

Quantum dynamics in multiple baths: Initial baths separability condition for noninterference

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(Received 17 April 2025; accepted 15 July 2025; published 1 August 2025)

We use a master equation to provide a proof of interference between noninteracting baths on a system's reduced dynamics, agreeing with a result in *Phys. Rev. A* **89**, 042117 (2014). Here, we provide a formula for the details of leading-order interference. We illustrate the use of such interference toward suppressing (even freezing) decoherence with a simplest example and identify what determines the extent of decoherence suppression. However, we find that interference of baths vanishes (up to second order) as long as the baths are initially separable, thereby identifying a “low-bar” condition for noninterference. This finding provides strong justification for the widely used “additive assumption” in weak coupling regime. We discuss how this noninterference finding sheds light on the “puzzle” of vanishing second-order interference in the joint scenario of quantum and classical noises in [arXiv:2004.13901](https://arxiv.org/abs/2004.13901). We also provide a general proof of interference between two baths under stochastic coupling, in contrast to the study of deterministic coupling in the literature. We demonstrate decoherence suppression with an example of a central spin stochastically coupled to two environmental spins. We find that two baths will cease to interfere (up to second order) if the two baths are initially separable and the system's stochastic coupling strengths with the two baths are uncorrelated.

DOI: [10.1103/rd78-ywr8](https://doi.org/10.1103/rd78-ywr8)

I. INTRODUCTION

Open quantum systems are of importance to both technological and fundamental aspects of quantum physics and are extensively studied in the literature [1,2]. There are a wide variety of situations where an open quantum system is coupled to multiple baths [3–20]. In such situations, one may ask whether there is interference between different baths on the system's reduced dynamics [3]. This question may be formalized as follows.

Suppose a system is coupled to two baths [21], whereas the baths are not directly coupled to each other. The literature often treats the combined effect of the two baths on the system's reduced dynamics as a sum of the individual effects due to each bath [3]. That is, if a system interacting with bath I alone evolves as $d\rho_t/dt = \mathbb{L}_{1,t}(\rho_t)$ and that with bath II alone evolves as $d\rho_t/dt = \mathbb{L}_{2,t}(\rho_t)$, then the combined effect with both baths present is treated additively as $d\rho_t/dt = \mathbb{L}_{1,t}(\rho_t) + \mathbb{L}_{2,t}(\rho_t)$, where there is no interference (cross term) between the baths. This “additive assumption” [3] might appear intuitive at first sight. However, such a simplistic description may not be assumed valid *a priori*, and the condition(s) for its validity need to be specified and/or justified. For instance, this additive assumption is justified in Ref. [3] for

Markovian baths and is justified in Ref. [16] under weak coupling and for settings where the free system Hamiltonian and all system-environment interactions commute. On the other hand, there can be scenarios of interference between baths, where the additive assumption does not adequately capture the full physics [3,15,17–19]. In particular, the additive assumption may not always be valid for non-Markovian situations [3]. (See also Ref. [7] for previous work on systems coupled to multiple baths in non-Markovian settings.)

In this work, we will use the master equation formalism in Ref. [22] to study the general reduced dynamics of a system coupled to two quantum baths. Along the way, our work will provide answers to questions about interference of baths on the system's reduced dynamics, such as (in Sec. II) why we need to consider interference, how we calculate interference, whether and how we can use such interference for good, (in Sec. III) when we can ignore interference, what we may learn from such noninterference, and so on. Some of these questions were also addressed in Ref. [3], with similar or different answers depending on different regimes and approaches. Our answers will be valid in the weak coupling and/or short time regimes where second-order effects are dominant.

Besides the aforementioned scenario of deterministic coupling, a quantum system may also be stochastically coupled to baths [23]. In this paper, we will also study the average reduced dynamics of a system stochastically coupled to two baths. One may ask similar questions about the interference of baths under stochastic coupling like those that were asked above in the case of deterministic coupling, the answers to which can be found in Sec. IV. In particular, we will pay attention to the nontrivial differences between the stochastic and deterministic cases.

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A previous work [20] also studies the scenario where two noises are statistically independent. In that study, they observe unconventional mutual cancellation of noises when both independent relaxation and dephasing noises are present, which is an interesting finding. Their study is based on exact dynamics. It should be noted that their nonperturbative finding is not at odds with our perturbative result of noninterference herein.

We emphasize that our analysis focuses on weak system-bath coupling and/or short time regimes where second-order effects dominate. In such regimes, nonsecular coherences are retained at the leading order. For strongly coupled systems, higher-order corrections (e.g., fourth order in Refs. [24,25]) become essential, but this is beyond the scope of the current study.

It is important to note again that the concept of “additive assumption” is not an artifact of the language employed but is literally an assumption. This assumption can break down, as will be shown in Sec. II and again in Sec. IV for stochastic coupling. In other words, this assumption cannot be taken for granted, and the conditions for its validity need to be investigated, which is what we will do in this work.

II. INTERFERENCE OF BATHS

A. Main result: General proof of interference

Suppose a system interacts with two baths through a Hamiltonian

$$H_{SE}(t) = \sum_{1a} S_{1a}(t) \otimes E_{1a}(t) + \sum_{2b} S_{2b}(t) \otimes E_{2b}(t), \quad (1)$$

where $\{S_{1a}(t), S_{2b}(t)\}$ are operators on the system's Hilbert space, and $\{E_{1a}(t)\}$ and $\{E_{2b}(t)\}$ are operators on bath 1's and 2's Hilbert spaces, respectively. Note that the Hamiltonian has no interaction term between the two baths, meaning the two baths are not directly coupled. With the master equation formalism in Ref. [22], it can be shown that, to as low as second order, the resulting equation of motion (EOM) is

$$\frac{d}{dt} \rho_t = \mathbb{L}_{1,t}(\rho_t) + \mathbb{L}_{2,t}(\rho_t) + \mathbb{L}_{\text{interference},t}(\rho_t), \quad (2)$$

where $\mathbb{L}_{1,t}(\rho_t)$ represents the individual effect of bath 1 alone on the system's reduced dynamics, $\mathbb{L}_{2,t}(\rho_t)$ represents that of bath 2 alone, and $\mathbb{L}_{\text{interference},t}(\rho_t)$ represents the interference between the two baths.

Physically, this establishes the main results that (1) there is generally interference on the system's reduced dynamics even if the two baths are not directly interacting and (2) such interference can be as significant as a second-order effect. In other words, such interference can be of the same order as the individual leading-order decoherence effects [22] due to bath 1 and bath 2, respectively, and thus generally cannot be neglected.

This general proof of interference between baths on a system's reduced dynamics agrees with the result in the previous work [3].

B. Derivations

In this section, we will show the derivations leading to the general result of interference in Eq. (2).

