

## Rotation sensor based on an atom-cavity system

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We propose to use the photon emission near the superradiant phase transition of a transversely pumped atom-cavity platform as a rotation sensor. The atoms are held as an array of Bose-Einstein condensates, in the plane of the pump beam and the cavity mode, so that the array encloses a large interferometric area. The photon emission from the cavity indicates changes in the rotation frequency. To determine the dependence of the photon emission on the rotation frequency, we derive an analytical expression for the phase boundaries and use a semiclassical method to map out the phase diagram numerically, and, in particular, the photon emission as a function of pump intensity and rotation. Furthermore, we determine the Allan deviation and random walk numerically, to demonstrate that the sensitivity is competitive with current optical gyroscopes.

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Quantum sensing aims to take advantage of the accuracy and long-time stability of quantum platforms, such as ultracold atoms, for technological applications [1], such as gyroscopes used for navigation. While optical Sagnac interferometers are important tools in present-day navigation, they can fall short due to limited sensitivity and long-term stability [2]. Atom interferometry emerged as a promising platform to address these issues with high-precision rotation and acceleration measurements [3–8]. The applications range from testing fundamental physics [9–11], metrology [12], and absolute gravimetry [13,14] to inertial navigation [15,16]. So far, the usage of atom interferometers as rotation sensors to measure signals precisely, or even measure time-varying signals, has been challenging. Each measurement is done destructively, which means that measurements have to be repeated, starting from condensate formation. An interleaved operation can be used to increase the repetition rate [17]. However, an accurate rotational measurement with an atom-only interferometry remains challenging, but is crucial if the device is to be used for inertial navigation [18].

Rotating an ultracold atom system creates an artificial gauge field [19–21], which was proposed to study gauge field-driven atom dynamics in a controlled environment and to produce cat states for quantum metrology [20,22]. A similar setup described by a rotating Bose-Hubbard model has been proposed for rotation sensing [23].

Open quantum systems such as atom-cavity systems [24,25] allow for real-time monitoring of the photons emitted out of the cavity. This has been used to observe phase

transitions *in situ* [26–30]. The precise control of cold atom experiments combined with the inherent cooling of the system due to energy leaking out of the cavity makes it an ideal platform to study nonequilibrium dynamics [31–38], as well as for metrology [39–44].

In this letter, we propose to build a quantum rotation sensor that allows to measure rotational frequencies by incorporating atom interferometry, artificial gauge phases in ultracold atoms, and photon-emission-based readout of open quantum systems. Specifically, we propose to combine the impact of the rotational Peierls substitution on the hopping parameter  $J$  of neutral atoms and the superradiant (SR) phase transition that can be realized by coupling neutral atoms to a single-mode cavity with a photon loss rate  $\kappa$ . With this, we propose to create a quantum rotation sensor that features nondestructive detection of the external rotational frequency  $\Omega$  of the system.

Using a mean-field ansatz, we analytically show the dependence of the phase boundary and the intracavity photon number on the rotation, represented by a gauge phase. We further use a semiclassical method to numerically determine the phase diagram as a function of the rotation frequency and the light-matter coupling strengths. We compute the Allan deviation as well as the random walk to quantify the sensitivity of our proposed sensor. We give an example for the atom-light dynamics due to a slowly varying rotation frequency, to be characterized and explored elsewhere.

A sketch of our proposed sensor is shown in Fig. 1(a) and with a top view depicted in Fig. 1(c). We propose to split a Bose-Einstein condensate of  $^{87}\text{Rb}$  atoms into  $M$  one-dimensional (1D) tubes and organize them in a two-dimensional square array with a spacing of  $\lambda/2$ , where  $\lambda$  is the wavelength of the laser beam used to produce the light-matter coupling between the atoms and the cavity. To trap the atoms and produce the atomic array, a digital micromirror device (DMD) can be employed to construct a square annulus potential with barriers running along each side at a periodicity of  $\lambda/2$  to produce an atomic array [8,45]. This

