation processes, such as the formation of Fortuin-Kasteleyn-Coniglio-Klein (FKCK) clusters, provide a microscopic explanation for the increasing complexity of the accessible phase space in the presence of frustration. Below T_p , the temperature at which these clusters percolate, the dynamics slow down and relaxation becomes heterogeneous, even in fully frustrated models without explicit disorder [32]. In the presence of disorder, rare regions can locally order and give rise to a Griffiths phase, with a broad distribution of relaxation times. In both scenarios, the net effect is that the

*Contact author: v.gomez6@uniandes.edu.co†Contact author: gtellez@uniandes.edu.co‡Contact author: fegomezr@fis.uc3m.es

autocorrelation function deviates from a simple exponential, reflecting the coexistence of T_p multiple timescales and the onset of slow dynamics. Determining whether T^* coincides with the ferromagnetic Ising critical temperature T_C or with the percolation temperature T_p remains a challenging numerical problem, as the two scales typically lie in close proximity.

Analogous to what occurs in glass-forming liquids, the emergence of nonexponential relaxation in spin glasses is closely related to changes in how the system explores its potential energy landscape [33–35]. At high temperatures, the average energy decreases only weakly with decreasing T , indicating that the system explores shallow regions of the landscape. As the spin glass transition temperature T_{SG} is approached, however, the energy begins to decrease more rapidly, reflecting that the system becomes confined to progressively deeper minima. This regime of enhanced energy relaxation coincides with the temperature range where the spin autocorrelation function transitions from exponential to nonexponential decay, signaling the onset of glassy dynamics.

In experimental realizations of these systems, such as those implemented on quantum annealing hardware, the situation becomes even more intricate due to the unavoidable presence of finite operating temperatures. Practical implementations of quantum annealers necessarily operate at finite temperatures, which imposes intrinsic limitations on their performance as scalable optimizers [36,37]. Recent studies have shown that unless the operating temperature decreases appropriately with system size, the device cannot reliably sample ground states or exploit quantum tunneling effectively [38–40]. Consequently, the competition between quantum and thermal fluctuations becomes a key factor shaping the relaxation dynamics of spin glass systems realized on current annealing processors.

The central idea of QA is to gradually quench quantum fluctuations in order to minimize a target Hamiltonian that encodes a complex optimization problem [41]. In principle, the quantum wave function may tunnel through barriers in the free-energy landscape, suggesting a possible advantage over classical methods such as simulated annealing (SA), where thermal fluctuations are progressively reduced to reach low-energy states [42]. However, previous studies have suggested that, for spin glass systems defined on the Chimera topology, the structure of the energy landscape may actually favor thermal strategies like SA, since the spin glass phase is stable only at zero temperature [43,44].

From a practical quantum annealing perspective, these observations raise a fundamental question: How does the hardware connectivity itself constrain the emergence of collective correlations and dynamically complex landscapes at the finite temperatures where current annealers operate? Addressing this question is essential for assessing the robustness of QA against thermal noise and for identifying graph-level design principles that can enhance annealing performance beyond idealized zero-temperature limits.

In this work, we revisit the interplay between topology, frustration, and relaxation in spin glass models defined on D-Wave-inspired architectures. By explicitly linking dynamical slowing down to topology-dependent percolation phenomena, we show that finite-temperature performance limitations can be understood in terms of the fragmentation of the system-spanning FKCK cluster when thermally acti-

vated defects disrupt the percolating network. In this picture, thermal fluctuations progressively break the connectivity of frustration-mediated pathways, suppressing long-range correlations and restoring predominantly local relaxation. This provides a physically transparent framework in which characteristic percolation temperatures emerge as connectivity-resolved indicators of hardware resilience to thermal effects. By analyzing the emergence of nonexponential relaxation through classical Monte Carlo simulations and interpreting the results within the framework of percolation theory, we aim to clarify how network connectivity shapes dynamical slowing down.

Rather than viewing the absence of a finite-temperature spin glass phase as a limitation, our results demonstrate that topology-driven frustration percolation can still generate regimes of slow, collective dynamics that are directly relevant to annealing processes on realistic devices. This is particularly important given that quantum annealing relies on mapping hard combinatorial optimization problems onto spin glass Hamiltonians, whose solution requires navigating a complex and frustrated energy landscape that is constrained by the hardware interaction graph. By clarifying how graph-induced connectivity and embedding structures shape the finite-temperature landscape, our findings provide useful insight for the design of quantum annealers, complementing algorithmic considerations with graph-theoretic and statistical-physics insights. Our objective is not to demonstrate a quantum speedup—a question that remains open—but to deepen the understanding of frustration in complex networks and provide insight that may guide future developments in both classical and quantum optimization.

The structure of this article is as follows: In Sec. II, we begin by introducing the fundamental concepts of percolation of FKCK clusters in spin glasses and their relation to the phase space of glassy systems. In Sec. III, we describe the methodology employed to obtain the results. In Sec. IV, we present an analysis of the percolation properties of these clusters in the three-dimensional (3D) lattice, Chimera, Pegasus, and Zephyr topologies, together with a study of the associated spin-spin autocorrelations. Finally, we discuss the implications and relevance of these results for understanding glassy dynamics in quasi-two-dimensional networks and for exploring how topology and finite-temperature collective behavior may influence dynamics in architectures relevant to quantum annealing.

II. PERCOLATION TRANSITION

In the study of critical phenomena, the FKCK cluster formalism provides a geometric framework that links thermodynamic correlations to percolation properties [45]. In the ferromagnetic Ising model, this approach introduces bonds between parallel nearest-neighbor spins of coupling strength J , which are connected with a temperature (T)-dependent probability

$$p(T) = 1 - \exp\left[-\frac{2J}{k_B T}\right]. \quad (1)$$

By summing over spin configurations weighted by the Boltzmann factor, the partition function can be reformulated as a sum over bond configurations. Within this representation,

FKCK clusters describe correlated spin fluctuations that percolate precisely at the Ising critical point, exhibiting the correct critical exponents. This geometric framework has proven particularly powerful for analyzing how network topology shapes the onset and universality of critical behavior [46].

When extending this approach to disordered and frustrated models such as spin glasses, important modifications appear. Consider the Ising spin glass Hamiltonian

$$H = -J \sum_{\langle ij \rangle} \varepsilon_{ij} S_i S_j, \quad (2)$$

where the quenched random couplings $\varepsilon_{ij} = \pm 1$ represent ferromagnetic or antiferromagnetic interactions. Following the FKCK formalism, a bond is placed between neighboring spins that satisfy their local interaction ($\varepsilon_{ij} S_i S_j = +1$) with probability p . However, the essential difference from the ferromagnetic case arises from frustration: Along a closed loop for which $\prod_{(ij) \in \text{loop}} \varepsilon_{ij} = -1$, no spin configuration can simultaneously satisfy all interactions. Such loops are said to be frustrated, and bonds cannot be assigned in a way that closes them.