1. Reduced dynamics with individual baths

For a system interacting with one bath alone, say bath 1, the interaction Hamiltonian can be written as

$$H_{SE1}(t) = \sum_{1a} S_{1a}(t) \otimes E_{1a}(t), \quad (3)$$

where the single index $1a$ denotes the operators associated with the interaction between the system and bath 1, and $\{S_{1a}(t)\}$ and $\{E_{1a}(t)\}$ are operators on the system's and bath 1's Hilbert spaces, respectively. Note that we are already working in the interaction picture. Plugging the Hamiltonian in Eq. (3) into the second-order master equation [Eqs. (31)–(34)] in Ref. [22], we can obtain the EOM for this individual bath scenario. The first-order term is

$$\mathcal{K}_{1,t}(\rho) = -i \sum_{1a} \text{Tr}_E(E_{1a}(t) \rho_{E0}) [S_{1a}(t), \rho] \quad (4)$$

and the second-order term is

$$\begin{aligned} \mathcal{K}_{2,t}(\rho) = & - \sum_{1a} \sum_{1a'} \int_0^t dt' \{ C_{1a,1a'}(t, t') [S_{1a}(t), S_{1a'}(t') \rho] \\ & - C_{1a',1a}(t', t) [S_{1a}(t), \rho S_{1a'}(t')] \}, \end{aligned} \quad (5)$$

where the coefficients are

$$\begin{aligned} C_{jk}(t, t') \equiv & \text{Tr}_E(E_j(t) E_k(t') \rho_{E0}) \\ & - \text{Tr}_E(E_j(t) \rho_{E0}) \text{Tr}_E(E_k(t') \rho_{E0}). \end{aligned} \quad (6)$$

Let us first introduce a shorthand notation to facilitate the derivation:

$$\begin{aligned} \mathcal{Q}_{mn}(\rho) = & - \int_0^t dt' \{ C_{mn}(t, t') [S_m(t), S_n(t') \rho] \\ & - C_{nm}(t', t) [S_m(t), \rho S_n(t')] \}. \end{aligned} \quad (7)$$

For notational convenience, let us also introduce a superoperator on ρ :

$$\begin{aligned} \mathbb{L}_{k,t}(\rho) = & -i \sum_{kn} \text{Tr}_E(E_{kn}(t) \rho_{E0}) [S_{kn}(t), \rho] \\ & + \sum_{kn} \sum_{kn'} \mathcal{Q}_{kn, kn'}(\rho), \end{aligned} \quad (8)$$

where k denotes which bath this superoperator is associated with. With this notation, the EOM for the system interacting with bath 1 alone is simply

$$\frac{d}{dt} \rho_t = \mathbb{L}_{1,t}(\rho_t). \quad (9)$$

Similarly, it can be shown that, for a system interacting with bath 2 alone through the Hamiltonian

$$H_{SE2}(t) = \sum_{2b} S_{2b}(t) \otimes E_{2b}(t), \quad (10)$$

the EOM for the system's reduced dynamics is

$$\frac{d}{dt} \rho_t = \mathbb{L}_{2,t}(\rho_t), \quad (11)$$

where $\mathbb{L}_{2,t}(\rho_t)$ is defined as in Eq. (8).

2. Reduced dynamics with both baths

For a system interacting with both bath 1 and bath 2 through the Hamiltonian

$$H_{SE}(t) = \sum_{1a} S_{1a}(t) \otimes E_{1a}(t) + \sum_{2b} S_{2b}(t) \otimes E_{2b}(t), \quad (12)$$

where the two baths are not mutually interacting, we can again work out its reduced dynamics using Eqs. (31)–(34) in Ref. [22]. The first-order term is found to be

$$\begin{aligned} \mathcal{K}_{1,t}(\rho) = & -i \sum_{1a} \text{Tr}_E(E_{1a}(t)\rho_{E0})[S_{1a}(t), \rho] \\ & -i \sum_{2b} \text{Tr}_E(E_{2b}(t)\rho_{E0})[S_{2b}(t), \rho], \end{aligned} \quad (13)$$

and the second-order term is

$$\mathcal{K}_{2,t}(\rho) = \sum_m \sum_n Q_{mn}(\rho), \quad (14)$$

where the index m runs through all values of $\{1a\}$ and $\{2b\}$ to include both baths in this case, and so does n . Equivalently, it can be written in detail as

$$\begin{aligned} \mathcal{K}_{2,t}(\rho) = & \sum_{1a} \sum_{1a'} Q_{1a,1a'}(\rho) + \sum_{1a} \sum_{2b'} Q_{1a,2b'}(\rho) \\ & + \sum_{2b} \sum_{1a'} Q_{2b,1a'}(\rho) + \sum_{2b} \sum_{2b'} Q_{2b,2b'}(\rho). \end{aligned} \quad (15)$$

Therefore, the EOM for the system interacting with both baths is (up to second order)

$$\begin{aligned} \frac{d}{dt}\rho_t = & \mathcal{K}_{1,t}(\rho_t) + \mathcal{K}_{2,t}(\rho_t) \\ = & -i \sum_{1a} \text{Tr}_E(E_{1a}(t)\rho_{E0})[S_{1a}(t), \rho_t] \\ & -i \sum_{2b} \text{Tr}_E(E_{2b}(t)\rho_{E0})[S_{2b}(t), \rho_t] \\ & + \sum_{1a} \sum_{1a'} Q_{1a,1a'}(\rho_t) + \sum_{2b} \sum_{2b'} Q_{2b,2b'}(\rho_t) \\ & + \sum_{1a} \sum_{2b'} Q_{1a,2b'}(\rho_t) + \sum_{2b} \sum_{1a'} Q_{2b,1a'}(\rho_t). \end{aligned} \quad (16)$$

3. Conclusion

Now comparing Eq. (16) with Eq. (8), we see that the second-order EOM for a system interacting with two baths can be expressed as

$$\frac{d}{dt}\rho_t = \mathbb{L}_{1,t}(\rho_t) + \mathbb{L}_{2,t}(\rho_t) + \mathbb{L}_{\text{interference},t}(\rho_t), \quad (17)$$

where

$$\mathbb{L}_{\text{interference},t}(\rho) \equiv \sum_{1a} \sum_{2b'} Q_{1a,2b'}(\rho) + \sum_{2b} \sum_{1a'} Q_{2b,1a'}(\rho). \quad (18)$$

Comparing Eq. (17) with Eq. (9) for the system's reduced dynamics due to bath 1 alone and Eq. (11) due to bath 2 alone, we may conclude that in general the combined effect of two baths on a system's reduced dynamics is not equal to a simple sum of the individual effects due to the two baths, respectively.

In other words, there is generally interference between the two baths, as is captured by the term $\mathbb{L}_{\text{interference},t}(\rho)$ in Eq. (18).

In the above, we have used the master equation in Ref. [22] to provide a proof of interference, which is a result of perturbative treatment. There are also perturbative discussions on the question of interference in Ref. [3] from a diagrammatic perspective and from a Nakajima-Zwanzig projection operator approach.

C. Example: Suppressing decoherence of a central spin with interference between environmental spins

Quantum coherence is essential to quantum information [26] and quantum control [27], and decoherence is a major obstacle in this field [2]. There have been theoretical and experimental works on decoherence suppression through various different approaches [3,28–33]. In particular, Ref. [3] shows that interference between baths can suppress decoherence through an example of a three-level Λ system under Markovian atomic relaxation and non-Markovian dephasing. Similarly, here we will use a simpler (toy) model to illustrate the idea of using two-bath interference to suppress (and even freeze) decoherence, by considering the dynamics of a central spin coupled to two environmental spins.

