

Topological leptogenesis

Juven Wang^{*,†}*London Institute for Mathematical Sciences, Royal Institution, W1S 4BS, London, United Kingdom
and Center of Mathematical Sciences and Applications, Harvard University, Massachusetts 02138, Cambridge, USA*

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In the standard lore, the baryon asymmetry of the present universe is attributed to the leptogenesis from the sterile right-handed neutrino with heavy Majorana fermion mass decaying into the Standard Model's leptons in the very early universe—called the Majorana leptogenesis, while the electroweak sphaleron causes baryogenesis at a later time. In this work, we propose a mechanism, named topological leptogenesis, to explain the lepton asymmetry. Topological leptogenesis replaces some of the sterile neutrinos by introducing a gapped topological order sector (whose low energy exhibits topological quantum field theory with long-range entanglement) that can cancel the baryon minus lepton ($\mathbf{B} \pm \mathbf{L}$) mixed gauge-gravitational anomaly of the standard model. But unlike the Majorana mass gap that causes the continuous ($\mathbf{B} - \mathbf{L}$) and lepton number $\mathbf{L}$ symmetry breaking down to the fermion parity $\mathbb{Z}_2^F$, the gapped topological order preserves a discrete ($\mathbf{B} \pm \mathbf{L}$) symmetry, with an extra gauge group symmetry extension over the discrete ($\mathbf{B} \pm \mathbf{L}$). Then, the beyond-the-standard-model dark matter consists of topological quantum matter, such that the gapped nonparticle excitations of extended line and surface defects with fractionalization and anyon charges can decay into the Standard Model particles. In addition, gravitational leptogenesis can be regarded as an intermediate step (between Majorana particle leptogenesis and topological nonparticle leptogenesis) to mediate such decaying processes from the highly entangled gapped topological order excitations.

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Introduction and summary. Leptogenesis is a hypothetical physical process that produces lepton and antilepton number asymmetry in the very early universe. Baryogenesis is yet another hypothetical process during the early universe to produce baryon and antibaryon asymmetry. The two syntheses result in the present-day dominance of leptons over antileptons and baryon over antibaryons in the observed universe. Sakharov [1] proposed three necessary conditions to produce an imbalance of numbers for baryons and antibaryons for generating a baryon asymmetry (1) Baryon number $\mathbf{B}$ violation, i.e. processes with $\Delta\mathbf{B} \neq 0$, which in the Standard Model arise nonperturbatively through electroweak sphaleron transitions associated with the anomalous non-conservation of the continuous $U(1)_B$ current. (2) Discrete charge conjugation C symmetry and CP (or time-reversal T due to CPT theorem) symmetry violation. (3) Out of thermal equilibrium, ensuring that detailed balance does not wash out the generated asymmetry. In the Standard Model, CP violation from the CKM matrix is insufficient to account for the observed baryon asymmetry. Moreover, for the measured Higgs mass, the electroweak phase transition is a crossover rather than strongly first-order, and therefore does not provide the necessary departure from thermal equilibrium. Consequently,

successful baryogenesis mechanisms, require physics beyond the Standard Model (BSM), typically involving lepton number violation and additional sources of CP violation.

One of the most popular scenarios is the *Majorana leptogenesis* advocated by Fukugita and Yanagida [2], among others (see Ref. [3] and references therein). Majorana leptogenesis [2] introduces some hypothetical heavy sterile right-handed neutrinos ν_R that are sterile to the SM gauge force, while ν_R is paired with itself with Majorana mass term in the lagrangian form $M\nu_R^T\nu_R + \text{H.c.}$ that breaks the fermion number conservation $U(1)_F$ down to the fermion parity $\mathbb{Z}_2^F$ subgroup. In the electroweak unbroken phase (where the Higgs doublet $\langle\tilde{\Phi}_H\rangle = 0$ has no vacuum expectation value, thus does not condense), the heavy right-handed neutrinos ν_R decay into a Standard Model lepton doublet and a Higgs doublet via Yukawa interactions. After electroweak symmetry breaking (where the Higgs doublet $\langle\tilde{\Phi}_H\rangle = \frac{1}{\sqrt{2}}(0, v_{\text{vev}}) \neq 0$ has a vacuum expectation value, thus condenses), these states correspond to final states involving $U(1)_{\text{EM}}$ electrically charged leptons (electron e^- , muon μ^- , and tauon τ^-) and $U(1)_{\text{EM}}$ electrically neutral left-handed neutrinos ($\nu_{L,e}$, $\nu_{L,\mu}$, and $\nu_{L,\tau}$), together with the physical Higgs boson ϕ_H . Some of these leptons can further decay to quarks through the electroweak sphaleron [4,5] via the Adler-Bell-Jackiw (ABJ) anomaly [6,7] under the Yang-Mills field instanton [8,9] of the weak gauge force, which is a popular scenario of *baryogenesis*.

Another scenario of leptogenesis without necessarily introducing any sterile right-handed neutrino ν_R is the *gravitational leptogenesis* [10]. The curved spacetime ripple locally may give rise to a source of gravitational instanton globally. So, the mixed gravitational anomaly [11–13] between the

*Contact author: jw@lims.ac.uk†<https://www.sns.ias.edu/~juven/>

lepton number $U(1)_L$ and the gravitational source from the triangle Feynman diagram $U(1)_L$ -gravity² can generate an unbalanced lepton number out of the curved spacetime ripple. Namely, the lepton number nonconservation or violation can be attributed to the mixed gravitational anomaly.

The purpose of this work is to propose a leptogenesis mechanism—*topological leptogenesis*—by introducing a hidden BSM topological sector that couples to the SM via topological gauge interaction [14–17]. Precisely, the BSM topological sector is a topological quantum matter sector whose low energy has a *topological quantum field theory* (TQFT, in the sense of Schwarz-type Chern-Simons-Witten-like theory [18–21]) and whose zero temperature phase (in terms of quantum many-body condensed matter sense) is *topological order* [22] (here sometimes abbreviated as TO).

For a concise introduction to low-energy TQFT descriptions [23,24] of topological order in $d + 1D$ spacetime, we refer to quantum field theories (QFTs) that lack local pointlike operators in the infrared and instead are characterized solely by extended operators, such as line and surface operators. The line operator can be regarded as a worldline of an anyon. The surface operator can be regarded as a world sheet of an anyonic string. The list of physical measurable observables includes the following:

(1) The path integral Z expectation value of link invariants between the extended operators evaluates the *braiding statistical phase* of these fractionalized (anyon) excitations evolving under the world trajectories (see, e.g., Refs. [20,23]),

$$Z(S^{d+1}; [\text{Link}]) / Z(S^{d+1}) = \exp(i\theta_{\text{statistic}}).$$

(2) The dimension of Hilbert space $\mathcal{H}$ on the M^d space counts the topological ground state degeneracy (GSD) (e.g., Ref. [24]) that evaluates the path integral Z on the $M^d \times S^1$ spacetime:

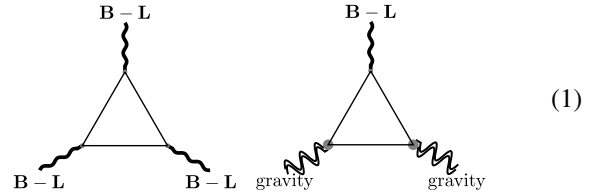
$$\dim(\mathcal{H})|_{M^d} = Z(M^d \times S^1) = \text{GSD}.$$

We propose that the fractionalized anyonlike excitations [25,26] with an energy E above the topological order gap Δ_{TO} (so $E \geq \Delta_{\text{TO}}$) can decay into the SM's “elementary” particles via topological discrete gauge interaction [later in Eq. (17)] or via the mixed baryon minus lepton ($\mathbf{B} - \mathbf{L}$) gauge-gravitational anomaly process [later in Eq. (16)].