As a result, the spin glass partition function can be expressed as

$$Z = \sum_{C^*} e^{\mu b(C)} 2^{N(C)}, \quad (3)$$

where $\mu = \ln[p/(1-p)]$, $b(C)$ is the number of bonds in the cluster configuration C and $N(C)$ is the total number of clusters in that configuration. The sum $\sum_{C^*}$ runs only over bond configurations that do not contain frustrated loops. At zero temperature, the ground state corresponds to the maximal number of bonds consistent with the no-no-frustration constraint, turning the problem into a geometrical optimization under frustration [32,47,48].

This distinction has significant consequences. In the ferromagnetic case, clusters directly mirror thermodynamic fluctuations, whereas in spin glasses correlations are mixed in sign: Two spins may be positively correlated if the path connecting them involves an even number of negative bonds, or negatively correlated if the path involves an odd number. These competing contributions partially cancel, meaning that clusters defined via connectivity no longer represent thermodynamic correlations. Nevertheless, connectivity is always non-negative, so clusters defined by bonds that do not close a frustrated loop can still percolate. This occurs at a temperature T_p higher than the actual spin glass critical temperature T_{SG} [49,50]. Once this percolating cluster forms, correlations extend across all length scales, and because the cluster connects spins belonging to frustrated regions of the interaction graph, collective rearrangements necessarily involve frustration, leading to slow, cooperative dynamics. In the three-dimensional binary Ising spin glass, numerical simulations show $T_p \approx 3.95J/k_B$ [51], while $T_{SG} \approx 1.11J/k_B$ [52]. This percolation threshold T_p can be mapped to the thermodynamic transition of a ferromagnetic $2s$ -state Potts model for $s \neq 1$ (see Refs. [29,32,47]). This exact equivalence between Potts thermodynamics and the percolation of correlated clusters allows one to interpret T_p as a Potts-like transition. In

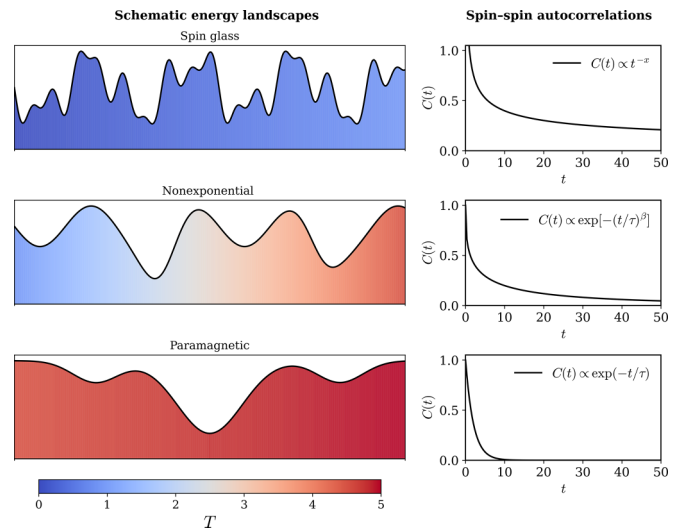


FIG. 1. Illustrative energy landscapes (left) and their corresponding spin-spin autocorrelations (right). Below the spin glass transition, the energy landscape becomes extremely rugged, with many deep valleys separated by high barriers; ergodicity is broken and spin autocorrelations decay slowly. Below the Potts (percolation) transition, frustration persists, while below the Griffiths temperature, disorder induces rare correlated regions—both mechanisms give rise to stretched-exponential relaxation. Above these phases, the system is paramagnetic, exhibiting exponential decay and a simple energy landscape.

the spin glass limit ($s = 1$) relevant to the present work, the existence of such a thermodynamic singularity has not been established analytically.

The phase diagram of a spin glass can be interpreted as follows. Below the critical temperature T_{SG} , the system enters the spin-glass phase, where ergodicity is broken and spin autocorrelations decay slowly, often following a power-law form. Above T_{SG} , however, the system already exhibits nonexponential relaxation, which can arise from two distinct mechanisms. For temperatures below T_p , frustration generates a corrugated energy landscape that induces stretched or nonexponential relaxation, even in the absence of disorder (see Fig. 1). In contrast, below T_C —which coincides with the critical temperature of an Ising ferromagnet—quenched disorder becomes relevant, giving rise to rare, strongly correlated regions that define the Griffiths phase. Both the Potts transition at T_p and the Griffiths transition at T_C contribute to the emergence of nonexponential relaxation. Since typically $T_C < T_p$, the disorder-driven Griffiths transition dominates and tends to mask the effects of frustration. In the limiting case of a fully frustrated model without disorder, only the Potts transition at T_p remains, clearly signaling the onset of nonexponential relaxation [29].

III. METHODOLOGY

In this study, we investigate the percolation properties and dynamical behavior of spin glass models defined on network topologies relevant for QA: Chimera, Pegasus, and Zephyr graphs, as well as a three-dimensional cubic lattice for comparison. Specifically, we determine the percolation threshold

T_p for different system sizes in each topology. To model disorder and frustration, we employ the Edwards-Anderson (EA) spin glass with Gaussian couplings, which are preferred in the context of quantum annealing as they suppress extensive degeneracies and yield a more generic and experimentally realistic energy landscape. In this formulation, the Hamiltonian is given by

$$H = - \sum_{(i,j)} J_{ij} S_i S_j, \quad (4)$$

where $S_i = \pm 1$ are binary Ising spin variables to be optimized, J_{ij} are quenched random coupling strengths drawn from a Gaussian distribution with zero mean and unit variance, and $\langle i, j \rangle$ denotes the set of connected pairs of spins in the underlying graph of the model.

To characterize the geometric organization of the FKCK correlated clusters and estimate the percolation temperature T_p , we compute both the spanning probability $P_\infty(T)$ and the average path length (APL) of the activated network. The spanning probability is defined as the fraction of spins belonging to the largest connected cluster in the system and serves as an indicator of the emergence of large-scale connectivity. Clusters are constructed by activating bonds according to Eq. (1), explicitly incorporating the values of the couplings J_{ij} . This probability can be viewed as a function $P(T)$; for clarity, from now on we will work with the variable T under this transformation. In practice, the transition temperature T_p is estimated from the crossing point of the curves $P_\infty(T)$ obtained for different system sizes N [47].