1. Problem description

We will use a simple model where a central spin (the system) is coupled to two other spins (the environments) through the Hamiltonian

$$H_{\text{total}} = \frac{\omega_0}{2}\sigma_z + \frac{\omega_1}{2}\sigma_{z,1} + \frac{\omega_2}{2}\sigma_{z,2} + g_1\sigma_z\sigma_{z,1} + g_2\sigma_z\sigma_{z,2}. \quad (19)$$

Treating $H_{\text{perturb}} = g_1\sigma_z\sigma_{z,1} + g_2\sigma_z\sigma_{z,2}$ as a perturbation and switching to the “rotating frame” generated by $H_0 = (\omega_0/2)\sigma_z + (\omega_1/2)\sigma_{z,1} + (\omega_2/2)\sigma_{z,2}$, we have the perturbing Hamiltonian in the interaction picture [34]

$$\begin{aligned} H_{SE}^{(\text{int pic})} = & g_1\sigma_z\sigma_{z,1} + g_2\sigma_z\sigma_{z,2} \\ = & \sigma_z \otimes (g_1\sigma_{z,1}) + \sigma_z \otimes (g_2\sigma_{z,2}). \end{aligned} \quad (20)$$

Suppose the environment's initial state is not some direct product of the two individual spins's states, that is, the two environmental spins are not statistically independent,

$$\rho_{E,0} \neq \rho_{E1,0} \otimes \rho_{E2,0}. \quad (21)$$

Suppose further that both environmental spins have zero expectation of spin- z ,

$$\text{Tr}_E(\rho_{E,0}\sigma_{z,1}) = \text{Tr}_E(\rho_{E,0}\sigma_{z,2}) = 0. \quad (22)$$

One example of such an initial state could be $\rho_{E,0} = \frac{1}{2}|0\rangle\langle 0| \otimes |0\rangle\langle 0| + \frac{1}{2}|1\rangle\langle 1| \otimes |1\rangle\langle 1|$.

2. Reduced dynamics

Plugging the conditions above into the general master equation [Eqs. (31)–(34)] in Ref. [22], it can be shown that the first-order effective Hamiltonian vanishes,

$$H_{\text{eff}}^I(t) = g_1\text{Tr}_E(\rho_{E,0}\sigma_{z,1})\sigma_z + g_2\text{Tr}_E(\rho_{E,0}\sigma_{z,2})\sigma_z = 0, \quad (23)$$

and the second-order term of the EOM is found to be [22]

$$\mathcal{K}_{2,t}(\rho) = - \sum_{m=1,2} \sum_{n=1,2} \int_0^t dt' C_{mn}[\sigma_z, [\sigma_z, \rho]], \quad (24)$$

where the coefficients are to be calculated,

$$C_{11} = |g_1|^2 \text{Tr}_E(\sigma_{z,1} \sigma_{z,1} \rho_{E,0}) = |g_1|^2, \quad (25)$$

$$C_{22} = |g_2|^2, \quad (26)$$

$$\begin{aligned} C_{12} &= g_1 g_2 \text{Tr}_E(\sigma_{z,1} \sigma_{z,2} \rho_{E,0}) \\ &\equiv g_1 g_2 \langle \sigma_{z,1} \sigma_{z,2} \rangle_{E,0} \neq 0, \end{aligned} \quad (27)$$

$$C_{21} = g_1 g_2 \langle \sigma_{z,1} \sigma_{z,2} \rangle_{E,0} \neq 0; \quad (28)$$

thus, given that $g_1, g_2 > 0$, the second-order term can be written as

$$\begin{aligned} \mathcal{K}_{2,t}(\rho) &= (-t)(|g_1|^2 + |g_2|^2 + 2|g_1 g_2| \langle \sigma_{z,1} \sigma_{z,2} \rangle_{E,0}) \\ &\times [\sigma_z, [\sigma_z, \rho]]. \end{aligned} \quad (29)$$

Thus, we see that the central spin's decoherence rate due to its interaction with two environmental spins is

$$D_{\text{total}} = t(|g_1|^2 + |g_2|^2 + 2|g_1 g_2| \langle \sigma_{z,1} \sigma_{z,2} \rangle_{E,0}). \quad (30)$$

3. Decoherence suppression

On the other hand, suppose we “turn off” the central spin's interaction with one environmental spin, say $g_2 = 0$, so it only interacts with the other. Thus, setting $g_2 = 0$ in Eq. (29) yields

$$\mathcal{K}_{2,t}(\rho) = (-t)|g_1|^2 [\sigma_z, [\sigma_z, \rho]]. \quad (31)$$

The decoherence rate due to an individual environmental spin is thus

$$D_1 = t|g_1|^2. \quad (32)$$

The same can be said about the decoherence rate due to the other environmental spin,

$$D_2 = t|g_2|^2. \quad (33)$$

Comparing the overall decoherence rate in Eq. (30) with the individual decoherence rates in Eqs. (32) and (33), we see that $\langle \sigma_{z,1} \sigma_{z,2} \rangle_{E,0} \equiv \text{Tr}_E(\sigma_{z,1} \sigma_{z,2} \rho_{E,0})$ is a key quantity determining the overall decoherence rate. If $\langle \sigma_{z,1} \sigma_{z,2} \rangle_{E,0} > 0$, then $D_{\text{total}} > D_1 + D_2$, in which case decoherence is enhanced. If $\langle \sigma_{z,1} \sigma_{z,2} \rangle_{E,0} < 0$, then $D_{\text{total}} < D_1 + D_2$, which means decoherence is suppressed. In other words, the overall decoherence rate is less than the sum of the two individual decoherence rates in the latter case, which is desirable from the perspective of preserving coherence.

We further assume that the central spin is coupled to the two environmental spins at equal strengths, that is, $|g_1| = |g_2|$. Moreover, suppose the two environmental spins are negatively correlated in terms of their spin- z such that $\langle \sigma_{z,1} \sigma_{z,2} \rangle_{E,0} = -1$. This would be the case if the initial environmental state is $\rho_{E,0} = \frac{1}{2}|0\rangle\langle 0| \otimes |1\rangle\langle 1| + \frac{1}{2}|1\rangle\langle 1| \otimes |0\rangle\langle 0|$, which is consistent with the two assumptions in Eqs. (21) and (22). In this case, it can be shown that the total decoherence rate of the central spin vanishes,

$$D_{\text{total}} \propto |g_1|^2 + |g_1|^2 + 2|g_1|^2(-1) = 0. \quad (34)$$

Interestingly, in this ideal scenario, the central spin becomes free of decoherence (up to second order) despite the presence of two environmental spins. (However, achieving this ideality requires fine-tuning of the aforementioned parameters).