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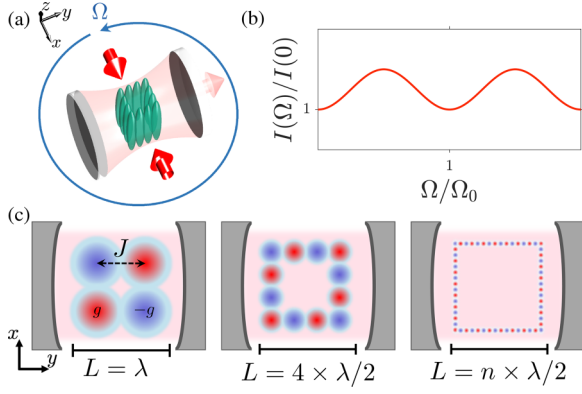


FIG. 1. (a) Sketch of the proposed system. 1D tubes of Bose-Einstein condensates are placed with a separation of  $\lambda/2$  in a high-finesse optical cavity. The atoms are transversely pumped by a red-detuned laser beam with wavelength  $\lambda$  that forms a standing wave potential. The effect of an applied rotation around the  $z$  axis at an angular frequency of  $\Omega$ , as sketched in blue, can be measured via the intensity of the photons transmitted out of the cavity at a rate  $\kappa$ , as shown in panel (b). (b) A schematic sketch of the measured light field intensity as a function of the rotational frequency  $\Omega$  rescaled by the intensity measured when the system is at rest. (c) Top view of the proposed setup. Red and blue circles denote the sign of the light-matter coupling  $\pm g$  at each site.  $J$  denotes the tunneling between neighboring sites. Using a DMD, a square annulus light potential is applied shown in light blue. From left to right: the proposed setup with increasing area enclosed by the atoms. This allows for an increased frequency sensitivity of the sensor, which scales with the number of array sites,  $1/M$ .

system is then placed inside a high-finesse cavity with a loss rate  $\kappa$ . In Fig. 1(c), we show schematic diagrams of the desired geometry of the atomic array including the confining potential. The enclosed area can be enlarged leading to an improved frequency sensitivity that scales with  $1/L$ , a typical strategy done in atom interferometers [7]. This sensor allows to measure the rotational frequency via the light intensity transmitted out of the cavity. The mechanism crucial for the operation is the following. For the SR phase to occur, the light-matter coupling strength needs to exceed the tunneling energy, which is minimized if all sites have the same population [see Eq. (6)]. However, the SR phase relies on an imbalance between neighboring sites as is depicted in red and blue in Fig. 1(c). Rotations of the system lead to an effective gauge field that modifies the tunneling amplitude. This leads to an effective reduction of the real part of the tunneling energy, which lowers the critical light-matter coupling needed to enter the SR phase. For a fixed light-matter strength, the rotational frequency can now be read off by the light field intensity leaking out of the cavity. This effect is used in our rotational sensor. We note that the sensitivity is further increased with an operation of the sensor close to the phase transition. This allows for small changes in the rotational frequency to induce large changes of the photon number, as demonstrated below, due to the strong dependence near the phase transition.

We assume that the entire setup is under external rotation and not only the trapping potential. Hence, after going to the rotating frame of the external rotation, the Hamiltonian is

static, as we discuss further in the Supplemental Material [46]. The atomic sector of our system is described by a bosonic field operator  $\hat{\Phi}(\mathbf{r})$  and the many-body Hamiltonian

$$\hat{H}_A = \int \hat{\Phi}^\dagger(\mathbf{r}) \left( -\frac{\hbar^2 \nabla^2}{2m} - \Omega \hat{L}_z + V(\mathbf{r}) \right) \hat{\Phi}(\mathbf{r}) d\mathbf{r}, \quad (1)$$

where the external  $\hat{L}_z$  is the angular momentum operator and  $V(\mathbf{r})$  is the external potential shaped by a DMD [8,45]. A DMD can be used to generate a square array of potential wells, such that the atoms are confined with a width of  $\lambda/2$  as sketched in Fig. 1(c). We further assume that the external potential has a periodicity of  $\lambda/2$  [8] along the  $x$  and  $y$  directions and is sufficiently weak, so that the system remains in the superfluid regime [47]. This condition places a constraint on the length of each side  $L$  to be an integer multiple of  $\lambda/2$ . The cavity photon annihilation (creation) operator is  $\hat{a}$  ( $\hat{a}^\dagger$ ) and the corresponding Hamiltonian is  $\hat{H}_C = \delta_C \hat{a}^\dagger \hat{a}$ . The atoms and photons interact according to [24]

