

Electric-magnetic geometric phase

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(Received 14 November 2025; accepted 25 March 2026; published 27 April 2026)

Geometric phases play an enormous role in optics and are generally associated with the evolution of light's polarization state on the Poincaré sphere, or its spin on the sphere of spin directions. Here, we put forward an optical geometric phase that appears exclusively in nonparaxial light, resulting from cyclic changes to the relative amplitude and phase between the electric and magnetic fields. This phase is naturally represented on a recently introduced “electric-magnetic” sphere.

DOI: [10.1103/qs1r-6g64](https://doi.org/10.1103/qs1r-6g64)

I. INTRODUCTION

That the coincident initial and final states of a wave system, evolved cyclically through parameter space, can be distinguished by a geometric phase is a key concept in modern physics [1–6]. It is difficult to overstate the importance of geometric phases in optics [7–9], underpinning the function of metasurfaces and spin-orbit coupling phenomena [10–13].

There are two basic types of optical geometric phases: (1) The Pancharatnam-Berry (PB) phase [14–16] emerges when the polarization of a paraxial beam of monochromatic light is changed cyclically by propagating through a suitable series of wave plates and/or polarizers; and (2) the spin-redirection geometric phase [17–20], which is accumulated during cyclic changes of the propagation direction of circularly polarized light. These two phases are represented by closed loops and solid angles on the *Poincaré* and *spin-direction* spheres, respectively. Once both the three-dimensional (3D) orientation and ellipticity of a polarization ellipse can vary, a more general approach involving a combined *Majorana-type* sphere [21,22] is needed. Furthermore, polarization-independent optical geometric phases can appear during (1) a cyclic evolution of the modal profile (transverse phase-intensity distribution) of a paraxial beam [23–25], which is naturally represented on the modal analog of the Poincaré sphere [25–27]; and (2) variations of the propagation direction of vortex beams carrying orbital angular momentum [28–31].

Each of the above kinds of optical geometric phase is described by evolutions of the polarization and intensity-phase distribution of the wave's *electric* field. Yet, just as the electric

field's polarization ellipse is transformed, so too is that of the *magnetic* field—and in a different fashion in general. Moreover, Maxwell's equations permit, in nonparaxial light, the electric and magnetic fields a surprising amount of freedom: Both the share of energy between the two fields and their relative phase can change. This is analogous to how the two components of a paraxial beam's polarization (Jones vector) change in amplitude and relative phase in the PB phase problem.

In this article, we show that such a relative evolution of the electric and magnetic fields in a nonparaxial optical field gives rise to an “*electric-magnetic*” (EM) geometric phase, described within a generalized theory of optical geometric phase for the six-component vector that combines the two fields. Most startlingly, we show that the EM geometric phase can even present itself when the polarization states of the electric and magnetic fields *do not change*, providing a simple example of this scenario: a standing wave modulated cyclically in time. In a second example, we show the remarkable emergence of the EM geometric phase in the longitudinal field components in a focused beam whose spatial structure evolves cyclically—a link between the modal geometric phase for paraxial beams and the strong-focusing regime. Notably, the geometric phase described in our work is of an Aharonov-Anandan type [32], which requires cyclic evolution of the state vector but not adiabaticity.

II. GENERAL THEORY

An electromagnetic field as a function of some parameters ρ can be expressed with a single vector in the $\mathbb{C}^2 \otimes \mathbb{C}^3 \otimes \mathbb{L}^2$ space [33–36]:

$$\vec{\Psi}(\rho) = \frac{1}{2} \begin{pmatrix} \sqrt{\varepsilon_0} \mathbf{E}(\rho) \\ \sqrt{\mu_0} \mathbf{H}(\rho) \end{pmatrix} = \sqrt{W(\rho)} e^{i\alpha(\rho)} \vec{\psi}(\rho), \quad (1)$$

where $\mathbf{E}(\rho)$ and $\mathbf{H}(\rho)$ are the complex electric and magnetic fields, $W = \vec{\Psi}^\dagger \cdot \vec{\Psi} = (\varepsilon_0 \mathbf{E}^* \cdot \mathbf{E} + \mu_0 \mathbf{H}^* \cdot \mathbf{H})/4$ is the electromagnetic energy density, $\vec{\psi}$ is a normalized vector

