

Reynolds number effect on dynamical state in a fully developed turbulent channel flow

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We investigate the Reynolds number effect on the dynamical state of a turbulent channel flow by means of orbital-instability-based forecasting method (OIFM) and ordinal partition transition network (OPTN). In the near-wall region, i.e., viscous sublayer and buffer layer, the coexistence of streaky structures with a spanwise spacing of about 100 wall units and large-scale structures with a spanwise spacing on the order of channel half-width decreases the predictive accuracy of streamwise velocity and increases the randomness of streamwise velocity. The low-dimensional chaotic state of streamwise velocity persists in the near-wall region regardless of the friction Reynolds number. In the far-wall region, the high-dimensional chaotic state of streamwise velocity emerges at low friction Reynolds numbers owing to the obvious low-Reynolds-number effect, whereas the low-dimensional chaotic state of streamwise velocity emerges at high friction Reynolds numbers owing to the formation of streamwise and spanwise large-scale structures, i.e., the large-scale motions. The chaotic state of the streamwise velocity at high friction Reynolds numbers is strongly associated with hierarchical structures. The formation of hierarchical structures can be clearly identified at high friction Reynolds numbers through the analyses of the premultiplied streamwise Reynolds stress spectrum and the instantaneous streamwise velocity fluctuations in the z - y plane, whereas it is hardly observed at low friction Reynolds numbers. The results of this study demonstrate that the OIFM and OPTN have high capabilities of elucidating the dynamics in turbulent flows and predicting their behavior.

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I. INTRODUCTION

Since the pioneering experiment conducted by Reynolds [1], wall-bounded turbulent flows, e.g., channel and pipe flows, have been widely investigated for more than 140 years. In particular, the marked improvement in computer performance over the past few decades enables us to study wall-bounded turbulent flows through direct numerical simulation (DNS), significantly promoting their understanding. Kim *et al.* [2] first performed the DNS of a turbulent channel flow at a relatively low Reynolds number of $Re_\tau = 180$, where Re_τ is the friction Reynolds number (explained in Sec. II). Subsequently, Moser *et al.* [3] conducted the DNS of turbulent channel flows up to $Re_\tau = 590$, showing that the obvious low-Reynolds-number effect vanishes at $Re_\tau = 590$. After these pioneering studies, the DNS of high-Reynolds-number turbulent channel flows has been energetically performed by

many researchers [4–14]: very recently, the DNS at the highest friction Reynolds number of $Re_\tau = 10\,000$ was conducted by Hoyas *et al.* [14].

These abovementioned DNSs have helped us understand the unique structures in high-Reynolds-number flows: the logarithmic law in the mean velocity profile and large-scale motions. The Reynolds number effect on the mean velocity profile U and the root mean square (RMS) of streamwise velocity fluctuations u_{rms} is shown in Fig. 1 using the DNS results obtained by Moser *et al.* [3] ($Re_\tau = 180$ – 590), Yamamoto and Tsuji [13] ($Re_\tau = 1000$ – 8000), and Hoyas *et al.* [14] ($Re_\tau = 10\,000$). As shown in Fig. 1(a), we observe a clear logarithmic region at $Re_\tau \geq 590$, extending the logarithmic region with the friction Reynolds number. In addition, u_{rms}^+ in the outer region increases with the friction Reynolds number; in particular, we observe that u_{rms}^+ is significantly enhanced in the region of $Re_\tau \geq 590$. Note that superscript $+$ represents wall units nondimensionalized by the friction velocity u_τ and the kinematic viscosity ν . According to Hoyas *et al.* [14], the second maximum of u_{rms} may appear around $y^+ \approx 120$ at $Re_\tau \approx 13\,500$ under the hypothesis that it exists. The trends for the mean velocity profile and RMS of streamwise velocity fluctuations have been considered to be due to large-scale motions. The role of large-scale motions in high-Reynolds-number flows has been a hot topic over many

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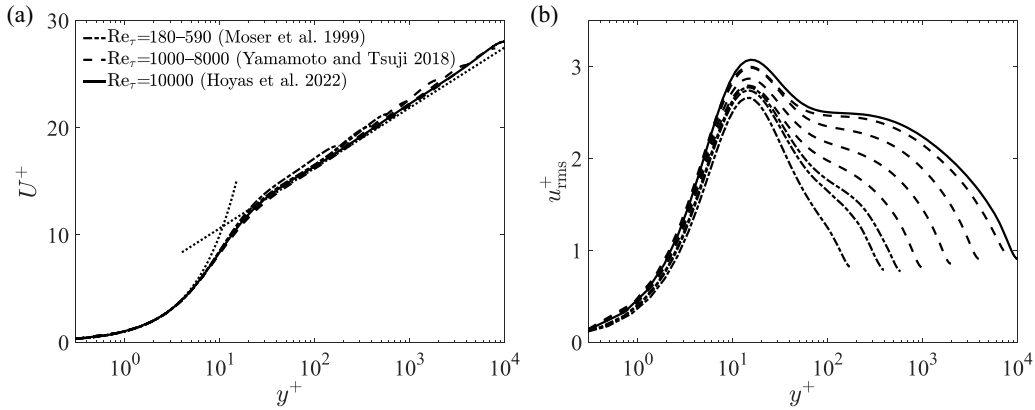


FIG. 1. Reynolds number effect on (a) mean velocity profile U and (b) RMS of streamwise velocity fluctuations u_{rms} in the uncontrolled flow: dash-dotted line, $Re_\tau = 180-590$ [3]; dashed line, $Re_\tau = 1000-8000$ [13]; solid line, $Re_\tau = 10000$ [14].

years [12,14–18]. However, these studies have not investigated how the large-scale motions are significantly related to the dynamical state of turbulent channel flows, focusing on the Reynolds number effect.

Nonlinear time series analysis inspired by the theories of chaos, fractal, and information has established a firm footing as a significant promising tool for extracting the deterministic nature inherent in complex aperiodic fluctuations arising from a wide variety of nonlinear systems [19–21]. The advent of nonlinear time series analysis has promoted the rapid growth of nonlinear physics and data science. Nonlinear time series analysis in accordance with the mathematical foundation of the delay-coordinate embedding approach [22] enables the quantification of important physical metrics responsible for the orbital instability and self-similar structure in phase space, e.g., the Lyapunov exponents and fractal dimension, involving various types of entropy in terms of dynamical complexity. The short-range forecastability and long-range unforecastability of a trajectory in the phase space due to the strong dependence on the initial conditions are the well-recognized characteristics of deterministic chaos due to orbital instability. On this basis, Gotoda *et al.* [23–25] proposed an orbital-instability-based forecasting method (OIFM), placing the emphasis on the conceptual framework of short-range forecastability and long-range unforecastability. This method has been shown to be effective for distinguishing between chaos and stochastic dynamics in noisy nonlinear extended systems describing phase turbulence [25,26]. In parallel with the development of nonlinear time series analysis, complex-network analysis [27,28] incorporating the framework from graph theory, which enables the quantification of a topological structure with connectivity patterns between nodes, has recently made progress toward an in-depth understanding and interpretation of computationally obtained complex turbulent dynamics [29–44]. These studies have led to the important disciplinary fusion between turbulent dynamics and network science. In relation to dynamical systems, a trajectory pattern in phase space can be linked to a finite string of symbolics [45]. Ordinal patterns considering the Markov partition-based symbolic string representations, which are defined by the rank order relationships between continuous points of trajectories, have been a focus of attention in network science [46]. The

ordinal partition transition network (OPTN) entropy proposed by Small [47] and McCullough *et al.* [48] has been shown to be a useful metric for quantifying the randomness of aperiodic fluctuations in various turbulent flows [33,40,41,49]. To reveal the nonlinear dynamics in a channel flow, we have recently investigated the dynamical state of a turbulent channel flow by OIFM and OPTN [40]. Low- and high-dimensional chaotic states emerge in the near-wall and far-wall regions, respectively [40]. Although this finding largely contributes to the understanding of nonlinear dynamics in wall-bounded flows, our previous study is restricted to a low-Reynolds-number condition: the obvious low-Reynolds-number effect is observed at $Re_\tau = 180$ [3]. Our main interest in this study is to reveal how the dynamical state of turbulent channel flows changes in the Reynolds number region where the obvious low-Reynolds-number effect vanishes, i.e., $Re_\tau \geq 590$. This important and challenging subject has been explored in not only many DNS studies on wall turbulence [2–14] but also various disciplines of fluid physics and nonlinear science.

On the basis of this background, the main objective of this study is to investigate the Reynolds number effect on the dynamical state of turbulent channel flows by OIFM and OPTN. In particular, we focus mainly on the effects of the coherent structures in the region near the wall and the large-scale structures in the region far from the wall on the dynamical state of streamwise velocity at relatively low to moderately high Reynolds numbers.