$$\hat{H}_{LM} = \int \hat{\Phi}^\dagger(\mathbf{r}) \eta \cos(kx) \cos(ky) (a + a^\dagger) \hat{\Phi}(\mathbf{r}) d\mathbf{r}, \quad (2)$$

where  $\eta$  is the strength of the coupling. We further assume that the potential  $V(\mathbf{r})$  is deep enough to neglect next-nearest neighbor tunneling and that the band gap is larger than the rotational energy. With this, we expand the atomic field operators in Wannier orbitals [21,48]

$$\tilde{W}_i = \exp \left( -\frac{im}{\hbar} \int_{\mathbf{r}_i}^{\mathbf{r}_{i+1}} \mathbf{A}(\mathbf{r}') d\mathbf{r}' \right) W_i(\mathbf{r}), \quad (3)$$

where  $\mathbf{A}(\mathbf{r}) = \Omega \mathbf{z} \times \mathbf{r}$  is the effective vector potential induced by the rotation. With this, the effective Hamiltonian for the system is given by

$$\begin{aligned} \hat{H} = & \delta_C \hat{a}^\dagger \hat{a} - g(\hat{a}^\dagger + \hat{a}) \sum_{i=1}^M (-1)^i \hat{n}_i \\ & - J \exp(i\theta(\Omega)) \sum_{i=1}^M \hat{b}_i^\dagger \hat{b}_{i+1} + \text{H.c.}, \end{aligned} \quad (4)$$

with the periodic boundary condition  $\hat{b}_{M+1} = \hat{b}_1$ .  $J$  is the tunneling energy between neighboring condensates and  $g$  is the light-matter coupling for the Wannier orbitals. These are defined as  $J = \int d\mathbf{r} W_i^* \left[ -\frac{\hbar^2 \nabla^2}{2m} + V(\mathbf{r}) \right] W_{i+1}$  and  $g = \eta \int d\mathbf{r} W_i^* \cos(kx) \cos(ky) W_i$ . The number operator at site  $i$  is  $\hat{n}_i = \hat{b}_i^\dagger \hat{b}_i$ , and  $\theta$  is an effective phase generated by the gauge field  $\theta = \int_{x_i}^{x_{i+1}} \mathbf{A}(\mathbf{r}) d\mathbf{r} = \pi^2 n_s \Omega / \omega_{\text{rec}}$ . The recoil frequency due to the pump is  $\omega_{\text{rec}} = \hbar k^2 / 2m$ , and we define the number of sites on each side as  $n_s = M/4 + 1$ . For the square geometry depicted in Fig. 1(c), the phase  $\theta$  is the same for all bonds between neighboring sites. We note that other geometries, e.g., rectangular potentials instead of a square potential, can be used as well. Here, the angle  $\theta$  will be different for the  $x$  and  $y$  axes. However, the functionality is retained for this modification as well. Other geometries, which result in values of  $\theta$  that differ substantially from bond to bond, are not desirable for sensing. Due to the geometry of the potential and the all-to-all coupling mediated by the cavity light field, all atoms on the even sites experience the same light-field

interaction strength, as do the atoms on the odd sites. We determine the phase diagram for the square-shaped four-site realization with a mean-field ansatz. We consider a product state of coherent states as an ansatz

$$|\Psi\rangle = (|\psi_{-}\rangle|\psi_{+}\rangle)^{M/2}|\alpha\rangle, \quad (5)$$

where  $\psi_{\pm}$  and  $\alpha$  are  $c$  numbers representing the amplitude of the coherent states.  $\psi_{+}$  ( $\psi_{-}$ ) corresponds to the condensate on the odd (even) sites. We compute the energy  $E = \langle\Psi|H|\Psi\rangle$ , define  $\psi_{\pm} = \sqrt{(N_A \pm \Delta)/M}$ , with  $N_A$  the total particle number and  $|\Delta| < N_A$ , and find

$$E = \delta_C |\alpha|^2 - 2g\alpha_r \Delta - 2J \cos \theta \sqrt{N_A^2 - \Delta^2}, \quad (6)$$

where we denote  $\alpha_r \equiv \text{Re}(\alpha)$ . By minimizing the energy with regard to the photon amplitude  $\alpha$  and the density imbalance  $\Delta$ , we find the critical light-matter coupling strength to be  $g_{\text{crit}} = \sqrt{J\delta_C \cos(\theta)/N_A}$ , with  $\theta \equiv \text{mod}(\theta - \pi/2, \pi) - \pi/2 \in (-\pi/2, \pi/2)$ . For an open system with dissipation rate  $\kappa$ , we obtain a modified critical coupling strength,

$$g_{\text{crit}} = \sqrt{\frac{J \cos \theta (\delta_C^2 + \kappa^2)}{N_A \delta_C}}. \quad (7)$$