A more detailed characterization of the correlated connectivity structure is obtained from the weighted APL. For a system of N spins, the APL is defined as

$$\langle \ell \rangle = \frac{2}{N(N-1)} \left[\left\langle \sum_{i < j} \ell_{ij} \right\rangle_T \right]_J, \quad (5)$$

where ℓ_{ij} denotes the shortest path between spins i and j . The brackets $\langle \cdots \rangle_T$ and $[\cdots]_J$ represent thermal and disorder averages, respectively, taken over independent realizations of temperature histories and quenched couplings. In disconnected graphs, only connected components contribute to the sum, and if the network is completely disconnected, the APL is defined to be zero.

The paths are constructed according to the bond-activation probability defined in Eq. (1). The metric is weighted using $1/J_{ij}$ as the distance between connected spins, so that stronger bonds correspond to shorter effective paths. As a result, this observable probes not only geometric connectivity, but also the preferential propagation of strongly correlated and frustration-mediated interactions across the graph. This becomes particularly relevant in heterogeneous and highly connected architectures, where collective relaxation and long-range connectivity depend sensitively on the existence of robust pathways sustained by strong couplings.

As the temperature is varied, the competition between bond activation and network fragmentation produces a pronounced maximum in the APL, reflecting the reorganization of long-distance connectivity across the correlated cluster structure. The position of this maximum defines the characteristic temperature $T_p(N)$.

After identifying the percolation temperature T_p , we compute the spin-spin autocorrelation function for each network. This observable is defined as

$$C(t) = \frac{1}{N} \sum_{i=1}^N [\langle S_i(0) S_i(t) \rangle_T]_J, \quad (6)$$

where $S_i(0)$ denotes the initial configuration, and the average is taken over both thermal histories and quenched disorder realizations drawn from the Gaussian EA distribution. The correlations were measured using a classical single-spin Monte Carlo flip dynamics after ensuring that the system had reached equilibrium. All temperatures are reported in units of σ_J/k_B , where σ_J is the standard deviation of the coupling distribution.

IV. RESULTS

A. Spin glass percolation transition

Figure 2 shows the spanning probability $P_\infty(T)$ and the finite-size scaling (FSS) collapse of the APL for each of the four network topologies, allowing the extraction of the exponents α and ϕ , which, respectively, characterize the growth of the APL peak amplitude with system size and the sharpening of the crossover region around the APL maximum. The corresponding percolation temperatures were obtained from the position of the APL peak before the collapse for each system size and subsequently extrapolated to the thermodynamic limit (see Fig. 3). The extrapolation of $T_p(N)$ was performed using quadratic fits in $1/N$, which provide numerically stable estimates of the thermodynamic-limit values $T_p(\infty)$ while effectively incorporating subleading finite-size corrections. These quadratic forms should not be interpreted as fundamental asymptotic scaling laws, but rather as practical extrapolation procedures motivated by the systematic curvature observed in the numerical data. The results are reported in Table I, together with a comparison to the other characteristic phase transition temperatures (T_{SG} and T_C).

The scaling collapse was constructed assuming an anomalous FSS form, where the shift and rounding of the pseudocritical region scale with different exponents, unlike the conventional case in which both are controlled by the same scaling exponent [55,56]. Within this framework, the size-dependent pseudocritical temperature $T_p(N)$, defined from the peak of the APL, satisfies

$$|T_p(N) - T_p(\infty)| \sim N^{-\lambda}, \quad (7)$$

while the width of the crossover region scales as

$$\delta T \sim N^{-\phi}, \quad (8)$$

where λ and ϕ denote the shift and rounding exponents, respectively. The corresponding scaling form of the observable is then given by

$$\begin{aligned} O(T, N) &= N^\alpha F[(T - T_p(N))N^\phi] \\ &= N^\alpha F[(T - T_p(\infty))N^\phi + AN^{\phi-\lambda}]. \end{aligned} \quad (9)$$

Figure 3 includes an inset showing the pseudocritical drift of the APL. The shift exponent λ is extracted through log-log fits of Eq. (9), yielding $\lambda_{3D} \approx 1.02$, $\lambda_{\text{Chimera}} \approx 0.87$, λ_{Pegasus}

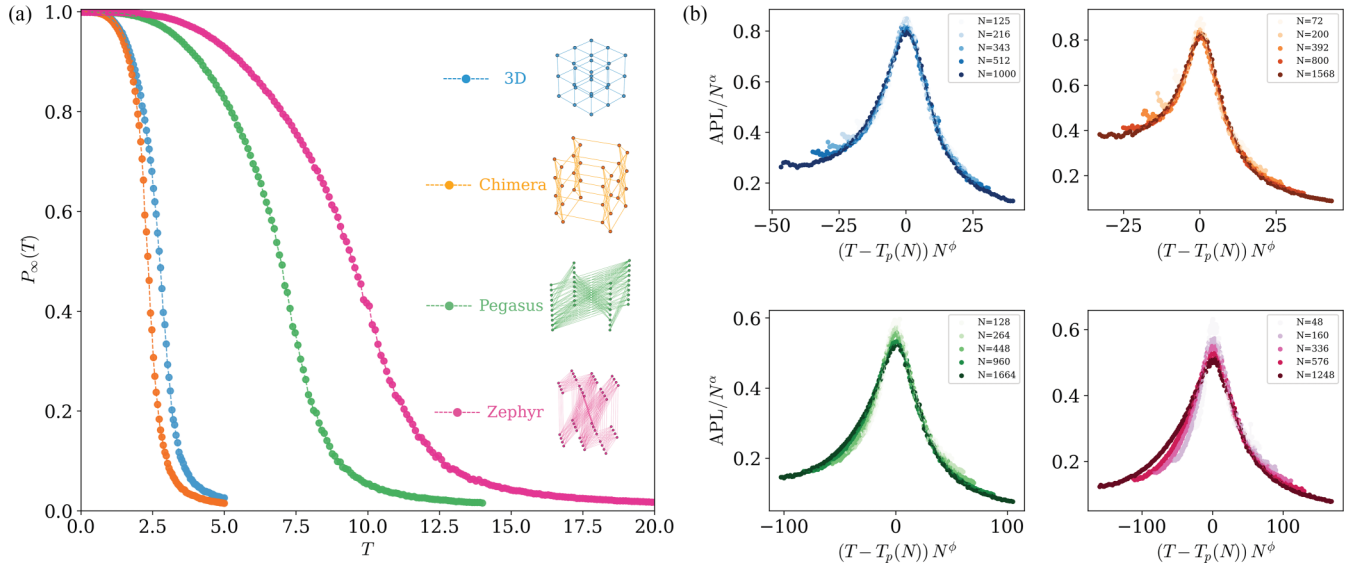


FIG. 2. (a) Spanning probability $P_\infty(T)$ for the largest system size of each topology: blue—3D ($N = 1000$), orange—Chimera ($N = 1568$), green—Pegasus ($N = 1664$), and pink—Zephyr ($N = 1248$). (b) Finite-size scaling collapse of the average path length for all system sizes studied in each network, based on the anomalous scaling form discussed in the main text. The collapse enables the extraction of the scaling exponents α and ϕ reported in Table I. The APL was computed using distances weighted by the coupling strengths J_{ij} ; this weighting modifies the absolute magnitude of the APL but does not affect the position of its maximum.

