

Implementing a nonpairwise three-body interaction of superconducting qubits and its applications in realizing three-qubit entanglement and quantum gates

Tong Liu ^{*}

*Quantum Information Research Center and Jiangxi Province Key Laboratory of Applied Optical Technology,
Shangrao Normal University, Shangrao 334001, China*



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Nonpairwise multibody interactions have recently attracted much attention due to their promising applications in the realm of quantum information science and technology. In this work, we propose a method to synthesize a nonpairwise three-body interaction of superconducting qubits using a coupler qubit dispersively coupled to two spatially separated qubits and a microwave pulse. This interaction describes physical processes of an excitation from the coupler qubit to two spatially separated qubits and the reverse process. We show that the interaction can be utilized to deterministically generate three-body Greenberger-Horne-Zeilinger (GHZ) entangled states, transfer the state of the coupler qubit into a two-qubit entangled state, and construct three-qubit controlled-controlled-phase and Toffoli gates. Our proposal only requires a coupler qubit and a single-step operation; thus, the system architecture and method are quite simple. As examples, we numerically simulate the dynamics of the system, and the numerical results demonstrate that high-fidelity preparation of the proposed three-body GHZ entangled state and quantum gates is feasible with current superconducting circuit technology. This approach is general and applicable to a wide range of quantum systems.

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I. INTRODUCTION

The interactions of many-body quantum systems play essential roles in quantum computing, quantum information processing, and quantum simulation [1]. Up to now, the majority of quantum tasks have relied on pairwise interactions that have been realized in various quantum physical systems. Recently, nonpairwise multibody interactions have attracted considerable interest. Unpairwise multibody interactions refer to interactions that involve multiple particles without pairwise interactions between individual particles. These interactions play a significant role in quantum information processing and quantum computing. They could enable the performance of more complex tasks, such as generating multipartite entanglement and constructing multiqubit quantum gates [2–4]. In recent years, a number of theoretical proposals have been presented for synthesizing the unpairwise multibody interactions [2–9]. For example, Refs. [2,3] have realized a three-body interaction between a single cavity or atom and two atoms in the ultrastrong-coupling regime. This interaction demonstrates that one single photon or atom can simultaneously excite two atoms. Reference [6] has shown that the three-qubit interaction proposed by Ref. [3] can also be synthesized without the ultrastrong coupling assisted by a cavity coupled to three qutrits. Motivated by the theoretical work [2], the implementation of a three-body interaction between a

superconducting cavity and two superconducting qubits in the ultrastrong-coupling regime has been experimentally reported [10]. In experiments, a three-, four-, and five-body interaction between a qudit and qubits has been synthesized in superconducting circuits [11,12]. In addition, multibody interactions between spins have been implemented by theoretical and experimental approaches [13–16].

On the other hand, Greenberger-Horne-Zeilinger (GHZ) entangled states are pivotal in a range of applications, including quantum teleportation [17], quantum error correction [18], quantum communication [19], quantum metrology [20], and entanglement swapping [21], and violate the Bell inequality [22]. Over the years, numerous theoretical proposals have been put forward for generating GHZ states in different physical systems [23–40]. Experimentally, the preparation of GHZ states has been demonstrated with photonic qubits [41], nitrogen-vacancy (NV) centers [42], atomic qubits [43], ionic qubits [44], and superconducting circuits [45–48].

Recently, there has been significant focus on the investigation of the three-qubit controlled-controlled-phase (CCPHASE) gate and Toffoli gate. The CCPHASE gate and Toffoli gate play crucial roles in various applications such as quantum algorithms [49,50], quantum error correction [51–53], quantum cloning [54], elementary arithmetic [55], quantum Fourier transform [56,57], and quantum entanglement preparation [58], among others. To date, CCPHASE gates and Toffoli gates have been constructed in various physical platforms, including nuclear magnetic resonance [52], linear optics [59], trapped ions [53,60], neutral atoms [61], quantum dots [62], and silicon spin qubits [63]. Moreover, numerous theoretical and experimental approaches have been proposed for realizing CCPHASE gates and Toffoli gates in superconducting circuits [64–70].

^{*}Contact author: liutong7500755@163.com

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Superconducting circuits have been considered as one of the promising physical platforms for studying and realizing many-body interactions due to their scalability and tunability [71–74]. In this work, we propose an approach to realize a nonpairwise three-body interaction among three superconducting qubits (1, c , 2), in which qubits 1 and 2 each couple with a center coupler qubit c . This system has been extensively studied both theoretically and experimentally recently [75–81]. For instance, the strong coupling has been experimentally reported between a superconducting transmon qubit and two transmon qubits [75,80], between a superconducting Xmon qubit and two Xmon qubits [78], between a superconducting transmon qubit and two Xmon qubits [79], and between a superconducting transmon qubit and two fluxonium qubits [81]. Moreover, the strong coupling between a superconducting flux qubit and two flux qubits has been experimentally demonstrated previously [82]. While three-body ZZZ interactions have been proposed in such circuit system [13–15], the realization of nonpairwise exchange interactions between three superconducting qubits has not been put forward. This proposal has the following advantages and features: (1) This three-body interaction can be synthesized using only a single-step operation. (2) The circuit system's architecture is compact and simple because only a single couple is required. (3) The three-body interaction can be described as a physical process involving an excitation from the coupler qubit to two spatially separated qubits. It allows the transfer of the state of the coupler qubit into a two-qubit entangled state and enables the generation of three-qubit GHZ states. (4) The three-body interaction can be used to direct implementation of CCPHASE and Toffoli gates, which can significantly reduce the complications and overhead. (5) This proposal is quite general and can be applied to other solid-state devices.

The paper is organized as follows. In Sec. II, we introduce the system and give a derivation of the nonpairwise three-body interaction Hamiltonian between three superconducting qubits. In Sec. III, we show that this three-body interaction Hamiltonian can be utilized to realize the state transfer, three-qubit entanglement, controlled-controlled-phase gates, and Toffoli gates. In Sec. IV, we investigate the experimental feasibility of our proposal. A brief concluding summary is given in Sec. V.

II. MODEL AND HAMILTONIAN

As schematically shown in Fig. 1, we consider a system consisting of three superconducting qubits (1, 2, c), where qubits 1 and 2 are dispersively coupled to qubit c (coupler), which is driven by a microwave pulse. The ground state and the excited state of qubit j are labeled as $|g\rangle_j$ and $|e\rangle_j$ ($j = 1, 2, c$).

In the interaction picture and after making the rotating-wave approximation (RWA), the total Hamiltonian of the system is given by (hereafter $\hbar = 1$)

$$H_I = g_1(e^{i\delta_1 t}\sigma_c^+\sigma_1^- + e^{-i\delta_1 t}\sigma_c^-\sigma_1^+) + g_2(e^{i\delta_2 t}\sigma_2^+\sigma_c^- + e^{-i\delta_2 t}\sigma_2^-\sigma_c^+) + \Omega[\sigma_c^+e^{i(\delta_p t - \phi)} + \sigma_c^-e^{-i(\delta_p t - \phi)}], \quad (1)$$

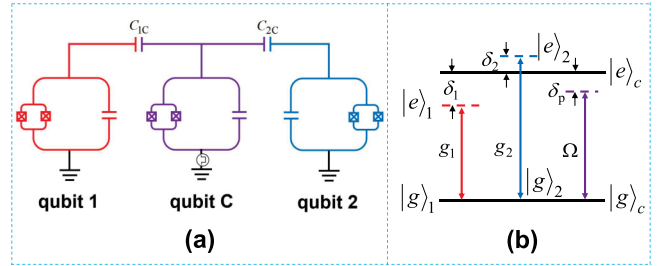


FIG. 1. (a) Schematic diagram of the setup for superconducting qubits 1 and 2 coupled to a coupler qubit c via capacitors C_{1c} and C_{2c} . (b) Diagram of the energy levels of the three qubits. Coupler c is dispersively coupled to qubit 1 (2) with coupling strengths g_1 (g_2) and detuning δ_1 (δ_2). Meanwhile, the coupler is dispersively coupled to a microwave pulse with Rabi frequency Ω and detuning δ_p .

where $\sigma_j^- = |g\rangle_j\langle e|$ and $\sigma_j^+ = |e\rangle_j\langle g|$ are the lowering and raising operators of qubit j with transition frequency ω_{qj} ($j = 1, 2, c$); g_1 (g_2) is the coupling strength between qubit 1 (2) and coupler qubit c ; $\delta_1 = \omega_{qc} - \omega_{q1} > 0$, $\delta_2 = \omega_{q2} - \omega_{qc} > 0$, and $\delta_p = \omega_{qc} - \omega_p > 0$ are the frequency detunings; and ω_p and ϕ are the Rabi frequency and the initial phase of the microwave pulse.

Under the large detuning conditions $\delta_1 \gg g_1$, $\delta_2 \gg g_2$, and $\delta_p \gg \Omega$, one can obtain the effective Hamiltonian [83–85]

$$H_{\text{eff}} \approx H_{\text{eff}}^{(2)} + H_{\text{eff}}^{(3)}, \quad (2)$$

with

$$H_{\text{eff}}^{(2)} = \frac{1}{i}H_I(t) \int^t H_I(t')dt', \quad (3)$$

$$H_{\text{eff}}^{(3)} = -H_I(t) \int^t H_I(t_1) \int^{t_1} H_I(t_2)dt_2dt_1, \quad (4)$$

where $H_{\text{eff}}^{(2)}$ and $H_{\text{eff}}^{(3)}$ are the second-order and third-order effective Hamiltonians, respectively. Because the coefficients of higher-order effective Hamiltonians are small, here the higher-order effective Hamiltonians are neglected.