Hence, the critical light-matter coupling strength can be significantly reduced via rotation of the setup, due to the dependence on  $\cos(\theta)$ . This dependence is the origin of the functionality as a rotation sensor, due to the dependence  $\theta = \theta(\Omega)$ . A change of  $\Omega$  results in a change of the emitted light intensity without a destructive measurement. In the following, we determine the dynamics of the four-site realization via a numerical implementation of the open Truncated Wigner Approximation (TWA) method [29,49,50]. The TWA is a semiclassical phase space method, which uses an ensemble of initial states sampled over the corresponding Wigner distribution to predict the quantum dynamics. For the cavity mode, we sample from a Wigner distribution corresponding to a coherent state with  $\langle\alpha\rangle = 0$ . For the atoms on each site, we sample from a Wigner distribution of a coherent state with  $\langle n_i \rangle = N_A/M$ . Due to the dissipative nature of the system, we propagate the initial states via stochastic differential equations. The equations of motion are given by the Heisenberg-Langevin equations

$$\frac{d\hat{b}_i}{dt} = i[\hat{H}, \hat{b}_i], \quad (8)$$

$$\frac{d\hat{a}}{dt} = i[\hat{H}, \hat{a}] - \kappa\hat{a} + \xi, \quad (9)$$

where  $\kappa$  is the cavity dissipation rate and  $\xi$  represents the noise associated with the dissipation. The noise fulfills the relation  $\langle \xi^*(t')\xi(t) \rangle = \kappa\delta(t - t')$ . In the following, we use experimentally realistic parameters [51]. In particular, we consider  $^{87}\text{Rb}$  atoms and choose a particle number of  $N_A = 60 \times 10^3$  and a recoil energy of  $\omega_{\text{rec}} = 2\pi \times 3.5$  kHz, which corresponds to a wavelength of  $\lambda \approx 800$  nm. We further assume to be in or near the good cavity or recoil-resolved regime  $\omega_{\text{rec}} \approx \kappa = 2\pi \times 5$  kHz and choose  $\delta_C = 2\pi \times 10$  kHz. However, we want to stress that our proposed sensor is not limited to operation in this regime. Using a larger  $\kappa$  is also feasible but with the trade-off that the number

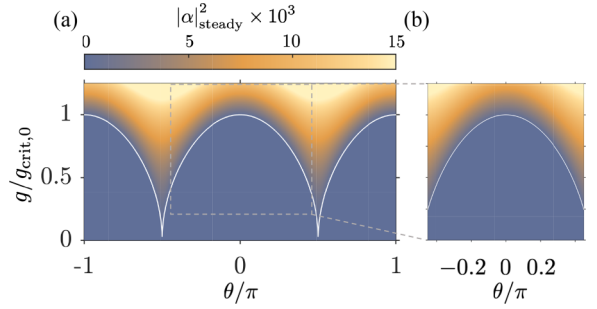


FIG. 2. (a) The analytical steady-state light field intensity  $|\alpha|^2_{\text{steady}}$  as a function of the rotation frequency and light-matter coupling strength. The light-matter coupling strength is rescaled by the critical value for the nonrotating case. The white curve corresponds to the analytically derived phase boundary in Eq. (7). The dashed box represents the area we show using TWA results in panel (b). (b) TWA results covering a larger region are shown in the Supplemental Material [46]. For each data point, we average over  $2 \times 10^2$  TWA trajectories.

of photons detected in real time would be less than that for smaller  $\kappa$ . For the tunneling rate, we assume  $J = 2\pi \times 2$  kHz.