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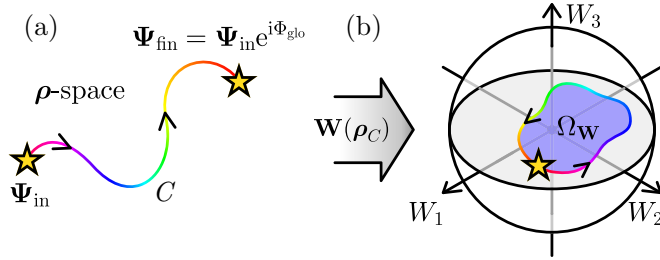


FIG. 1. General geometric phase scenario. (a) The field $\vec{\Psi}$ [Eq. (1)] evolves over a curve C in the parameter ρ space. At the two ends of C , the field differs only by the phase $\Phi_{\text{glo}} = \Phi_{\text{loc}} + \Phi_{\text{g}}$ [Eqs. (2), (4), and (5)]. (b) If the electric and magnetic polarization states do not vary along C , then $\Phi_{\text{g}} = -\Omega_{\text{W}}/2$ is the EM geometric phase, proportional to the solid angle enclosed by the evolution loop on a Poincaré-like EM sphere [36] [Eqs. (10) and (11)].

($\vec{\psi}^\dagger \cdot \vec{\psi} = 1$), and the overall phase α is to be chosen. Here and in what follows, superscripts $\dagger$ and $\top$ mean conjugate transpose and transpose, respectively, bold characters like $\mathbf{E}$ are $\mathbb{C}^3$ vectors, while those with arrow accents like $\vec{\Psi}$ are $\mathbb{C}^2 \otimes \mathbb{C}^3$ vectors, and explicit expression of the ρ dependence of quantities in Eq. (1) is to be avoided except for useful emphasis. All phase angles in this work are defined within the interval $[0, 2\pi)$ —Any expression for a phase angle is assumed to be taken modulo 2π .

Suppose that there is an open curve C in parameter ρ space, along which $\vec{\Psi}$ is varied between an initial value $\vec{\Psi}_{\text{in}} = \vec{\Psi}(\rho_{\text{in}})$ and a final value $\vec{\Psi}_{\text{fin}} = \vec{\Psi}(\rho_{\text{fin}})$, distinguishable only by a *global* phase: $\vec{\Psi}_{\text{fin}} = \vec{\Psi}_{\text{in}} \exp(i\Phi_{\text{glo}})$ [32] [see Fig. 1(a)]. It is straightforward to see that this phase factor is consistent with Pancharatnam’s definition [14,15] of the phase difference between two nonorthogonal field vectors:

$$\Phi_{\text{glo}} = \text{Arg}(\vec{\Psi}_{\text{in}}^\dagger \cdot \vec{\Psi}_{\text{fin}}) = \Delta\alpha + \text{Arg}(\vec{\psi}_{\text{in}}^\dagger \cdot \vec{\psi}_{\text{fin}}), \quad (2)$$

where $\Delta\alpha = \alpha(\rho_{\text{fin}}) - \alpha(\rho_{\text{in}})$. Since the field evolves continuously over C , it is also possible to record *local* phase accumulation. An incremental change in phase between neighboring points in ρ space is, using Pancharatnam’s definition (2),

$$d\Phi_{\text{loc}} = \text{Arg}[\vec{\Psi}^\dagger \cdot (\vec{\Psi} + d\vec{\Psi})] = \underbrace{\frac{1}{W} \text{Im}[\vec{\Psi}^\dagger \cdot (\nabla_\rho) \vec{\Psi}]}_{\mathbf{k}_\rho} \cdot d\rho. \quad (3)$$