In summary, we show that it is possible to use interference between environmental spins to suppress (even freeze, in the ideal case) decoherence effect. This is a toy model and in reality environments causing decoherence of the system are generally more complicated [1–3]. Nevertheless, our model, albeit simpler and less realistic than the Λ system example in Ref. [3], does capture the gist of using environmental interference to suppress decoherence, by identifying the two-point correlation $\langle \sigma_{z,1} \sigma_{z,2} \rangle_{E,0}$ as a simple determinant for the degree of decoherence suppression. We learn straightforwardly that, if we can initialize the environmental spins such that $\langle \sigma_{z,1} \sigma_{z,2} \rangle_{E,0} < 0$, we can readily attain suppressed decoherence in the central spin. It should be noted that the applicability domain of our result is the weak coupling and/or short time regimes where second-order effects dominate. We do not extrapolate to other regimes where the second-order perturbative treatment may not suffice and higher-order contributions may need to be considered.

On the other hand, we add that this “opportunity” of suppressing decoherence apparently brings with itself some “risk.” If the environmental spins were so initialized that $\langle \sigma_{z,1} \sigma_{z,2} \rangle_{E,0} > 0$, then we would be faced with enhanced decoherence, which is even more detrimental. (A similar concern was also briefly mentioned in Ref. [3].) Thus, for this idea of decoherence suppression to work, we apparently need to have adequate control over the environmental initial state so as to avoid the detrimental scenario.

III. NONINTERFERENCE OF BATHS

A. Counterexample to interference: Atomic decay in two photonic baths

We will now apply the second-order EOM in Ref. [22] to a Hamiltonian in Ref. [3] describing a two-level system decaying into two uncoupled photonic reservoirs (in Schrodinger picture),

$$\begin{aligned} H_{\text{total}}^{(\text{Schrodinger picture})} &= \frac{\omega_0}{2} \sigma_z + \sum_j \omega_{1j} b_{1j}^\dagger b_{1j} + \sum_k \omega_{2k} b_{2k}^\dagger b_{2k} \\ &+ \sum_j g_{1j} (b_{1j}^\dagger \sigma_- + b_{1j} \sigma_+) \\ &+ \sum_k g_{2k} (b_{2k}^\dagger \sigma_- + b_{2k} \sigma_+), \end{aligned} \quad (35)$$

where ω_0 is the energy splitting of the two-level system, ω_{1j} (ω_{2k}) is the frequency of a photonic mode of reservoir I (II), b_{1j} and $b_{1j}^\dagger$ (b_{2k} and $b_{2k}^\dagger$) are annihilation and creation operators of the respective mode, and g_{1j} (g_{2k}) is the coupling of a photonic mode of reservoir I (II) to the two-level system.

Switching to the “rotating frame” generated by the unperturbed Hamiltonian $H_0 = (\omega_0/2)\sigma_z + \sum_j \omega_{1j} b_{1j}^\dagger b_{1j} + \sum_k \omega_{2k} b_{2k}^\dagger b_{2k}$, we obtain the system-environment interaction

Hamiltonian in the interaction picture [1,34],

$$\begin{aligned}
 H_{\text{SE}}^{(\text{interaction picture})}(t) = & \sigma_- \otimes \left(\sum_j g_{1j} e^{i(\omega_{1j} - \omega_0)t} b_{1j}^\dagger \right) \\
 & + \sigma_+ \otimes \left(\sum_j g_{1j} e^{-i(\omega_{1j} - \omega_0)t} b_{1j} \right) \\
 & + \sigma_- \otimes \left(\sum_k g_{2k} e^{i(\omega_{2k} - \omega_0)t} b_{2k}^\dagger \right) \\
 & + \sigma_+ \otimes \left(\sum_k g_{2k} e^{-i(\omega_{2k} - \omega_0)t} b_{2k} \right), \quad (36)
 \end{aligned}$$

which is of the form as in Eq. (12). Thus, following the derivation in Eqs. (12)–(18), we surprisingly find that the resultant EOM (up to second order) is of the simple additive form

$$\frac{d}{dt} \rho_t = \mathbb{L}_{1,t}(\rho_t) + \mathbb{L}_{2,t}(\rho_t), \quad (37)$$

where $\mathbb{L}_{1,t}(\rho_t)$ describes the individual effect of reservoir I, $\mathbb{L}_{2,t}(\rho_t)$ describes that of reservoir II, and the interference term $\mathbb{L}_{\text{interference},t}(\rho_t)$ as defined by Eq. (18) is found to vanish under the Hamiltonian in Eq. (36) with the reservoirs initially in the vacuum state.

This does not necessarily contradict with Ref. [3], as the nonadditive result in Eqs. (6)–(9) therein is not perturbative. Nevertheless, our result indicates that any interference effect found therein should be of higher order in nature. If the system-bath interaction is sufficiently weak and/or if the time regime of interest is sufficiently short, in which case second-order effects are dominant, then interference effect may be insignificant.

This finding also prompts us to examine more generally the condition(s) for the noninterference of baths on a system's reduced dynamics. Such condition(s), if found, may also provide justification for previous works in the open system literature that are based on the additive assumption.

B. Main result: “initial baths separability” condition for noninterference

It can in fact be shown that if two baths are initially separable, then there is no interference between the two baths on the system's reduced dynamics up to second order. This initial baths separability condition can be formally expressed as

$$\rho_{E,0} = \rho_{E1,0} \otimes \rho_{E2,0}, \quad (38)$$

where $\rho_{E,0} \equiv \rho_E(t_0)$ is the initial density matrix of the whole bath (i.e., the entirety of bath 1 and bath 2), $\rho_{E1,0} \equiv \rho_{E1}(t_0)$ is the initial density matrix of bath 1, and $\rho_{E2,0} \equiv \rho_{E2}(t_0)$ is that of bath 2 [35].

More precisely, we find that, under the initial baths separability condition in Eq. (38), the system's reduced dynamics obeys an EOM (up to second order) of the form

$$\frac{d}{dt} \rho_t = \mathbb{L}_{1,t}(\rho_t) + \mathbb{L}_{2,t}(\rho_t), \quad (39)$$

where $\mathbb{L}_{1,t}(\rho_t)$ describes the effect of bath 1 alone on the system's reduced dynamics and $\mathbb{L}_{2,t}(\rho_t)$ describes that of bath 2 alone. In other words, there is no interference between the two baths beyond their individual contributions.

Note that this issue of noninterference (i.e., the “additive assumption”) is also investigated in Ref. [3], wherein the Markovian condition is needed for their additive result to hold. In contrast, our additive result in Eq. (39) is not subject to this Markovian restriction, though it should be noted that its domain of validity is the weak coupling and/or short time regimes where second-order effects dominate.

C. Derivations

In this section, we will show the derivation for the noninterference result as in Eq. (39) from the initial separability of two baths as in Eq. (38).

First, note that the system's reduced dynamics due to an individual bath 1 alone has been worked out in Eq. (9) and that due to bath 2 alone in Eq. (11). All we need to do in this case is to work out the reduced dynamics in the presence of both baths under the initial baths separability condition in Eq. (38).