Regardless of which types of leptogenesis out of the three (Majorana, gravitational, or topological) scenarios that we decide to study, the three scenarios can be motivated by the same common theme on the quantum anomaly structure of the SM. In other words, subconsciously or inadvertently, all three leptogenesis scenarios try to either implement or resolve the following anomaly facts of the SM:

(1) The continuous baryon $\mathbf{B}$ minus lepton $\mathbf{L}$ number symmetry, $U(1)_{\mathbf{B}-\mathbf{L}}$, is preserved within the SM. More precisely, to have a properly quantized charge, it is better to scale $U(1)_{\mathbf{B}-\mathbf{L}}$ by a factor of color number N_c as the quark $\mathbf{Q}$ number minus N_c lepton $\mathbf{L}$ number symmetry, $U(1)_{\mathbf{Q}-N_c\mathbf{L}}$, where the color number is $N_c = 3$ in the SM. So, the properly quantized charge is indeed $q_{\mathbf{Q}-N_c\mathbf{L}} = N_c q_{\mathbf{B}-\mathbf{L}}$. Namely, when we mention $U(1)_{\mathbf{B}-\mathbf{L}}$ symmetry and anomaly, we really mean $U(1)_{\mathbf{Q}-N_c\mathbf{L}}$ symmetry and anomaly. But the $U(1)_{\mathbf{Q}-N_c\mathbf{L}}$ symmetry has a 't Hooft anomaly in four-dimensional (4d)

spacetime captured by two triangle Feynman diagrams:

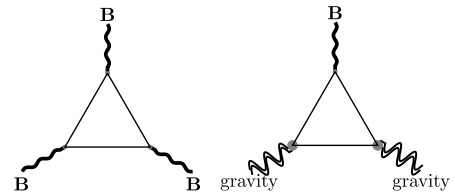


The 4d 't Hooft anomaly is also captured by a five-dimensional (5d) invertible topological field theory (iTFT) in one extra dimension with the following invertible complex $U(1)$ valued functional via the anomaly inflow [13,27–29]:

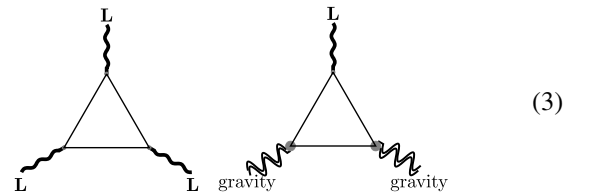
$$\exp(iS_5) \equiv \exp \left[i \int_{M^5} (-N_f + n_{v_R}) A \wedge \left(N_c^3 \frac{1}{6} \frac{dA}{2\pi} \wedge \frac{dA}{2\pi} + N_c \frac{1}{24} \frac{1}{8\pi^2} \text{Tr}[R \wedge R] \right) \right], \quad (2)$$

where the anomaly index $-N_f + n_{v_R}$ counts the difference between the family $N_f = 3$ and the total number of the types of right-hand neutrinos $n_{v_R} \equiv \sum_I n_{v_{I,R}} \cdot n_{v_R}$ is so far undetermined by the experiments, so n_{v_R} can be equal to, smaller than, or larger than 3 (here $I = 1, 2, 3, \dots$ for $e, \mu, \tau, \dots$ type of neutrinos). A is the $U(1)_{\mathbf{Q}-N_c\mathbf{L}}$ gauge field connection (locally a 1-form), and R is the spacetime curvature, locally a 2-form. The $U(1)^3$ anomaly in 4d matches with the 5d $A \wedge dA \wedge dA$ term, and the $U(1)$ -gravity² anomaly in 4d matches with the 5d $A \wedge \text{Tr}[R \wedge R]$ term.

We shall beware that the two Feynman diagrams from $U(1)_{\mathbf{B}}^3$ and $U(1)_{\mathbf{B}}\text{-gravity}^2$ contribute zero anomaly indices, schematically:



So, the only contribution of Eqs. (1) and (2) is from $U(1)_L^3$ and $U(1)_L\text{-gravity}^2$ anomalies:



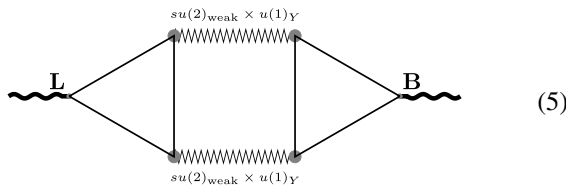
The 4d anomaly of Eq. (3) is captured by the 5D iTFT similar to Eq. (2) but a rescaling with $A_L = -N_c A$. The reason we prefer to focus on $U(1)_{\mathbf{Q}-N_c\mathbf{L}}$ instead of $U(1)_L$ symmetry is because only the former $U(1)_{\mathbf{Q}-N_c\mathbf{L}}$ has mixed anomaly-free with the SM gauge group but the latter $U(1)_L$ is anomalous with the SM gauge group. This relates to the next anomaly fact.

(2) $U(1)_{\mathbf{B}-\mathbf{L}}$ (or more precisely $U(1)_{\mathbf{Q}-N_c\mathbf{L}}$) is mixed anomaly-free with the SM Lie algebra $\mathcal{G}_{\text{SM}} \equiv su(3) \times su(2) \times u(1)_{\bar{Y}}$, or any of the four versions of the SM gauge group $G_{\text{SM}} \equiv (SU(3) \times SU(2) \times U(1)_{\bar{Y}})/\mathbb{Z}_q$ gauge group for

$q = 1, 2, 3, 6$ [30]. But $U(1)_B$ or $U(1)_L$ alone, each individually, does have mixed anomaly with $su(2) \times u(1)_Y$. We write $N_f = 3$ families of 15 or 16 Weyl fermions (spin- $\frac{1}{2}$ Weyl spinor is in the 2_L^C representation of the spacetime symmetry $\text{Spin}(1, 3)$ as a left-handed 15- or 16-plet (ψ_L) in the following $\mathcal{G}_{\text{SM}}$ representation:

$$(\psi_L)_I = (\bar{d}_R \oplus l_L \oplus q_L \oplus \bar{u}_R \oplus \bar{e}_R)_I \oplus n_{\nu_{L,R}} \bar{\nu}_{L,R} \sim ((\bar{3}, 1)_2 \oplus (1, 2)_{-3} \oplus (3, 2)_1 \oplus (\bar{3}, 1)_{-4} \oplus (1, 1)_6)_I \oplus n_{\nu_{L,R}} (1, 1)_0 \quad (4)$$

for each family. The family index is in the roman font $I = 1, 2, 3$, with ψ_{L1} for u, d, e type, ψ_{L2} for c, s, μ type, and ψ_{L3} for t, b, τ type of quarks and leptons. Thus, the following Feynman diagram consists of two nonvanishing ABJ anomaly contributions:



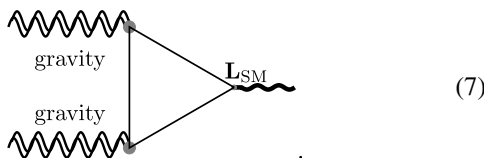
which can convert between $U(1)_B$ and $U(1)_L$ number violation through their anomalous current formulas:

$$\begin{aligned} d \star j_Q &= -N_c N_f \left(18 \frac{c_1(U(1)_Y)^2}{2} + c_2(SU(2)) \right), \\ d \star j_L &= -N_f \left(18 \frac{c_1(U(1)_Y)^2}{2} + c_2(SU(2)) \right). \end{aligned} \quad (6)$$

The anomalous current requires either nontrivial instanton number from the second Chern class c_2 globally of $su(2)$ (the sphaleron) or the first Chern class c_1^2 of $u(1)$: Equation (5) schematically illustrates the *electroweak sphaleron baryogenesis* conversion process between **B** and **L** [31].