The rest of this paper is organized into three sections. In Sec. II, we summarize the numerical method of the present DNS and the analytical methods of OIFM and OPTN. In Sec. III, we investigate the Reynolds number effect on the dynamical state of turbulent channel flows and discuss how the coherent structures in the near-wall region and the large-scale structures in the far-wall region affect the dynamical state of streamwise velocity at $Re_\tau = 90-1000$. Finally, we conclude the present study in Sec. IV.

II. NUMERICAL AND ANALYTICAL METHODS

A. Direct numerical simulation

In this study, we consider a fully developed turbulent channel flow. The governing equations are the incompressible

TABLE I. Computational conditions in the present DNSs.

Re_τ	$L_x \times L_y \times L_z$	$N_x \times N_y \times N_z$	Δx^+	Δy^+	Δz^+	Δt^+
90	$8\pi \times 2 \times 4\pi$	$256 \times 96 \times 256$	8.8	0.5–3.0	4.4	0.036
180	$4\pi \times 2 \times 2\pi$	$256 \times 96 \times 256$	8.8	0.9–6.0	4.4	
590	$2\pi \times 2 \times \pi$	$384 \times 256 \times 384$	9.7	0.9–7.6	4.8	
1000	$2\pi \times 2 \times \pi$	$512 \times 384 \times 512$	11.1	0.7–8.8	6.1	

continuity and Navier-Stokes equations, i.e.,

$$\frac{\partial u_i}{\partial x_i} = 0, \quad (1)$$

$$\frac{\partial u_i}{\partial t} = -\frac{\partial(u_i u_j)}{\partial x_j} - \frac{\partial p}{\partial x_i} + \frac{1}{Re_\tau} \frac{\partial^2 u_i}{\partial x_j \partial x_j} - \frac{dP}{dx_1} \delta_{i1}, \quad (2)$$

where u_i ($i = 1, 2, 3$) and p denote the velocity components and the pressure, respectively. In addition, subscripts 1, 2, and 3 denote the streamwise, wall-normal, and spanwise directions, respectively. All variables are made dimensionless by the friction velocity u_τ^* , the channel half-width δ^* , and the fluid density ρ^* , where the superscript $*$ denotes dimensional quantities. The friction Reynolds number in Eq. (2) is defined as $Re_\tau = u_\tau^* \delta^* / \nu^*$, where ν^* is the kinematic viscosity. Hereafter, for convenience, (x_1, x_2, x_3) and (u_1, u_2, u_3) are interchangeably denoted by (x, y, z) and (u, v, w) .

The periodic boundary condition is applied in the streamwise (x) and spanwise (z) directions, whereas the no-slip boundary condition, i.e., $u = v = w = 0$, is imposed on the walls.

The DNS code used in this study is based on the channel flow code developed by Fukagata *et al.* [50], which has been extended by Nabae and Fukagata [51] and Nabae *et al.* [52]. The governing equations are spatially discretized using the energy-conserving fourth-order central difference scheme [53] in the streamwise and spanwise directions and the second-order central difference scheme on the staggered grid system. The time integration is performed using the low-storage, third-order Runge-Kutta/Crank-Nicolson scheme [54] with the high-order SMAC-like velocity-pressure coupling scheme [55]. The pressure Poisson equation is solved using the fast Fourier transform in the streamwise and spanwise directions and the tridiagonal matrix algorithm in the wall-normal direction.

All simulations in this study are performed under a constant pressure gradient condition. We consider four different friction Reynolds numbers, i.e., $Re_\tau = 90, 180, 590$, and 1000. The computational conditions are summarized in Table I. Note that L_i , N_i , and Δx_i represent the length of the computational domain, the number of grid points, and the grid spacing, respectively, in the i th direction, and Δt is the time resolution.

To validate the present DNS code, we compare the turbulence statistics in the uncontrolled flow with those in the reference DNS by Moser *et al.* [3] ($Re_\tau = 180$ and 590), Lee and Moser [12] ($Re_\tau = 1000$), and Yamamoto and Tsuji [13] ($Re_\tau = 1000$). Figure 2 shows the mean velocity profile and RMS of velocity fluctuations in the uncontrolled flow at three different friction Reynolds numbers, together with the reference DNS results. The present results are in good agreement

with each reference DNS result. Therefore, the computational condition in this study is determined to be sufficient.

B. Orbital-instability-based forecasting method

In this study, we utilize the OIFM introduced by Gotoda *et al.* [23], which is an extended version of the Sugihara-May algorithm [56]. This method has been shown to be valid for extracting the short-range predictability and long-range unpredictability of chaotic dynamics in various complex nonlinear phenomena [24,25,33,40,57].

The computational conditions of OIFM in this study are basically the same as those utilized by Mamori *et al.* [40]. First, the data of time series are divided into the library and reference data. The library and reference data correspond to the time series of streamwise velocity in $0 \leq t^+ \leq 5000$ and $5000 \leq t^+ \leq 9000$, respectively. We construct the D_e -dimensional phase space consisting of a vector given as $\mathbf{u}_p = [u(t_i^+), u(t_i^+ + \tau^+), \dots, u(t_i^+ + \tau^+(D_e - 1))]^T$ (τ is the delay time in the phase space) using the library data and then obtain a vector of the predicted point after t_p^+ of a trajectory in the phase space $\hat{\mathbf{u}}_p(t_f^+ + t_p^+)$ as

$$\hat{\mathbf{u}}_p(t_f^+ + t_p^+) = \frac{\sum_{k=1}^K \mathbf{u}_p(t_k^+ + t_p^+) \exp(-\|\mathbf{u}_p(t_f^+) - \mathbf{u}_p(t_k^+)\|)}{\sum_{k=1}^K \exp(-\|\mathbf{u}_p(t_f^+) - \mathbf{u}_p(t_k^+)\|)}, \quad (3)$$

where $\mathbf{u}_p(t_k^+)$ is a point near the final point $\mathbf{u}_p(t_f^+)$ of the trajectory in the phase space and K is the number of nearby points. In this study, the library data are updated to capture the dynamics in the time evolution of streamwise velocity, maintaining the size of the library data constant. Here, t_p and τ correspond to the predictable time and the time of sampling interval, respectively. To investigate the short-range predictability and long-range unpredictability characteristics of streamwise velocity, we calculate the correlation coefficient C between the predicted data $\hat{\mathbf{u}}_p(t_f^+ + t_p^+)$ and the reference data $\mathbf{u}_p(t_f^+ + t_p^+)$ as a function of t_p^+ or τ at each y . Mamori *et al.* [40] determined D_e on the basis of a preliminary study on missing patterns of a time series [58] and the false nearest neighboring method [59]. In addition, we have confirmed the little effect of K on C in the range of $20 \leq K \leq 200$ (see Appendix A for details). Therefore, we set D_e to 5 and K to 50, which are the same as those used by Mamori *et al.* [40].

C. Ordinal partition transition network

To quantitatively assess the degree of randomness in the time series of streamwise velocity, we utilize the information entropy in OPTN, i.e., the OPTN entropy [47,48]. The OPTN entropy S_t can be calculated as

$$S_t = -\frac{\sum_{i=1}^{D_t!} \sum_{j=1}^{D_t!} p_t(\Pi_i \rightarrow \Pi_j) \ln p_t(\Pi_i \rightarrow \Pi_j)}{\ln(D_t!^2)}, \quad (4)$$

where $p_t(\Pi_i \rightarrow \Pi_j)$ is the transition probability from Π_i ($i = 1, 2, \dots, D_t!$) to Π_j ($j = 1, 2, \dots, D_t!$) for each velocity component with Π_i denoting the permutation pattern. To obtain a suitable value of D_t , Mamori *et al.* [40] and Iseki *et al.* [41] utilized the method proposed by Nomi *et al.* [57],