Interpreting $\mathbf{k}_\rho$ as a “local wave vector” over ρ [22,34,37], we might initially expect that its integral along C would recover Φ_{glo} [Eq. (2)]. But when substituting Eq. (1), its integral,

$$\Phi_{\text{loc}} = \int_C \mathbf{k}_\rho \cdot d\rho = \Delta\alpha - i \int_C \vec{\psi}^\dagger \cdot (\nabla_\rho) \vec{\psi} \cdot d\rho, \quad (4)$$

is *not* generally equal to Φ_{glo} . Indeed, the difference between Eqs. (2) and (4) is a *geometric phase*: $\Phi_{\text{g}} = \Phi_{\text{glo}} - \Phi_{\text{loc}}$. Geometric phase is what effects “global change without local change” [38]. The sum effect of local change, Φ_{loc} , can be identified with *dynamical phase*.

We have yet to exercise our freedom to select the phase angle α . While the difference $\Phi_{\text{g}} = \Phi_{\text{glo}} - \Phi_{\text{loc}}$ is invariant

with respect to changes of α (i.e., $U(1)$ gauge transformations), its choice can be leveraged to obtain a more satisfying expression in terms of the normalized field $\vec{\psi}$. Since α is an overall phase ascribed to all six field components, it is natural to define it via an associated complex scalar quadratic field $\Psi = \vec{\Psi}^\top \cdot \vec{\Psi}$: $\alpha = \text{Arg}(\Psi)/2$ [22,37,39]. Then, it follows from Eq. (1) that $\text{Arg}(\vec{\psi}^\dagger \cdot \vec{\psi}) = 0$, and subsequently in Eq. (2), $\text{Arg}(\vec{\psi}_{\text{in}}^\dagger \cdot \vec{\psi}_{\text{fin}}) = N\pi$ [see Eqs. (S4)–(S7) of the Supplemental Material [40]]. Here, $N = 0$ or 1 is a $\mathbb{Z}_2$ topological index that encodes the even or odd number of half-turns made by principal semiaxes of the “six-dimensional EM polarization ellipse” [41] of $\vec{\psi}$ over C . For a closed contour C , $N = 1$ corresponds to *polarization Möbius strips* [22,42–45].

Subtracting now Eq. (4) from Eq. (2), we obtain

$$\Phi_{\text{g}} = i \int_C \vec{\psi}^\dagger \cdot (\nabla_\rho) \vec{\psi} \cdot d\rho + N\pi. \quad (5)$$

Equation (5) is the central general expression in our work, describing the optical geometric phase that incorporates both electric and magnetic fields, valid for both paraxial and nonparaxial waves, and alluded to in Refs. [22,37]. Note, however, that previous electric-only geometric phase descriptions [22,37] associate the integrated electric local wave vector with *total* phase, the electric-only equivalent of $\Delta\alpha$ with *dynamical* phase, and their difference with *geometric* phase, definitions that are not applicable to field evolutions over open curves in parameter space such as in Fig. 1. We have instead provided Eqs. (2), (4), and (5) as *total*, *dynamical*, and *geometric* phases, respectively, which are analogous to the definitions of Aharonov and Anandan [32].

A natural next step is to attempt to divide Φ_{g} into electric and magnetic contributions, as is customarily done for many quantities in electromagnetism [33–35,37,46,47]. We express the normalized state vector $\vec{\psi}$ as

$$\vec{\psi} = \begin{pmatrix} \cos \frac{\theta}{2} e^{i\chi_e} \hat{\mathbf{e}} \\ \sin \frac{\theta}{2} e^{i\chi_m} \hat{\mathbf{h}} \end{pmatrix}, \quad (6)$$

with $\theta = 2 \arctan(\sqrt{\mu_0} |\mathbf{H}| / \sqrt{\epsilon_0} |\mathbf{E}|)$ characterizing the relative amplitude of the electric and magnetic components, and the complex unit polarization vectors $\hat{\mathbf{e}} = \exp[-i(\alpha + \chi_e)] \mathbf{E} / |\mathbf{E}|$ and $\hat{\mathbf{h}} = \exp[-i(\alpha + \chi_m)] \mathbf{H} / |\mathbf{H}|$, whose phases χ_e and χ_m are added to α such that $\text{Arg}(\hat{\mathbf{e}} \cdot \hat{\mathbf{e}}) = \text{Arg}(\hat{\mathbf{h}} \cdot \hat{\mathbf{h}}) = 0$. This fixes the real and imaginary parts of $\hat{\mathbf{e}}$ ($\hat{\mathbf{h}}$) at right angles, aligned to the semimajor and semiminor axes of the electric (magnetic) polarization ellipse, respectively [22,39]. Surprisingly, substituting Eq. (6) into Eq. (5) yields two kinds of geometric phase terms:

$$\Phi_{\text{g}} = \Phi_{\text{gI}} + \Phi_{\text{gII}} + N\pi, \quad (7)$$

where

$$\Phi_{\text{gI}} = i \int_C \left[\cos^2 \frac{\theta}{2} \hat{\mathbf{e}}^* \cdot (\nabla_\rho) \hat{\mathbf{e}} + \sin^2 \frac{\theta}{2} \hat{\mathbf{h}}^* \cdot (\nabla_\rho) \hat{\mathbf{h}} \right] \cdot d\rho \quad (8)$$

is a weighted sum of the electric and magnetic terms, each similar in structure to the integral term in Eq. (5), which describes geometric phase arising from variations of the electric and magnetic polarization ellipses along C . This type of geometric phase has been described in Ref. [22]

(for closed-contour evolutions in real space); it accounts for both the PB phase in two-dimensional paraxial waves, where $\theta = \pi/2$ and $\Phi_{\text{gl}} + N\pi = \Phi_{\text{PB}}$, and the spin-redirection phase for 3D evolutions of polarization. Having fixed $\text{Arg}(\hat{\mathbf{e}} \cdot \hat{\mathbf{e}}) = \text{Arg}(\hat{\mathbf{h}} \cdot \hat{\mathbf{h}}) = 0$ means that $\Phi_{\text{gl}} = 0$ if the electric and magnetic polarization states are uniform over C .

Our main result is what we term the *EM geometric phase*:

$$\Phi_{\text{gII}} = - \int_C \left[\cos^2 \frac{\theta}{2} d\chi_e + \sin^2 \frac{\theta}{2} d\chi_m \right]. \quad (9)$$

It originates from the incorporation of the $\mathbb{C}^2$ EM space in $\tilde{\Psi}$ and arises *exclusively in nonparaxial light*, i.e., $\Phi_{\text{gII}} = 0$ for paraxial waves [see the Supplemental Material [40]]. Perhaps, the most striking feature of Φ_{gII} is its independence from the $\mathbb{C}^3$ polarization vectors $\hat{\mathbf{e}}$ and $\hat{\mathbf{h}}$, enabled by our choice of χ_e and χ_m —In nonparaxial light, a geometric phase may emerge even when the electric and magnetic polarization states are constant; i.e., neither $\hat{\mathbf{e}}$ nor $\hat{\mathbf{h}}$ evolve over C . When, moreover, the electric and magnetic fields share the same polarization state throughout, $\hat{\mathbf{e}} \equiv \hat{\mathbf{h}}$, the phase Φ_{gII} may be related to the trajectory of the tip of a Stokes-like vector $\mathbf{W} = \tilde{\Psi}^\dagger \cdot (\hat{\sigma}) \tilde{\Psi} / W$ [35,36]:

$$\mathbf{W} = \frac{1}{4W} \begin{pmatrix} 2\text{Re}\{\mathbf{E}^* \cdot \mathbf{H}\}/c \\ 2\text{Im}\{\mathbf{E}^* \cdot \mathbf{H}\}/c \\ \epsilon_0|\mathbf{E}|^2 - \mu_0|\mathbf{H}|^2 \end{pmatrix} = \begin{pmatrix} \sin \theta \cos \phi \\ \sin \theta \sin \phi \\ \cos \theta \end{pmatrix}, \quad (10)$$

where Re and Im denote real and imaginary parts, $\hat{\sigma} = (\hat{\sigma}_1, \hat{\sigma}_2, \hat{\sigma}_3)$ is the vector of Pauli matrices, and $\phi = \text{Arg}(\mathbf{E}^* \cdot \mathbf{H}) = \chi_m - \chi_e$. In this case, it can be shown that [see the Supplemental Material [40]]