In the following sections, by using the above two anomaly facts, we can streamline the known scenarios of *gravitational leptogenesis* and *Majorana leptogenesis* section, and then we propose the *topologic leptogenesis* section. Table II summarizes, compares, and contrasts the three scenarios.

Gravitational leptogenesis. Gravitational leptogenesis [10,32] can be simply explained by the following Feynman diagram:



In the pure gravitational leptogenesis, we have

$$N_f = 3 \quad \text{and} \quad n_{\nu_R} = 0.$$

The anomalous current nonconservation (or the violation of the $U(1)$ number conservation for the lepton number **L** or

Q – $N_c \mathbf{L}$ of the SM) obeys

$$\begin{aligned} d \star j_L &= (-N_f + n_{\nu_R}) \frac{p_1(TM)}{24}, \\ d \star j_{Q-N_c L} &= -(-N_f + n_{\nu_R}) N_c \frac{p_1(TM)}{24}. \end{aligned} \quad (8)$$

The lepton number **L** here in Eq. (8) is the $\mathbf{L}_{\text{SM}}$ of SM in Eq. (7). These equations can also be solved from the variation of the bulk-boundary coupled equation of Eq. (2). The first Pontryagin class p_1 can be defined globally when integrating over $\text{Tr}[R \wedge R]$ locally over the spacetime 4-manifold, by taking into account the transition functions between overlapping patches. p_1 is also related to the gravitational Chern-Simons 3-form:

$$\begin{aligned} p_1 &:= -\frac{1}{8\pi^2} \text{Tr}[R \wedge R] = -\frac{1}{8\pi^2} R_b^a \wedge R_a^b \\ &= -\frac{1}{8\pi^2} \frac{1}{2^2} \tilde{\epsilon}^{\mu\nu\alpha\beta} R_{b\mu\nu}^a R_{a\alpha\beta}^b d^4x \equiv -\frac{1}{2\pi} d\text{GCS}, \end{aligned} \quad (9)$$

$$\begin{aligned} \text{GCS} &:= \frac{1}{4\pi} \text{Tr} \left[\omega \wedge d\omega + \frac{2}{3} \omega \wedge \omega \wedge \omega \right] \\ &= \frac{1}{4\pi} \tilde{\epsilon}^{\nu\alpha\beta} \left(\omega_v^a \partial_a \omega_\beta^b + \frac{2}{3} \omega_v^a \omega_\alpha^b \omega_\beta^c \right) d^3x. \end{aligned} \quad (10)$$

In Eq. (8), the local nonzero $\text{Tr}[R \wedge R]$ can make $d \star j_L$ nonzero locally. However, to achieve the lepton number violation globally in the early universe, we need to have the spacetime topology changing process so that the gravitational instanton number p_1 changes (such as creating a K3 surface with $p_1(K3) = -16$ or $p_1(\mathbb{CP}^2) = 1$ in the Euclidean signature). So, the lepton number changing from the time t_i to t_f via a topology changing ΔM^4 is

$$L(t_f) - L(t_i) \equiv \Delta L = \int_{\Delta M^4} d \star j_L = (-N_f + n_{\nu_R}) \frac{\Delta p_1}{24}. \quad (11)$$

There are a few challenges for this scenario:

(1) Gravitational leptogenesis by itself [10] originally does not provide light neutrino mass. The light neutrino mass here requires a new mechanism. Proposals along this direction can be found in Refs. [33,34].

(2) The continuous $U(1)$ of $(\mathbf{B} - \mathbf{L})$ is unbroken from the IR into the deep UV; thus, it has to be dynamically gauged to be consistent with the quantum gravity argument (see Ref. [35] and references therein). But a new $U(1)$ photon of dynamically gauged gauge $(\mathbf{B} - \mathbf{L})$ contradicts experiments in nature.

Precision electroweak measurements at the Large Electron-Positron Collider (LEP), including Z-pole observables, fermion pair production at LEP-2, and the invisible Z width, place strong constraints on any additional neutral gauge boson such as a $U(1)_{B-L}$'s Z' boson, severely limiting its mass, coupling, and mixing with the Standard Model Z [36–38].

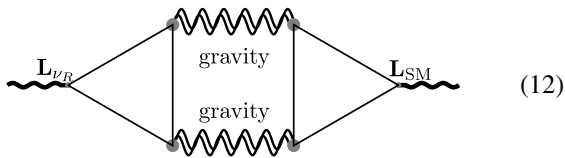
(3) The model itself is gravitational anomalous, so this proposal alone may not be a final story in a dynamical gravity theory or in full quantum gravity. The full theory is better to be fully anomaly-free thus in a trivial cobordism class [35] in the sense of Refs. [28,29].

Majorana leptogenesis can rescue some of these challenges.

Majorana leptogenesis. The popular Majorana leptogenesis [2,3] provides a UV completion of gravitational leptogenesis by canceling the gravitational anomaly via adding ν_R . For a pure Majorana leptogenesis to cancel the gravitational anomaly, we have

$$N_f = 3 \quad \text{and} \quad n_{\nu_R} = 3.$$

The gravitational anomaly was not emphasized or motivated in the original Fukugita-Yanagida model [2]; thus, it is common that the traditional leptogenesis does not demand $n_{\nu_R} = 3$ [3]. A schematic Feynman diagram that evolves Majorana leptogenesis through gravitational anomaly can be given as follows:



But this gravitational involved effect can be a much less significant order effect, namely, a higher-order effect [39].

Indeed, the heavy ν_R and its lepton number current of $\mathbf{L}_{\nu_R}$ can decay to Higgs boson and other experimentally observed less-heavy leptons—including neutral left-handed neutrinos $\nu_{L,e}$, $\nu_{L,\mu}$, and $\nu_{L,\tau}$, and the Higgs can further decay to other quarks and leptons via the following terms: Eqs. (13) and (14) [2]. A typical seesaw mechanism introduces Dirac mass term with the SM Higgs ϕ_H and with Yukawa-Higgs coupling λ_v ,

$$\begin{aligned} & -\lambda_v(\nu_L^\dagger \phi_H^*)\nu_R - \lambda_v^* \nu_R^\dagger(\phi_H \nu_L) \quad \text{from} \\ & -\lambda_v(\bar{\ell}_L \varepsilon \bar{\Phi}_H^\dagger)\nu_R - \lambda_v^* \bar{\nu}_R(\bar{\Phi}_H \varepsilon \bar{\ell}_L) \end{aligned} \quad (13)$$

(which formally comes from the lagrangian involving Higgs SU(2) doublet $\bar{\Phi}_H$ and the lepton SU(2) doublet $\bar{\ell}_L$) and Majorana mass term among ν_R ,

$$-\frac{iM}{2}(\nu_R^T \sigma^2 \nu_R + \nu_R^{\dagger} \sigma^2 \nu_R^*). \quad (14)$$

Here, ε and $i\sigma^2$ are alternating tensors on the internal SU(2) and the Lorentz SU(2) subgroup, respectively.