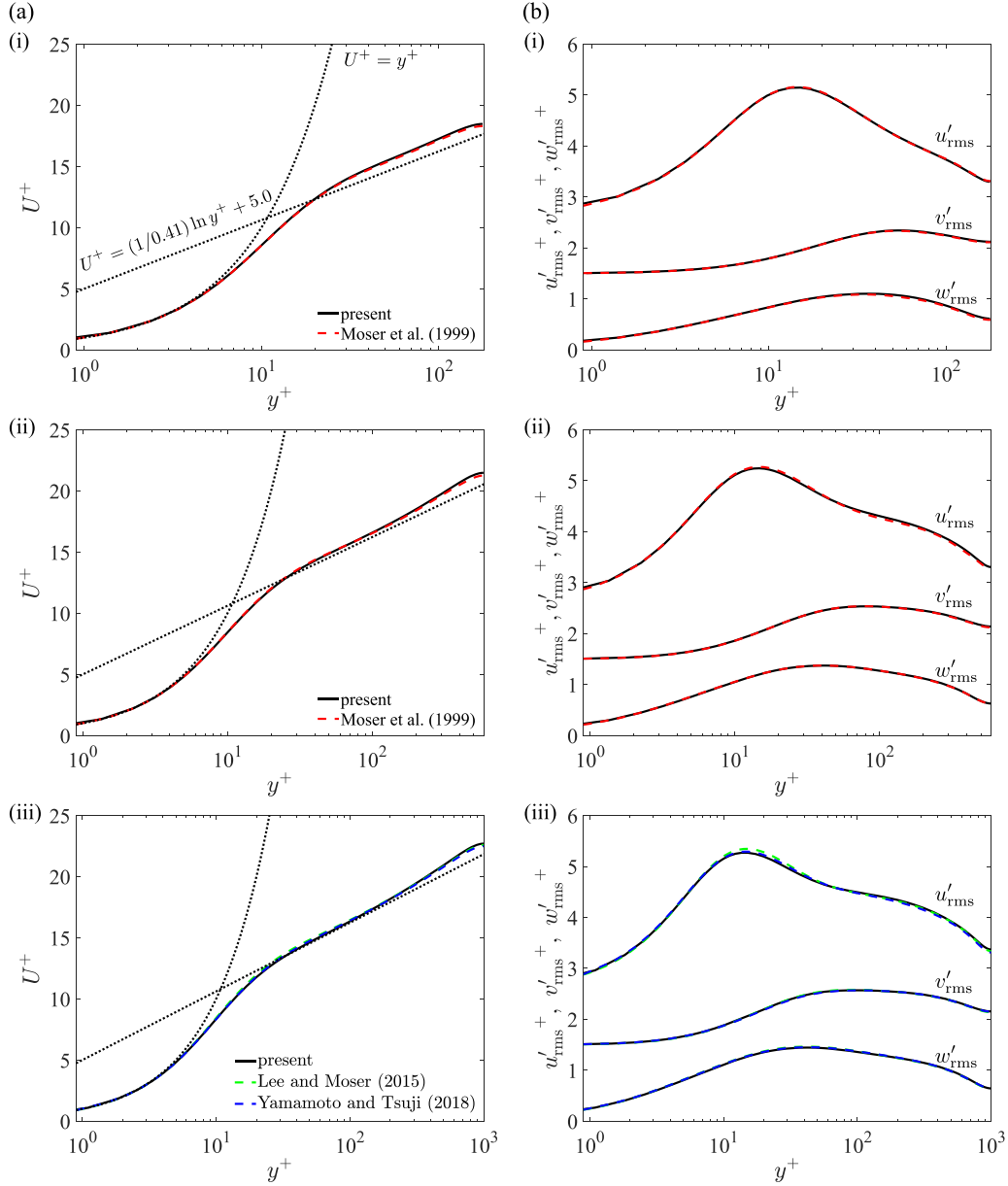


FIG. 2. (a) Mean velocity profile and (b) RMS of velocity fluctuations in the uncontrolled flow: (i) $\text{Re}_\tau = 180$; (ii) $\text{Re}_\tau = 590$; and (iii) $\text{Re}_\tau = 1000$. Black line, present; red line, Moser *et al.* [3]; green line, Lee and Moser [12]; blue line, Yamamoto and Tsuji [13]. To be easily viewable, 1.5 and 2.5 are added to v'^+_{rms} and u'^+_{rms} , respectively.

which considers missing permutation patterns of a time series [60]. According to Mamori *et al.* [40] and Iseki *et al.* [41], the missing permutation pattern of a time series appears at $D_t \geq 4$ for Gaussian noise and fractional Brownian motion, which means that the value at $D_t \geq 4$ is not suitable for the construction of OPTN. Thus, we set D_t to 3 in this study. The OPTN entropy S_t takes the value in the range of $0 \leq S_t \leq 1$. In the case where the temporal evolution of streamwise velocity is monotonically increasing or decreasing, S_t takes zero: only one transition permutation pattern is observed. In the case where the temporal evolution of streamwise velocity is governed by a completely random process, S_t takes unity: a uniform transition probability distribution for all permutation patterns is formed.

III. RESULTS AND DISCUSSION

On the basis of a very classical estimation by Pope [61], we define the near-wall region and far-wall region as $y^+ \lesssim 30$ and $y^+ > 30$, respectively. The near-wall region corresponds to the viscous sublayer and buffer layer, whereas the far-wall region corresponds to the logarithmic layer and outer layer. Note that the logarithmic layer exists only when $\text{Re}_\tau \gtrsim 500$ [62,63].

Figure 3 shows the time history of streamwise velocity and the streak structures at three different wall-normal locations, i.e., $y^+ \approx 4, 20$, and 400 in the case of $\text{Re}_\tau = 1000$. Here, we utilize the temporal evolution of streamwise velocity u at the center of each x - z plane, i.e., $x = L_x/2$, $0 \leq y < 1$, and $z = L_z/2$. Note that we have confirmed little dependence of spatial

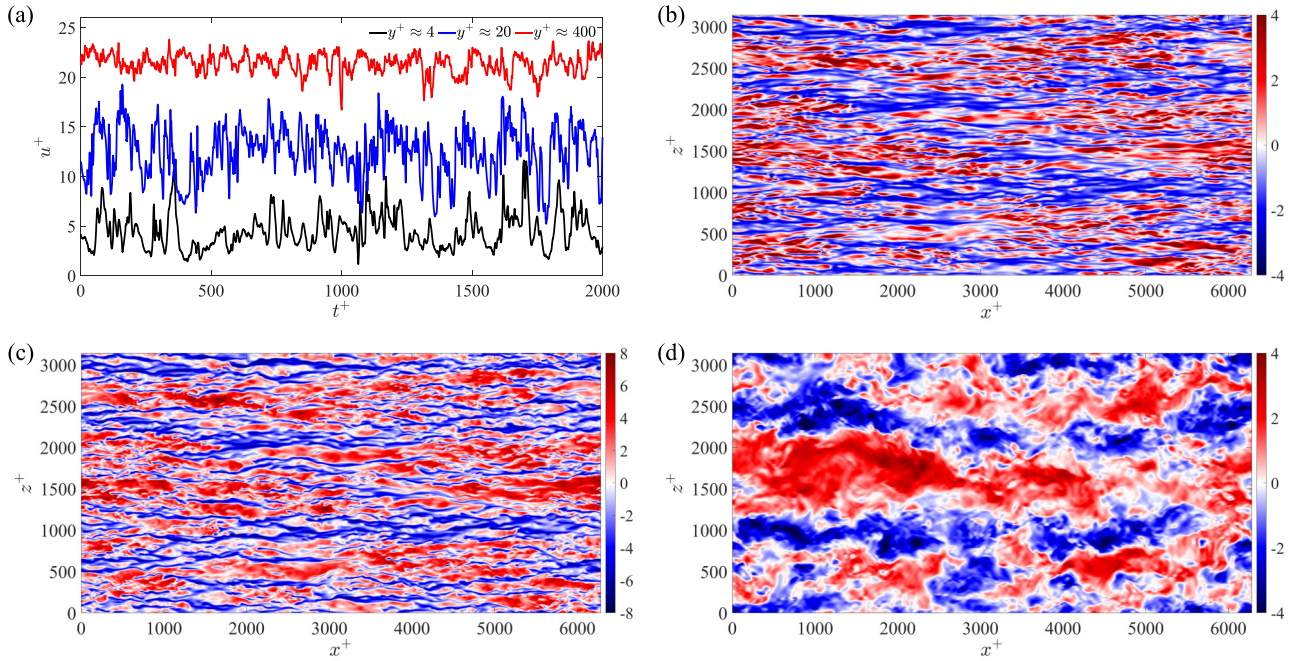


FIG. 3. (a) Time history of streamwise velocity u^+ (black, $y^+ \approx 4$; blue, $y^+ \approx 20$; red, $y^+ \approx 400$), and (b)–(d) instantaneous streamwise velocity fluctuations u'^+ (i.e., streak structures) in the x – z plane at $\text{Re}_\tau = 1000$: (b) $y^+ \approx 4$; (c) $y^+ \approx 20$; (d) $y^+ \approx 400$.

location obtaining the time evolution of u on the results (not shown here). As shown in Fig. 3(a), the streamwise velocity u varies irregularly regardless of y^+ . In the near-wall region, i.e., the viscous sublayer and buffer layer [Figs. 3(b) and 3(c)], we observe the well-known streak structures with a streak spacing of about 100 wall unit length. The amplitude of u' in the buffer layer is larger than that in the viscous sublayer owing to active turbulent motions such as the regeneration cycle [64] in the buffer layer. The Reynolds number effect on these near-wall coherent structures is relatively small, so that the similar trend is observed at $\text{Re}_\tau = 90$ –590 (not shown here). In contrast, in the far-wall region [Fig. 3(d)], both the streamwise scale of streaks and the streak spacing in the spanwise direction are much larger than those in the region near the wall: the streak spacing is approximately equal to the channel half-width (i.e., Re_τ in wall unit). These large-scale structures, which are the so-called large-scale motions (LSMs), are clearly observed in the Reynolds number region where the obvious low-Reynolds-number effect is significantly smaller: Moser *et al.* [3] showed that the obvious low-Reynolds-number effect vanishes at $\text{Re}_\tau = 590$. As similarly observed by Moser *et al.* [3], the LSMs are clearly formed at $\text{Re}_\tau = 590$ and 1000.