In Fig. 2(a), we show the analytical results of the intra-cavity light-field obtained via our mean-field approach, in particular, the steady-state photon number for varying the gauge phase  $\theta$  and light-matter coupling  $g$  rescaled by the critical light-matter coupling for the system at rest,  $g_{0,\text{crit}} = g_{\text{crit}}(\theta = 0)$ . We observe a periodicity of  $\pi$  for the external rotation  $\theta$ . The white curve corresponds to the analytical critical light-matter coupling strength in Eq. (7). We observe that the photon number strongly depends on the phase and external rotations can push the system to undergo a  $\mathbb{Z}_2$  symmetry-breaking phase transition between the normal phase (NP), in which there are no cavity photons and no imbalance between the population on the even and odd sites, and the SR phase, in which photons are scattered into the cavity and there is population imbalance between the even and odd sites. In Fig. 2(b), we present TWA results obtained using  $2 \times 10^2$  trajectories for each data point. We find good agreement between the mean-field prediction and our numerical results. In the supplemental material [46], we show an extended TWA phase diagram, and find very slow convergence for small values of  $g$  and values of  $\theta$  close to  $n \times \pi/2$  with  $n \in \mathbb{N}$ . We suggest to not use the sensor in this regime. Hence, we will use the rotation dependence of the photon number as the signal for the external rotation.

Next, we discuss the sensitivity of our proposed sensor, which we quantify via

$$\sigma(T_i) = \Delta N_p / \frac{dN_p}{d\Omega}, \quad (10)$$

where  $N_p$  is the number of photons emitted from the cavity during an interrogation time  $T_i$ , i.e.,  $N_p = T_i \kappa \langle a^\dagger a \rangle$ . When interpreted as a function of  $T_i$ , this is the Allan deviation, displaying a characteristic  $1/\sqrt{T_i}$  decay. The sensitivity converts the uncertainty of the measured quantity, i.e., of the photon number  $N_p$ , into the desired uncertainty of the quantity to be inferred, i.e., the rotation frequency  $\Omega$ , (also see Ref. [52]).

For the mean-field approach described above, we obtain for it

$$\sigma(T_i) \propto \frac{\omega_{\text{rec}}}{M\sqrt{N_A}} \sqrt{\frac{\delta_C^3}{(\delta_C^2 + \kappa^2)T_i\kappa J}}, \quad (11)$$

as discussed in the Supplemental Material [46]. As expected, we find that the Allan deviation scales as  $T_i^{-1/2}$  with the interrogation time. Regarding the dependence on the system properties, it scales inversely with the number of sites. Hence, the sensitivity can be increased by adding more sites to the array, as discussed above, [see Fig. 1(c)]. Further, the Allan deviation scales as  $1/\sqrt{N_A}$  with the particle number. This is a second parameter to increase the sensitivity. We further note that the Allan deviation scales with  $J^{-1/2}$ , which suggest that increasing the tunneling amplitude is beneficial. In our proposed setup with an implementation for ultracold atoms, this implies to use a shallow potential between neighboring sites to increase the tunneling amplitude. However,  $J$  has to be kept sufficiently small to avoid exciting atoms into higher bands. Another route to increase the sensitivity is to use a regime with  $\delta_C < \kappa$  by increasing the photon loss rate  $\kappa$ . We note that, while it is beneficial to choose  $\delta_C$  small for sensing, it should be sufficiently large to ensure that backaction from a shift in the frequency due to the localization of atoms can be neglected.

As can be seen from the Allan deviation in Eq. (11), another choice is to change the platform for implementing the discrete model studied here. Specifically, the spacing between different sites could be increased to decrease  $\omega_{\text{rec}}$ , which means that the system no longer operates in the optical regime but in the near-infrared regime instead, for example. This suggests that this mechanism to detect changes in rotational frequencies is not limited to atom-cavity systems, but can be implemented in other light-matter coupled systems, with a different recoil energy of the atomic sector and to enclose even larger areas. This makes our approach tunable to the desired frequency domain in which the sensor is to operate.

Next, we discuss the random walk angle RW, which we define as

$$\text{RW} = \sigma(T_i)\sqrt{T_i} \propto \frac{\omega_{\text{rec}}}{M\sqrt{N_A}} \sqrt{\frac{\delta_C^3}{(\delta_C^2 + \kappa^2)\kappa J}}. \quad (12)$$

It describes the rate at which the sensitivity improves with the inverse square root of the interrogation time; i.e., it is the prefactor of the  $1/\sqrt{T_i}$  dependence. It is commonly presented in  $\text{deg}/\sqrt{\text{h}}$ .