By substituting the Hamiltonian H_I into formula (3) and neglecting rapidly oscillating terms through the RWA, we can obtain the second-order effective Hamiltonian (see Appendix) [83–85]

$$H_{\text{eff}}^{(2)} = \lambda_1(\sigma_c^{ee}\sigma_1^{gg} - \sigma_c^{gg}\sigma_1^{ee}) + \lambda_p\sigma_c^z + \lambda_2(\sigma_c^{gg}\sigma_2^{ee} - \sigma_c^{ee}\sigma_2^{gg}), \quad (5)$$

where $\lambda_1 = \frac{g_1^2}{\delta_1}$, $\lambda_2 = \frac{g_2^2}{\delta_2}$, $\lambda_p = \frac{\Omega^2}{\delta_p}$, $\sigma_j^{gg} = |g\rangle_j\langle g|$, $\sigma_j^{ee} = |e\rangle_j\langle e|$ ($j = 1, 2, c$), and $\sigma_c^z = |e\rangle_c\langle e| - |g\rangle_c\langle g|$.

Under the large detuning conditions, we assume that $\delta_1 \gg \delta_p > \delta_2$ and that $\delta_1 \approx \delta_p + \delta_2$ is satisfied. Thus, one can obtain the third-order effective Hamiltonian (see Appendix) [83–85]

$$H_{\text{eff}}^{(3)} = \chi[e^{i(\Delta t + \phi)}\sigma_c^+\sigma_1^-\sigma_2^- + e^{i(\Delta t + \phi)}\sigma_c^-\sigma_1^+\sigma_2^+], \quad (6)$$

where $\chi = g_1g_2\Omega[\frac{1}{\delta_2(\delta_2 + \delta_p)} + \frac{1}{\delta_1(\delta_1 - \delta_p)}]$ and $\Delta = \delta_1 - \delta_2 - \delta_p$.

Accordingly, the effective Hamiltonian (2) can be written as

$$H_{\text{eff}} \approx H_0 + H_I, \quad (7)$$

with

$$H_0 = \lambda_1(\sigma_c^{ee}\sigma_1^{gg} - \sigma_c^{gg}\sigma_1^{ee}) + \lambda_p\sigma_c^z + \lambda_2(\sigma_c^{gg}\sigma_2^{ee} - \sigma_c^{ee}\sigma_2^{gg}), \quad (8)$$

$$H_i = \chi[e^{i(\Delta t + \phi)}\sigma_c^+\sigma_1^-\sigma_2^- + e^{i(\Delta t + \phi)}\sigma_c^-\sigma_1^+\sigma_2^+]. \quad (9)$$

The Hamiltonian H_i represents a nonpairwise three-body interaction, which shows that the superconducting coupler qubit (c) is coupled to two spatially separated qubits (1 and 2) simultaneously with the trilinear coupling strength χ . In the next section, we discuss how to use the Hamiltonian (7) to implement excitation and state transfer, three-qubit entanglement, controlled-controlled-phase gates, and Toffoli gates.

III. APPLICATIONS

We now show that the three-body interaction Hamiltonian (7) can be employed to describe physical processes of a coupler qubit can simultaneously excite two spatially separated qubits and the reverse process. The Hamiltonian (7) also enables implementation of three-qubit GHZ entangled states, transfer of the coupler qubit's state into a two-qubit entangled state, and construction of three-qubit CPHASE and Toffoli gates.

A. Simultaneous excitation of two qubits by a coupler qubit

Turning to the interaction picture with respect to H_0 , the Hamiltonian therefore reads

$$H'_{\text{eff}} = e^{iH_0 t} H_i e^{-iH_0 t} = \chi[e^{i(\varphi t + \phi)}\sigma_c^+\sigma_1^-\sigma_2^- + e^{-i(\varphi t + \phi)}\sigma_c^-\sigma_1^+\sigma_2^+], \quad (10)$$

where $\varphi = 2(\lambda_1 + \lambda_p - \lambda_2) + \Delta$. We set $\varphi = 0$, i.e.,

$$2\lambda_1 + 2\lambda_p + \Delta = 2\lambda_2; \quad (11)$$

thus, the Hamiltonian (10) reduces to

$$H'_{\text{eff}} = \chi(e^{i\phi}\sigma_c^+\sigma_1^-\sigma_2^- + e^{-i\phi}\sigma_c^-\sigma_1^+\sigma_2^+). \quad (12)$$

It is obvious that this Hamiltonian is a standard three-body interaction Hamiltonian for $\phi = 2k\pi$ (k is an integer).

Under the Hamiltonian (12), one can obtain state evolution

$$|e\rangle_c |g\rangle_1 |g\rangle_2 \rightarrow \cos(\chi t) |e\rangle_c |g\rangle_1 |g\rangle_2 - ie^{-i\phi} \sin(\chi t) |g\rangle_c |e\rangle_1 |e\rangle_2, \quad (13)$$

$$|g\rangle_c |e\rangle_1 |e\rangle_2 \rightarrow \cos(\chi t) |g\rangle_c |e\rangle_1 |e\rangle_2 - ie^{i\phi} \sin(\chi t) |e\rangle_c |g\rangle_1 |g\rangle_2. \quad (14)$$

When the evolution time $t = \pi/|2\chi|$ and the phase $\phi = 3\pi/2$, one can obtain a transition $|e\rangle_c |g\rangle_1 |g\rangle_2 \rightarrow |g\rangle_c |e\rangle_1 |e\rangle_2$. This indicates that the coupler qubit makes a transition from the excite state $|e\rangle_c$ to the ground state $|g\rangle_c$ while qubits 1 and 2 are excited, resulting in the transition $|g\rangle_1 |g\rangle_2 \rightarrow |e\rangle_1 |e\rangle_2$. Thus, one can see that the coupler qubit can simultaneously excite qubits 1 and 2. This process is reversible; Eq. (14) shows for $t = \pi/|2\chi|$ and $\phi = \pi/2$, qubits 1 and 2 undergo a transition $|e\rangle_1 |e\rangle_2 \rightarrow |g\rangle_1 |g\rangle_2$ while the coupler is excited from the ground state $|g\rangle_c$ to excite state $|e\rangle_c$. The three-body interaction Hamiltonian (12) describes physical processes of

an excitation from the coupler qubit to two spatially separated qubits and the reverse process.

B. Transfer the state of coupler qubit into a two-qubit entangled state of separated qubits

We suppose that the coupler qubit is initially in an arbitrary superposition state $\alpha|g\rangle_c + \beta|e\rangle_c$. This can be achieved by applying a microwave pulse resonant to the coupler when the coupler is in state $|g\rangle_c$. In addition, we assume that qubits 1 and 2 are initially in the ground states $|g\rangle_1$ and $|g\rangle_2$, respectively. Based on Eq. (13), one has $|e\rangle_c |g\rangle_1 |g\rangle_2 \rightarrow |g\rangle_c |e\rangle_1 |e\rangle_2$ for $\chi t = \pi/2$ and $\phi = 3\pi/2$. Accordingly, one can obtain

$$(\alpha|g\rangle_c + \beta|e\rangle_c) |g\rangle_1 |g\rangle_2 \rightarrow |g\rangle_c (\alpha|g\rangle_1 |g\rangle_2 + \beta|e\rangle_1 |e\rangle_2), \quad (15)$$

which implies that an arbitrary state of coupler qubit is fully transferred to a two-qubit entangled state of qubits 1 and 2. This operation has valuable applications in quantum repetition codes, which are used for error correction [3].

C. Generation of Greenberger-Horne-Zeilinger entangled states

Assume that the coupler qubit is initially in the excited state $|e\rangle_c$, which can be realized by driving a microwave π pulse resonant to the coupler in the state $|g\rangle_c$. In addition, we suppose that qubits 1 and 2 are initially in the ground states $|g\rangle_1$ and $|g\rangle_2$, respectively. Thus, the initial state of the whole system is $|e\rangle_c |g\rangle_1 |g\rangle_2$. Based on Eq. (13), for $t = \pi/|4\chi|$ and $\phi = 3\pi/2$, one can prepare a three-qubit entangled GHZ state

$$\frac{1}{\sqrt{2}}(|e\rangle_c |g\rangle_1 |g\rangle_2 + |g\rangle_c |e\rangle_1 |e\rangle_2). \quad (16)$$

If we perform a rotation on coupler c that realizes the conversions $|g\rangle_c \rightarrow -i|e\rangle_c$ and $|e\rangle_c \rightarrow -i|g\rangle_c$, then one can obtain another type of three-qubit GHZ state

$$\frac{1}{\sqrt{2}}(|g\rangle_c |g\rangle_1 |g\rangle_2 + |e\rangle_c |e\rangle_1 |e\rangle_2), \quad (17)$$

where a common factor $-i$ is omitted. Here, the three-qubit GHZ states can be deterministically generated because there is no need to measure the states of the qubits. It is well known that GHZ entangled states have important applications in quantum information processing, quantum computation, and quantum communication [17–22].

D. Implementation of a three-qubit controlled-controlled-phase gate and Toffoli gate

We introduce an auxiliary energy level $|f\rangle_c$ of coupler qubit to realize a controlled-controlled-phase gate. Suppose that the coupler qubit, qubit 1, and qubit 2 are, respectively, prepared in superposition states $(\sin\theta_c|g\rangle_c + \cos\theta_c|f\rangle_c)$, $(\sin\theta_1|g\rangle_1 + \cos\theta_1|e\rangle_1)$, and $(\sin\theta_2|g\rangle_2 + \cos\theta_2|e\rangle_2)$, which can be achieved by applying a microwave pulse to each qubit.