≈ 0.85 , and $\lambda_{\text{Zephyr}} \approx 0.83$, while the collapse shown in Fig. 2 provides the corresponding rounding exponent ϕ (see Table I). Notably, in all cases we find $\lambda < \phi$, implying that the correction term $N^{\phi-\lambda}$ vanishes in the thermodynamic limit.

We employ this non-standard FSS framework because the hardware-inspired graphs considered here do not possess a simple Euclidean geometry or conventional boundary conditions. These architectures may also exhibit high and heterogeneous connectivity, leading to complex nonlocal path

structures. The APL is defined on FKCK correlated clusters, so the effective paths contributing to the observable depend not only on the underlying hardware topology, but also on the temperature-dependent connectivity of the correlated cluster structure. Even for regular lattices, these paths may differ substantially from the shortest geometric paths of the bare graph because they are constrained by correlated percolation processes.

For the Zephyr network, the critical temperature T_C was additionally estimated using the same FSS analysis applied to the APL, but considering the ferromagnetic case, i.e., with all couplings set to $J_{ij} = 1$. Due to the relatively small network sizes accessible in our simulations, the reported values may slightly underestimate their true thermodynamic limits. Nonetheless, the extracted trends provide a reliable reference for comparing the effective behavior across architectures.

As expected, the percolation temperature T_p and the Griffiths transition temperature T_C increase with the overall connectivity of the network. Among the studied topologies, Chimera exhibits the lowest T_p , consistent with its relatively low average degree and modular connectivity pattern. Interestingly, Chimera also displays the highest APL values before the fragmentation of the percolating cluster, indicating that information or correlations must traverse longer paths to maintain global connectivity. This behavior reflects the presence of sparse intercell couplings, which create local bottlenecks and enhance the effective distance between distant regions of the network. In contrast, Zephyr, with its denser and more uniform interqubit connections, shows the largest T_p and T_C , consistent with a more robust and strongly interconnected structure that delays fragmentation.

Importantly, the scaling analysis of the APL is formulated in terms of the total system size N rather than a well-defined linear dimension L . For the regular 3D lattice, one may formally use $N = L^3$, allowing the scaling variable to be

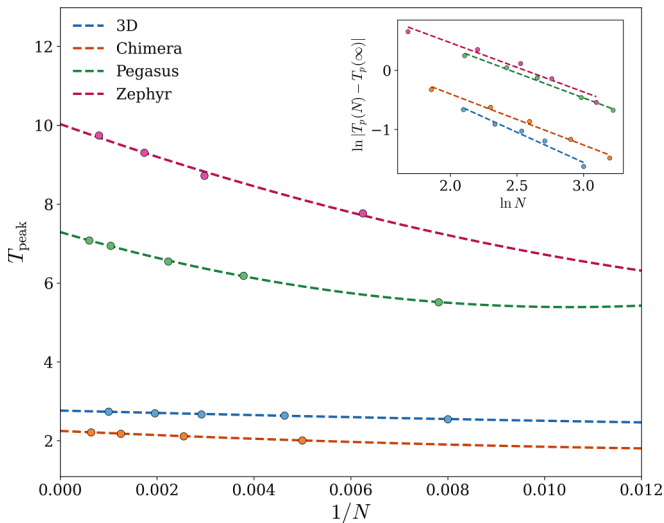


FIG. 3. Temperature $T_p(N)$ corresponding to the maximum of the APL for the four network topologies plotted as a function of $1/N$. Dashed lines indicate quadratic fits used to extrapolate the thermodynamic-limit values $T_p(\infty)$ reported in Table I. The inset shows the log-log plots of $|T_p(N) - T_p(\infty)|$ vs N , used to extract the shift exponent λ from Eq. (9).

TABLE I. Summary of characteristic temperatures and FSS exponents for each topology. T_{SG} denotes the spin glass transition temperature, T_p the Potts (percolation) transition temperature, and T_C the Griffiths (ferromagnetic) transition temperature. α and ϕ correspond, respectively, to the amplitude and rounding exponents obtained from the FSS collapse of the APL.

Graph	N	T_{SG}	T_p	T_C	α	ϕ
3D	1000	0.95 ^a	2.76 ^b	4.51 ^c	0.508 ^b	0.416 ^b
Chimera	1568	0.00 ^d	2.25 ^b	4.16 ^e	0.512 ^b	0.373 ^b
Pegasus	1664	0.00 ^f	7.29 ^b	12.60 ^g	0.535 ^b	0.366 ^b
Zephyr	1248	0.00 ^h	10.02 ^b	16.23 ^b	0.537 ^b	0.393 ^b

^aReference [52].

^bThis study.

^cReference [53].

^dReferences [8,44,54].

^eReference [44].

^fReferences [8,54].

^gReference [8].

^hReference [54].

rewritten in the more conventional form $(T - T_p)L^{1/\nu}$. Within this correspondence, the measured value $\phi \approx 0.416$ yields an effective exponent $\nu \approx 0.80$. However, for the hardware-inspired architectures such a definition becomes ambiguous due to their heterogeneous and anisotropic connectivity structure. Within this interpretation, the extracted exponents should therefore be regarded as effective geometric scaling exponents characterizing the finite-size organization of the FKCK correlated cluster structure rather than conventional thermodynamic critical exponents. Given the limited range of system sizes analyzed, these values may slightly shift with access to larger systems, especially for the most connected and heterogeneous networks, such as Zephyr.

B. Relaxation analysis

The relaxation behavior is characterized by examining the decay of the spin-spin autocorrelation function defined in Eq. (6) for each network topology. Simulations were performed using classical single-spin flip Metropolis Monte Carlo dynamics with random sequential updates. For each temperature, $N_{\text{conf}} = 50$ independent disorder realizations were done. Each simulation consisted of 10^6 Monte Carlo steps per spin (MCS), with observables measured every MCS.