Figures 4 and 5 show the correlation coefficient C between the original and predicted streamwise velocities as a function of the predictable time t_p^+ and the time histories of the original and predicted streamwise velocities at the representative predictable time, i.e., $t_p^+ = 1.08$, 10.8, and 108, respectively, when the friction Reynolds numbers are $\text{Re}_\tau = 180$ and 1000. Here, similarly to Mamori *et al.* [40], the sampling interval in Figs. 4–7 is set to $\tau^+ = 1.08$. As shown in Fig. 4, in the case of $\text{Re}_\tau = 180$, C at $t_p^+ = \tau^+ = 1.08$ is approximately unity in the near-wall region, i.e., $y^+ \approx 4$ and 20. Subsequently, C decreases exponentially as t_p^+ increases, representing the

region of short-range forecastability and long-range unforecastability. This trend is a typical characteristic of chaos based on orbital instability in phase space. In the far-wall region, i.e., $y^+ \approx 150$, C markedly decreases in the entire region of t_p^+ compared with that in the near-wall region ($y^+ \approx 4$ and 20), indicating that the OIFM is difficult to predict the streamwise velocity even in the case of small t_p^+ . These mean that the dynamical states of streamwise velocity are the low- and high-dimensional chaotic states in the region near the wall and far from the wall, respectively, which are the similar results reported by Mamori *et al.* [40]. In the case of $\text{Re}_\tau = 1000$, the trends of C in the near-wall region, i.e., the viscous sublayer and buffer layer ($y^+ \approx 4$ and 20), are similar to those in the case of $\text{Re}_\tau = 180$. On the other hand, C in the far-wall region is larger than that in the buffer layer, showing the short-range forecastability and long-range unforecastability of the streamwise velocity in the far-wall region at $\text{Re}_\tau = 1000$. As shown in Fig. 5, in the case of $t_p^+ = 1.08$, the OIFM can well predict the original u regardless of the wall-normal location y^+ and the friction Reynolds number Re_τ , whereas we observe the significant difference between the original and predicted u in the case of $\tau_p^+ = 108$. In the case of $t_p^+ = 10.8$, although the failure of predicting u is observed at $\text{Re}_\tau = 180$, the OIFM can reasonably predict the original u at $\text{Re}_\tau = 1000$.

To investigate the Reynolds number effect on the dynamical state of streamwise velocity in more detail, Fig. 6 shows the contour of the correlation coefficient C between the original and predicted streamwise velocities as a function of t_p^+ and y^+ at four different friction Reynolds numbers, i.e., $\text{Re}_\tau = 90$, 180, 590, and 1000. Here, the weights in Eq. (3) are defined by the exponential function of the distance between state vectors, so that the predicted results may depend on the scaling of the streamwise velocity. To evaluate this sensitivity,

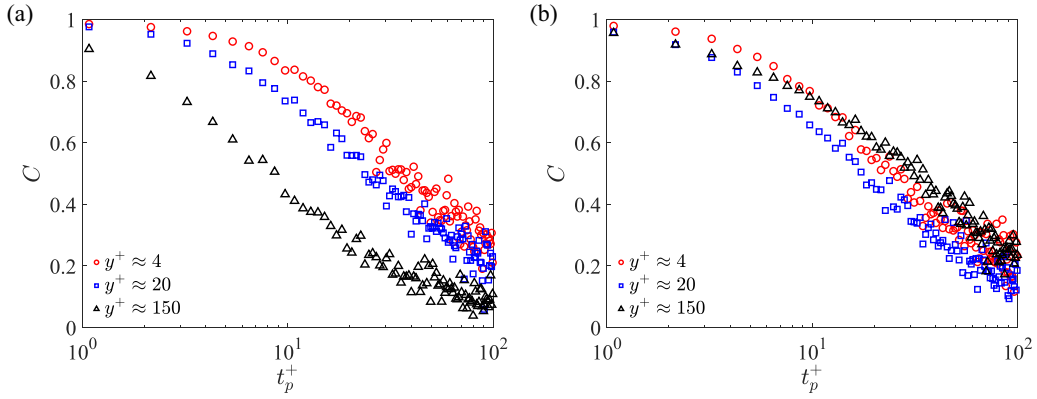


FIG. 4. Correlation coefficient C between the original and predicted streamwise velocities at $y^+ \approx 4, 20$, and 150 as a function of the predictable time t_p^+ : (a) $\text{Re}_\tau = 180$; (b) $\text{Re}_\tau = 1000$. The red, blue, and black symbols represent $y^+ \approx 4, 20$, and 150 , respectively. Here, the sampling interval is $\tau^+ = 1.08$.

we consider an alternative definition in which the streamwise velocity is normalized by the local mean streamwise velocity at each y^+ . The results obtained with this normalization exhibit trends consistent with those shown in Fig. 6, demonstrating that the main conclusions are relatively robust with respect to the choice of velocity scaling. As shown in Fig. 6, at lower friction Reynolds numbers, i.e., $\text{Re}_\tau = 90$ and 180 ,

we observe the high short-range forecastable region near the wall, i.e., $y^+ \lesssim 10$. Subsequently, this region exponentially decreases from the buffer layer to the outer layer, indicating the short- and long-range unforecastability of u far from the wall. Mamori *et al.* [40] reported similar results at $\text{Re}_\tau = 180$, indicating that the Reynolds number effect on the dynamical state of u is small in the Reynolds number region where the

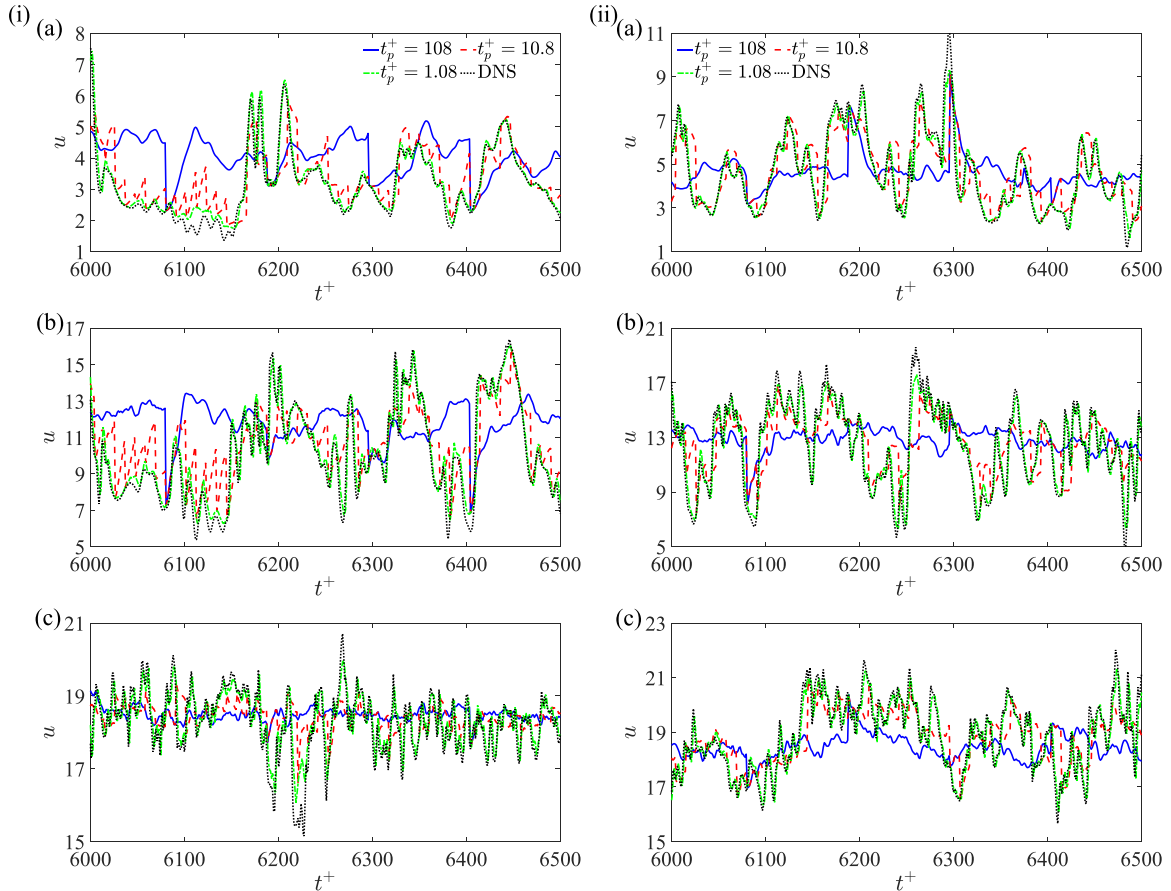


FIG. 5. Time histories of the original and predicted streamwise velocities at $y^+ \approx 4, 20$, and 150 : (i) $\text{Re}_\tau = 180$; (ii) $\text{Re}_\tau = 1000$; (a) $y^+ \approx 4$; (b) $y^+ \approx 20$; (c) $y^+ \approx 150$. The green, red, and blue lines represent the predicted u in the cases of $t_p^+ = 1.08, 10.8$, and 108 , respectively, and the black line represents the original u obtained by the DNS. Here, the sampling interval is $\tau^+ = 1.08$.