We adjust the frequency of the coupler qubit such that its $|g\rangle_c \leftrightarrow |e\rangle_c$ transition is dispersively coupled to qubits 1 and 2, while the $|e\rangle_c \leftrightarrow |f\rangle_c$ transition is decoupled from both qubits 1 and 2. The effective Hamiltonian and the state evolution are given by Eqs. (12) and (14), respectively. In the eight computational basis states $\{|g\rangle_c |g\rangle_1 |g\rangle_2, |g\rangle_c |g\rangle_1 |e\rangle_2, |g\rangle_c |e\rangle_1 |g\rangle_2, |g\rangle_c |e\rangle_1 |e\rangle_2,$

$|f\rangle_c|g\rangle_1|g\rangle_2, |f\rangle_c|g\rangle_1|e\rangle_2, |f\rangle_c|e\rangle_1|g\rangle_2, |f\rangle_c|e\rangle_1|e\rangle_2\}$, according to Eq. (14), one can obtain the transition $|g\rangle_c|e\rangle_1|e\rangle_2 \rightarrow -|g\rangle_c|e\rangle_1|e\rangle_2$ for $t = \pi/|\chi|$. Moreover, under the Hamiltonian (12), one can easily find that the other states are unaffected, i.e., $e^{-iH_{\text{eff}}t}|k\rangle_c|l\rangle_1|m\rangle_2 \rightarrow |k\rangle_c|l\rangle_1|m\rangle_2$, where $|k\rangle_c|l\rangle_1|m\rangle_2 \in \{|g\rangle_c|g\rangle_1|g\rangle_2, |g\rangle_c|g\rangle_1|e\rangle_2, |g\rangle_c|e\rangle_1|g\rangle_2, |f\rangle_c|g\rangle_1|g\rangle_2, |f\rangle_c|g\rangle_1|e\rangle_2, |f\rangle_c|e\rangle_1|g\rangle_2, |f\rangle_c|e\rangle_1|e\rangle_2\}$. Thus, one has the following state transformation:

$$\begin{aligned} |g\rangle_c|g\rangle_1|g\rangle_2 &\rightarrow |g\rangle_c|g\rangle_1|g\rangle_2, & |g\rangle_c|g\rangle_1|e\rangle_2 &\rightarrow |g\rangle_c|g\rangle_1|e\rangle_2, \\ |g\rangle_c|e\rangle_1|g\rangle_2 &\rightarrow |g\rangle_c|e\rangle_1|g\rangle_2, & |g\rangle_c|e\rangle_1|e\rangle_2 &\rightarrow -|g\rangle_c|e\rangle_1|e\rangle_2, \\ |f\rangle_c|g\rangle_1|g\rangle_2 &\rightarrow |f\rangle_c|g\rangle_1|g\rangle_2, & |f\rangle_c|g\rangle_1|e\rangle_2 &\rightarrow |f\rangle_c|g\rangle_1|e\rangle_2, \\ |f\rangle_c|e\rangle_1|g\rangle_2 &\rightarrow |f\rangle_c|e\rangle_1|g\rangle_2, & |f\rangle_c|e\rangle_1|e\rangle_2 &\rightarrow |f\rangle_c|e\rangle_1|e\rangle_2, \end{aligned} \quad (18)$$

which results in a phase shift of π for the coupler qubit's state $|g\rangle_c$ only if qubits 1 and 2 are in the state $|e\rangle_1|e\rangle_2$. This means that a three-qubit CCPHASE gate is achieved using only a single-step operation. In such gate, the coupler qubit acts as a target qubit, while qubits 1 and 2 are treated as two control qubits. After this operation, we need to adjust the level configuration of the coupler qubit such that it is decoupled from qubits 1 and 2.

Now, we show the implementation of a three-qubit Toffoli gate. We apply a Hadamard gate operation to the coupler qubit such that $|g\rangle_c \rightarrow |+\rangle_c$ and $|f\rangle_c \rightarrow |-\rangle_c$, with $|+\rangle_c = (|g\rangle_c + |f\rangle_c)/\sqrt{2}$ and $|-\rangle_c = (|f\rangle_c - |g\rangle_c)/\sqrt{2}$. Thus, according to the initial state given above, one can obtain the state transformation

$$\begin{aligned} |g\rangle_c|g\rangle_1|g\rangle_2 &\rightarrow |+\rangle_c|g\rangle_1|g\rangle_2, & |g\rangle_c|g\rangle_1|e\rangle_2 &\rightarrow |+\rangle_c|g\rangle_1|e\rangle_2, \\ |g\rangle_c|e\rangle_1|g\rangle_2 &\rightarrow |+\rangle_c|e\rangle_1|g\rangle_2, & |g\rangle_c|e\rangle_1|e\rangle_2 &\rightarrow |+\rangle_c|e\rangle_1|e\rangle_2, \\ |f\rangle_c|g\rangle_1|g\rangle_2 &\rightarrow |-\rangle_c|g\rangle_1|g\rangle_2, & |f\rangle_c|g\rangle_1|e\rangle_2 &\rightarrow |-\rangle_c|g\rangle_1|e\rangle_2, \\ |f\rangle_c|e\rangle_1|g\rangle_2 &\rightarrow |-\rangle_c|e\rangle_1|g\rangle_2, & |f\rangle_c|e\rangle_1|e\rangle_2 &\rightarrow |-\rangle_c|e\rangle_1|e\rangle_2. \end{aligned} \quad (19)$$

Then, we perform a CCPHASE gate operation on qubits 1, 2, and c . The corresponding state transformation is given by Eq. (18). Therefore, from Eqs. (18) and (20), one can obtain state transformations $|+\rangle_c|e\rangle_1|e\rangle_2 \rightarrow |-\rangle_c|e\rangle_1|e\rangle_2$ and $|-\rangle_c|e\rangle_1|e\rangle_2 \rightarrow |+\rangle_c|e\rangle_1|e\rangle_2$, while the other states remain unchanged.

By performing another Hadamard gate operation on the coupler qubit, one can realize $|g\rangle_c \rightarrow \frac{1}{\sqrt{2}}(|g\rangle_c - |f\rangle_c)$ and $|f\rangle_c \rightarrow \frac{1}{\sqrt{2}}(|g\rangle_c + |f\rangle_c)$. Accordingly, one has $|+\rangle_c \rightarrow |g\rangle_c$ and $|-\rangle_c \rightarrow |f\rangle_c$. We can eventually obtain the following state transformation:

$$\begin{aligned} |g\rangle_c|g\rangle_1|g\rangle_2 &\rightarrow |g\rangle_c|g\rangle_1|g\rangle_2, & |g\rangle_c|g\rangle_1|e\rangle_2 &\rightarrow |g\rangle_c|g\rangle_1|e\rangle_2, \\ |g\rangle_c|e\rangle_1|g\rangle_2 &\rightarrow |g\rangle_c|e\rangle_1|g\rangle_2, & |g\rangle_c|e\rangle_1|e\rangle_2 &\rightarrow |f\rangle_c|e\rangle_1|e\rangle_2, \\ |f\rangle_c|g\rangle_1|g\rangle_2 &\rightarrow |f\rangle_c|g\rangle_1|g\rangle_2, & |f\rangle_c|g\rangle_1|e\rangle_2 &\rightarrow |f\rangle_c|g\rangle_1|e\rangle_2, \\ |f\rangle_c|e\rangle_1|g\rangle_2 &\rightarrow |f\rangle_c|e\rangle_1|g\rangle_2, & |f\rangle_c|e\rangle_1|e\rangle_2 &\rightarrow |g\rangle_c|e\rangle_1|e\rangle_2, \end{aligned} \quad (20)$$

which shows that if and only if control qubits 1 and 2 are in the state $|e\rangle_1|e\rangle_2$, the $|g\rangle_c$ and $|f\rangle_c$ states of the target qubit (coupler c) are swapped. Thus, a Toffoli gate is

implemented by using a CCPHASE gate and two Hadamard gates. The Toffoli and CCPHASE gates have many significant applications in quantum computing and quantum information processing [49–58]. Typically, the Toffoli or CCPHASE gate is constructed by decomposing it into a sequence of single- and two-qubit gates. It has been demonstrated that the simplest decomposition of the Toffoli gate requires at least five two-qubit gates [86]. Our proposal can significantly reduce the complexity and overhead compared to the conventional gate decomposition method.

Before ending this subsection, several points need to be addressed as follows:

(1) We assume that the coupler qubit's $|g\rangle_c \leftrightarrow |e\rangle_c$ ($|e\rangle_c \leftrightarrow |f\rangle_c$) transition can be coupled to (decoupled from) qubits 1 and 2; this coupling and decoupling process can be achieved by tuning the level spacings of the coupler qubit. For superconducting qubits, their level spacings can be rapidly adjusted within the range of 1–3 ns [71–74].

(2) For the realization of the Hadamard gate, one can use a microwave pulse resonant with the $|g\rangle_c \leftrightarrow |f\rangle_c$ transition of the coupler qubit. The corresponding Hamiltonian is described as

$$H_p = \Omega_p(e^{-i\phi_p}\sigma_{fg}^+ + e^{i\phi_p}\sigma_{fg}^-), \quad (21)$$

where Ω_p and ϕ_p are the Rabi frequency and the initial phase of the pulse, respectively. Under the Hamiltonian (21), the state evolution is given by

$$\begin{aligned} |g\rangle &\rightarrow \cos(\Omega_p t)|g\rangle - ie^{-i\phi_p}\sin(\Omega_p t)|f\rangle, \\ |f\rangle &\rightarrow \cos(\Omega_p t)|f\rangle - ie^{i\phi_p}\sin(\Omega_p t)|g\rangle. \end{aligned} \quad (22)$$

By choosing $t = \pi/(4\Omega_p)$ and $\phi_p = -\pi/2$ (or $\phi_p = \pi/2$), a fast Hadamard gate operation can be performed: $|g\rangle_c \rightarrow \frac{1}{\sqrt{2}}(|g\rangle_c + |f\rangle_c)$, $|f\rangle_c \rightarrow \frac{1}{\sqrt{2}}(|f\rangle_c - |g\rangle_c)$ [or $|g\rangle_c \rightarrow \frac{1}{\sqrt{2}}(|g\rangle_c - |f\rangle_c)$, $|f\rangle_c \rightarrow \frac{1}{\sqrt{2}}(|g\rangle_c + |f\rangle_c)$].