Relaxation in spin glass systems has been extensively investigated. Analytical work by Randeria *et al.* [28] demonstrated that, even above the spin glass transition, the decay of correlations under local spin-flip dynamics cannot be purely exponential, revealing an intrinsic constraint that reflects the presence of large, nearly unfrustrated spin clusters dominating the long-time dynamics. Complementary numerical studies by Ogielski [30] showed that this slow relaxation is well captured by empirical, temperature-dependent forms of the type

$$C(t) = c t^{-x} \exp(-\omega t^\beta), \quad (10)$$

where all parameters c , x , ω , and β vary with temperature.

Figure 4 shows the relaxation of the spin-spin autocorrelation function for a 3D network at different system sizes. The simulation time was measured in units of single-spin flips ($\delta t = 1$) rather than full Monte Carlo sweeps, so that

one unit of time corresponds to the attempted update of a single spin instead of all N spins. This definition provides a much finer temporal resolution and allows us to capture early-time relaxation processes, which is particularly useful for highly connected networks where correlations evolve rapidly. To compare different system sizes, the horizontal axis was rescaled as t/N since each Monte Carlo sweep would correspond, on average, to N single-spin updates. With this rescaling, the relaxation curves for different N collapse onto a single master curve.

At very short times, only a few spins have flipped and the system remains close to its initial configuration; correlations then decay slowly because relaxation is limited to local rearrangements that do not yet involve collective modes. The physically relevant behavior emerges in the long-time tails of the autocorrelation, where collective rearrangements driven by frustration and disorder dominate. For $T < T_{\text{SG}}$, the relaxation follows a power-law behavior, for $T \lesssim T_C$ it is well described by Eq. (10), and for $T > T_C$ it reduces to a simple exponential decay.

Given the relaxation behavior identified in Fig. 4, we extended the analysis to all network topologies using the largest available system sizes to test whether similar scaling trends persist across architectures. Figure 5 displays the relaxation dynamics at different temperatures for all of the graphs. The autocorrelation function [Eq. (6)] is rescaled in the form $\ln[-\ln(C(t)/C(0))]$, so that if $C(t) \propto \exp[-(t/\tau)^\beta]$, the data should collapse onto a straight line with slope β . As the temperature increases, the slope approaches $\beta = 1$, corresponding to a simple exponential decay. In intermediate regimes, the relaxation is better described by a stretched-exponential form with an algebraic prefactor as in Eq. (10), consistent with the behavior reported by Ogielski [30].

Accurately extracting the exponents β and x from this form is challenging because the fitting function involves four free parameters: x , β , τ , and the prefactor amplitude c . These parameters are not independent, and significant correlations arise between them during the optimization process. In particular, variations in β can be partially compensated by changes in c .

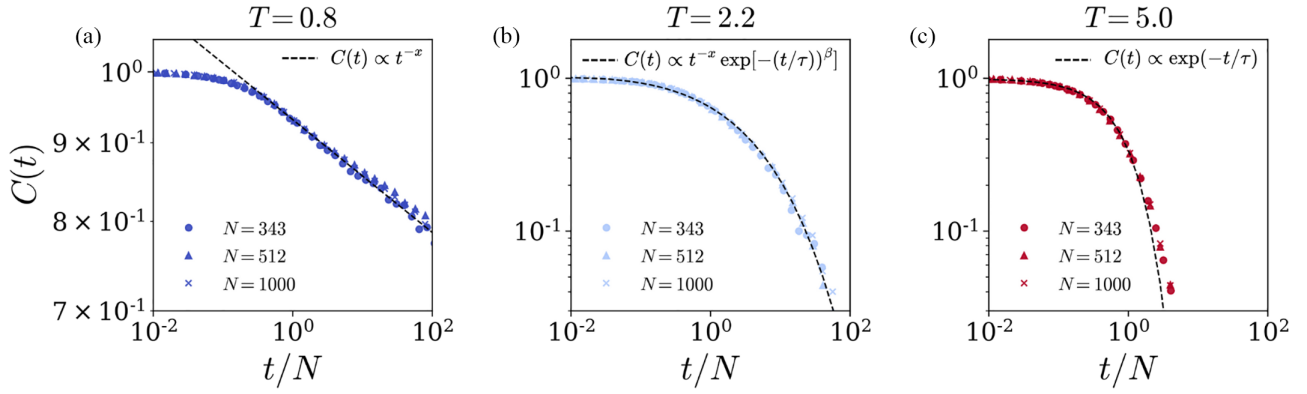


FIG. 4. Log-log plots of the spin-spin autocorrelation function [Eq. (6)] for different system sizes on a 3D network. (a) $T = 0.8$, spin glass phase. (b) $T = 2.2$, nonexponential relaxation regime. (c) $T = 5.0$, paramagnetic phase.

Notably, above the percolation temperature T_p , the behavior observed in Fig. 5 gradually approaches a straight line, highlighting a progressive change in the spin-spin autocorrelations from a power-law decay to a simple exponential form. While this does not correspond to an abrupt transition, it clearly signals a smooth crossover in the relaxation dynamics. As the temperature increases beyond T_p , the influence of frustrated regions diminishes, and the dynamics become

dominated by more uniform, unfrustrated spin domains. By the time the system approaches the critical temperature T_C , the curves nearly collapse onto each other, exhibiting almost identical behavior. This progressive crossover, rather than a sharp transition, is consistent with theoretical expectations: The effects of frustration and quenched disorder gradually lose relevance, and the system enters a regime characterized by simple paramagnetic relaxation, well captured by a single