We present numerical TWA results of the RW in the range from  $0.2g_{\text{crit}}$  to  $1.2g_{\text{crit}}$  and  $\theta \in (-0.45\pi, 0.45\pi)$ . As discussed above, the regime with very slow convergence for small  $g$  close to  $\theta = \pi/2 \pm n\pi$  with  $n \in \mathbb{N}$  is not beneficial for sensing applications and we will only focus on the area centered around  $\theta = 0$ . Figure 3 presents the numerical RW on a  $\log_{10}$  scale, for the same parameter as above. For regions in the normal phase, this value is not meaningful and we plot these points in dark blue. We find the smallest values of the RW close to the phase transition in the superradiant phase. With this set of parameters, we find values of RW down to  $-1.5$  on a  $\log_{10}$  scale.

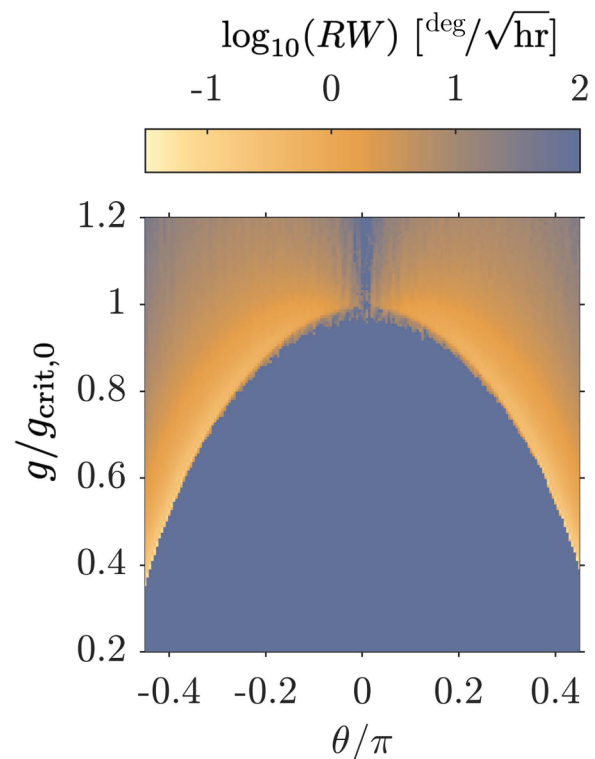


FIG. 3. Diagram of random walk RW for  $\theta \in (-0.45\pi, 0.45\pi)$  and the light-matter interaction  $g$  obtained with TWA using  $2 \times 10^2$  each trajectories for each point. The RW is plotted on a logarithmic scale and we set values within the NP to  $10^2$ .

In conclusion, we have presented an atom-cavity based mechanism for rotational sensing. Our proposed system utilizes the artificial gauge field that neutral atoms experience due to external rotation, and combines it with the superradiant phase transition of an array of atoms coupled to a single-mode cavity. This allows for a high-precision measurement of the rotation frequency due to the continuous, nondestructive measurement of photon emission. We have shown that, with realistic parameters, values of down to  $10^{-1.5}$  can be reached for the RW. We have further analytically shown that the Allan deviation scales as  $T_i^{-1/2}$  with the interrogation time due to white gaussian noise in the system. We further highlight the possibility that the sensitivity can be increased by increasing the enclosed area of the quantum sensor and note that by reaching  $n_s \approx 10^3$  and  $N_A \approx 10^5$  our proposed sensor reaches a random walk stability as commercial optical gyroscopes. In Fig. S2 of the Supplemental Material [46], we show an example for a slowly varying rotational frequency, where we note that real-time usage of our sensor mechanism will be explored elsewhere. In Fig. S3 of the Supplemental Material [46], we give an example for the effect of particle fluctuations.

We note that the influence of rotation on the superradiant phase transition can be observed in current state-of-the-art atom-cavity experiments [51,53] using an actuator giving a dynamical twist to the optical table. Our proposal puts forth to create quantum rotational sensors for static high-precision frequency measurements, with the goal of inertial measurements and navigation.

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The data displayed in this paper is available upon reasonable request from the authors.

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