IV. EXPERIMENTAL FEASIBILITY

In this section, we investigate the experimental feasibility of our proposal by using three superconducting flux qubits [82,87–89]. We will first study the impact of qubits' decoherence on operation fidelity, and then investigate the effect of unwanted interactions induced by the coupler's $|f\rangle_c$ on fidelity.

A. Decoherence

When the decoherence of three qubits is considered, the dynamics of the lossy system is described by the Markovian master equation

$$\begin{aligned} \frac{d\rho}{dt} = & -i[H_I, \rho] + \sum_{j=1,2,c} (\gamma_j \mathcal{L}[\sigma_j^-] + \gamma_j^{fe} \mathcal{L}[\sigma_j^{fe-}] \\ & + \gamma_j^{fg} \mathcal{L}[\sigma_j^{fg-}] + \gamma_{\varphi_j} \mathcal{L}[\sigma_j^{ee}] + \gamma_{\varphi_j}^f \mathcal{L}[\sigma_j^{ff}]), \end{aligned} \quad (23)$$

where ρ is the density operator of the system; H_I is given by Eq. (1); $\mathcal{L}[\Lambda] = \Lambda\rho\Lambda^\dagger - \Lambda^\dagger\Lambda\rho/2 - \rho\Lambda^\dagger\Lambda/2$ with $\Lambda = \sigma_j^-$, σ_j^{fe-} , σ_j^{fg-} , σ_j^{ee} , σ_j^{ff} ; $\sigma_j^{fe-} = |e\rangle_j\langle f|$, $\sigma_j^{fg-} = |g\rangle_j\langle f|$, $\sigma_j^{ff} = |f\rangle_j\langle f|$ ($j = 1, 2, c$). In addition, γ_j , γ_j^{fe} ,

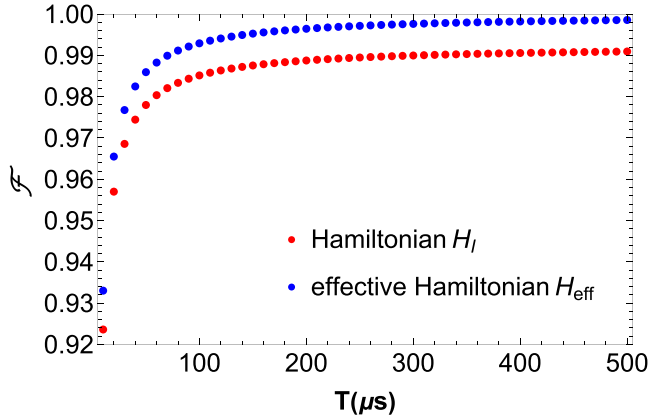


FIG. 2. Fidelity $\mathcal{F}$ vs T for the GHZ entangled state. The values of the parameters in the numerical simulations are given in the main text.

and γ_j^{fg} are, respectively, the energy relaxation rates of qubit j for the decay paths $|e\rangle_j \rightarrow |g\rangle_j$, $|f\rangle_j \rightarrow |e\rangle_j$, and $|f\rangle_j \rightarrow |g\rangle_j$; γ_{φ_j} and $\gamma_{\varphi_j}^f$ are, respectively, the dephasing rates of the states $|e\rangle_j$ and $|f\rangle_j$ of qubit j ($j = 1, 2, c$).

The fidelity of the operation is calculated by $\mathcal{F} = \sqrt{\langle \psi_{\text{id}} | \rho | \psi_{\text{id}} \rangle}$, where $|\psi_{\text{id}}\rangle$ is the ideal target state. Numerical calculations are coded in Python by using the QuTiP library [90,91]. In our numerical simulation, we set $\gamma_{\varphi_j}^{-1} = \gamma_{\varphi_j}^f = T$, $\gamma_j^{-1} = \gamma_j^{fe} = 2T$, and $\gamma_j^{fg} = 10T$. We choose $g_2 \approx 1.14g_1$, $\Omega \approx 1.8g_1$, $\delta_2 \approx 7.72g_1$, $\delta_1 \approx 4\delta_2 \approx 31.48g_1$, and $\delta_p \approx 3\delta_2 \approx 23.59g_1$, with $g_1/2\pi \approx 1.62$ MHz. The coupling strengths used are available in experiments [82,89]. Moreover, the Rabi frequency here is easily available in experiments [92,93]. Typically, the transition frequency between adjacent energy levels of a flux qubit ranges from 1 to 20 GHz [73,74,82,94,95]. We take the frequency of coupler qubit as $\omega_{qc}/2\pi = 5.0$ GHz. Thus, the frequencies of qubits 1, 2, and microwave pulse are $\omega_{q1}/2\pi \approx 4.949$ GHz, $\omega_{q2}/2\pi \approx 5.012$ GHz, and $\omega_p/2\pi \approx 4.962$ GHz, respectively. By solving the Markovian master equation (23), we numerically calculate the operation fidelity for the GHZ state preparation. The ideal target state $|\psi_{\text{id}}\rangle$ is given by Eq. (16), and the initial state is $|e\rangle_c|g\rangle_1|g\rangle_2$. Figure 2 shows the fidelity $\mathcal{F}$ as a function of T . From the red dots in Fig. 2, one can see that for $T > 10$ μs , the fidelity can surpass 92.36%. To validate the effectiveness of the approximations made, we also calculate the fidelity based on the effective Hamiltonian (7), as shown by the blue dots. When $T > 10$ μs , the fidelity is greater than 93.30% based on the effective Hamiltonian. For $T = 500$ μs , the fidelities are 99.10% and 99.88%, respectively. One can see that the fidelity based on the Hamiltonian (1) slightly decreased by 0.78% to 0.94% compared to using the effective Hamiltonian. Thus, the approximations made for the effective Hamiltonian are reasonable. The coherence time here is available in experiments because the decoherence times of 70 μs –1 ms have been experimentally reported for flux qubits [89]. Thus, we will choose $T = 500$ μs for the following numerical simulations.

We numerically calculate the mean excitation probabilities of qubits 1, 2, and c , and the correlation function of

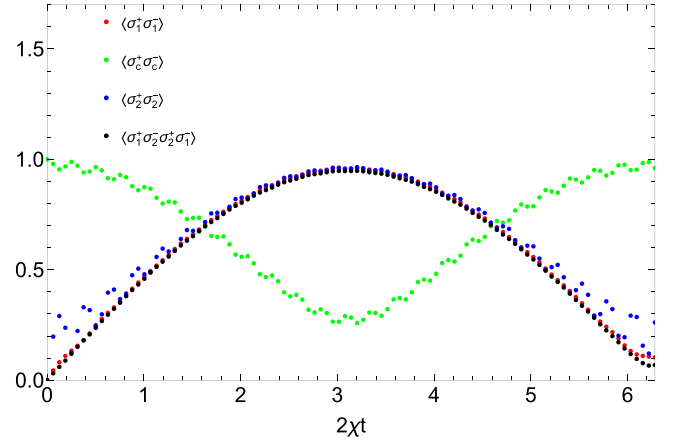


FIG. 3. Time evolution of the mean excitation numbers of qubits 1, 2, and c , $\langle \sigma_1^+ \sigma_1^- \rangle$, $\langle \sigma_2^+ \sigma_2^- \rangle$, and $\langle \sigma_c^+ \sigma_c^- \rangle$, and the correlation function $\langle \sigma_1^+ \sigma_2^+ \sigma_2^- \sigma_1^- \rangle$ of qubits 1 and 2.

qubits 1 and 2. The initial state and the ideal target state are given by $|e\rangle_c|g\rangle_1|g\rangle_2$ and $|g\rangle_c|e\rangle_1|e\rangle_2$, respectively. Figure 3 shows the time evolution of the mean excitation numbers $\langle \sigma_1^+ \sigma_1^- \rangle$, $\langle \sigma_2^+ \sigma_2^- \rangle$, and $\langle \sigma_c^+ \sigma_c^- \rangle$, and the correlation function $\langle \sigma_1^+ \sigma_2^+ \sigma_2^- \sigma_1^- \rangle$ as a function of $2\chi t$. Figure 3 displays the reversible excitation exchange between the coupler qubit and qubits 1 and 2. When $2\chi t = \pi$, the mean excitation numbers of qubits 1 and 2 reach a maximum value at around 0.959, while the coupler's excitation number reaches its minimum value at around 0.283, which means that the excitation is transferred from coupler qubit to qubits 1 and 2. Moreover, the qubit 1's mean excitation number is almost consistent with the two-qubit correlation function $\langle \sigma_1^+ \sigma_2^+ \sigma_2^- \sigma_1^- \rangle$. We notice that using larger detunings can further improve or decrease the maximum or minimum value of the mean excitation numbers for qubits. However, this requires a longer decoherence time T in numerical simulations, which may pose a challenge in experiments. From Fig. 3, it can be observed that there is a slight oscillation in the mean excitation numbers of qubit 2 and coupler c , while no such oscillation is observed in the population of qubit 1. This is because, under the current choice of parameters, the detuning δ_1 is significantly larger than δ_2 and δ_p , which allows the interaction between qubit 1 and the coupler qubit c to better satisfy the large detuning condition and effectively avoid interference from other frequencies. On the other hand, the detuning between qubit 2 and coupler c is minimal, and the oscillation may be caused by errors. Additionally, the coupler c is also affected by the fluctuations of qubit 2, resulting in synchronized oscillations. We find that increasing the ratio δ_2/g_2 (and correspondingly δ_1/g_1 and δ_p/Ω) can effectively eliminate the oscillations of qubits 2 and c . For instance, by tuning the detuning δ_2 to three times its current value, the mean excitation numbers of qubits 2 and c exhibit almost no oscillation, as shown in Fig. 4.