$$\Phi_g = \Phi_{\text{gII}} + N\pi = -\frac{1}{2} \int_C (1 - \cos \theta) d\phi = -\frac{1}{2} \Omega_{\mathbf{W}}, \quad (11)$$

where $\Omega_{\mathbf{W}}$ is the solid angle swept out by $\mathbf{W}$ [see Fig. 1(b)]. The vector $\mathbf{W}$ can be interpreted as a Stokes vector for an *EM ellipse* [41], represented on a Poincaré-like *EM sphere*, originally proposed in Ref. [36] [the sphere in Fig. 1(b) is rotated compared to that in Ref. [36] because of our use of an electric-magnetic basis for $\tilde{\Psi}$]. In this representation, the geometric phase (11) takes on the standard PB-like form, associated with the $\text{SU}(2)$ evolution of the $\mathbb{C}^2$ EM vector.

Now is an appropriate time to highlight a crucial property of our geometric phase formalism, namely, its behavior under gauge transformations of $\tilde{\Psi}$. We have so far worked tacitly within an EM basis for $\tilde{\Psi}$, meaning its entries [as in Eq. (1)] are the electric and magnetic fields. Yet, $\tilde{\Psi}$ carries an $\text{SU}(2)$ gauge freedom that we have not exercised—We could have derived Eqs. (1)–(11) using a transformed spinor $\tilde{\Psi}_{\text{uv}} = (\mathbf{F}_u, \mathbf{F}_v)/2$, where $\mathbf{F}_u$ and $\mathbf{F}_v$ are linear combinations of $\mathbf{E}$ and $\mathbf{H}$, and arrived at the same results. Such a transformation to $\tilde{\Psi}$ includes, for example, replacing $\mathbf{E}$ and $\mathbf{H}$ with right- and left-handed Riemann-Silberstein vectors $\mathbf{F}_{\text{R/L}} = (\sqrt{\epsilon_0}\mathbf{E} \pm i\sqrt{\mu_0}\mathbf{H})/\sqrt{2}$, such that $\tilde{\Psi}_{\text{RL}} = (\mathbf{F}_R, \mathbf{F}_L)/2$. Doing so does not change any physical properties of the field; its energy, momentum, and spin densities (as a sum of contributions of $\mathbf{F}_u$ and $\mathbf{F}_v$) are unaltered [36]. The total, dynamical, and overall geometric Φ_g phases that can be accumulated

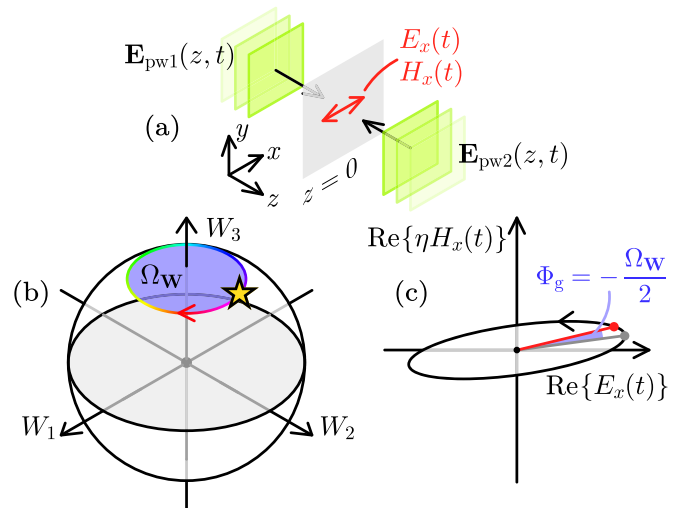


FIG. 2. EM geometric phase emerging in a time-modulated standing wave. (a) Diagram of scenario, designed so that $\mathbf{E}$ and $\mathbf{H}$ are $\hat{\mathbf{x}}$ polarized at $z = 0$ [Eq. (13)]. (b) Trajectory of $\mathbf{W}$ (the star denotes $\mathbf{W}(0) = \mathbf{W}(T)$), corresponding to Eq. (14) with $\theta_0 = \pi/6$ and $\chi_0 = 2\pi/3$ and enclosing solid angle $\Omega_{\mathbf{W}} \simeq -0.27\pi$ sr. (c) Initial/final EM ellipse [41] at $z = 0$. The red line is the instantaneous vector ($\text{Re}\{E_x(T)\}$, $\text{Re}\{\eta H_x(T)\}$) of the standing wave having acquired a phase of $-\omega_0 T + \Phi_g$. The gray line is that same vector for a reference standing wave that gains the phase $-\omega_0 T$.