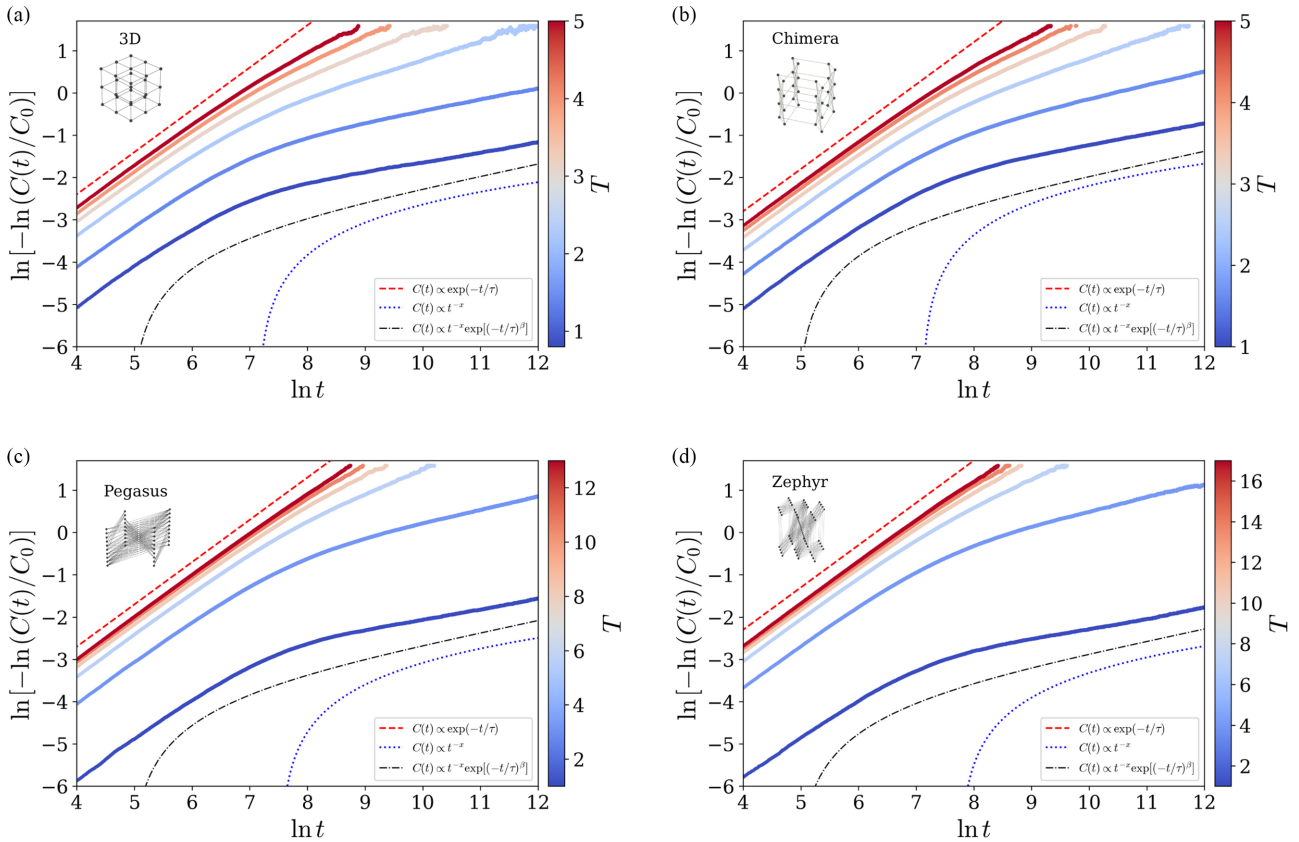


FIG. 5. Spin-spin autocorrelations [Eq. (6)] for different network topologies: (a) 3D lattice with $N = 1000$ and $T_p = 2.84$, (b) Chimera network with $N = 1568$ and $T_p = 2.27$, (c) Pegasus network with $N = 1664$ and $T_p = 7.18$, and (d) Zephyr network with $N = 1248$ and $T_p = 9.78$. Here, t corresponds to a single-spin Monte Carlo flip attempt rather than a full-lattice update, in order to obtain higher temporal resolution. The reference power-law curves were obtained from fits to the lowest-temperature data, yielding exponents $x_{3D} = 0.067$, $x_{Chimera} = 0.104$, $x_{Pegasus} = 0.050$, and $x_{Zephyr} = 0.040$.

exponential decay. This trend is consistently observed across all the networks studied.

In Chimera, Pegasus, and Zephyr graphs, although no true spin glass transition is reported in the thermodynamic limit, the low-temperature dynamics exhibit spin-spin autocorrelations remarkably similar to those observed in conventional spin glass systems. For example, the darkest blue line in panel (A) of Fig. 5, corresponding to the 3D case, is located at $T = 0.8$, where this graph is still in a spin glass phase (see Table I). In Ref. [54], Jaumà *et al.* showed that, although Chimera, Pegasus, and Zephyr graphs do not undergo a true spin glass transition in the thermodynamic limit, a pseudocritical temperature can still be defined for finite systems. This pseudocritical temperature is obtained from the Binder cumulant, which gradually rises from zero in the paramagnetic regime to a finite value in the spin glass regime, with the maximum of its derivative indicating the pseudocritical temperature. However, it does not scale consistently with system size. For the same sizes we studied here, they reported $T_{SC}^{\text{Chimera}} = 0.904$, $T_{SC}^{\text{Pegasus}} = 2.130$, and $T_{SC}^{\text{Zephyr}} = 2.661$.

For the EA spin glasses studied in this work, the difference between the disorder-driven crossover temperature T_C and the percolation temperature T_p is significantly larger than in the case of a binary ($\pm J$) spin glass with equal probability couplings. For example, in a binary 3D graph, $T_p \approx 3.95$. This is expected because the Gaussian distribution of couplings introduces a broader range of interaction strengths, increasing the effective disorder in the system. Consequently, energy barriers and local minima in the landscape are more heterogeneous, which delays the onset of uniform relaxation and shifts T_C to higher values relative to T_p .

V. DISCUSSION

The results show that the topology of the interaction network strongly influences the energy landscape of spin glass systems. Different graphs—3D, Chimera, Pegasus, and Zephyr—display distinct characteristic temperatures (T_{SG} , T_p , T_C), highlighting how connectivity controls cluster formation, the spatial distribution of frustration, and the overall ruggedness of the landscape. More highly connected networks generally exhibit higher T_p and T_C . Below T_C , the dynamics are dominated by slow relaxation due to the combined effects of frustration, nearly unfrustrated regions, and disorder, whereas above T_C these effects are largely suppressed, leading to more uniform relaxation. Networks with higher T_p and T_C , such as Pegasus and Zephyr, are particularly advantageous for quantum annealing, since device noise and finite operating temperatures can hinder exploration of the spin glass regime in weakly connected architectures.

From the perspective of quantum annealing at finite temperature, our results provide a geometric interpretation of the onset of frustration-dominated dynamics. The percolation temperature T_p marks the formation of a system-spanning FKCK cluster, where correlations extend across all length scales and collective rearrangements necessarily involve frus-

trated regions of the interaction graph. Above T_p , thermal fluctuations fragment this percolating structure, suppressing long-range frustration propagation and restoring predominantly local relaxation. Since T_p is explicitly topology dependent, it naturally serves as a connectivity-resolved metric that quantifies how resilient a given hardware graph is to thermally induced simplification of the energy landscape.