We now numerically calculate the fidelity of the construction of CPHASE or Toffoli gate by solving the master equation (23) based on the Hamiltonian (1) and the effective Hamiltonian (7). The initial state is $(\sin \theta_c |g\rangle_c + \cos \theta_c |f\rangle_c)(\sin \theta_1 |g\rangle_1 + \cos \theta_1 |e\rangle_1)(\sin \theta_2 |g\rangle_2 + \cos \theta_2 |e\rangle_2)$, and the ideal target state can be easily obtained from Eq. (18)

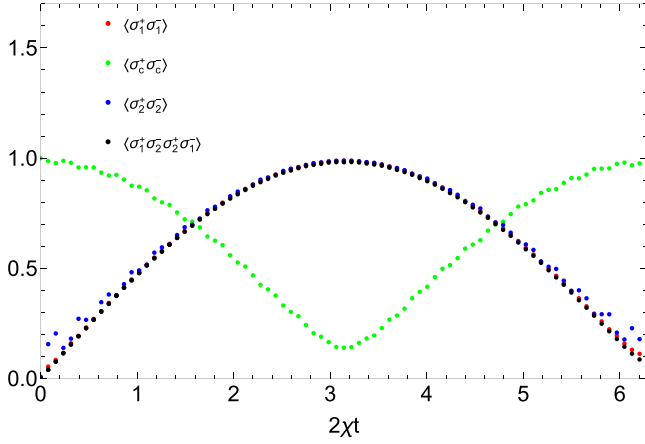


FIG. 4. Time evolution of the mean excitation numbers of qubits 1, 2, and c , $\langle \sigma_1^+ \sigma_1^- \rangle$, $\langle \sigma_2^+ \sigma_2^- \rangle$, and $\langle \sigma_c^+ \sigma_c^- \rangle$, and the correlation function $\langle \sigma_1^+ \sigma_2^+ \sigma_2^- \sigma_1^- \rangle$ of qubits 1 and 2. The detunings here are three times larger compared to those in Fig. 3.

or (20). For example, for the CCPHASE gate, the ideal target state is obtained by

$$\begin{aligned} & c_1 |g\rangle_c |g\rangle_1 |g\rangle_2 + c_2 |g\rangle_c |g\rangle_1 |e\rangle_2 + c_3 |g\rangle_c |e\rangle_1 |g\rangle_2 \\ & - c_4 |g\rangle_c |e\rangle_1 |e\rangle_2 + c_5 |f\rangle_c |g\rangle_1 |g\rangle_2 + c_6 |f\rangle_c |g\rangle_1 |e\rangle_2 \\ & + c_7 |f\rangle_c |e\rangle_1 |g\rangle_2 + c_8 |f\rangle_c |e\rangle_1 |e\rangle_2, \end{aligned} \quad (24)$$

where $c_1 = \sin \theta_c \sin \theta_1 \sin \theta_2$, $c_2 = \sin \theta_c \sin \theta_1 \cos \theta_2$, $c_3 = \sin \theta_c \cos \theta_1 \sin \theta_2$, $c_4 = \sin \theta_c \cos \theta_1 \cos \theta_2$, $c_5 = \cos \theta_c \sin \theta_1 \sin \theta_2$, $c_6 = \cos \theta_c \sin \theta_1 \cos \theta_2$, $c_7 = \cos \theta_c \cos \theta_1 \sin \theta_2$, and $c_8 = \cos \theta_c \cos \theta_1 \cos \theta_2$. For simplicity, we choose $\theta_c = \theta_1 = \theta_2 = \theta \in [0, 2\pi]$ in numerical simulations.

Figure 5 shows the fidelity $\mathcal{F}$ versus θ for the CCPHASE or Toffoli gate operation. Figure 5 displays that when $\theta \in [0, 2\pi]$, the fidelities are greater than 98.48% and 99.50% for the Hamiltonians (1) and (7), respectively. For $\theta = \pi/4$, the fidelities can reach 99.13% and 99.59%, respectively. Thus, the approximations made for the effective Hamiltonian are reasonable.

From Fig. 5, one can see that the fidelity exhibits state-dependent behavior, which is caused by the following aspects: The initial and ideal target states change with θ , and the

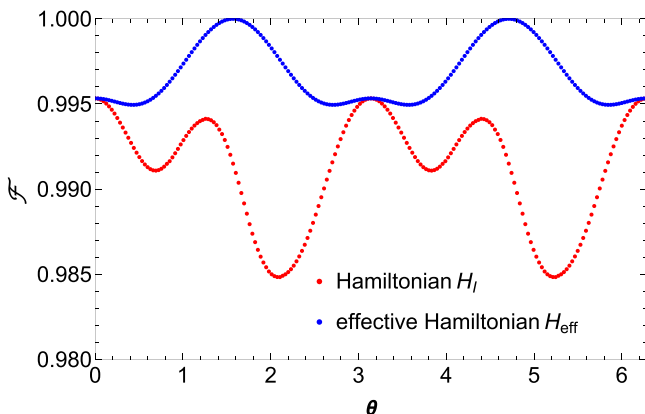


FIG. 5. Fidelity $\mathcal{F}$ vs θ for gate construction.

fidelity is calculated by the overlap between the numerically evolved density matrix and the ideal target state, resulting in the fidelity being state dependent. In addition, during the gate operation, some computational basis states containing excited states suffer different amounts of noise. When numerically simulating based on the effective Hamiltonians (7), the fidelity reaches its maximum values at the peaks of the wave for $\theta = \pi/2$, while it reaches the minimum fidelity values at the troughs of the wave for $\theta = 0, \pi, \pi/4$, and $3\pi/4$. The choice $\theta = \pi/2$ corresponds to the initial state $|g\rangle_c |g\rangle_1 |g\rangle_2$, which is unaffected by decoherence and therefore allows the gate to operate with unit fidelity. In contrast, $\theta = 0$ or π corresponds to the states $\pm |f\rangle_c |e\rangle_1 |e\rangle_2$, which is in the highest excitation state, where the influence of the relaxation process reaches its maximum, leading to a minimum in fidelity with 99.53%. Additionally, $\theta = \pi/4$ or $3\pi/4$ corresponds to the state $(1/\sqrt{2})^3 (|g\rangle_c + |f\rangle_c) (|g\rangle_1 + |e\rangle_1) (|g\rangle_2 + |e\rangle_2)$, where each qubit is in a maximum superposition state that is maximally sensitive to dephasing, resulting in another minimum fidelity of 99.59%.

When numerically simulating based on the Hamiltonian (1), the fidelity reaches its maximum values of 99.53%, 99.53%, and 99.41% at the peaks of the wave, corresponding to $\theta = 0, \pi$, and 0.4π , while at the troughs of the wave, i.e., $\theta = 0.22\pi$, and 0.66π , the minimum fidelity values are 99.11% and 98.48%. One can see that, compared to the effective Hamiltonian, the extrema points have experienced slight shifts. This is due to that under the large detuning conditions, in addition to the Stark shift terms induced by levels $|g\rangle$ and $|e\rangle$, the involving level $|f\rangle$ can also leak and generate induced Stark shift terms, which leads to each level producing an additional phase. The superposition state is highly susceptible to phase inhomogeneities and decoherence effects caused by approximations, large detuning conditions, and level leakage, thus becoming a point of low fidelity. However, at $\theta = 0$ or π , the generated phase serves as a global phase for $|f\rangle_c |e\rangle_1 |e\rangle_2$, and since the coherence time is chosen to be sufficiently long and the dissipation is insensitive to the overall phase, these points become the maxima of fidelity compared to others. In the following, we will quantitatively investigate the impact of level $|f\rangle_c$ leakage on operation fidelity.

B. Full Hamiltonian with unwanted interactions

Since the quantum gate operation involves the level $|f\rangle_c$ of the coupler, we consider the impact of the coupler's level $|f\rangle_c$ on the operation fidelity. When the effect of third level $|f\rangle_c$ is taken into account, unwanted interactions may occur: between qubit c 's $|e\rangle \leftrightarrow |f\rangle$ transition and qubit 1's (or 2's) $|g\rangle \leftrightarrow |e\rangle$ transition; and between the pulse and qubit c 's $|e\rangle \leftrightarrow |f\rangle$ transition. Note that for a superconducting flux qubit, the coupling associated with the $|g\rangle \leftrightarrow |f\rangle$ transition can be ignored [87]. Thus, the Hamiltonian H_I is modified as

$$\begin{aligned} H_I' = & H_I + g_1' (e^{i\delta_1' t} \sigma_c^{fe+} \sigma_1^- + e^{-i\delta_1' t} \sigma_c^{fe-} \sigma_1^+) \\ & + g_2' (e^{i\delta_2' t} \sigma_c^{fe+} \sigma_2^- + e^{-i\delta_2' t} \sigma_c^{fe-} \sigma_2^+) \\ & + \Omega' [\sigma_c^{fe+} e^{i(\delta_p' t - \phi)} + \sigma_c^{fe-} e^{-i(\delta_p' t - \phi)}], \end{aligned} \quad (25)$$

where g_1' (g_2') is the coupling strength between the $|g\rangle \leftrightarrow |e\rangle$ transition of qubit 1 (2) and the $|e\rangle \leftrightarrow |f\rangle$ transition