Equation (14) explicitly breaks the neutrino number continuous symmetry of U(1) down to the discrete fermion parity $\mathbb{Z}_2^F$. If the ν_R is heavy with a large Majorana mass M , the nearly massless energy eigenstate has $\frac{m^2}{M} \sim \frac{|\lambda_v \langle \phi_H \rangle|^2}{M}$. The low-energy physics by integrating out the heavy ν_R gives rise to the dimension-5 Weinberg operator

$$\begin{aligned} & \sim \frac{1}{2M}(\lambda_v^2 \times (\nu_L^\dagger \phi_H^*)(\phi_H^* \nu_L^*) + \lambda_v^{*2}(\nu_L^T \phi_H)(\phi_H \nu_L)) \quad \text{from} \\ & \frac{1}{2M}(\lambda_v^2 \times (\bar{\ell}_L \varepsilon \bar{\Phi}_H^\dagger)(\bar{\ell}_L \varepsilon \bar{\Phi}_H^\dagger) + \lambda_v^{*2} \times (\bar{\Phi}_H \varepsilon \bar{\ell}_L)(\bar{\Phi}_H \varepsilon \bar{\ell}_L)) \end{aligned} \quad (15)$$

that gives the effective Majorana mass to ν_L .

In *Majorana leptogenesis* [2,3], ν_R replaces the role of the curved spacetime ripple with nonzero $\text{Tr}[R \wedge R]$ in gravitational leptogenesis. But there are some curious facts about Majorana leptogenesis that may pose challenges to it:

(1) The ν_R Majorana mass term already breaks the U(1) version of $\mathbf{B} - \mathbf{L}$ (namely, $\text{U}(1)_{\mathbf{Q}-N_c\mathbf{L}}$) down to fermion parity $\mathbb{Z}_2^F$ at its mass energy scale M at UV. So, it is curious how that the broken $\mathbf{B} - \mathbf{L}$ re-emerge at the SM scale as a good nearly anomaly-free continuous global symmetry $\text{U}(1)_{\mathbf{Q}-N_c\mathbf{L}}$ back in the SM at IR.

From the renormalization group and IR effective field theory viewpoint in Eq. (15), we may say that $\text{U}(1)_{\mathbf{Q}-N_c\mathbf{L}}$ is never an exact global symmetry but only an approximate symmetry, broken by the ratio of $O(\frac{E}{M})$ at the energy scale E .

(2) Once the U(1) version of $\mathbf{B} - \mathbf{L}$ (namely, $\text{U}(1)_{\mathbf{Q}-N_c\mathbf{L}}$) symmetry is broken by ν_R Majorana mass term at UV down to fermion parity $\mathbb{Z}_2^F$, there is no direct IR constraint to demand the 16th Weyl fermion ν_R to cancel the SM's $\mathbf{B} - \mathbf{L}$ anomaly, because there is no $\mathbf{B} - \mathbf{L}$ anomaly at UV when $\mathbf{B} - \mathbf{L}$ is already broken from U(1) down to $\mathbb{Z}_2^F$ at UV.

So, it seems to have an alternative ($\mathbf{B} - \mathbf{L}$)-preserving scenario at UV with an alternative energetically gapped state, but without introducing ν_R .

In contrast, *Dirac leptogenesis* [40,41] conserves the continuous U(1) symmetry, namely, does not break $\text{U}(1)_{\mathbf{B}-\mathbf{L}}$ nor break the lepton $\text{U}(1)_{\mathbf{L}}$ symmetry. Thus, the above two challenges about breaking a lepton $\text{U}(1)_{\mathbf{L}}$ symmetry at UV and re-emerging the $\text{U}(1)_{\mathbf{L}}$ at IR only apply to Majorana leptogenesis (lepton number change $\Delta\mathbf{L} = 0 \pmod{2}$) but do not apply to Dirac leptogenesis ($\Delta\mathbf{L} = 0$ at the Lagrangian level).

Still, these curious facts prompt the author to think of an alternative scenario: topological leptogenesis. The author pursues the alternative story such that the discrete version of $\mathbf{B} - \mathbf{L}$ can be exact, anomaly-free, and gaugeable in the next section. Table 1 summarizes the symmetry-preserving versus symmetry-breaking patterns for these scenarios.

Topological leptogenesis. Indeed, when the $\mathbf{B} - \mathbf{L}$ is discrete, such as a discrete $\mathbb{Z}_{4,X}$ subgroup, where $X \equiv 5(\mathbf{B} - \mathbf{L}) - \frac{2}{3}\tilde{Y} = \frac{5}{N_c}(\mathbf{Q} - N_c\mathbf{L}) - \frac{2}{3}\tilde{Y}$ with properly integer quantized hypercharge $\tilde{Y}$ [42,43], the perturbative mixed $\mathbf{B} - \mathbf{L}$ -gravitational anomaly with a $\mathbb{Z}$ classification becomes a nonperturbative mixed $\mathbb{Z}_{4,X}$ -gravitational anomaly with a $\mathbb{Z}_{16}$ classification for this anomaly in 4d spacetime [44–48]. There are new ways to cancel the $\mathbb{Z}_{16}$ anomaly by introducing 4d TQFT or 4d CFT and 5d invertible TQFT [14–17]. Importantly, these new 4d TQFT or 4d CFT and 5d invertible TQFT preserve the $\mathbb{Z}_{4,X}$ entirely without $\mathbb{Z}_{4,X}$ breaking. The underlying mechanism of these interacting sectors is based on the symmetry extension [49], instead of the symmetry breaking of the traditional Nambu-Goldstone-Anderson-Higgs or Landau-Ginzburg symmetry-breaking-type theory. So, topological leptogenesis introduces some potential $\mathbb{Z}_{4,X}$ -symmetric gapped topological order with low-energy 4d TQFT. The characteristic properties of topological order are as follows:

(1) The energy spectrum above the TQFT ground state is gapped, with an energy gap size denoted by Δ_{TO} . The energetic excitations with energy E above the gap ($E \geq \Delta_{\text{TO}}$) contain fractionalized excitations with anyon statistics [26] (that can be neither boson nor fermion statistics).

(2) The ground-state degeneracy (the number of ground states) of TO/TQFT depends on the spatial topology. The ground states of TO/TQFT exhibit long-range entanglement

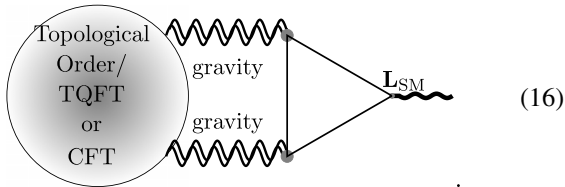
TABLE I. Comparison of four different mass generating physics, and their symmetry-preserving or symmetry-breaking patterns. SSB stands for the conventional “spontaneous symmetry breaking.” The comment on the “Spontaneous” only applies to those terms with $\langle\phi_H\rangle$ SSB for the energy scale below the electroweak scale $E \lesssim \Lambda_{EW}$.

	Neutrino Yukawa-Higgs Dirac mass (13)	ν_R Majorana mass (14)	ν_L dimension-5 Majorana mass (15)	$\mathbb{Z}_{4,X}$ -symmetric TO/TQFT or CFT [14–17]
Continuous $U(1)_Y$	Explicit: preserved. Spontaneous: broken by $\langle\phi_H\rangle$ SSB whose $\tilde{Y} = 3$ below $E \lesssim \Lambda_{EW}$.	Explicit: preserved.	Explicit: preserved. Spontaneous: broken by $\langle\phi_H\rangle$ SSB below $E \lesssim \Lambda_{EW}$.	Explicit: preserved. Spontaneous: preserved.
Continuous $(\mathbf{B} - \mathbf{L})$ or $U(1)_{Q-N_{cL}}$	Explicit: preserved. Spontaneous: preserved.	Explicit: broken, down to $\mathbb{Z}_2^F$.	Explicit: broken, down to $\mathbb{Z}_2^F$.	Explicit: no continuous $(\mathbf{B} - \mathbf{L})$ symmetry.
Continuous $U(1)_X$	Explicit: preserved. Spontaneous: broken by $\langle\phi_H\rangle$ SSB whose $X = -2$ below $E \lesssim \Lambda_{EW}$ down to $\mathbb{Z}_2^F$.	Explicit: broken, down to $\mathbb{Z}_2^F$.	Explicit: broken, down to $\mathbb{Z}_2^F$. Spontaneous: broken already, but also by $\langle\phi_H\rangle$ SSB whose $X = -2$ below $E \lesssim \Lambda_{EW}$.	Explicit: no continuous $U(1)_X$ symmetry.
Discrete $\mathbb{Z}_{4,X}$	Explicit: preserved. Spontaneous: broken by $\langle\phi_H\rangle$ SSB whose $X = 2$ below $E \lesssim \Lambda_{EW}$ down to $\mathbb{Z}_2^F$.	Explicit: broken, down to $\mathbb{Z}_2^F$.	Explicit: broken, down to $\mathbb{Z}_2^F$. Spontaneous: broken already, but also by $\langle\phi_H\rangle$ SSB whose $X = 2$ below $E \lesssim \Lambda_{EW}$.	Explicit: preserved. Discrete $\mathbb{Z}_{4,X}$ is exact, anomaly-free gaugeable for the full SM + BSM [16], and dynamically gauged in a full quantum gravity theory [28,29,35].