Understanding how topology shapes the energy landscape is therefore essential from both technical and thermodynamic perspectives. Observables derived from percolation frameworks, such as the weighted APL, provide complementary information about how strongly correlated and frustration-mediated interactions organize and propagate across the graph, offering additional insight into the geometric structure underlying collective relaxation processes and the rugged energy landscapes that quantum annealing architectures are designed to explore.

Importantly, the absence of a conventional finite-temperature spin glass transition in these hardware-relevant graphs does not imply a trivial phase space. Instead, our results show that topology-driven frustration percolation can still generate regimes of slow, collective dynamics that persist at finite temperature and remain relevant for both classical and quantum annealing processes. This provides a concrete mechanism through which dynamically complex energy landscapes may emerge even when conventional equilibrium thermodynamic signatures are weak or absent.

Rugged landscapes with multiple metastable states serve as natural test beds for assessing the performance of quantum annealers and for identifying conditions under which a genuine quantum speedup may emerge. When optimization problems are mapped onto spin glass Hamiltonians, the structure of energy barriers and frustration patterns directly encodes the computational hardness of the instance. By relating these features to graph-dependent percolation scales, our framework may provide a useful baseline for understanding how different hardware topologies sustain complex collective behavior and nontrivial energy landscapes, and for guiding future studies of topology-dependent effects in quantum annealing performance.

ACKNOWLEDGMENTS

We thank Exacore HPC Uniandes for providing high-performance computing time. G.T. acknowledges support from Fondo de Investigaciones, Facultad de Ciencias, Universidad de los Andes INV-2023-176-2951. F.J.G.-R. acknowledges financial support from the Spanish Government via the Project No. PID2024-161371NB-C21 (MCIU/AEI/FEDER, EU). F.J.G.-R. thanks the Spanish Supercomputing Network (Red Española de Supercomputación, RES) for providing computational resources under Project No. FI-2026-1-0056.

DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

- [1] I. M. Georgescu, S. Ashhab, and F. Nori, Quantum simulation, *Rev. Mod. Phys.* **86**, 153 (2014).
- [2] A. Ekert and R. Jozsa, Quantum computation and Shor's factoring algorithm, *Rev. Mod. Phys.* **68**, 733 (1996).
- [3] K. Bharti, A. Cervera-Lierta, T. H. Kyaw, T. Haug, S. Alperin-Lea, A. Anand, M. Degroote, H. Heimonen, J. S. Kottmann, T. Menke, W.-K. Mok, S. Sim, L.-C. Kwek, and A. Aspuru-Guzik, Noisy intermediate-scale quantum algorithms, *Rev. Mod. Phys.* **94**, 015004 (2022).
- [4] S. McArdle, S. Endo, A. Aspuru-Guzik, S. C. Benjamin, and X. Yuan, Quantum computational chemistry, *Rev. Mod. Phys.* **92**, 015003 (2020).
- [5] T. Kadowaki and H. Nishimori, Quantum annealing in the transverse Ising model, *Phys. Rev. E* **58**, 5355 (1998).
- [6] D. Aharonov, W. van Dam, J. Kempe, Z. Landau, S. Lloyd, and O. Regev, Adiabatic quantum computation is equivalent to standard quantum computation, in *45th Annual IEEE Symposium on Foundations of Computer Science* (IEEE Computer Society Press, Los Alamitos, CA, 2004), pp. 42–51.
- [7] T. Albash and D. A. Lidar, Adiabatic quantum computation, *Rev. Mod. Phys.* **90**, 015002 (2018).
- [8] K. Boothby, P. Bunyk, J. Raymond, and A. Roy, Next-generation topology of D-Wave quantum processors, Technical Report, D-Wave Systems Inc., 2019.
- [9] K. Boothby, A. D. King, and J. Raymond, Zephyr topology of D-Wave quantum processors, Technical Report, D-Wave Systems Inc., 2021.
- [10] E. Pelofske, Comparing three generations of D-Wave quantum annealers for minor embedded combinatorial optimization problems, *Quantum Sci. Technol.* **10**, 025025 (2025).
- [11] A. D. King *et al.*, Beyond-classical computation in quantum simulation, *Science* **388**, 199 (2025).
- [12] A. D. King *et al.*, Quantum critical dynamics in a 5,000-qubit programmable spin glass, *Nature (London)* **617**, 61 (2023).
- [13] A. D. King *et al.*, Coherent quantum annealing in a programmable 2,000 qubit Ising chain, *Nat. Phys.* **18**, 1324 (2022).
- [14] Y. Bando, Y. Susa, H. Oshiyama, N. Shibata, M. Ohzeki, F. J. Gómez-Ruiz, D. A. Lidar, S. Suzuki, A. del Campo, and H. Nishimori, Probing the universality of topological defect formation in a quantum annealer: Kibble-Zurek mechanism and beyond, *Phys. Rev. Res.* **2**, 033369 (2020).
- [15] W. H. Zurek, Cosmological experiments in superfluid helium? *Nature (London)* **317**, 505 (1985); Cosmic strings in laboratory superfluids and the topological remnants of other phase transitions, *Acta Phys. Pol. B* **24**, 1301 (1993); Cosmological experiments in condensed matter systems, *Phys. Rep.* **276**, 177 (1996).
- [16] T. W. B. Kibble, Topology of cosmic domains and strings, *J. Phys. A: Math. Gen.* **9**, 1387 (1976); Some implications of a cosmological phase transition, *Phys. Rep.* **67**, 183 (1980).
- [17] A. del Campo, T. W. B. Kibble, and W. H. Zurek, Causality and non-equilibrium second-order phase transitions in inhomogeneous systems, *J. Phys.: Condens. Matter* **25**, 404210 (2013).
- [18] F. J. Gómez-Ruiz, J. J. Mayo, and A. del Campo, Full counting statistics of topological defects after crossing a phase transition, *Phys. Rev. Lett.* **124**, 240602 (2020).
- [19] G. E. Santoro, R. Martoňák, E. Tosatti, and R. Car, Theory of quantum annealing of an Ising spin glass, *Science* **295**, 2427 (2002).
- [20] D. Chowdhury, *Spin Glasses and Other Frustrated Systems* (Co-Published with Princeton University Press, Princeton, NJ and World Scientific Publishing Co. Pte. Ltd., Singapore, 1986).
- [21] S. Knysh, Zero-temperature quantum annealing bottlenecks in the spin-glass phase, *Nat. Commun.* **7**, 12370 (2016).
- [22] A. Grabarits, K. R. Swain, M. S. Heydari, P. Chandarana, F. J. Gómez-Ruiz, and A. del Campo, Quantum chaos in random Ising networks, *Phys. Rev. Res.* **7**, 013146 (2025).
- [23] M. Bernaschi, I. González-Adalid Pemartín, V. Martín-Mayor, and G. Parisi, The quantum transition of the two-dimensional Ising spin glass, *Nature (London)* **631**, 749 (2024).
- [24] B. Heim, T. F. Rønnow, S. V. Isakov, and M. Troyer, Quantum versus classical annealing of Ising spin glasses, *Science* **348**, 215 (2015).
- [25] D. Sherrington and S. Kirkpatrick, Solvable model of a spin-glass, *Phys. Rev. Lett.* **35**, 1792 (1975).
- [26] K. Gunnarsson, P. Svedlindh, P. Nordblad, L. Lundgren, H. Aruga, and A. Ito, Dynamics of an Ising spin-glass in the vicinity of the spin-glass temperature, *Phys. Rev. Lett.* **61**, 754 (1988).
- [27] R. R. Singh and S. Chakravarty, Critical behavior of an Ising spin-glass, *Phys. Rev. Lett.* **57**, 245 (1986).
- [28] M. Randeria, J. P. Sethna, and R. G. Palmer, Low-frequency relaxation in Ising spin-glasses, *Phys. Rev. Lett.* **54**, 1321 (1985).
- [29] G. Franzese and A. Coniglio, Precursor phenomena in frustrated systems, *Phys. Rev. E* **59**, 6409 (1999).
- [30] A. T. Ogielski, Dynamics of three-dimensional Ising spin glasses in thermal equilibrium, *Phys. Rev. B* **32**, 7384 (1985).
- [31] U. Pezzella and A. Coniglio, Spin glasses and frustrated percolation: A renormalization group approach, *Physica A* **237**, 353 (1997).
- [32] G. Franzese and A. Coniglio, Phase transitions in the Potts spin-glass model, *Phys. Rev. E* **58**, 2753 (1998).
- [33] S. C. Glotzer, N. Jan, and P. H. Poole, The potential energy landscape of the $\pm J$ Ising spin glass, *J. Phys.: Condens. Matter* **12**, 6675 (2000).
- [34] S. Sastry, P. G. Debenedetti, and F. H. Stillinger, Signatures of distinct dynamical regimes in the energy landscape of a glass-forming liquid, *Nature (London)* **393**, 554 (1998).
- [35] H. Jónsson and H. C. Andersen, Icosahedral ordering in the Lennard-Jones liquid and glass, *Phys. Rev. Lett.* **60**, 2295 (1988).
- [36] T. Albash, V. Martin-Mayor, and I. Hen, Temperature scaling law for quantum annealing optimizers, *Phys. Rev. Lett.* **119**, 110502 (2017).
- [37] A. Abbas *et al.*, Challenges and opportunities in quantum optimization, *Nat. Rev. Phys.* **6**, 718 (2024).
- [38] E. J. Crosson and D. A. Lidar, Prospects for quantum enhancement with diabatic quantum annealing, *Nat. Rev. Phys.* **3**, 466 (2021).
- [39] V. Mehta, H. De Raedt, K. Michielsen, and F. Jin, Understanding the physics of D-Wave annealers: From Schrödinger to Lindblad to Markovian dynamics, *Phys. Rev. A* **112**, 032616 (2025).
- [40] H. Munoz-Bauza and D. Lidar, Scaling advantage in approximate optimization with quantum annealing, *Phys. Rev. Lett.* **134**, 160601 (2025).
- [41] A. Rajak, S. Suzuki, A. Dutta, and B. K. Chakrabarti, Quantum annealing: An overview, *Philos. Trans. R. Soc. A* **381**, 20210417 (2023).