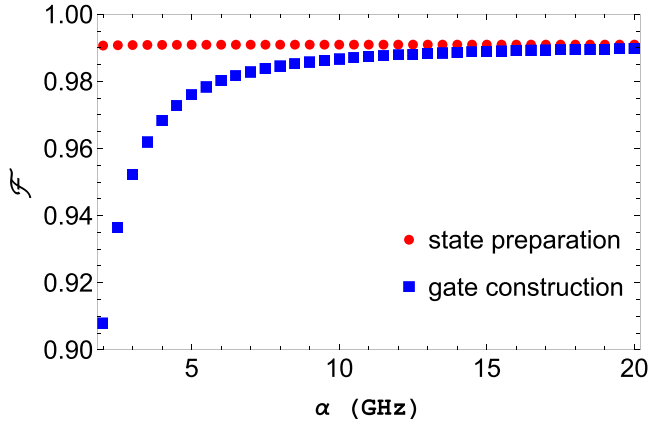


FIG. 6. Fidelity $\mathcal{F}$ vs anharmonicity α for GHZ state preparation and gate construction.

of qubit c ; Ω' is the pulse Rabi frequency associated with the $|e\rangle \leftrightarrow |f\rangle$ transition of the coupler c ; $\sigma_c^{fe-} = |e\rangle_c \langle f|$; $\sigma_c^{fe+} = |f\rangle_c \langle e|$; and $\delta'_1 = \omega_{fe,c} - \omega_{q1} = \delta_1 + \omega_{fe,c} - \omega_{qc}$, $\delta'_2 = \omega_{q2} - \omega_{fe,c} = \delta_2 + \omega_{qc} - \omega_{fe,c}$, and $\delta'_p = \omega_{fe,c} - \omega_p = \delta_p + \omega_{fe,c} - \omega_{qc}$ are the detunings. Here, $\omega_{fe,c}$ represents the $|e\rangle \leftrightarrow |f\rangle$ transition frequency of qubit c .

Due to the involvement of the level $|f\rangle_c$, under the large detuning conditions $\delta'_1 \gg g'_1$, $\delta'_2 \gg g'_2$, and $\delta'_p \gg \Omega'$, a second-order effective Hamiltonian is induced: $\lambda'_1(\sigma_c^{ee}\sigma_1^{ee} - \sigma_c^{ff}\sigma_1^{gg}) + \lambda'_2(\sigma_c^{ee}\sigma_2^{ee} - \sigma_c^{ff}\sigma_2^{gg}) + \lambda'_p(\sigma_c^{ee} - \sigma_c^{ff})$, where $\lambda'_1 = \frac{g_1^2}{\delta_1'}$, $\lambda'_2 = \frac{g_2^2}{\delta_2'}$, and $\lambda'_p = \frac{\Omega'^2}{\delta_p'}$. Additionally, a third-order effective Hamiltonian is induced: $\chi_a[e^{i(\Delta_a t - \phi)}\sigma_c^{fe+}\sigma_1^-\sigma_2^- + \text{H.c.}]$, where $\chi_a \sim \frac{g_1 g_2 \Omega'}{\delta_2'(\delta_2' - \delta_p')}$ and $\Delta_a = \delta'_1 + \delta'_2 - \delta'_p$. To minimize the influence of the level $|f\rangle_c$, one of the following conditions needs to be satisfied: (1) $\lambda_l \gg \lambda'_l$ for $l \in \{1, 2, p\}$ and $\chi \gg \chi_a$; (2) $\delta'_1 \gg g_1^2$, $\delta'_2 \gg g_2^2$, and $\delta'_p \gg \Omega'^2$; and (3) $\lambda_l \gg \lambda'_l$ and Δ_a is large enough to neglect the fast oscillating terms with the factor $e^{\pm i\Delta_a t}$. These conditions can be achieved if δ'_l ($l \in \{1, 2, p\}$) is sufficiently large. Therefore, the anharmonicity $\alpha = \omega_{fe,c} - \omega_{qc}$ of the coupler should be chosen to be large enough so that these detunings are as large as practically possible in experiments.

Now, we quantitatively calculate the impact of the inefficient interactions induced by the level $|f\rangle_c$ on the operational fidelity. We numerically calculate the operation fidelity by replacing the Hamiltonian H_I with H'_I in the master equation (23). By properly designing the flux qubit, the $|g\rangle \leftrightarrow |e\rangle$ and $|e\rangle \leftrightarrow |f\rangle$ transition dipole matrix elements can be made to be on the same order of magnitude [87]. Therefore, we set $g'_1 = g_1$, $g'_2 = g_2$, and $\Omega' = \Omega$.

Figure 6 illustrates the fidelity $\mathcal{F}$ as a function of α for the GHZ state preparation and gate construction, with the results represented by the red and blue dots, respectively. Figure 6 shows that when $\alpha \geq 2.0$ GHz, the fidelity exceeds 90.79% and 99.07%, respectively. For $\alpha = 8.0$ GHz, the fidelities are approximately 99.07% and 98.46%, respectively. Compared to using the Hamiltonian (7), the fidelities decrease slightly by 0.03% and 0.67%, respectively. From Fig. 6, one can see that the fidelity of entangled state preparation is insensitive to small anharmonicity α , while the fidelity of quantum gate

operation increases with larger α . This is because quantum state preparation does not involve populating the $|f\rangle_c$ state, so its overall effect is likely limited to a Stark shift on the remaining energy levels, which can easily be compensated for by small frequency shifts in the drives. In contrast, during gate construction, the $|f\rangle_c$ state is populated, and it is assumed that this population does not evolve under the drives, which only holds in the large-anharmonicity limit. Typically, a flux qubit can exhibit large anharmonicity, and the transition frequency between adjacent energy levels ranges from 1 to 20 GHz [73,74,82,94,95]. The anharmonicity ~ 5 –9 GHz of a flux qubit have been reported in experiments [94,95]. Thus, the numerical simulations demonstrate the feasibility of achieving a high-fidelity GHZ state and quantum gate with current experimental technology.

We also numerically calculate the effect of small operational time errors on the fidelity by choosing $\alpha = 8.0$ GHz for the GHZ state preparation and gate construction, respectively. We take the actual operational time to be $(\epsilon + 1)t$, where t is the optimal operation time. We observe that when $\epsilon \in [-0.01, 0.01]$, the fidelities exceed 95.91% and 96.06%, respectively. Thus, the GHZ entangled state and quantum gates can be realized with high fidelity even in the presence of small operational time errors.

C. Discussion

We note that three-qubit GHZ states using three-body interactions have been proposed in cavity-qubit systems [3,4,12]. In experiments, GHZ states for three superconducting qubits have been created with high fidelities of 62% [45], 96% [46], and 95% [11]. In addition, three-qubit CPHASE or Toffoli gates have been experimentally constructed in superconducting circuits with high fidelities of 98% [64], 70% [65], 69% [66], and 98% [67]. The system and physical mechanisms employed in our work are different from previous works, and numerical results demonstrate that high-fidelity preparation of the three-qubit GHZ state or quantum gates is feasible within current superconducting circuit technology.

We notice that if there is no microwave drive in Hamiltonian (1), one can easily obtain the reduced effective Hamiltonian

$$H_{\text{eff}} \approx \lambda_1(\sigma_c^{ee}\sigma_1^{gg} - \sigma_c^{gg}\sigma_1^{ee}) + \lambda_2(\sigma_c^{gg}\sigma_2^{ee} - \sigma_c^{ee}\sigma_2^{gg}), \quad (26)$$

which still contains two exchange terms (with λ_1 and λ_2). This implies that the dispersive interaction, without the need for a microwave drive, could potentially contribute to the construction of phase gates. For example, we assume that the coupler qubit, qubit 1, and qubit 2 are prepared in states $(1/\sqrt{2})(|g\rangle_c + |f\rangle_c)$, $(1/\sqrt{2})(|g\rangle_1 + |e\rangle_1)$, and $(1/\sqrt{2})(|g\rangle_2 + |e\rangle_2)$, respectively. Under the Hamiltonian (26), one can obtain the following three-qubit phase gate:

$$\begin{aligned} |g\rangle_c |g\rangle_1 |g\rangle_2 &\rightarrow |g\rangle_c |g\rangle_1 |g\rangle_2, \\ |g\rangle_c |g\rangle_1 |e\rangle_2 &\rightarrow e^{-i\varphi_2} |g\rangle_c |g\rangle_1 |e\rangle_2, \\ |g\rangle_c |e\rangle_1 |g\rangle_2 &\rightarrow e^{i\varphi_1} |g\rangle_c |e\rangle_1 |g\rangle_2, \\ |g\rangle_c |e\rangle_1 |e\rangle_2 &\rightarrow e^{i(\varphi_1 - \varphi_2)} |g\rangle_c |e\rangle_1 |e\rangle_2, \\ |f\rangle_c |g\rangle_1 |g\rangle_2 &\rightarrow |f\rangle_c |g\rangle_1 |g\rangle_2, \quad |f\rangle_c |g\rangle_1 |e\rangle_2 \rightarrow |f\rangle_c |g\rangle_1 |e\rangle_2, \\ |f\rangle_c |e\rangle_1 |g\rangle_2 &\rightarrow |f\rangle_c |e\rangle_1 |g\rangle_2, \quad |f\rangle_c |e\rangle_1 |e\rangle_2 \rightarrow |f\rangle_c |e\rangle_1 |e\rangle_2, \end{aligned} \quad (27)$$

where $\varphi_1 = \lambda_1 t$ and $\varphi_2 = \lambda_2 t$. When setting $\varphi_1 = \varphi_2 = \pi$, one obtains $|g\rangle_c |g\rangle_1 |e\rangle_2 \rightarrow -|g\rangle_c |g\rangle_1 |e\rangle_2$, $|g\rangle_c |e\rangle_1 |g\rangle_2 \rightarrow -|g\rangle_c |e\rangle_1 |g\rangle_2$, and other states remain unchanged. This implies that if the control qubit c is in the state $|g\rangle_c$, a sign flip occurs in the state $|e\rangle_j$ of the target qubit j ($j = 1, 2$). Thus, a three-qubit controlled phase gate is implemented. It consists of two two-qubit phase gates, each having a shared control qubit c but a different target qubit 1 or 2. In addition, the exchange terms with λ_1 and λ_2 limit the efficiency of transferring excitations and operation fidelities between the coupler qubit and qubits 1 and 2. Condition (11) should be strictly met to reduce the influence of the exchange terms.