by $\tilde{\Psi}$ are likewise gauge invariant. This holds, for example, in a paraxial wave propagating along $\hat{\mathbf{k}}$, where in the EM basis $\Phi_g = N\pi + \Phi_{\text{gl}}$ and we associate geometric phase with evolution of the unit polarization vectors $\hat{\mathbf{e}}$ and $\hat{\mathbf{h}} = \hat{\mathbf{k}} \times \hat{\mathbf{e}}$ (i.e., it is the PB phase). Instead of electric and magnetic fields, one could, as has been studied recently [48], track the individual evolution of the Riemann-Silberstein vectors $\mathbf{F}_R$ and $\mathbf{F}_L$, which is generally different from that of $\mathbf{E}$ and $\mathbf{H}$, yet $\Phi_g \equiv \Phi_{\text{PB}}$ is ultimately unchanged.

Unlike the sum $\Phi_g = N\pi + \Phi_{\text{gl}} + \Phi_{\text{gII}}$, the distribution of phase between Φ_{gl} and Φ_{gII} is not generally gauge invariant, except special situations. If the evolution of $\tilde{\Psi}$ takes place with $\hat{\mathbf{e}} \equiv \hat{\mathbf{h}}$, that is, both electric and magnetic fields have identical polarization vectors, then any arbitrary pair of fields $\mathbf{F}_u$ and $\mathbf{F}_v$ must also share the same unit polarization vectors, and hence Φ_{gl} and Φ_{gII} become gauge invariant. If $\hat{\mathbf{e}} \equiv \hat{\mathbf{h}} = \text{const}$, then we can unambiguously identify any observed geometric phase with the electric-magnetic geometric phase Φ_{gII} . We next propose two scenarios in which this is possible.

III. EXAMPLES OF EM GEOMETRIC PHASE

A. Standing wave

Whereas the usual PB and spin-redirection geometric phases can be measured with a standard interferometry setup for paraxial light, the EM geometric phase can only manifest in nonparaxial light and would require measurement of interference at a localized point in space. Here, we consider the simplest example of a nonparaxial field: a standing wave formed by two counterpropagating plane waves along the z

axis [Fig. 2(a)]. We choose time as the parameter that drives the evolution: $\rho = t$.

The electric and magnetic fields of such a time-varying standing wave at $z = 0$ can be written as

$$\begin{aligned} \mathbf{E}(t) &= e^{-i\omega_0 t} [(A_x(t) + B_x(t))\hat{\mathbf{x}} + (A_y(t) + B_y(t))\hat{\mathbf{y}}], \\ \mathbf{H}(t) &= \frac{e^{-i\omega_0 t}}{\eta} [(B_y(t) - A_y(t))\hat{\mathbf{x}} + (A_x(t) - B_x(t))\hat{\mathbf{y}}], \end{aligned} \quad (12)$$

where ω_0 is the central frequency of the waves, $A_i(t)$ ($B_i(t)$) are the complex electric field amplitudes of the plane wave propagating in the $+z$ ($-z$) direction, which are modulated in time, and $\eta = \sqrt{\mu_0/\epsilon_0}$. By enforcing $A_x(t) = B_x(t)$ and $A_y(t) = -B_y(t)$, both electric and magnetic fields in the standing wave become purely $\hat{\mathbf{x}}$ polarized at $z = 0$ for any value of t . Then, the field (1) becomes