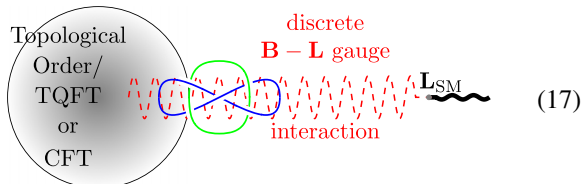
that cannot be deformed to a trivial gapped vacuum (with a single ground state) via any local quantum unitary transformation.

We shall name this scenario topological leptogenesis, due to its long-range entanglement topological property of TO/TQFT. Topological leptogenesis resolves the following:

(1) A UV completion of gravitational leptogenesis without ν_R as this schematic process:



(2) Furthermore, without using gravity as an intermediate mediator, there is also the discrete $\mathbf{B} - \mathbf{L}$ gauge topological force (the $\mathbb{Z}_{4,X}$ gauge force) to mediate between the $\mathbb{Z}_{4,X}$ -charged quarks and leptons in the SM and the $\mathbb{Z}_{4,X}$ -Cheshire-charged extended TO/TQFT link configurations (that stores the $\mathbb{Z}_{4,X}$ charge nonlocally in the link in the Cheshire-charge way):



Following Ref. [47], here we show the Whitehead link that can detect a similar kind of Arf invariants [50] and Arf-Brown-Kervrie invariant [51,52] appearing in the 4d TQFT construction [14–17].

(3) Different low-energy physics at IR: This scenario cannot generate the dimension-5 Weinberg operator (15), because that operator breaks the continuous and discrete X symmetry ($U(1)_X$ and $\mathbb{Z}_{4,X}$) explicitly. Also, the TQFT/TO have fundamentally different properties than the defects studied in Refs. [33,34].

The three scenarios studied here are general and universal in the sense of anomaly cancellation consideration, given the systematic classification of anomalies in Refs. [45,48,54] and [14–16,28,29], respectively. Regardless of which of the three (Majorana, gravitational, or topological) leptogenesis scenarios that nature may choose (or not), the three scenarios will need to welcome new experimental signatures for verification.

Future directions. Let us conclude with some future directions:

(1) The electroweak sphaleron process preserves $U(1)_{\mathbf{B}-\mathbf{L}}$, but it can change the baryon and lepton numbers, so they can be nonzero but equal:

$$\Delta \mathbf{B} = \Delta \mathbf{L}. \quad (18)$$

The sphaleron process also preserves the discrete $\mathbf{B} + \mathbf{L}$ symmetry $\mathbb{Z}_{2N_f, \mathbf{B}+\mathbf{L}} = \mathbb{Z}_{6, \mathbf{B}+\mathbf{L}}$ (e.g., Refs. [29,55]), an order-6 discrete finite abelian symmetry; hence,

$$\Delta \mathbf{B} + \Delta \mathbf{L} = 0 \pmod{2N_f = 6}. \quad (19)$$

TABLE II. Comparison of three different leptogenesis scenarios. When we refer to $\mathbf{B} - \mathbf{L}$, we really mean $\mathbf{Q} - N_c \mathbf{L}$, while its U(1) version is the $U(1)_{\mathbf{Q}-N_c \mathbf{L}}$ with properly quantized charges. We refer to the X symmetry as a chiral symmetry with $X \equiv 5(\mathbf{B} - \mathbf{L}) - 4Y \equiv 5(\mathbf{B} - \mathbf{L}) - \frac{2}{3}\tilde{Y} = \frac{5}{N_c}(\mathbf{Q} - N_c \mathbf{L}) - \frac{2}{3}\tilde{Y}$ that acts chirally on the SM Weyl fermions. We consider the energy scale E with respect to various physically pertinent energy: Majorana mass M and topological order gap Δ_{TO} , or the electroweak Higgs Λ_{EW} scale. SSB stands for the conventional “spontaneous symmetry breaking.”

	Majorana leptogenesis [2]	Gravitational leptogenesis [10]	Topological leptogenesis [16]
ν_R	Require ν_R	No ν_R needed	No ν_R needed
UV symmetry (E around M or Δ_{TO})	$(\mathbf{B} - \mathbf{L})$ and $U(1)_X$ alike, and $\mathbb{Z}_{4,X}$ explicitly broken by ν_R 's Majorana mass term.	$(\mathbf{B} - \mathbf{L})$, $U(1)_X$, and $\mathbb{Z}_{4,X}$ preserved (no Majorana mass term).	$(\mathbf{B} - \mathbf{L})$, $U(1)_X$, and $\mathbb{Z}_{4,X}$ preserved. An X -symmetric gapped topological order with a low-energy TQFT (no Majorana mass term). Discrete X becomes dynamically gauged [53] due to no global symmetry in quantum gravity [35].
IR symmetry (E below Λ_{EW})	Continuous and also discrete $(\mathbf{B} - \mathbf{L})$ are fully preserved in the SM. But the continuous and discrete X are spontaneously broken down to $\mathbb{Z}_2^F$ due to Higgs condensate $\langle \phi_H \rangle$ carrying X and electroweak hyper Y charges.		
Anomaly matching or cancellation	(1) Adding ν_R cancels the continuous $(\mathbf{B} - \mathbf{L})$ -grav ² and $(\mathbf{B} - \mathbf{L})^3$ anomalies, and the discrete $\mathbb{Z}_{4,X}$ -gravity anomaly.	(1) The continuous $(\mathbf{B} - \mathbf{L})$ -grav ² and $(\mathbf{B} - \mathbf{L})^3$ anomalies, and the discrete $\mathbb{Z}_{4,X}$ -gravity anomaly are <i>not</i> canceled.	(1) The continuous $U(1)$ of $(\mathbf{B} - \mathbf{L})$ or X is <i>not</i> exact <i>nor</i> dynamically gauged at UV.
	(2) But ν_R 's Majorana mass <i>already breaks</i> the $(\mathbf{B} - \mathbf{L})$ and X symmetries at UV.	(2) Gravitational anomaly generates leptogenesis.	(2) The discrete X (say, $\mathbb{Z}_{4,X}$) is exact and dynamically gauged at UV. (3) The $\mathbb{Z}_{16}$ class of $\mathbb{Z}_{4,X}$ -gravity anomaly with index $-3 \bmod 16$ in the SM is canceled by the new BSM TQFT or CFT.
Dark matter candidate	heavy ν_R with Majorana mass	additional axion φ like in $\varphi \text{Tr}[R \wedge R]$ [10]	(1) Fractionalized zero-dimensional (0d) particle [one-dimensional (1d) worldline] and anyon 1d loop [two-d (2D) world sheet] gapped excitations above Δ_{TO} in the 4d spacetime. (2) Gapped SPTs with low-energy iTFT in the 5d spacetime. (3) Gapped topological order with low-energy noninvertible TQFT in the 5d spacetime when $\mathbb{Z}_{4,X}$ becomes dynamically gauged at UV high energy. (4) Possible unparticle CFT excitations [16] if we use interacting CFT to cancel the $\mathbb{Z}_{4,X}$ -gravity anomaly.
Baryogenesis	All scenarios can use the sphaleron from the electroweak $su(2)_w \times u(1)_Y$ sector to generate baryon asymmetry. (1) Typically through the sphaleron doing the conversion from the anomalous lepton current via $\mathbf{L}$ - $su(2)_w^2$ anomaly to the anomalous baryon current via $\mathbf{B}$ - $su(2)_w^2$ anomaly in the flat spacetime. (2) It is possible to have the conversion from the anomalous lepton current via $\mathbf{L}$ - $u(1)_Y^2$ anomaly to the anomalous baryon current via $\mathbf{B}$ - $u(1)_Y^2$ anomaly in the nontrivial spacetime topology. (3) $(\mathbf{B} - \mathbf{L})$ current is conserved and nonanomalous, because $(\mathbf{B} - \mathbf{L})$ - $su(2)_w^2$ and $(\mathbf{B} - \mathbf{L})$ - $u(1)_Y^2$ are anomaly-free, at this energy scale range.		