If there is a drive term in Hamiltonian (1), one can obtain the effective Hamiltonian

$$H_{\text{eff}} \approx \lambda_1 (\sigma_c^{ee} \sigma_1^{gg} - \sigma_c^{gg} \sigma_1^{ee}) + \lambda_p \sigma_c^z + \lambda_2 (\sigma_c^{gg} \sigma_2^{ee} - \sigma_c^{ee} \sigma_2^{gg}). \quad (28)$$

In the dressed-state basis: $|g\rangle_c |g\rangle_1 |g\rangle_2$, $|g\rangle_c |g\rangle_1 |e\rangle_2$, $|g\rangle_c |e\rangle_1 |g\rangle_2$, $|g\rangle_c |e\rangle_1 |e\rangle_2$, $|e\rangle_c |g\rangle_1 |g\rangle_2$, $|e\rangle_c |g\rangle_1 |e\rangle_2$, $|e\rangle_c |e\rangle_1 |g\rangle_2$, and $|e\rangle_c |e\rangle_1 |e\rangle_2$, under the Hamiltonian (28), one can obtain the following three-qubit phase gate:

$$\begin{aligned} |g\rangle_c |g\rangle_1 |g\rangle_2 &\rightarrow e^{i\varphi_p} |g\rangle_c |g\rangle_1 |g\rangle_2, \\ |g\rangle_c |g\rangle_1 |e\rangle_2 &\rightarrow e^{i(\varphi_p - \varphi_2)} |g\rangle_c |g\rangle_1 |e\rangle_2, \\ |g\rangle_c |e\rangle_1 |g\rangle_2 &\rightarrow e^{i(\varphi_1 + \varphi_p)} |g\rangle_c |e\rangle_1 |g\rangle_2, \\ |g\rangle_c |e\rangle_1 |e\rangle_2 &\rightarrow e^{i(\varphi_1 - \varphi_2 + \varphi_p)} |g\rangle_c |e\rangle_1 |e\rangle_2, \\ |e\rangle_c |g\rangle_1 |g\rangle_2 &\rightarrow e^{-i(\varphi_1 - \varphi_2 + \varphi_p)} |e\rangle_c |g\rangle_1 |g\rangle_2, \\ |e\rangle_c |g\rangle_1 |e\rangle_2 &\rightarrow e^{-i(\varphi_1 + \varphi_p)} |e\rangle_c |g\rangle_1 |e\rangle_2, \\ |e\rangle_c |e\rangle_1 |g\rangle_2 &\rightarrow e^{-i(\varphi_p - \varphi_2)} |e\rangle_c |e\rangle_1 |g\rangle_2, \\ |e\rangle_c |e\rangle_1 |e\rangle_2 &\rightarrow e^{-i\varphi_p} |e\rangle_c |e\rangle_1 |e\rangle_2, \end{aligned} \quad (29)$$

where $\varphi_p = \lambda_p t$. It can be observed that under these bases, it is not possible to implement common three-qubit gates using Hamiltonian (28) alone. Moreover, when considering the effective third-order Hamiltonian (9) simultaneously, even if we satisfy condition (11) after performing a frame rotation to eliminate the effect of Hamiltonian (28), during single-qubit operations, when the $|g\rangle$ state flips to $|e\rangle$, it also causes the $|e\rangle$ state to flip to $|g\rangle$. Therefore, common quantum phase gates still cannot be implemented under these state bases.

It should be mentioned that the choice of basis states is related to the quantum gate to be implemented. In this work, we mainly focus on the implementation of common three-qubit gates, such as the CCPhase and Toffoli gates, as described by Eqs. (18) and (20). Thus, we introduce an auxiliary level $|f\rangle$ of the coupler qubit to realize these gates. From Eq. (13), it can be seen that the purpose of selecting the initial state of the coupler qubit as $(\sin \theta_c |g\rangle_c + \cos \theta_c |f\rangle_c)$ is to achieve $|g\rangle_c |e\rangle_1 |e\rangle_2 \rightarrow -|g\rangle_c |e\rangle_1 |e\rangle_2$ while avoiding $|e\rangle_c |g\rangle_1 |g\rangle_2 \rightarrow -|e\rangle_c |g\rangle_1 |g\rangle_2$, so that the CCPhase gate can be implemented in a single step. In addition, consider the effect of Hamiltonian (28). Similar to Eq. (29), the basis states related to $|g\rangle_c$ (i.e., $|g\rangle_c |g\rangle_1 |g\rangle_2$, $|g\rangle_c |g\rangle_1 |e\rangle_2$, $|g\rangle_c |e\rangle_1 |g\rangle_2$, and $|g\rangle_c |e\rangle_1 |e\rangle_2$) will cause phase accumulation, which can also affect the gate operation fidelity. We minimize this impact as much as possible by setting condition (11).

V. CONCLUSION

We have proposed a method for synthesizing a nonpairwise three-body interaction between a superconducting coupler qubit and two spatially separated qubits. This interaction involves physical processes including an excitation from the coupler qubit to two spatially separated qubits and the reverse process. Our work demonstrates that the three-body interaction can be used to deterministically prepare three-body GHZ entangled states, transfer the state of the coupler qubit into a two-qubit entangled state, and construct three-qubit controlled-controlled-phase and Toffoli gates. The simplicity of the method, requiring only a coupler qubit and a single-step operation, can significantly reduce the experimental difficulty. Numerical simulations demonstrated that by using current superconducting circuit technology, a three-qubit GHZ state or quantum gate can be synthesized with high fidelity. This proposal is quite general and applicable to a wide range of physical systems. Our findings may have other potential applications in large-scale quantum computation, quantum information processing, and quantum simulation.

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DATA AVAILABILITY

The data that support the findings of this article are openly available [96].

APPENDIX: DERIVATION OF THE EFFECTIVE HAMILTONIANS

In this Appendix, we utilize the method [83,84] to derive the second-order and third-order effective Hamiltonians as presented in the main text. In the interaction picture, the Hamiltonian of the whole system is given by

$$H_I(t) = \sum_m \hat{h}_m \exp(i\omega_m t) + \hat{h}_m^\dagger \exp(-i\omega_m t). \quad (A1)$$

Under the large detuning conditions, assume that all of the frequencies ω_m are distinct and the algebraic sum of any three frequencies is equal to zero or distinct from zero. Referring to the works of Refs. [83,84], one can obtain the second-order and third-order effective Hamiltonians by using the RWA:

$$\begin{aligned} H_{\text{eff}}^{(2)} &= \frac{1}{i} H_I(t) \int^t H_I(t') dt' \\ &= \sum_m \frac{1}{\omega_m} [\hat{h}_m, \hat{h}_m^\dagger], \\ H_{\text{eff}}^{(3)} &= -H_I(t) \int^t H_I(t_1) \int^{t_1} H_I(t_2) dt_2 dt_1 \\ &= \sum_{l,m,n} \left\{ \frac{1}{\omega_n(\omega_n - \omega_m)} [\hat{h}_l \hat{h}_m^\dagger \hat{h}_n e^{i(\omega_l - \omega_m + \omega_n)t} \right. \end{aligned} \quad (A2)$$

$$\begin{aligned}
& + \hat{h}_l^\dagger \hat{h}_m \hat{h}_n^\dagger e^{i(-\omega_l + \omega_m - \omega_n)t} + \hat{h}_l \hat{h}_m \hat{h}_n^\dagger e^{i(\omega_l + \omega_m - \omega_n)t} \\
& + \hat{h}_l^\dagger \hat{h}_m \hat{h}_n e^{i(-\omega_l - \omega_m + \omega_n)t}] \\
& + \frac{1}{\omega_n(\omega_n + \omega_m)} [\hat{h}_l^\dagger \hat{h}_m \hat{h}_n e^{i(-\omega_l + \omega_m + \omega_n)t} \\
& + \hat{h}_l \hat{h}_m \hat{h}_n^\dagger e^{i(\omega_l - \omega_m - \omega_n)t}] \}. \quad (\text{A3})
\end{aligned}$$

In our main text, the interaction Hamiltonian of whole system is given by Eq. (1), i.e.,

$$\begin{aligned}
H_I = & g_1 (e^{i\delta_1 t} \sigma_c^+ \sigma_1^- + e^{-i\delta_1 t} \sigma_c^- \sigma_1^+) \\
& + g_2 (e^{i\delta_2 t} \sigma_2^+ \sigma_c^- + e^{-i\delta_2 t} \sigma_2^- \sigma_c^+) \\
& + \Omega [\sigma_c^+ e^{i(\delta_p t - \phi)} + \sigma_c^- e^{-i(\delta_p t - \phi)}]. \quad (\text{A4})
\end{aligned}$$

We now consider the second-order case in the dispersive regimes $\delta_1 \gg g_1$, $\delta_2 \gg g_2$, and $\delta_p \gg \Omega$. According to Eqs. (A1) and (A2), we take $\hat{h}_1 = g_1 \sigma_c^+ \sigma_1^-$, $\omega_1 = \delta_1$; $\hat{h}_2 = g_2 \sigma_2^+ \sigma_c^-$, $\omega_2 = \delta_2$; and $\hat{h}_3 = \Omega \sigma_c^+ e^{-i\phi}$, $\omega_3 = \delta_p$. Thus, one has

$$\begin{aligned}
[\hat{h}_1, \hat{h}_1^\dagger] &= g_1^2 (\sigma_c^{ee} \sigma_1^{gg} - \sigma_c^{gg} \sigma_1^{ee}), \\
[\hat{h}_2, \hat{h}_2^\dagger] &= g_2^2 (\sigma_c^{gg} \sigma_2^{ee} - \sigma_c^{ee} \sigma_2^{gg}), \\
[\hat{h}_3, \hat{h}_3^\dagger] &= \Omega^2 \sigma_c^z, \quad (\text{A5})
\end{aligned}$$

where $\sigma_j^{gg} = |g\rangle_j \langle g|$, $\sigma_j^{ee} = |e\rangle_j \langle e|$ ($j = 1, 2, c$), and $\sigma_c^z = |e\rangle_c \langle e| - |g\rangle_c \langle g|$.