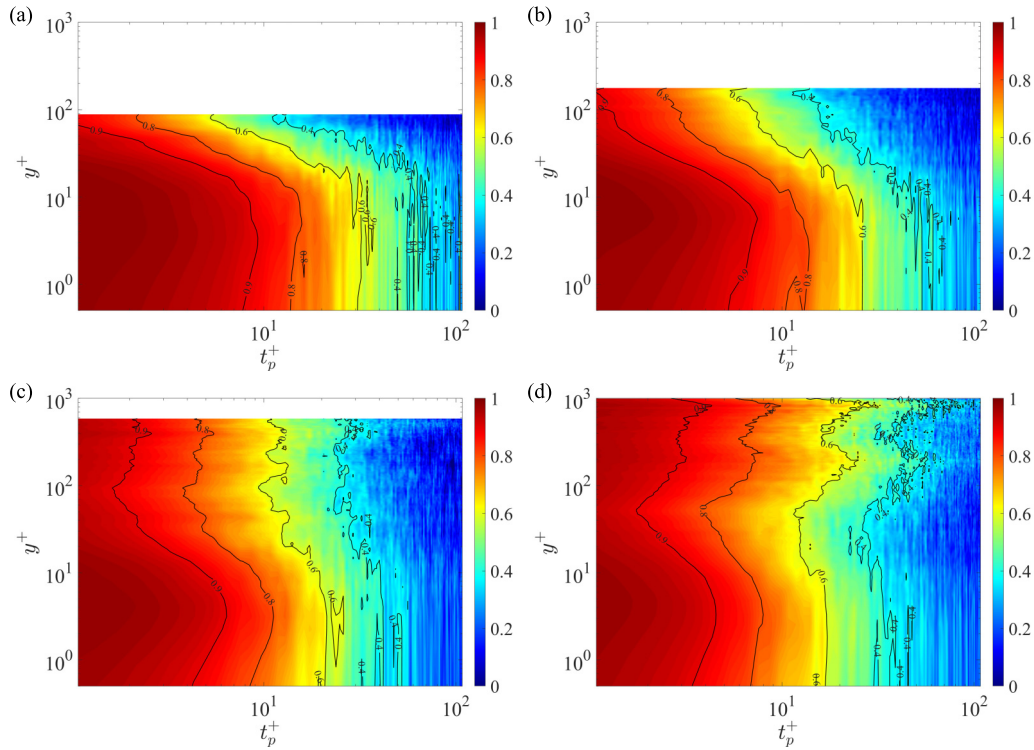


FIG. 6. Contours of the correlation coefficient C between the original and predicted streamwise velocities as functions of the predictable time t_p^+ and the wall-normal coordinate y^+ : (a) $\text{Re}_\tau = 90$; (b) $\text{Re}_\tau = 180$; (c) $\text{Re}_\tau = 590$; (d) $\text{Re}_\tau = 1000$. Here, the sampling interval is $\tau^+ = 1.08$.

low-Reynolds-number effect is clearly observed [3]. At higher friction Reynolds numbers, i.e., $\text{Re}_\tau = 590$ and 1000 , the short-range forecastable region near the wall decreases compared with those in the cases of $\text{Re}_\tau = 90$ and 180 . Although this region decreases from the buffer layer to the beginning of the far-wall region similarly to the cases of $\text{Re}_\tau = 90$ and 180 , we observe its extension in the far-wall region at $y^+ \gtrsim 100$. This increase in C means that u in the far-wall region at a higher Re_τ is more predictable than that at a lower Re_τ . In other words, the results shown in Figs. 4–6 imply that the low-dimensional chaotic state of streamwise velocity emerges in the far-wall region at higher Reynolds numbers.

Figure 7 shows the critical predictable time $t_{p,c}^+$ and OPTN entropy S_t for the original streamwise velocity as functions of y^+ at $\text{Re}_\tau = 90$ – 1000 . Similarly to the previous study [40], we define the critical predictable time $t_{p,c}^+$ as the predictable time at $C \approx 0.9$. Note that the trend of $t_{p,c}^+$ in the case where the threshold of correlation coefficient is lower, i.e., $C \approx 0.8$ and 0.85 , is similar to that in the case where the threshold is set to $C \approx 0.9$. In the near-wall (far-wall) region, $t_{p,c}^+$ decreases (increases) as Re_τ increases, which corresponds to the decrease (increase) in the short-range forecastable region shown in Fig. 6. In addition, in the region near the wall (especially, in the buffer layer), S_t at higher Reynolds numbers (i.e.,

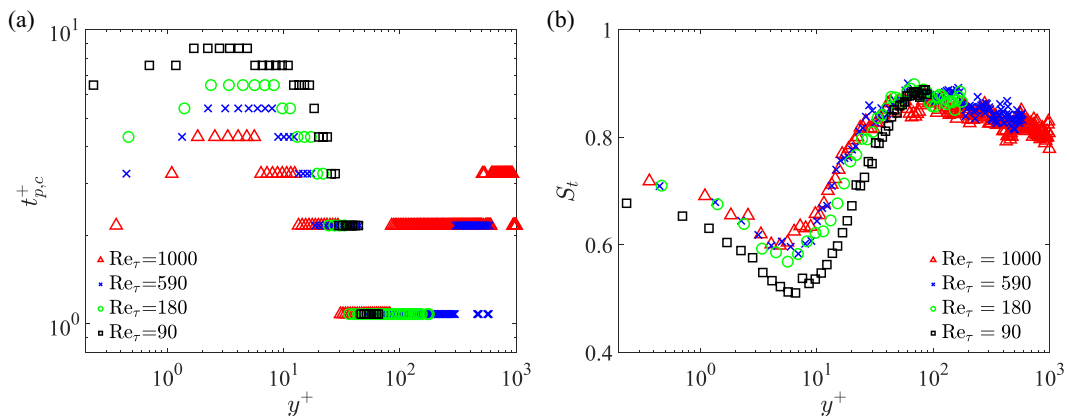


FIG. 7. (a) Critical predictable time $t_{p,c}^+$ and (b) OPTN entropy S_t for the original streamwise velocity as functions of y^+ at $\text{Re}_\tau = 90$ – 1000 . Black, $\text{Re}_\tau = 90$; green, $\text{Re}_\tau = 180$; blue, $\text{Re}_\tau = 590$; red, $\text{Re}_\tau = 1000$. Here, the sampling interval is $\tau^+ = 1.08$.

$Re_\tau = 590$ and 1000) is larger than that at lower Reynolds numbers (i.e., $Re_\tau = 90$ and 180). According to the previous studies for a high-Reynolds-number turbulent channel flow (i.e., $Re_\tau > 1000$) [7,65–70], the streaky structures with a spanwise spacing of about 100 wall units exist only near the wall ($y^+ < 30$), whereas the large-scale structures with a spanwise spacing on the order of channel half-width are observed in a wide range from the channel center ($y/\delta \approx 1$) to the near-wall region ($y^+ \approx 30$). Subsequently, this coexistence of streaky and large-scale structures in the region near the wall increases the amplitude of u_{rms} near the wall [also shown in Fig. 1(b)], resulting in a decrease in the predictive accuracy of u and an increase in the randomness of u . In the buffer layer, S_i and $t_{p,c}^+$ rapidly increase and decrease with y^+ , respectively. According to previous studies [41,71,72], streaks actively coalesce and separate in the buffer layer. The active coalescence and separation of streaks make the flow field complex, leading to the decrease in the predictive accuracy of u and the increase in the randomness of u . In the far-wall region, S_i at $Re_\tau = 90$ and 180 remains nearly unchanged, whereas that at $Re_\tau = 590$ and 1000 mildly decreases with increasing y^+ . In addition, roughly speaking, S_i in the far-wall region decreases as Re_τ increases. In the Reynolds number region where the obvious low-Reynolds-number effect vanishes, we clearly observe the formation of the logarithmic layer (Fig. 1) and LSMs in the region far from the wall (Fig. 3). These findings imply that the clear formations of the logarithmic layer and the LSMs decrease the randomness of u in the far-wall region, leading to the low-dimensional chaotic state of u in the far-wall region at higher friction Reynolds numbers.