$$\tilde{\Psi}(t) = \sqrt{W} e^{-i\omega_0 t} \begin{pmatrix} \cos \frac{\theta(t)}{2} e^{i\chi_x(t)} \hat{\mathbf{x}} \\ -\sin \frac{\theta(t)}{2} e^{i\chi_y(t)} \hat{\mathbf{x}} \end{pmatrix}, \quad (13)$$

where $\theta(t) = 2 \arctan(|A_y(t)|/|A_x(t)|)$ and $\chi_{x,y}(t) = \text{Arg}(A_{x,y}(t))$. Since modulation is carried out over time, the “local wave vector” (3) with $\nabla_\rho = \partial/\partial t$ is related to the instantaneous angular frequency of the field: $k_t = \partial\Phi_{\text{loc}}/\partial t = -\omega(t)$.

The geometric phase can be measured against an unmodulated reference wave that accumulates a dynamical phase of $-\omega_0 T$ over some interval $t \in [0, T]$. To ensure that the modulated wave has the same dynamical phase, we require $\partial\Phi_{\text{loc}}/\partial t = -\omega_0$, for which one simple solution is

$$\chi_x(t) = \frac{2\pi t}{T} \sin^2 \frac{\theta_0}{2}, \quad \chi_y(t) = -\frac{2\pi t}{T} \cos^2 \frac{\theta_0}{2} + \chi_0, \quad (14)$$

with $\theta(t) = \theta_0$ kept constant, and where χ_0 determines the initial phase offset between the electric and magnetic fields. Substituting Eq. (14) into Eq. (13) and calculating the global phase (2) of the field between $t = 0$ and $t = T$, we derive

$$\text{Arg}(\tilde{\Psi}^\dagger(0) \cdot \tilde{\Psi}(T)) = \underbrace{-\omega_0 T}_{\Phi_{\text{loc}}} + \underbrace{(1 - \cos \theta_0)\pi}_{\Phi_g}. \quad (15)$$

This clearly contains a dynamical phase Φ_{loc} and a geometric phase Φ_g that is independent of the modulation time interval T and corresponds to the solid angle enclosed by the evolution contour on the EM $\mathbf{W}$ -sphere [Fig. 2(b)]. It is straightforward also to obtain Φ_g using the integral (11) over $t \in [0, T]$, with $\theta = \theta_0$ and $d\phi = d\chi_m - d\chi_e \equiv d\chi_y - d\chi_x = -(2\pi/T) dt$. In Fig. 2(c), Φ_g is visualized as a phase gained by an EM ellipse [41], whose shape before and after modulation is identical, against a reference standing wave.

We emphasize that the geometric phase in Eq. (15) arises despite both electric and magnetic fields having constant $\hat{\mathbf{x}}$ polarization. Moreover, Eq. (15) is exact, as in the Aharonov-Anandan approach [32], and does not require time modulation to be slow. This is in contrast to the adiabatic geometric phase [1], which is just the zero-order term in an infinite series over $1/T$ [49,50].

B. Focused beams

Another manifestation of the EM geometric phase can be found in the longitudinal components of a strongly