By inserting Eq. (A5) into Eq. (A2), one can obtain the second-order effective Hamiltonian

$$H_{\text{eff}}^{(2)} = \lambda_1 (\sigma_c^{ee} \sigma_1^{gg} - \sigma_c^{gg} \sigma_1^{ee}) + \lambda_p \sigma_c^z + \lambda_2 (\sigma_c^{gg} \sigma_2^{ee} - \sigma_c^{ee} \sigma_2^{gg}), \quad (\text{A6})$$

where $\lambda_1 = \frac{g_1^2}{\delta_1}$, $\lambda_2 = \frac{g_2^2}{\delta_2}$, and $\lambda_p = \frac{\Omega^2}{\delta_p}$. This Hamiltonian is the Hamiltonian (5) in the main text.

Then, we consider the third-order case in the dispersive regimes $\delta_1 \gg g_1$, $\delta_2 \gg g_2$, and $\delta_p \gg \Omega$. We assume that $\delta_1 > \delta_p \gg \delta_2$ (e.g., $\delta_1 \approx 4\delta_2$, $\delta_p = 3\delta_2$). Based on Eqs. (A1) and (A3), we have

$$\begin{aligned}
\hat{h}_1 \hat{h}_2^\dagger \hat{h}_3 &= 0, \quad \hat{h}_1^\dagger \hat{h}_2 \hat{h}_3^\dagger = 0, \quad \hat{h}_1 \hat{h}_2 \hat{h}_3^\dagger = 0, \quad \hat{h}_1^\dagger \hat{h}_2^\dagger \hat{h}_3 = 0, \\
\hat{h}_1^\dagger \hat{h}_2 \hat{h}_3 &= 0, \quad \hat{h}_1 \hat{h}_2^\dagger \hat{h}_3^\dagger = 0, \quad \hat{h}_1 \hat{h}_3^\dagger \hat{h}_2 = 0, \quad \hat{h}_1^\dagger \hat{h}_3 \hat{h}_2^\dagger = 0, \\
\hat{h}_1 \hat{h}_3 \hat{h}_2^\dagger &= 0, \quad \hat{h}_1^\dagger \hat{h}_3^\dagger \hat{h}_2 = 0, \quad \hat{h}_1^\dagger \hat{h}_3 \hat{h}_2 = g_1 g_2 \Omega e^{-i\phi} \sigma_c^- \sigma_1^+ \sigma_2^+, \\
\hat{h}_1 \hat{h}_3^\dagger \hat{h}_2^\dagger &= g_1 g_2 \Omega e^{i\phi} \sigma_c^+ \sigma_1^- \sigma_2^-, \quad \hat{h}_2 \hat{h}_1^\dagger \hat{h}_3 = 0, \quad \hat{h}_2^\dagger \hat{h}_1 \hat{h}_3^\dagger = 0,
\end{aligned}$$

$$\begin{aligned}
\hat{h}_2 \hat{h}_1 \hat{h}_3^\dagger &= g_1 g_2 \Omega e^{i\phi} \sigma_c^- \sigma_1^- \sigma_2^+, \quad \hat{h}_2^\dagger \hat{h}_1 \hat{h}_3 = 0, \quad \hat{h}_2 \hat{h}_3^\dagger \hat{h}_1 = 0, \\
\hat{h}_2^\dagger \hat{h}_1^\dagger \hat{h}_3 &= g_1 g_2 \Omega e^{-i\phi} \sigma_c^+ \sigma_1^+ \sigma_2^-, \quad \hat{h}_2 \hat{h}_1^\dagger \hat{h}_3^\dagger = 0, \quad \hat{h}_2^\dagger \hat{h}_3 \hat{h}_1^\dagger = 0, \\
\hat{h}_2 \hat{h}_3 \hat{h}_1^\dagger &= g_1 g_2 \Omega e^{-i\phi} \sigma_c^- \sigma_1^+ \sigma_2^+, \quad \hat{h}_2^\dagger \hat{h}_3 \hat{h}_1 = 0, \quad \hat{h}_2 \hat{h}_3^\dagger \hat{h}_1^\dagger = 0, \\
\hat{h}_2^\dagger \hat{h}_3^\dagger \hat{h}_1 &= g_1 g_2 \Omega e^{i\phi} \sigma_c^+ \sigma_1^- \sigma_2^-, \quad \hat{h}_3 \hat{h}_1^\dagger \hat{h}_2 = 0, \quad \hat{h}_3^\dagger \hat{h}_1 \hat{h}_2^\dagger = 0, \\
\hat{h}_3 \hat{h}_1 \hat{h}_2^\dagger &= 0, \quad \hat{h}_3^\dagger \hat{h}_1^\dagger \hat{h}_2 = 0, \quad \hat{h}_3^\dagger \hat{h}_1 \hat{h}_2 = g_1 g_2 \Omega e^{i\phi} \sigma_c^- \sigma_1^- \sigma_2^+, \\
\hat{h}_3 \hat{h}_1^\dagger \hat{h}_2^\dagger &= g_1 g_2 \Omega e^{-i\phi} \sigma_c^+ \sigma_1^+ \sigma_2^-, \quad \hat{h}_3 \hat{h}_2^\dagger \hat{h}_1 = 0, \quad \hat{h}_3^\dagger \hat{h}_2 \hat{h}_1^\dagger = 0, \\
\hat{h}_3 \hat{h}_2 \hat{h}_1^\dagger &= 0, \quad \hat{h}_3^\dagger \hat{h}_2^\dagger \hat{h}_1 = 0, \quad \hat{h}_3^\dagger \hat{h}_2 \hat{h}_1 = 0, \quad \hat{h}_3 \hat{h}_2^\dagger \hat{h}_1^\dagger = 0. \quad (\text{A7})
\end{aligned}$$

Substituting Eq. (A7) and ω_i ($i = 1, 2, 3$) into formula (A3), one can obtain the third-order effective Hamiltonian

$$\begin{aligned}
H_{\text{eff}}^{(3)} &= \frac{1}{\omega_2(\omega_2 + \omega_3)} [\hat{h}_1^\dagger \hat{h}_3 \hat{h}_2 e^{i(-\omega_1 + \omega_3 + \omega_2)t} \\
& + \hat{h}_1 \hat{h}_3^\dagger \hat{h}_2^\dagger e^{-i(-\omega_1 + \omega_3 + \omega_2)t}] \\
& + \frac{1}{\omega_3(\omega_3 - \omega_1)} [\hat{h}_2 \hat{h}_1 \hat{h}_3^\dagger e^{i(\omega_2 + \omega_1 - \omega_3)t} \\
& + \hat{h}_2^\dagger \hat{h}_1^\dagger \hat{h}_3 e^{-i(\omega_2 + \omega_1 - \omega_3)t}] \\
& + \frac{1}{\omega_1(\omega_1 - \omega_3)} [\hat{h}_2 \hat{h}_3 \hat{h}_1^\dagger e^{i(\omega_2 + \omega_3 - \omega_1)t} \\
& + \hat{h}_2^\dagger \hat{h}_3^\dagger \hat{h}_1 e^{-i(\omega_2 + \omega_3 - \omega_1)t}] \\
& + \frac{1}{\omega_2(\omega_2 + \omega_1)} [\hat{h}_3^\dagger \hat{h}_1 \hat{h}_2 e^{i(-\omega_3 + \omega_1 + \omega_2)t} \\
& + \hat{h}_3 \hat{h}_1^\dagger \hat{h}_2^\dagger e^{-i(-\omega_3 + \omega_1 + \omega_2)t}] \\
& = \chi' [e^{i(\Delta' t + \phi)} \sigma_2^+ \sigma_c^- \sigma_1^- + e^{-i(\Delta' t + \phi)} \sigma_2^- \sigma_c^+ \sigma_1^+] \\
& + \chi [e^{i(\Delta t + \phi)} \sigma_c^+ \sigma_1^- \sigma_2^- + e^{-i(\Delta t + \phi)} \sigma_c^- \sigma_1^+ \sigma_2^+], \quad (\text{A8})
\end{aligned}$$

where $\chi' = g_1 g_2 \Omega [\frac{1}{\delta_p(\delta_p - \delta_1)} + \frac{1}{\delta_2(\delta_1 + \delta_2)}]$, $\chi = g_1 g_2 \Omega [\frac{1}{\delta_2(\delta_2 + \delta_p)} + \frac{1}{\delta_1(\delta_1 - \delta_p)}]$, $\Delta' = \delta_2 + \delta_1 - \delta_p$, and $\Delta = \delta_1 - \delta_2 - \delta_p$.

When $\delta_1 \approx \delta_p + \delta_2$, the fast oscillating terms with the factor $e^{i\Delta' t}$ and $e^{-i\Delta' t}$ can be neglected by using the RWA. Keeping the slow oscillating terms with the factors $e^{i\Delta t}$ and $e^{-i\Delta t}$, the third-order effective Hamiltonian (A8) reduces to

$$H_{\text{eff}}^{(3)} = \chi [e^{i(\Delta t + \phi)} \sigma_c^+ \sigma_1^- \sigma_2^- + e^{-i(\Delta t + \phi)} \sigma_c^- \sigma_1^+ \sigma_2^+], \quad (\text{A9})$$

which is the Hamiltonian (6) in the main text.

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