The correlation dimension, which is a type of fractal dimension, is an important nonlinear metric for evaluating self-similarity in phase space as a key geometrical feature of chaos. The Grassberger-Procaccia (GP) method [73,74] is one of the most widely recognized methods for inferring the correlation dimension and has been applied in some studies of turbulent flows [75–78]. However, it often yields false estimates of the correlation dimension due to the difficulty of extracting a reliable scaling region even for the high-dimensional phase space. To resolve this problem, the prototype of OIFM was proposed by Sugihara and May [56], instead of the GP method. On the other hand, it is insufficient for fully understanding turbulent flow solely by a single fractal dimension. Turbulent flow possesses multifractal structures in the complex spatiotemporal flow field. A multifractal structure exhibits diverse self-similarities in phase space, which are commonly described by a singularity spectrum. The singularity spectrum representing the fractal dimensions of interwoven sets in the attractor allows us to comprehend the multifractal structures. In our next study, we aim to associate the predictability of trajectories in phase space with the multifractality by investigating how the singularity spectrum varies over a broad range of Reynolds numbers.

Figure 8 shows the contours of the OPTN entropy S_i for the original and predicted streamwise velocities as functions of τ^+ and y^+ at $Re_\tau = 90$ – 1000 , where the predictable time is equal to the sampling interval, i.e., $t_p^+ = \tau^+$. Note that the correlation coefficient C as a function of τ^+ and y^+ has the

similar trend shown in Fig. 6 (see Appendix B for details). Focusing on S_i for the original streamwise velocity shown in Fig. 8(i), the region where S_i is relatively small is extended in the far-wall region at $Re_\tau = 590$ and 1000 , whereas we observe no such extension of S_i at $Re_\tau = 90$ and 180 . In other words, the effect of coarse graining for u at higher Reynolds numbers is smaller than that at lower Reynolds numbers, implying the effect of the clear formations of the logarithmic layer and the LSMs on the dynamical state of u . Furthermore, the distribution of S_i for the original u is in reasonable agreement with that for the predicted u at all Re_τ , indicating that u predicted by the OIFM preserves the randomness of u obtained by the DNS. Thus, the OPTN entropy is useful for assessing the predictive accuracy of u in terms of randomness at both low and high friction Reynolds numbers.

To furthermore investigate the effect of LSMs in the region far from the wall, we compare the two-dimensional premultiplied spectra of the streamwise Reynolds stress normalized by the maximum value at each wall-normal coordinate $\phi_{uu}/\phi_{uu,max}$ in Fig. 9. The premultiplied spectrum of streamwise Reynolds stress is defined as

$$\phi_{uu}(\lambda_x, \lambda_z) = k_x k_z E_{uu}(\lambda_x, \lambda_z), \quad (5)$$

where λ_x (k_x) and λ_z (k_z) represent the wavelength (wave number) in the streamwise and spanwise directions, respectively. In addition, $E_{uu} = \text{Re}\langle \tilde{u}' \tilde{u}'^\# \rangle$ is the streamwise Reynolds stress spectrum, where the tilde represents the Fourier-transformed quantities and the superscript # represents the complex conjugate. Note that summation is not taken for Eq. (5). In the near-wall region, i.e., the viscous sublayer ($y^+ \approx 4$) and buffer layer ($y^+ \approx 20$), we observe the extension of the region where $\phi_{uu}/\phi_{uu,max}$ becomes relatively large with increasing Re_τ . This is due to the coexistence of streaky and large-scale structures at higher Reynolds numbers. In contrast, in the far-wall region ($y^+ \approx 150$ and 400), the distribution of $\phi_{uu}/\phi_{uu,max}$ shifts toward the region where the streamwise and spanwise wavelengths become large as Re_τ increases. This clearly shows the obvious effect of LSMs at higher Reynolds numbers. Figure 10 shows the instantaneous u' in the z - y plane at $Re_\tau = 90$ – 1000 . At higher Reynolds numbers, i.e., $Re_\tau = 590$ and 1000 , we qualitatively observe the streaky structures with a spanwise spacing of about 100 wall units only near the wall and the large-scale structures with a spanwise spacing on the order of channel half-width in a wide range from the channel center to the near-wall region, which corresponds to the hierarchical structures formed in a high-Reynolds-number turbulent channel flow [7]. On the other hand, no hierarchical structures are observed at lower Reynolds numbers, i.e., $Re_\tau = 90$ and 180 .

As shown in Figs. 3–10, the large-scale structures in a high-Reynolds-number flow significantly affect the dynamical state of the streamwise velocity in the entire region of a channel. In the near-wall region (the viscous sublayer and buffer layer), the coexistence of streaky and large-scale structures decreases the predictive accuracy of streamwise velocity and increases the randomness of streamwise velocity at higher Reynolds numbers. However, the low-dimensional chaotic state of streamwise velocity persists in the near-wall region

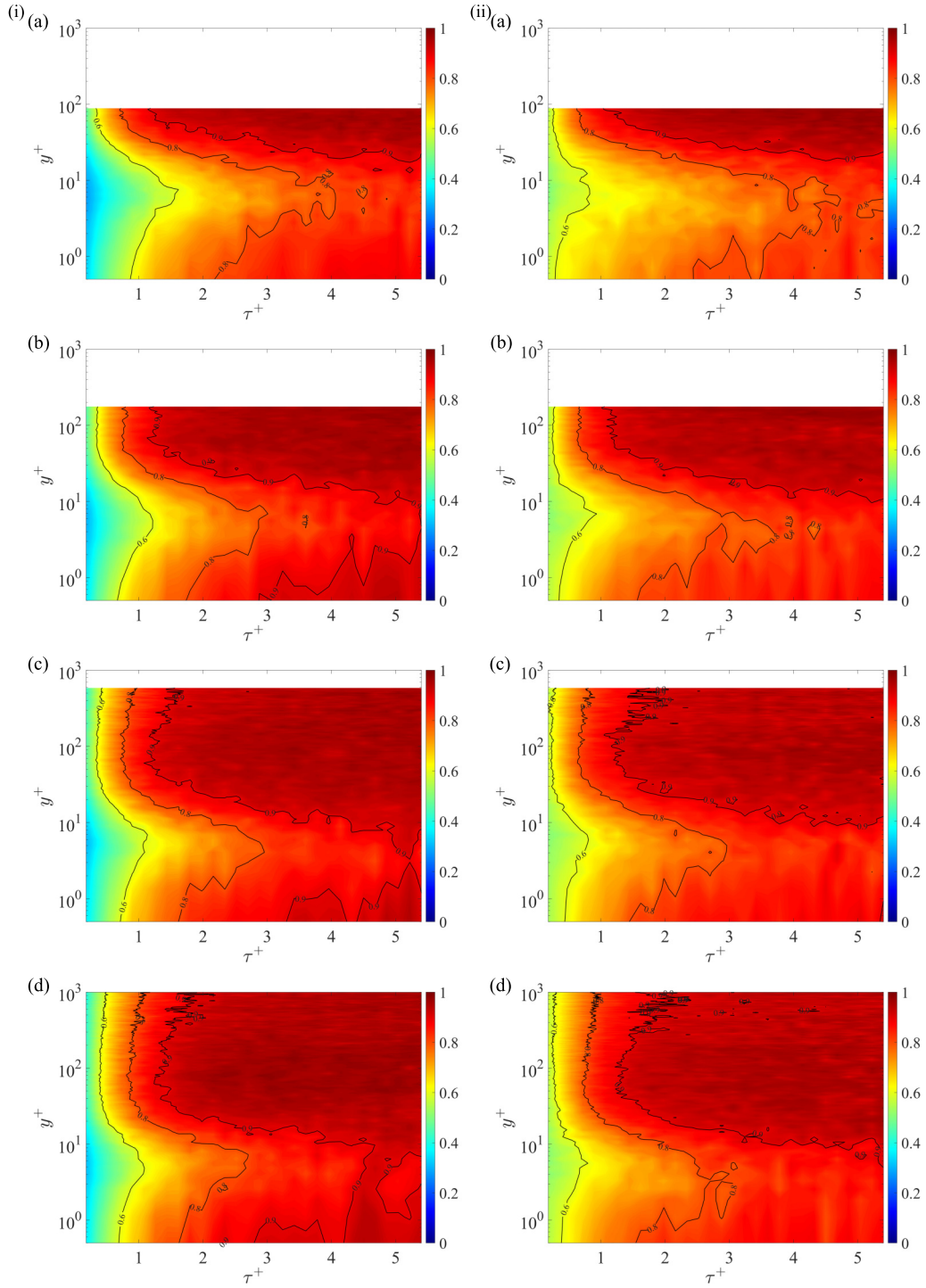


FIG. 8. Contours of the OPTN entropy S_t for the original and predicted streamwise velocities as functions of the sampling interval τ^+ and the wall-normal coordinate y^+ at $\text{Re}_\tau = 90$ –1000: (i) original; (ii) predicted; (a) $\text{Re}_\tau = 90$; (b) $\text{Re}_\tau = 180$; (c) $\text{Re}_\tau = 590$; (d) $\text{Re}_\tau = 1000$. Here, the predictable time is equal to the sampling interval, i.e., $t_p^+ = \tau^+$.

regardless of the friction Reynolds number. Focusing on the region far from the wall, the high-dimensional chaotic state of streamwise velocity emerges at lower friction Reynolds numbers owing to the obvious low-Reynolds-number effect. In contrast, the low-dimensional chaotic state of streamwise velocity emerges at higher friction Reynolds numbers owing

to the formation of the streamwise and spanwise large-scale structures, i.e., the LSMs. In summary, the chaotic state of the streamwise velocity at higher Reynolds numbers is strongly associated with the hierarchical structures, which are identified in a high-Reynolds-number turbulent channel flow [7].