TABLE II. (*Continued.*)

	Majorana leptogenesis [2]	Gravitational leptogenesis [10]	Topological leptogenesis [16]
ν_R	Require ν_R	No ν_R needed	No ν_R needed
Comments	<p><i>Pros:</i> Seesaw mechanism with tiny ν_L mass. Then, the sphaleron generates the baryon asymmetry.</p> <p><i>Cons:</i> (1) No $\mathbb{Z}_{16}$ anomaly cancellation at all at UV (due to $\mathbb{Z}_{4,X}$ already broken by Majorana mass) to constrain 16th Weyl ν_R.</p> <p>(2) Both continuous and discrete X ($U(1)_X$ and $\mathbb{Z}_{4,X}$) are <i>explicitly broken</i> at deep UV, but re-emerge at IR then to be SSB <i>again</i> by Higgs vev $\langle\phi_H\rangle$. $\Rightarrow$ More contrived symmetry pattern from UV to IR.</p>	<p><i>Pros:</i> Gravitational instanton/gravitational wave or curved spacetime background generates lepton asymmetry. Then, the sphaleron generates the baryon asymmetry.</p> <p><i>Cons:</i> (1) The continuous $U(1)$ of $(\mathbf{B} - \mathbf{L})$ is unbroken from IR to UV; thus, it has to be dynamically gauged to be consistent with quantum gravity [35]. But new gauge $U(1)$ photon of $(\mathbf{B} - \mathbf{L})$ contradicts experiments in nature.</p> <p>(2) The model itself is gravitational anomalous.</p>	<p><i>Pros:</i> (1) The $U(1)$ of $(\mathbf{B} - \mathbf{L})$ is <i>not</i> exact <i>nor</i> dynamically gauged. No additional new gauge $U(1)$ of $(\mathbf{B} - \mathbf{L})$; thus, no new photon is needed.</p> <p>(2) Consistent discrete $\mathbb{Z}_{4,X}$ gauge symmetry preserving from deep UV to IR, until below Λ_{EW} scale, the $\mathbb{Z}_{4,X}$ SSB by $\langle\phi_H\rangle$ condensate.</p> <p>(3) Consistent discrete $(\mathbf{B} - \mathbf{L})$ gauge symmetry preserving from deep UV to IR, even below Λ_{EW} scale, it is still preserved.</p> <p>(4) Both discrete X and discrete $(\mathbf{B} - \mathbf{L})$ are secretly dynamically gauged, although may behave as global symmetries at IR [53]. The model with SM + topological matter is anomaly-free.</p> <p><i>Cons:</i> Topological quantum matter as dark matter and its topological discrete gauge interactions with the SM are difficult to be detected by the conventional experiments.</p>

Thus, the above two conditions, due to the sphaleron process, combine into

$$\Delta\mathbf{B} = \Delta\mathbf{L} = 0 \pmod{N_f = 0 \pmod{3}},$$

$$\text{or simply } \Delta\mathbf{B} = \Delta\mathbf{L} \in 3\mathbb{Z}. \quad (20)$$

Furthermore, we consider the new generalized $U(1)_{X_n}$ symmetry of the SM discovered in Ref. [56] for any integer $n \in \mathbb{Z}$: $X_n \equiv n(\mathbf{B} - \mathbf{L}) + (1 - \frac{n}{N_c})\tilde{Y} \equiv \frac{n}{N_c}(\mathbf{Q} - N_c\mathbf{L}) + (1 - \frac{n}{N_c})\tilde{Y}$, which contains the baryon minus lepton symmetry $X_3 = N_c(\mathbf{B} - \mathbf{L}) = \mathbf{Q} - N_c\mathbf{L}$ and Wilczek-Zee's chiral symmetry [42] at $X_5 \equiv X \equiv 5(\mathbf{B} - \mathbf{L}) - \frac{2}{3}\tilde{Y}$. If the SM and BSM preserve $U(1)_{X_n}$ symmetry, then we have

$$\Delta X_n = 0. \quad (21)$$

It is also possible to impose a discrete X_n symmetry for coupling a dynamical gauge field, such as

$$\Delta X_5 \equiv \Delta X = 0 \pmod{4}. \quad (22)$$

In fact, the anomaly constraints on TQFT sectors for various discrete $\mathbf{B} \pm \mathbf{L}$ internal (gauge or global) symmetries are recently systematically examined in Refs. [57–59] and similarly appeared in the framework of Refs. [17,60–63]. Can these new TQFT sectors further constrain topological leptogenesis or provide a new source for baryogenesis?

(2) Further UV completions: Can one further embed the topologically ordered sector and TQFT together with the SM sector into quantum gravity or string-inspired constructions or frameworks?

(3) IR observables: Look for possible experimental particle-physics or cosmological signatures of topologically ordered sectors in the absence of neutrinoless double beta decay (No $\Delta L = 2$), but in the presence of $\Delta X = 4$ process induced by four-fermion interactions [57]

$$g_{abcd} \psi_a \psi_b \psi_c \psi_d$$

with a charge $X_\psi = 1 \pmod{4}$. For example, we can find possible discrete $\mathbf{B} \pm \mathbf{L}$ or X interactions in the cosmological

Lithium problem [64] or cosmic relics for dark sector couplings.