- [42] S. Kirkpatrick Jr., C. D. Gelatt Jr., and M. P. Vecchi, Optimization by simulated annealing, *Science* **220**, 671 (1983).
- [43] H. G. Katzgraber, F. Hamze, Z. Zhu, A. J. Ochoa, and H. Munoz-Bauza, Seeking quantum speedup through spin glasses: The good, the bad, and the ugly, *Phys. Rev. X* **5**, 031026 (2015).
- [44] H. G. Katzgraber, F. Hamze, and R. S. Andrist, Glassy chimeras could be blind to quantum speedup: Designing better benchmarks for quantum annealing machines, *Phys. Rev. X* **4**, 021008 (2014).
- [45] C. M. Fortuin and P. W. Kasteleyn, On the random-cluster model: I. Introduction and relation to other models, *Physica* **57**, 536 (1972).
- [46] V. Gómez and G. Téllez, Self-organized critical dynamic on the Sierpinski carpet, *Phys. Rev. E* **110**, 064141 (2024).
- [47] A. Coniglio, F. di Liberto, G. Monroy, and F. Peruggi, Cluster approach to spin glasses and the frustrated-percolation problem, *Phys. Rev. B* **44**, 12605 (1991).
- [48] A. de Candia, V. Cataudella, and A. Coniglio, Cluster formulation of spin glasses and the frustrated percolation model: Statics and dynamics, *J. Phys. A: Math. Gen.* **32**, 4817 (1999).
- [49] L. Münster and M. Weigel, Spin glasses and percolation, *Front. Phys.* **12**, 1448175 (2024).
- [50] L. Münster and M. Weigel, Cluster percolation in the two-dimensional Ising spin glass, *Phys. Rev. E* **107**, 054103 (2023).
- [51] L. de Arcangelis, A. Coniglio, and F. Peruggi, Percolation transition in spin glasses, *Europhys. Lett.* **14**, 515 (1991).
- [52] H. G. Katzgraber, M. Körner, and A. P. Young, Universality in three-dimensional Ising spin glasses: A Monte Carlo study, *Phys. Rev. B* **73**, 224432 (2006).
- [53] A. M. Ferrenberg, J. Xu, and D. P. Landau, Pushing the limits of Monte Carlo simulations for the three-dimensional Ising model, *Phys. Rev. E* **97**, 043301 (2018).
- [54] G. Jaumà, J. J. García-Ripoll, and M. Pino, Exploring quantum annealing architectures: A spin glass perspective, *Adv. Quantum Technol.* **7**, 2300245 (2024).
- [55] J. Rudnick, G. Gaspari, and V. Privman, Effect of boundary conditions on the critical behavior of a finite high-dimensional Ising model, *Phys. Rev. B* **32**, 7594 (1985).
- [56] A. P. Young, Violations of hyperscaling in finite-size scaling above the upper critical dimension, *Entropy* **26**, 509 (2024).