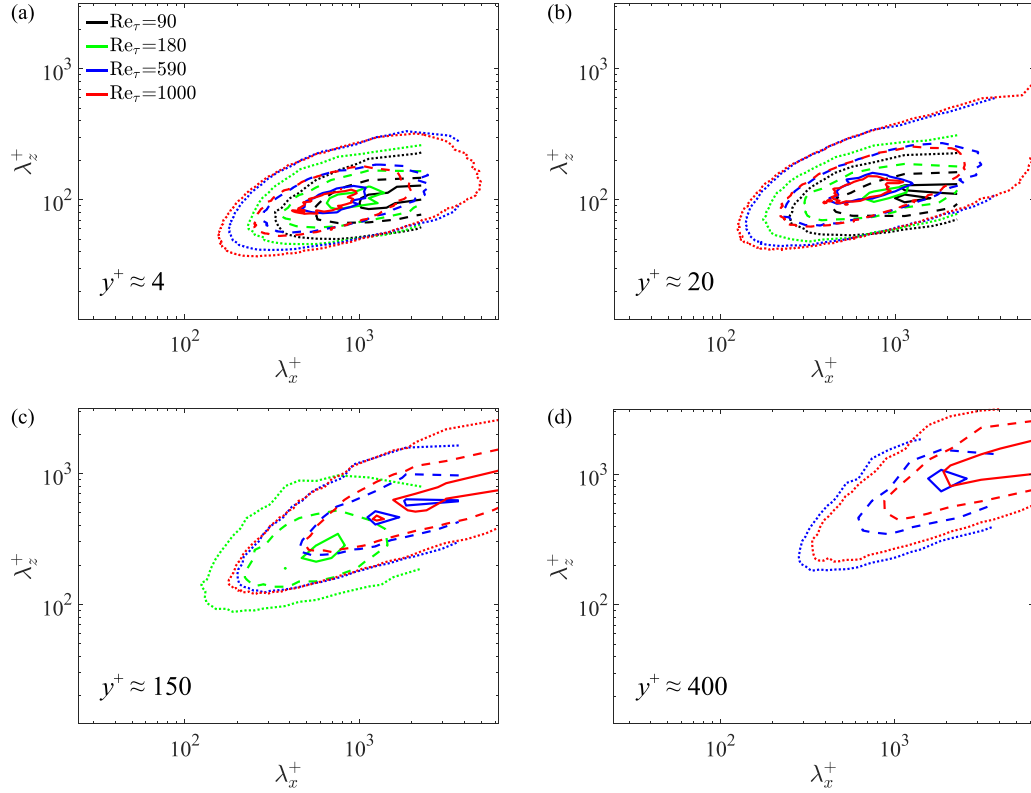


FIG. 9. Premultiplied streamwise Reynolds stress spectra normalized by the maximum value at each wall-normal coordinate $\phi_{uu}/\phi_{uu,\max}$: (a) $y^+ \approx 4$; (b) $y^+ \approx 20$; (c) $y^+ \approx 150$; (d) $y^+ \approx 400$. Black, $\text{Re}_\tau = 90$; green, $\text{Re}_\tau = 180$; blue, $\text{Re}_\tau = 590$; red, $\text{Re}_\tau = 1000$. The solid, dashed, and dotted lines represent $\phi_{uu}/\phi_{uu,\max} = 0.9, 0.6$, and 0.3 , respectively.

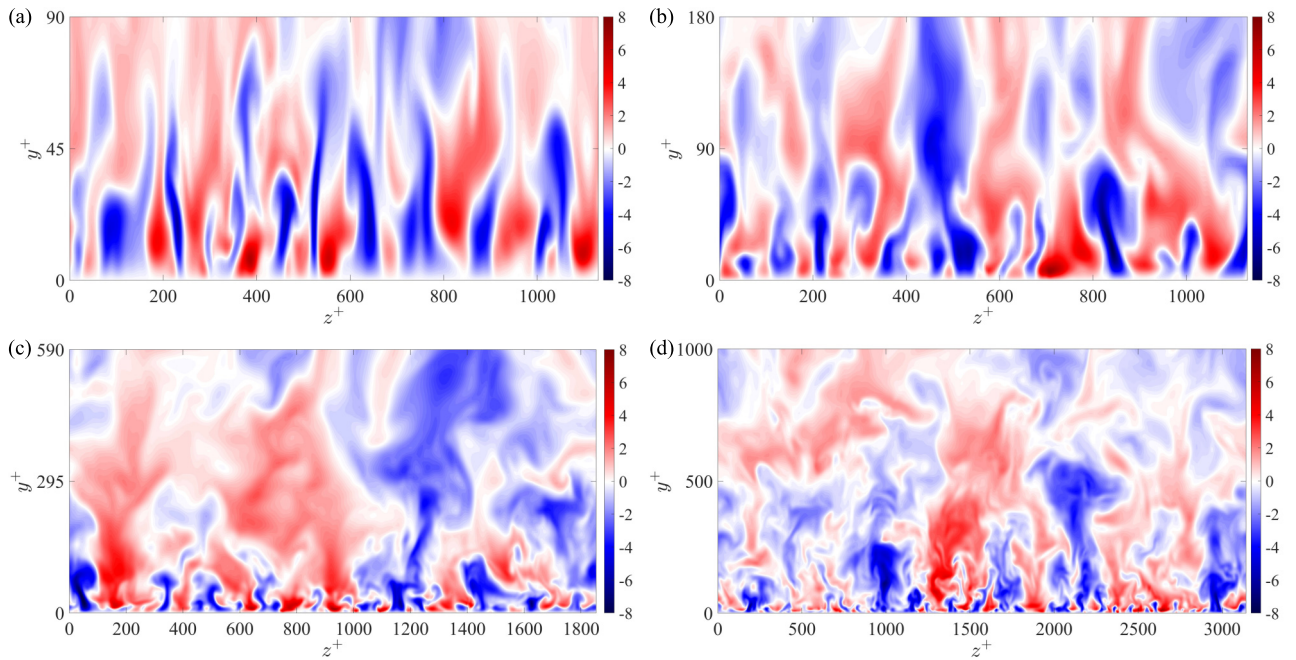


FIG. 10. Instantaneous streamwise velocity fluctuations u' in the z - y plane at $\text{Re}_\tau = 90$ – 1000 : (a) $\text{Re}_\tau = 90$; (b) $\text{Re}_\tau = 180$; (c) $\text{Re}_\tau = 590$; (d) $\text{Re}_\tau = 1000$.