1. Reduced dynamics with both baths

For a system interacting with two baths via a Hamiltonian of the form

$$H_{\text{SE}}(t) = \sum_{1a} S_{1a}(t) \otimes E_{1a}(t) + \sum_{2b} S_{2b}(t) \otimes E_{2b}(t), \quad (40)$$

we have already shown in Eqs. (17) and (18) that the general EOM is (up to second order)

$$\begin{aligned}
 \frac{d}{dt} \rho_t = & \mathbb{L}_{1,t}(\rho_t) + \mathbb{L}_{2,t}(\rho_t) \\
 & + \sum_{1a} \sum_{2b'} Q_{1a,2b'}(\rho_t) + \sum_{2b} \sum_{1a'} Q_{2b,1a'}(\rho_t), \quad (41)
 \end{aligned}$$

where $\mathbb{L}_{1,t}(\rho_t)$ and $\mathbb{L}_{2,t}(\rho_t)$ are as defined in Eq. (8) and $Q_{mn}(\rho)$ is as defined in Eq. (7).

Now, under the initial baths separability condition $\rho_{E,0} = \rho_{E1,0} \otimes \rho_{E2,0}$, for the third term in Eq. (41), we can calculate the summand

$$\begin{aligned}
 Q_{1a,2b'}(\rho) = & - \int_0^t dt' \{ \mathcal{C}_{1a,2b'}(t, t') [S_{1a}(t), S_{2b'}(t') \rho] \\
 & - \mathcal{C}_{2b',1a}(t', t) [S_{1a}(t), \rho S_{2b'}(t')] \}, \quad (42)
 \end{aligned}$$

where the coefficient of the first term is

$$\begin{aligned}
 \mathcal{C}_{1a,2b'}(t, t') = & \text{Tr}_E(E_{1a}(t) E_{2b'}(t') \rho_{E0}) \\
 & - \text{Tr}_E(E_{1a}(t) \rho_{E0}) \text{Tr}_E(E_{2b'}(t') \rho_{E0}) \\
 = & \text{Tr}_E(E_{1a}(t) \rho_{E1,0} \otimes E_{2b'}(t') \rho_{E2,0}) \\
 & - \text{Tr}_E(E_{1a}(t) \rho_{E1,0}) \otimes \rho_{E2,0} \\
 & \times \text{Tr}_E(\rho_{E1,0} \otimes E_{2b'}(t') \rho_{E2,0}) \\
 = & \text{Tr}_{E1}(E_{1a}(t) \rho_{E1,0}) \text{Tr}_{E2}(E_{2b'}(t') \rho_{E2,0}) \\
 & - \text{Tr}_{E1}(E_{1a}(t) \rho_{E1,0}) \text{Tr}_{E2}(\rho_{E2,0}) \\
 & \times \text{Tr}_{E1}(\rho_{E1,0}) \text{Tr}_{E2}(E_{2b'}(t') \rho_{E2,0})
 \end{aligned}$$

$$\begin{aligned}
&= \text{Tr}_{E1}(E_{1a}(t)\rho_{E1,0})\text{Tr}_{E2}(E_{2b'}(t')\rho_{E2,0}) \\
&\quad - \text{Tr}_{E1}(E_{1a}(t)\rho_{E1,0})\text{Tr}_{E2}(E_{2b'}(t')\rho_{E2,0}) \\
&= 0,
\end{aligned} \tag{43}$$

and the coefficient of the second term is

$$\begin{aligned}
\mathcal{C}_{2b',1a}(t',t) &= \text{Tr}_E(E_{2b'}(t')E_{1a}(t)\rho_{E0}) \\
&\quad - \text{Tr}_E(E_{2b'}(t')\rho_{E0})\text{Tr}_E(E_{1a}(t)\rho_{E0}) \\
&= \text{Tr}_E(E_{1a}(t)E_{2b'}(t')\rho_{E0}) \\
&\quad - \text{Tr}_E(E_{1a}(t)\rho_{E0})\text{Tr}_E(E_{2b'}(t')\rho_{E0}) \\
&= \mathcal{C}_{1a,2b'}(t,t') \\
&= 0.
\end{aligned} \tag{44}$$

The second equality in Eq. (44) holds because $[E_{1a}(t), E_{2b'}(t')] = 0$, as $E_{1a}(t)$ acts on the Hilbert space of bath 1 and $E_{2b'}(t')$ acts on that of bath 2. Thus, we find $Q_{1a,2b'}(\rho) = 0$ holds for arbitrary a and b' , so the third term in Eq. (41) vanishes,

$$\sum_{1a} \sum_{2b'} Q_{1a,2b'}(\rho) = 0. \tag{45}$$

Similarly, it can be shown that the initial baths separability condition implies $Q_{2b,1a'}(\rho) = 0$ for arbitrary a' and b ; thus, the fourth term in Eq. (41) vanishes as well,

$$\sum_{2b} \sum_{1a'} Q_{2b,1a'}(\rho) = 0. \tag{46}$$

2. Conclusion

Now, we are left with the first two terms in Eq. (41); thus, the EOM for a system interacting with two initially separable baths is (up to second order)

$$\frac{d}{dt}\rho_t = \mathbb{L}_{1,t}(\rho_t) + \mathbb{L}_{2,t}(\rho_t). \tag{47}$$

Comparing with the EOMs for the system's reduced dynamics due to bath 1 alone and that due to bath 2 alone in Eqs. (9) and (11), we may conclude that, under the initial baths separability condition, the combined effect of two baths on a system's reduced dynamics is a simple sum of the individual effects of the two baths, respectively. In other words, the initial separability of two baths is a sufficient condition for noninterference of the two baths (up to second order).

D. Implications

1. Wide applicability of the noninterference finding

First, the noninterference result holds regardless of the type of system-bath interaction. As long as the baths satisfy the said condition, system-bath interactions of all kinds will lead to the same noninterference result (within second order).

Second, note that the baths under consideration are not directly coupled. And one may ask, if two things are not coupled, what is the chance of them being correlated from the outset, after all? Unless experimentalists purposely prepare the baths in some correlated state, we could arguably expect the separability condition be met in most (naturally occurring) cases. Previous works such as Refs. [3,6,7] have assumed the same separability condition between baths.

Third, note that we only require “initial” separability of baths for the noninterference result to hold. As the system-bath interaction is turned on, the baths may subsequently become correlated, but that does not affect our result.

Arguably, the three considerations above imply the wide applicability of the “initial baths separability” condition, and thus the wide applicability of the noninterference result. Its applicability is within the weak coupling and/or short time regimes, where second-order effects dominate.

2. Physical implications

In light of the noninterference finding, when we identify scenarios of an open system coupled to initially separable baths, we can readily be assured of the validity of the additive assumption in describing its physics. As an example, we can revisit the simple model of a central spin coupled to two environmental spins to illustrate the usefulness of the noninterference finding. Suppose that the environmental spins are known to be initially separable, and that the decoherence rates due to individual spins are also known to be $D_1 = t|g_1|^2$ and $D_2 = t|g_2|^2$. Then, without the need of any calculation, we can readily know that when the central spin is *simultaneously* coupled to both environmental spins, it will decohere at a rate of $D_{\text{total}} = D_1 + D_2 = t|g_1|^2 + t|g_2|^2$. In other words, we know beforehand that no decoherence enhancement or suppression will occur (up to second order) between the environmental spins, and that their respective contributions to the central spin's decoherence will simply add up.