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- [1] A. D. Sakharov, Violation of CP Invariance, C asymmetry, and baryon asymmetry of the universe, *Pis'ma Zh. Eksp. Teor. Fiz.* **5**, 32 (1967).
 - [2] M. Fukugita and T. Yanagida, Baryogenesis without grand unification, *Phys. Lett. B* **174**, 45 (1986).
 - [3] S. Davidson, E. Nardi, and Y. Nir, Leptogenesis, *Phys. Rep.* **466**, 105 (2008).
 - [4] F. R. Klinkhamer and N. S. Manton, A saddle-point solution in the weinberg-salam theory, *Phys. Rev. D* **30**, 2212 (1984).
 - [5] V. A. Kuzmin, V. A. Rubakov, and M. E. Shaposhnikov, On the anomalous electroweak baryon number nonconservation in the early universe, *Phys. Lett. B* **155**, 36 (1985).
 - [6] S. L. Adler, Axial vector vertex in spinor electrodynamics, *Phys. Rev.* **177**, 2426 (1969).
 - [7] J. S. Bell and R. Jackiw, A PCAC puzzle: $\pi^0 \rightarrow \gamma\gamma$ in the σ model, *Nuovo Cimento A* **60**, 47 (1969).
 - [8] G. 't Hooft, Symmetry breaking through Bell-Jackiw anomalies, *Phys. Rev. Lett.* **37**, 8 (1976).
 - [9] A. A. Belavin, A. M. Polyakov, A. S. Schwartz, and Yu. S. Tyupkin, Pseudoparticle solutions of the Yang-Mills equations, *Phys. Lett. B* **59**, 85 (1975).
 - [10] S. H.-S. Alexander, M. E. Peskin, and M. M. Sheikh-Jabbari, Leptogenesis from gravity waves in models of inflation, *Phys. Rev. Lett.* **96**, 081301 (2006).
 - [11] T. Eguchi and P. G. O. Freund, Quantum gravity and world topology, *Phys. Rev. Lett.* **37**, 1251 (1976).
 - [12] L. Alvarez-Gaume and E. Witten, Gravitational anomalies, *Nucl. Phys. B* **234**, 269 (1984).
 - [13] P. Putrov and J. Wang, Categorical symmetry of the standard model from gravitational anomaly, *Phys. Rev. D* **110**, 125028 (2024).
 - [14] J. Wang, Anomaly and cobordism constraints beyond the standard model: Topological force, [arXiv:2006.16996](https://arxiv.org/abs/2006.16996) [hep-th].
 - [15] J. Wang, Anomaly and cobordism constraints beyond grand unification: Energy hierarchy, [arXiv:2008.06499](https://arxiv.org/abs/2008.06499) [hep-th].
 - [16] J. Wang, Ultra Unification: Unified model beyond grand unification, *Phys. Rev. D* **103**, 105024 (2021).
 - [17] M. Cheng, J. Wang, and X. Yang, Boundary topological orders of $(4+1)d$ fermionic $\mathbb{Z}_{2N}^F$ symmetry protected topological states, *Phys. Rev. B* **113**, 125105 (2026).
 - [18] S.-S. Chern and J. Simons, Characteristic forms and geometric invariants, *Ann. Math.* **99**, 48 (1974).
 - [19] A. S. Schwarz, The partition function of degenerate quadratic functional and ray-singer invariants, *Lett. Math. Phys.* **2**, 247 (1978).
 - [20] E. Witten, Quantum field theory and the jones polynomial, *Commun. Math. Phys.* **121**, 351 (1989).
 - [21] M. Atiyah, Topological quantum field theories, *Inst. Hautes Etudes Sci. Publ. Math.* **68**, 175 (1989).
 - [22] X.-G. Wen, Zoo of quantum-topological phases of matter, *Rev. Mod. Phys.* **89**, 041004 (2017).
 - [23] P. Putrov, J. Wang, and S.-T. Yau, Braiding statistics and link invariants of bosonic/fermionic topological quantum matter in $2+1$ and $3+1$ dimensions, *Ann. Phys.* **384**, 254 (2017).
 - [24] J. Wang, K. Ohmori, P. Putrov, Y. Zheng, Z. Wan, M. Guo, H. Lin, P. Gao, and S.-T. Yau, Tunneling topological vacua via extended operators: (Spin-)TQFT spectra and boundary deconfinement in various dimensions, *Prog. Theor. Exp. Phys.* **2018**, 053A01 (2018).
 - [25] F. Wilczek, Quantum mechanics of fractional spin particles, *Phys. Rev. Lett.* **49**, 957 (1982).
 - [26] Fractional statistics and anyon superconductivity, edited by F. Wilczek (World Scientific, 1990).
 - [27] C. G. Callan, Jr. and J. A. Harvey, Anomalies and fermion zero modes on strings and domain walls, *Nucl. Phys. B* **250**, 427 (1985).
 - [28] J. Wang, Z. Wan, and Y.-Z. You, Cobordism and deformation class of the standard model, *Phys. Rev. D* **106**, L041701 (2022).
 - [29] J. Wang, Z. Wan, and Y.-Z. You, Proton stability: From the standard model to beyond grand unification, *Phys. Rev. D* **106**, 025016 (2022).
 - [30] D. Tong, Line operators in the standard model, *J. High Energy Phys.* **07** (2017) 104.
 - [31] c_1 is obtained by integrating over $c_1 \equiv \int \frac{1}{2\pi} \text{Tr } F$ locally with nontrivial transition functions between patches. c_2 is obtained by integrating over $c_2 \equiv \int -\frac{1}{8\pi^2} \text{Tr}(F \wedge F) + \frac{1}{2}(\frac{1}{2\pi} \text{Tr } F)^2$ locally with nontrivial transition functions between patches. It is more common to consider the $su(2)$ sphaleron because the instanton number on a compact space like four-sphere S^4 can still be nonzero. But the $u(1)$ instanton number on S^4 is trivial; only that on those such as $S^2 \times S^2$ with nontrivial lower homotopy can be nonzero.
 - [32] See Refs. [65,66] for the gravitational leptogenesis realization in Einstein gravity versus modified gravity. See Ref. [67] for discussions on Dirac or Majorana neutrino mass scenarios for such leptogenesis.
 - [33] G. Dvali and L. Funcke, Small neutrino masses from gravitational θ -term, *Phys. Rev. D* **93**, 113002 (2016).
 - [34] G. Dvali, L. Funcke, and T. Vachaspati, Time- and space-varying neutrino mass matrix from soft topological defects, *Phys. Rev. Lett.* **130**, 091601 (2023).
 - [35] J. McNamara and C. Vafa, Cobordism classes and the swampland, [arXiv:1909.10355](https://arxiv.org/abs/1909.10355) [hep-th].
 - [36] ALEPH, DELPHI, L3, and OPAL Collaborations, Precision electroweak measurements on the Z resonance, *Phys. Rep.* **427**, 257 (2006).
 - [37] J. Erler and P. Langacker, Constraints on extended neutral gauge structures, *Phys. Rev. Lett.* **84**, 212 (2000).
 - [38] P. Langacker, The physics of heavy z' gauge bosons, *Rev. Mod. Phys.* **81**, 1199 (2009).

[39] Let us examine the order of magnitude of gravitational contribution of

$$\begin{aligned} d \star j_L &= (-N_f + n_{\nu_R}) \frac{p_1(TM)}{24} \\ &= -(3 + n_{\nu_R}) \frac{1}{24 \times 8\pi^2} \frac{1}{2^2} \tilde{\epsilon}^{\mu\nu\alpha\beta} R^a_{b\mu\nu} R^b_{a\alpha\beta} d^4x. \end{aligned}$$