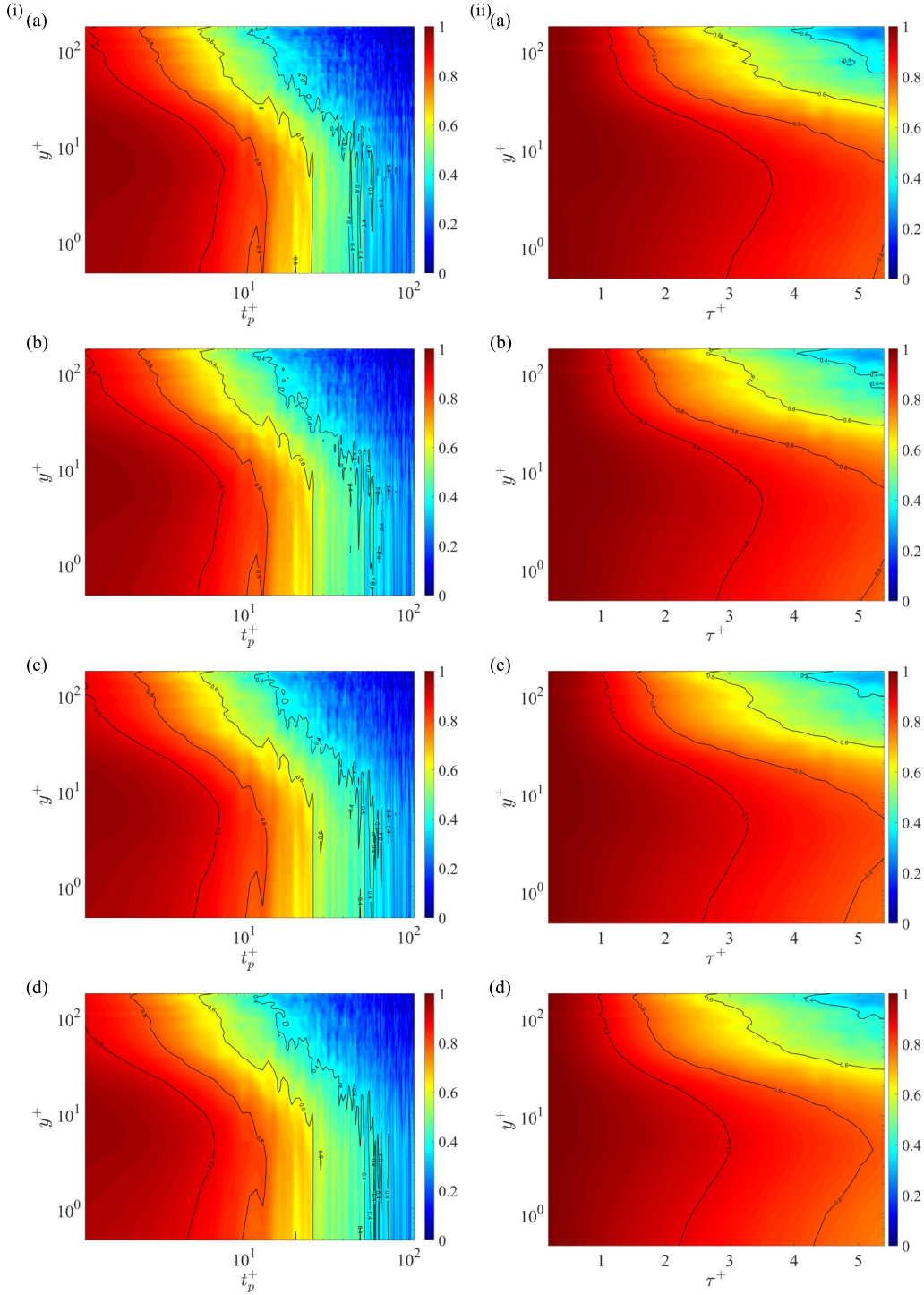


FIG. 11. Effect of K on the correlation coefficient C between the original and predicted streamwise velocities at $\text{Re}_\tau = 180$ and all y^+ locations as a function of (i) the predictable time t_p^+ and (ii) the sampling interval τ^+ : (a) $K = 20$; (b) $K = 50$; (c) $K = 100$; (d) $K = 200$. Here, the sampling interval in panel (i) is $\tau^+ = 1.08$ and the predictable time in panel (ii) is equal to the sampling interval, i.e., $t_p^+ = \tau^+$.

IV. CONCLUSIONS

We have investigated the Reynolds number effect on the dynamical state of the streamwise velocity in a turbulent channel flow computed by direct numerical simulation using two sophisticated analytical methods: the orbital-instability-based nonlinear forecasting method and the ordinal partition transition network.

In the near-wall region, i.e., the viscous sublayer and the logarithmic layer, we observe the coexistence of the streaky structures with a spanwise spacing of about 100 wall units and the large-scale structures with a spanwise spacing on the order of channel half-width, resulting in a decrease in the predictive accuracy of streamwise velocity and an increase in the randomness of streamwise velocity. The randomness

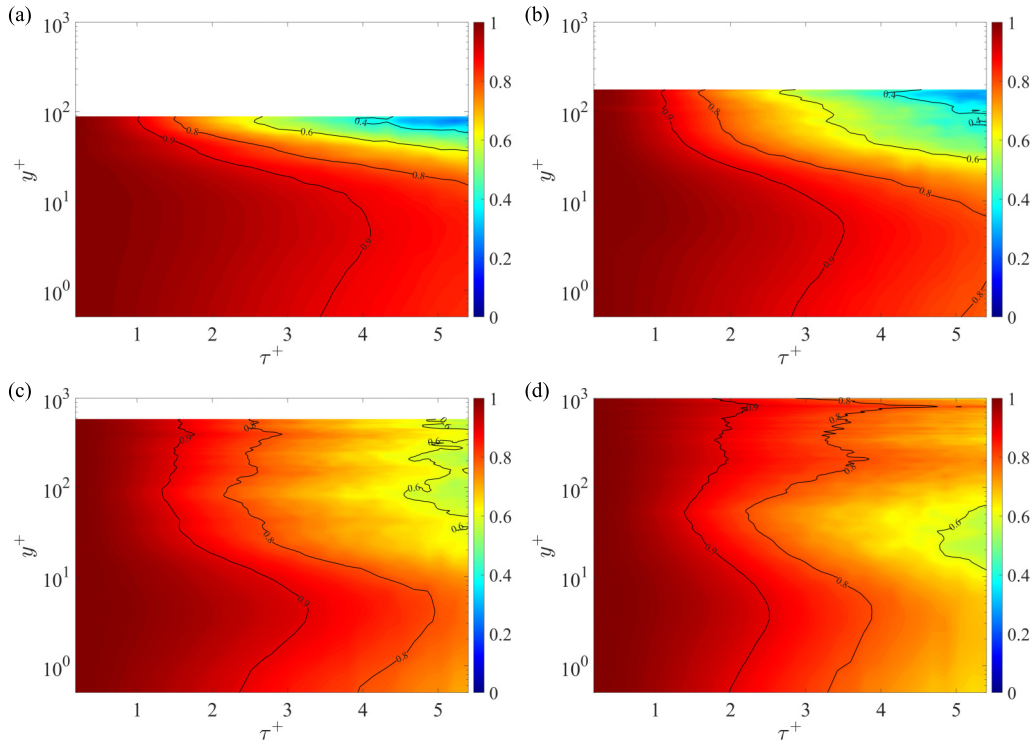


FIG. 12. Contours of the correlation coefficient C between the original and predicted streamwise velocities as functions of the sampling interval τ^+ and the wall-normal coordinate y^+ : (a) $\text{Re}_\tau = 90$; (b) $\text{Re}_\tau = 180$; (c) $\text{Re}_\tau = 590$; (d) $\text{Re}_\tau = 1000$. Here, the predictable time is equal to the sampling interval, i.e., $t_p^+ = \tau^+$.

near the wall increases with the friction Reynolds number, whereas the low-dimensional chaotic state of streamwise velocity persists in the near-wall region regardless of the friction Reynolds number. In the far-wall region, the high-dimensional chaotic state of streamwise velocity emerges at lower friction Reynolds numbers owing to the obviously low-Reynolds-number effect, whereas the low-dimensional chaotic state of streamwise velocity emerges at higher friction Reynolds numbers owing to the formation of the streamwise and spanwise large-scale structures, i.e., the so-called large-scale motions. The emergence of the low-dimensional chaotic state at high Reynolds numbers leads to a decrease in the randomness of the streamwise velocity with the friction Reynolds number. As similarly reported by Mamori *et al.* [40], OPTN entropy is useful for assessing the predictive accuracy of streamwise velocity in terms of randomness at both low and high friction Reynolds numbers.

The chaotic state of streamwise velocity at higher Reynolds numbers is strongly associated with hierarchical structures [7]. The formation of hierarchical structures can be clearly identified at higher Reynolds numbers through the analyses of the premultiplied streamwise Reynolds stress spectrum and the instantaneous streamwise velocity fluctuations in the z - y plane, whereas it is hardly observed at lower Reynolds numbers.

The findings obtained in this study demonstrate that the OIFM and OPTN provide a powerful framework for elucidating the dynamics of turbulent flows in terms of their predictability properties. Furthermore, the predictability

of turbulence afforded by nonlinear forecasting may open avenues for the development of innovative flow control strategies aimed at addressing environmental problems such as exhaustion of energy resources. Advancing these applications remains an important direction for future work.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

APPENDIX A: EFFECT OF THE NUMBER OF THE NEARBY POINTS K ON THE CORRELATION COEFFICIENT C

We investigate the effect of the number of nearby points K on the correlation coefficient C at $\text{Re}_\tau = 180$ in Fig. 11. As

shown in this figure, we hardly observe the dependence of the number of nearby points K on the correlation coefficient C as a function of the predictable time t_p^+ and the sampling interval τ^+ for all y^+ locations. Therefore, similarly to Ref. [40], we set K to 50 in this study.

APPENDIX B: EFFECT OF THE SAMPLING INTERVAL τ ON THE CORRELATION COEFFICIENT C

Figure 12 shows the contours of the correlation coefficient C between the original and predicted streamwise velocities as functions of τ^+ and y^+ at $Re_\tau = 90$ –1000. Similarly to

Fig. 8, the predictable time in Fig. 12 is equal to the sampling interval, i.e., $t_p^+ = \tau^+$. Regardless of the wall-normal location y^+ and friction Reynolds numbers Re_τ , C decreases as τ^+ increases, indicating that the coarse graining of u leads to the decrease in the predictability of u . In the near-wall region, i.e., the viscous sublayer and buffer layer, we observe the high short-range forecastable region. Subsequently, it decreases from the buffer layer to the beginning of the far-wall region. However, at the higher friction Reynolds numbers, i.e., $Re_\tau = 590$ and 1000, we observe the extension of the high short-range forecastable region in the far-wall region at $y^+ \gtrsim 100$.

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