On the other hand, we also learn from this work that initial correlation of baths could possibly lead to the breakdown of the additive assumption [36]. This may help us identify scenarios where the additive assumption is insufficiently justified and/or inappropriately invoked in the existing literature, in which case our formalism may also help find corrections to such (oversimplistic) additive treatments.

3. Further work

It should be noted that here we have shown the noninterference result (up to second order) under the initial separability of noninteracting baths, thereby establishing it as a sufficient condition for noninterference. This does not establish it as a necessary condition, however. Strictly speaking, there might still exist the possibility of other situations of noninterference, which can be a subject of further exploration. For example, a noninterference result is obtained in Ref. [3] for Markovian baths, and is obtained in Ref. [16] under weak coupling and for settings where the free system Hamiltonian and all system-environment interactions commute.

It may also be interesting to investigate into third- and/or higher-order effects of multiple baths on the system's reduced dynamics under the same initial baths separability condition to see whether (and how far beyond) the noninterference result would still hold. Though in practice, this may be of less importance in weak coupling and/or short time regimes [22], where second-order perturbative description usually suffices.

E. Discussion: Vanishing interference of quantum and classical noises

It was found in Ref. [37] that second-order interference between quantum and classical noises on the system's dynamics vanishes identically. This comes as a surprise because we do not seem to have *a priori* reason to think that quantum and classical noises do not interfere in second order. Therein, one naturally asks how to physically (intuitively) explain the unconditional vanishing of the second-order interference. Here, with our finding that second-order interference of two quantum baths vanishes under the condition of their initial separability, we may have a hint on the origin of the second-order noninterference of quantum and classical noises in Ref. [37].

Suppose we are to interpret the classical noise in Ref. [37] as some “classical limit” of a quantum bath as in our current paper. Then, our noninterference finding herein would imply that, provided that the classical noise does not have initial correlation with the quantum noise in Ref. [37], we would not have second-order interference between classical and quantum noises on the system's dynamics. However, notice that this condition is automatically satisfied, as the very way in which the quantum and classical sources of noise are formulated in Ref. [37] renders them independent. This (automatic) independence thus results in the second-order noninterference between quantum and classical noises in Ref. [37], in light of the second-order noninterference finding herein.

We stress that the comparison and discussion above are by no means rigorous, but are meant to provide some heuristics in an attempt to “understand” the second-order noninterference result in Ref. [37], which we hope could provide some intuition on the issue and serve as some starting point for further discussion.

IV. AVERAGE REDUCED DYNAMICS IN MULTIPLE BATHS UNDER STOCHASTIC COUPLING

A. General proof of interference

In the following, we will study the general dynamics of a quantum system stochastically coupled to two environments, focusing on whether the two (noninteracting) environments interfere with one other on the system's dynamics.

1. Reduced dynamics with individual baths

For comparison, we will first study the general dynamics of a quantum system stochastically coupled to a single environment through the coupling Hamiltonian

$$H_{\text{SE},1}^{(r)}(t) = \sum_{1a} g_{1a}^{(r)}(t) S_{1a}(t) \otimes E_{1a}(t), \quad (48)$$

where $g_{1a}^{(r)}(t)$ are the stochastic coupling strengths. Plugging Eq. (48) into the formalism in Ref. [23], we find that the first-order term of the EOM is

$$\mathcal{K}_{1,t}(\rho) = -i \sum_{1a} \overline{g_{1a}(t)} \text{Tr}_E(E_{1a}(t) \rho_{E0}) [S_{1a}(t), \rho], \quad (49)$$

and the second-order term can be written as

$$\mathcal{K}_{2,t}(\rho) = \sum_{1a} \sum_{1a'} P_{1a,1a'}(\rho), \quad (50)$$

where $P_{mn}(\rho)$ is defined as

$$\begin{aligned} P_{mn}(\rho) \equiv & \int_0^t dt' \{ \\ & \overline{\{g_m(t) g_n(t')\}} \langle E_m(t) E_n(t') \rangle_{E0} \\ & - \overline{g_m(t)} \overline{g_n(t')} \langle E_m(t) \rangle_{E0} \langle E_n(t') \rangle_{E0} \} \\ & \times (-S_m(t) S_n(t') \rho + S_n(t') \rho S_m(t)) \\ & + \{ \overline{g_m(t) g_n(t')} \langle E_n(t') E_m(t) \rangle_{E0} \\ & - \overline{g_m(t)} \overline{g_n(t')} \langle E_m(t) \rangle_{E0} \langle E_n(t') \rangle_{E0} \} \\ & \times (-\rho S_n(t') S_m(t) + S_m(t) \rho S_n(t')). \end{aligned} \quad (51)$$

To facilitate our investigation, we introduce the notation

$$\begin{aligned} \mathbb{L}_{k,t}(\rho) = & -i \sum_{kn} \overline{g_{kn}(t)} \text{Tr}_E(E_{kn}(t) \rho_{E0}) [S_{kn}(t), \rho] \\ & + \sum_{kn} \sum_{kn'} P_{kn,kn'}(\rho), \end{aligned} \quad (52)$$

with which the second-order EOM for the Hamiltonian in Eq. (48) can be written as

$$\frac{d}{dt} \rho_t = \mathbb{L}_{1,t}(\rho_t). \quad (53)$$

Similarly, for a system stochastically interacting with a second environment through the Hamiltonian

$$H_{\text{SE},2}^{(r)}(t) = \sum_{2b} g_{2b}^{(r)}(t) S_{2b}(t) \otimes E_{2b}(t), \quad (54)$$

it can be shown that the second-order EOM is

$$\frac{d}{dt} \rho_t = \mathbb{L}_{2,t}(\rho_t). \quad (55)$$

2. Reduced dynamics with both baths

Now, we turn to the case of a system stochastically coupled to two environments via the coupling Hamiltonian

$$\begin{aligned} H_{\text{SE}}^{(r)}(t) = & \sum_{1a} g_{1a}^{(r)}(t) S_{1a}(t) \otimes E_{1a}(t) \\ & + \sum_{2b} g_{2b}^{(r)}(t) S_{2b}(t) \otimes E_{2b}(t), \end{aligned} \quad (56)$$

where $\{E_{1a}(t)\}$ and $\{E_{2b}(t)\}$ are operators of the two environments, respectively. Note that the two environments are not directly coupled. Plugging the coupling Hamiltonian in Eq. (56) into the formalism in Ref. [23], we find that the second-order EOM is

$$\begin{aligned} \frac{d}{dt} \rho_t = & -i \sum_{1a} \overline{g_{1a}(t)} \text{Tr}_E(E_{1a}(t) \rho_{E0}) [S_{1a}(t), \rho_t] \\ & -i \sum_{2b} \overline{g_{2b}(t)} \text{Tr}_E(E_{2b}(t) \rho_{E0}) [S_{2b}(t), \rho_t] \\ & + \sum_{1a} \sum_{1a'} P_{1a,1a'}(\rho_t) + \sum_{2b} \sum_{2b'} P_{2b,2b'}(\rho_t) \\ & + \sum_{1a} \sum_{2b'} P_{1a,2b'}(\rho_t) + \sum_{2b} \sum_{1a'} P_{2b,1a'}(\rho_t), \end{aligned} \quad (57)$$

with $P_{mn}(\rho)$ as defined in Eq. (51).