In the Friedmann-Lemaître-Robertson-Walker (FLRW) metric that describes a homogeneous isotropic expanding universe, the Pontryagin class p_1 vanishes. So, there is no classical gravitational anomaly source in FLRW. The nonzero p_1 requires CP-odd elliptically polarized chiral gravitational waves [10], inflationary tensor backgrounds [68–70], etc. The Hubble expansion rate is $H = \frac{\dot{a}}{a}$ for a dimensionless scale factor a in a radiation-dominated universe. The Friedmann equation simplifies to $H^2 = \frac{8\pi G}{3} \rho$ with the energy density of a relativistic thermal plasma $\rho = \frac{\pi^2}{30} g^* T^4$, with g^* effective number of relativistic degrees of freedom and some, so $H \simeq \sqrt{\frac{8\pi^3}{90} g^*} \frac{T^2}{M_{\text{Planck}}}$ with $M_{\text{Planck}} \sim 10^{19}$ GeV and $g^* \sim 109$. For Type I seesaw leptogenesis, $T \sim M_{\text{Majorana}} \gtrsim 10^{10}$ GeV due to Davidson-Ibarra bound [71], so $H \simeq 15 \times \frac{T^2}{M_{\text{Planck}}} \gtrsim 10 \times \frac{10^{10 \times 2}}{10^{19}} \sim 100$ GeV. Let us assume an appropriate classical gravitational background. Thus, the gravitational anomaly can contribute to the lepton number violation of order $\frac{dn_L}{dt} \propto R\tilde{R} \propto H^4 \sim 10^8$ GeV⁴. Consider the lepton number density n_L over entropy density $s \sim T^3$, for Type I Majorana seesaw leptogenesis [2] $\frac{n_L}{s} \sim \frac{\frac{dn_L}{dt} \times \Delta t}{s} \sim \frac{H^4 \times H^{-1}}{T^3} \sim \frac{H^3}{T^3} \sim \frac{10^6}{10^{30}} \sim 10^{-24}$. Reference [10] under a classical chiral gravitational wave background obtains $\frac{n_L}{s} \sim 10^{-22}$. However, this scale is much smaller (about 10^{-12} to 10^{-14} magnitude smaller) in comparison of the experimental lepton asymmetry n_L or baryon asymmetry n_B relative to the entropy density s , which is experimentally measured as $\frac{n_L}{s}$; also, $Y_B \equiv \frac{n_B}{s} \sim 10^{-10}$. Let us further evaluate the suppression of the three-loop effect of Eq. (12) involving two fermionic triangles and one box involving graviton propagators. Each graviton vertex introduces a factor $\frac{1}{M_{\text{Planck}}}$, so four graviton vertices contribute $\frac{1}{M_{\text{Planck}}^4}$; taking into account two graviton propagator $(\frac{1}{k^2})^2 \sim T^{-4}$ and fermion loop integrals $(\int d^4k)^2 \sim T^8$, we get a suppression of $\frac{T^4}{M_{\text{Planck}}^4}$ for Eq. (12) diagram. This is indeed a rather small contribution compared with the usual order of Type I leptogenesis [2].

- [40] K. Dick, M. Lindner, M. Ratz, and D. Wright, Leptogenesis with Dirac neutrinos, *Phys. Rev. Lett.* **84**, 4039 (2000).
- [41] H. Murayama and A. Pierce, Realistic Dirac leptogenesis, *Phys. Rev. Lett.* **89**, 271601 (2002).
- [42] F. Wilczek and A. Zee, Operator analysis of nucleon decay, *Phys. Rev. Lett.* **43**, 1571 (1979).
- [43] F. Wilczek and A. Zee, Conservation or Violation of $B - L$ in Proton Decay, *Phys. Lett. B* **88**, 311 (1979).
- [44] Y. Tachikawa and K. Yonekura, Why are fractional charges of orientifolds compatible with Dirac quantization? *SciPost Phys.* **7**, 058 (2018).
- [45] I. Garcia-Etxebarria and M. Montero, Dai-Freed anomalies in particle physics, *J. High Energy Phys.* **08** (2019) 003.
- [46] C.-T. Hsieh, Discrete gauge anomalies revisited, [arXiv:1808.02881](https://arxiv.org/abs/1808.02881) [hep-th].
- [47] M. Guo, K. Ohmori, P. Putrov, Z. Wan, and J. Wang, Fermionic finite-group gauge theories and interacting symmetric/crystalline orders via cobordisms, *Commun. Math. Phys.* **376**, 1073 (2020).
- [48] Z. Wan and J. Wang, Beyond standard models and grand unifications: Anomalies, topological terms, and dynamical constraints via cobordisms, *J. High Energy Phys.* **07** (2020) 062.
- [49] J. Wang, X.-G. Wen, and E. Witten, Symmetric gapped interfaces of SPT and SET states: Systematic constructions, *Phys. Rev. X* **8**, 031048 (2018).
- [50] C. Arf, Untersuchungen über quadratische Formen in Körpern der Charakteristik 2, I, *J. Reine Angew. Math.* **1941**, 148 (1941).
- [51] E. H. Brown, Generalizations of the Kervaire invariant, *Ann. Math.* **95** 368 (1972).
- [52] A. Kapustin, R. Thorngren, A. Turzillo, and Z. Wang, Fermionic symmetry protected topological phases and cobordisms, *J. High Energy Phys.* **12** (2015) 052.
- [53] L. M. Krauss and F. Wilczek, Discrete gauge symmetry in continuum theories, *Phys. Rev. Lett.* **62**, 1221 (1989).
- [54] J. Davighi, B. Gripaios, and N. Lohitsiri, Global anomalies in the Standard Model(s) and Beyond, *J. High Energy Phys.* **07** (2020) 232.
- [55] S. Koren, A note on proton stability in the standard model, *Universe* **8**, 308 (2022).
- [56] Z. Wan, J. Wang, and Y.-Z. You, Topological responses of the standard model gauge group, [arXiv:2412.21196](https://arxiv.org/abs/2412.21196) [hep-th].
- [57] J. Wang, Topological quantum dark matter via global anomaly cancellation, [arXiv:2502.21319](https://arxiv.org/abs/2502.21319) [hep-th].
- [58] Z. Wan and J. Wang, Anomalous (3+1)d fermionic topological quantum field theories via symmetry extension, [arXiv:2512.25038](https://arxiv.org/abs/2512.25038) [hep-th].
- [59] Z. Wan, J. Wang, and S.-T. Yau, Three Families and Pontryagin Class Mod 3: Topological order via symmetry extension, 2025, <https://lims.ac.uk/paper/solve-three-generations/>, <https://lims.ac.uk/documents/solve-three-generations.pdf>
- [60] C. Cordova and K. Ohmori, Anomaly constraints on gapped phases with discrete chiral symmetry, *Phys. Rev. D* **102**, 025011 (2020).
- [61] A. Debray, W. Ye, and M. Yu, How to build anomalous (3 + 1)d topological quantum field theories, [arXiv:2510.24834](https://arxiv.org/abs/2510.24834) [math-ph].
- [62] A. Kapustin and R. Thorngren, Anomalies of discrete symmetries in three dimensions and group cohomology, *Phys. Rev. Lett.* **112**, 231602 (2014).
- [63] A. Debray, M. Yu, and W. Ye, Symmetric gapped states and symmetry-enforced gaplessness in 3-dimension, [arXiv:2602.12335](https://arxiv.org/abs/2602.12335) [hep-th].
- [64] S. Koren, Cosmological lithium solution from discrete gauged $B - L$, *Phys. Rev. Lett.* **131**, 091003 (2023).
- [65] A. Maleknejad, Chiral gravity waves and leptogenesis in inflationary models with non-abelian gauge fields, *Phys. Rev. D* **90**, 023542 (2014).
- [66] A. Maleknejad, Gravitational leptogenesis in axion inflation with SU(2) gauge field, *J. Cosmol. Astropart. Phys.* **2016**, 027 (2016).
- [67] P. Adshead, A. J. Long, and E. I. Sfakianakis, Gravitational leptogenesis, reheating, and models of neutrino mass, *Phys. Rev. D* **97**, 043511 (2018).
- [68] D. H. Lyth and Y. Rodriguez, The Inflationary prediction for primordial non-Gaussianity, *Phys. Rev. Lett.* **95**, 121302 (2005).

- [69] D. H. Lyth, Generating the curvature perturbation at the end of inflation, *J. Cosmol. Astropart. Phys.* **2005**, 006 (2005).
- [70] R. R. Caldwell and C. Devulder, Axion gauge field inflation and gravitational leptogenesis: A lower bound on B modes from the matter-antimatter asymmetry of the universe, *Phys. Rev. D* **97**, 023532 (2018).
- [71] S. Davidson and A. Ibarra, A Lower bound on the right-handed neutrino mass from leptogenesis, *Phys. Lett. B* **535**, 25 (2002).