3. Conclusion

Comparing Eq. (57) with Eq. (52), we can conclude that the second-order EOM for a system stochastically coupled to two environments can be written as

$$\frac{d}{dt}\rho_t = \mathbb{L}_{1,t}(\rho_t) + \mathbb{L}_{2,t}(\rho_t) + \mathbb{L}_{\text{interference},t}(\rho_t), \quad (58)$$

where the first two terms are individual effects of the two environments on the system's average reduced dynamics, respectively, and the interference term is

$$\mathbb{L}_{\text{interference},t}(\rho) \equiv \sum_{1a} \sum_{2b'} P_{1a,2b'}(\rho) + \sum_{2b} \sum_{1a'} P_{2b,1a'}(\rho). \quad (59)$$

This interference term is not to be found in the EOM's for the case of a system stochastically coupled to individual environments in Eqs. (53) and (55).

Thus, we (1) prove that in general two environments interfere with each other on the system's dynamics under stochastic coupling even if the two environments are not directly interacting and (2) find the details of the interference term as in Eqs. (51) and (59). Note that such interference is of second order in nature and thus as significant as individual decoherence effects.

B. Example: Suppressing decoherence of a central spin stochastically coupled to two environmental spins

1. Problem description

We will now study an illustrative example of a central spin stochastically coupled to two environmental spins through the Hamiltonian (in Schrodinger picture)

$$H_{\text{total}}^{(r)} = \frac{\omega_0}{2}\sigma_z + \frac{\omega_1}{2}\sigma_{z,1} + \frac{\omega_2}{2}\sigma_{z,2} + g_1^{(r)}\sigma_z\sigma_{z,1} + g_2^{(r)}\sigma_z\sigma_{z,2}, \quad (60)$$

where $g_1^{(r)}$ and $g_2^{(r)}$ are stochastic coupling strengths. Switching to the "rotating frame," we have the interaction picture Hamiltonian

$$H_{\text{SE}}^{(r)} = g_1^{(r)}\sigma_z\sigma_{z,1} + g_2^{(r)}\sigma_z\sigma_{z,2}. \quad (61)$$

We assume that the environmental spins are initially not statistically independent,

$$\rho_{E,0} \neq \rho_{E1,0} \otimes \rho_{E2,0}. \quad (62)$$

For simplicity, we also assume that

$$\text{Tr}_E(\rho_{E,0}\sigma_{z,1}) = \text{Tr}_E(\rho_{E,0}\sigma_{z,2}) = 0. \quad (63)$$

2. Reduced dynamics

Plugging the stochastic coupling Hamiltonian in Eq. (61) into the formalism in Ref. [23], we find that the first-order effective Hamiltonian vanishes,

$$H_{\text{eff}}^I(t) = \overline{g_1}\text{Tr}_E(\rho_{E,0}\sigma_{z,1})\sigma_z + \overline{g_2}\text{Tr}_E(\rho_{E,0}\sigma_{z,2})\sigma_z = 0, \quad (64)$$

and that, with $[\sigma_{z,m}, \sigma_{z,n}] = 0$, the second-order term of the EOM is

$$\begin{aligned} \mathcal{K}_{2,t}(\rho) &= - \sum_{m=1,2} \sum_{n=1,2} \int_0^t dt' (\overline{g_m g_n} \langle \sigma_{z,m} \sigma_{z,n} \rangle_{E0}) [\sigma_z, [\sigma_z, \rho]] \\ &= -t(\overline{g_1^2} + \overline{g_2^2} + 2\overline{g_1 g_2} \langle \sigma_{z,1} \sigma_{z,2} \rangle_{E0}) [\sigma_z, [\sigma_z, \rho]]. \end{aligned} \quad (65)$$

Thus, the total decoherence rate of the central spin due to the stochastic coupling with two environmental spins is

$$D_{\text{total}} = t(\overline{g_1^2} + \overline{g_2^2} + 2\overline{g_1 g_2} \langle \sigma_{z,1} \sigma_{z,2} \rangle_{E0}). \quad (66)$$

3. Decoherence suppression (and enhancement)

The first two terms of Eq. (66) represent the decoherence rates due to the individual environmental spins, whereas the last term represents the interference effect between the two environmental spins on the central spin. The total decoherence rate in the presence of both environmental spins is not a simple sum of the two individual decoherence rates. Depending on the sign of the interference term, the decoherence effect may be enhanced or suppressed.

Note that even if we assume $\overline{g_1} > 0$ and $\overline{g_2} > 0$ like what we did in the case of deterministic coupling, the quantity $\overline{g_1 g_2}$ can still be positive or negative. So, the situation in this case is different from what we had previously. Here, two factors jointly determine whether decoherence is enhanced or suppressed, namely, $\overline{g_1 g_2}$ and $\langle \sigma_{z,1} \sigma_{z,2} \rangle_{E0}$, whereas in the case of deterministic coupling we have $\langle \sigma_{z,1} \sigma_{z,2} \rangle_{E0}$ as the single determinant. In particular, if $\overline{g_1 g_2} < 0$ and $\langle \sigma_{z,1} \sigma_{z,2} \rangle_{E0} < 0$, we would have decoherence enhancement rather than suppression, a result that might be somewhat counterintuitive.

That being the case, provided that $\overline{g_1 g_2} > 0$ and $\langle \sigma_{z,1} \sigma_{z,2} \rangle_{E0} < 0$, or that $\overline{g_1 g_2} < 0$ and $\langle \sigma_{z,1} \sigma_{z,2} \rangle_{E0} > 0$, we would have a total decoherence rate less than the sum of the two individual decoherence rates. Such a scenario of decoherence suppression is desirable for quantum information processing. It should be cautioned, however, that we apparently need to have adequate control over the environmental initial state and the coupling strength correlation in order for this idea of decoherence suppression to work.

C. Conditions for noninterference

It has been shown that the interference effect of two baths on a system's dynamics under stochastic coupling is described by

$$\mathbb{L}_{\text{interference},t}(\rho) = \sum_{1a} \sum_{2b'} P_{1a,2b'}(\rho) + \sum_{2b} \sum_{1a'} P_{2b,1a'}(\rho), \quad (67)$$

where $P_{mn}(\rho)$ is defined as in Eq. (51). We will now investigate under what conditions such interference under stochastic coupling vanishes, so we are left with a simple additive scenario.

To look at the term $P_{1a,2b'}(\rho)$ in detail, we first note that $[E_{1a}(t), E_{2b'}(t')] = 0$, so that we have

$$\begin{aligned} P_{1a,2b'}(\rho) &= \int_0^t dt' \{ \overline{g_{1a}(t) g_{2b'}(t')} \langle E_{1a}(t) E_{2b'}(t') \rangle_{E0} \\ &\quad - \overline{g_{1a}(t) g_{2b'}(t')} \langle E_{1a}(t) \rangle_{E0} \langle E_{2b'}(t') \rangle_{E0} \} \\ &\quad \times [S_{1a}(t), [\rho, S_{2b'}(t')]]. \end{aligned} \quad (68)$$

Supposing that the two baths are initially statistically independent, namely,

$$\rho_{E,0} = \rho_{E1,0} \otimes \rho_{E2,0}, \quad (69)$$

we have the environmental correlation being

$$\begin{aligned} \langle E_{1a}(t)E_{2b'}(t') \rangle_{E0} &= \text{Tr}_E((E_{1a}(t) \otimes E_{2b'}(t'))(\rho_{E1,0} \otimes \rho_{E2,0})) \\ &= \text{Tr}_E(E_{1a}(t)\rho_{E1,0} \otimes E_{2b'}(t')\rho_{E2,0}) \\ &= \text{Tr}_{E1}(E_{1a}(t)\rho_{E1,0})\text{Tr}_{E2}(E_{2b'}(t')\rho_{E2,0}) \\ &= \text{Tr}_E((E_{1a}(t) \otimes \mathbb{I}_2)(\rho_{E1,0} \otimes \rho_{E2,0})) \\ &\quad \times \text{Tr}_E((\mathbb{I}_1 \otimes E_{2b'}(t'))(\rho_{E1,0} \otimes \rho_{E2,0})) \\ &= \langle E_{1a}(t) \rangle_{E0} \langle E_{2b'}(t') \rangle_{E0}, \end{aligned} \quad (70)$$

so we are left with

$$\begin{aligned} P_{1a,2b'}(\rho) &= \int_0^t dt' (\overline{g_{1a}(t)g_{2b'}(t')} - \overline{g_{1a}(t)}\overline{g_{2b'}(t')}) \\ &\quad \times \langle E_{1a}(t) \rangle_{E0} \langle E_{2b'}(t') \rangle_{E0} [S_{1a}(t), [\rho, S_{2b'}(t')]]. \end{aligned} \quad (71)$$

Suppose further that the system's coupling strengths with the two baths are uncorrelated, that is, for arbitrary a and b' ,

$$\overline{g_{1a}(t)g_{2b'}(t')} = \overline{g_{1a}(t)}\overline{g_{2b'}(t')}, \quad (72)$$

then the aforementioned term vanishes for all a and b' ,

$$\sum_{1a} \sum_{2b'} P_{1a,2b'}(\rho) = 0. \quad (73)$$

It can be shown that, under the same conditions,

$$\sum_{2b} \sum_{1a'} P_{2b,1a'}(\rho) = 0. \quad (74)$$

Thus, we have

$$\mathbb{L}_{\text{interference},t}(\rho) = 0. \quad (75)$$

Thus, we show that the two conditions in Eqs. (69) and (72) are jointly sufficient for the interference to vanish under stochastic coupling between the system and baths (up to second order). In other words, under the two aforementioned conditions, the combined effect of two baths on the system's average reduced dynamics is equal to a simple sum of the individual effects of the two baths, respectively, in the scenario of stochastic coupling, namely,

$$\frac{d}{dt}\rho_t = \mathbb{L}_{1,t}(\rho_t) + \mathbb{L}_{2,t}(\rho_t). \quad (76)$$

Equation (71) implies that the initial baths separability condition $\rho_{E,0} = \rho_{E1,0} \otimes \rho_{E2,0}$ alone does not suffice for noninterference. This is a nontrivial finding, as it is different from the case of deterministic coupling we study previously. In that case, the initial baths separability condition alone suffices for noninterference, regardless of the details of system-bath interaction.

V. CONCLUSIONS

We use a time-local master equation approach in Ref. [22] to provide a proof that quantum baths, albeit not directly coupled, generally can interfere on a system's reduced dynamics, confirming a previous result in Ref. [3] that is obtained using a diagrammatic approach and an integro-differential

equation approach. This means in principle multiple quantum baths may not be treated additively in terms of their dynamical effects on the system, at least not without adequate justifications. Specifically, interference effects between baths can be properly taken into account (up to second perturbative order) by using Eqs. (6), (7), (17), and (18). By studying an example of a central spin coupled to two environmental spins, which is much simpler than the three-level Λ system example in Ref. [3], we readily demonstrate the idea of using interference between baths to suppress decoherence of the system. Using the second-order master equation in Ref. [22] toward the example of a two-level system decaying into two photonic reservoirs in Ref. [3], we find that the second-order interference vanishes in this case. This finding differs from, but does not contradict with, the interference result in Ref. [3], which was not obtained perturbatively; nevertheless, this counterexample to interference prompts us to study more generally the condition for noninterference. We find that interference of two baths on the system's dynamics vanishes (up to second order) as long as the baths are initially separable (i.e., statistically independent), thereby identifying a “low-bar” condition for noninterference. Considering the prevalence of initially separable baths, this finding thus also serves to justify simple additive treatments of multiple baths usually practiced in the literature, especially in weak coupling and/or short time regimes where second-order effects dominate. Note that our additive result holds even in non-Markovian regimes, in contrast to the additive result in Ref. [3] that is obtained under the Markovian condition. We also discuss how this work on the conditional noninterference of quantum baths may contribute to the understanding of a counterintuitive finding in Ref. [37] for the scenario of quantum and classical noises, namely, that their second-order interference vanishes identically. We also extend the study of dynamics in multiple baths to the case of stochastic coupling. We use the formalism in Ref. [23] to prove that two baths generally interfere with one other on the system's average reduced dynamics under stochastic coupling. We use a simple example of a central spin stochastically coupled to two environmental spins to illustrate the idea of decoherence suppression (and enhancement), and show that the degree of decoherence suppression or enhancement depends on both $\overline{g_1 g_2}$ and $\langle \sigma_{z,1} \sigma_{z,2} \rangle_{E0}$ under stochastic coupling. We also find that two baths do not interfere with one other on a system's dynamics if (1) the two baths are initially statistically independent and (2) the system's stochastic coupling strengths with the two baths are uncorrelated. In the future, the question of higher-order interference (under deterministic or stochastic coupling) can be further investigated by working out higher-order terms of the master equations in Refs. [22,23] and applying them to the case of multiple baths.

ACKNOWLEDGMENT

L.Y. would like to thank Prof. Susanne Yelin for valuable discussions.

DATA AVAILABILITY

No data were created or analyzed in this study.

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- [35] More precisely, we mean that the baths are statistically independent at initial time. Thus, there is neither quantum correlation nor classical correlation between them. Note that the term “separability” sometimes refers to the absence of quantum correlation without regard to classical correlation, for example, in the quantum information literature, but that is not the same as what we mean here.
- [36] To be precise, what we mean is, without the condition of initially separable baths as in Eq. (38), we are not guaranteed the validity of the additive assumption and may not assume it *a priori*. Therefore, initial nonseparability of baths may, but does not necessarily, lead to the failure of the additive assumption.
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