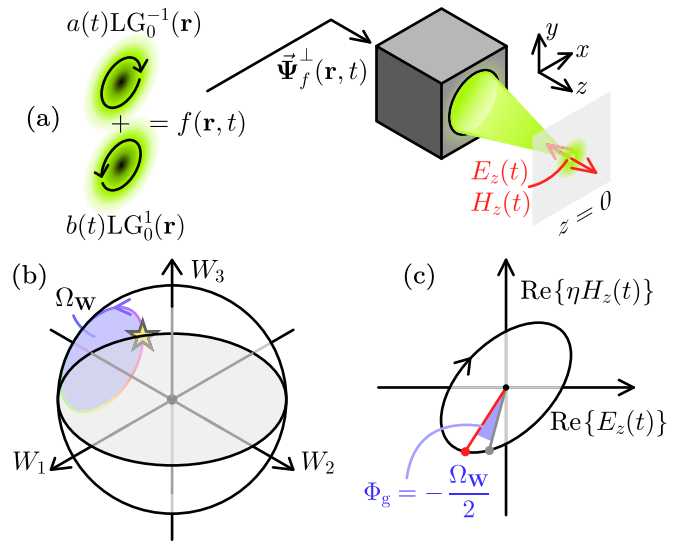


FIG. 3. EM geometric phase arising in the longitudinal components of a time-evolving focused beam carrying a first-order modal structure. (a) Schematic, where a linearly polarized beam whose mode profile f is modulated over t [Eq. (16)], is focused to produce longitudinal components (17) in the focal center $\mathbf{r} = \mathbf{0}$. (b) Closed trajectory of $\mathbf{W}$ corresponding to Eqs. (17) and (18) with $\theta_0 = \pi/6$ and $\chi_0 = \pi/2$. (c) Final state of the ellipse formed of the electric and magnetic z components in the beam focus, annotated similarly as in Fig. 2(c).

focused beam with mode order 1 (such as Hermite-Gauss HG_{10} and HG_{01} or Laguerre-Gauss $\text{LG}_0^{\pm 1}$ modes, and their superpositions), whose transverse spatial structure is transformed cyclically prior to focusing [Fig. 3(a)]. Cyclic mode transformations made to paraxial beams are well known to produce polarization-independent geometric phases [23–25] associated with closed paths on the modal Poincaré sphere [25–27]. A focused beam of light, however, develops *longitudinal* electric and magnetic field components that are not accounted for by the modal Poincaré sphere but that can be made to evolve in time ($\rho = t$ as in the previous example) by changing the input paraxial beam’s mode structure, and thereby producing the EM geometric phase.

A general first-order mode f can be presented as a linear superposition of $\text{LG}_0^{\pm 1}$ modes. The transverse field of a time-varying first-order beam is given by the four-vector

$$\tilde{\Psi}_f^\perp(\mathbf{r}, t) = e^{-i\omega_0 t} \tilde{\Psi}_0^\perp \underbrace{[a(t)\text{LG}_0^{-1}(\mathbf{r}) + b(t)\text{LG}_0^1(\mathbf{r})]}_{f(\mathbf{r}, t)}, \quad (16)$$

where $a(t)$ and $b(t)$ are complex time-varying coefficients satisfying $|a|^2 + |b|^2 = 1$ (e.g., $|a| = |b|$ corresponds to Hermite-Gauss modes), and the vector $\tilde{\Psi}_0^\perp = (\mathbf{F}_{R0}^\perp, \mathbf{F}_{L0}^\perp)/2$ determines the constant transverse polarization, here expressed in a basis of Riemann-Silberstein vectors [33,36] $\mathbf{F}_{R0,L0}^\perp = (\sqrt{\epsilon_0}\mathbf{E}_0^\perp \pm i\sqrt{\mu_0}\mathbf{H}_0^\perp)/\sqrt{2}$. Due to paraxiality, $\mathbf{F}_{R0,L0}^\perp = A_{R,L}^\perp(\hat{\mathbf{x}} \pm i\hat{\mathbf{y}})/\sqrt{2}$ are oppositely circularly polarized, where $A_{R,L}^\perp$ are complex constants.

Once strongly focused, the longitudinal ($\hat{\mathbf{z}}$ -polarized) field, $\tilde{\Psi}_f^\parallel$, strongest in the focal plane at the beam’s center $\mathbf{r} = \mathbf{0}$,

can be approximated as [51,52] $\vec{\Psi}_f^\parallel \simeq ik^{-1}(\nabla_\perp \cdot \vec{\Psi}_f^\perp)\hat{\mathbf{z}}$, which yields (see the Supplemental Material [40])

$$\vec{\Psi}_f^\parallel(\mathbf{0}, t) = \frac{1}{2} \begin{pmatrix} F_{Rz}(t)\hat{\mathbf{z}} \\ F_{Lz}(t)\hat{\mathbf{z}} \end{pmatrix} = \frac{iBe^{-i\omega_0 t}}{k\sqrt{2}} \begin{pmatrix} A_R^\perp a(t)\hat{\mathbf{z}} \\ A_L^\perp b(t)\hat{\mathbf{z}} \end{pmatrix}, \quad (17)$$

where B is a constant. Remarkably, Eq. (17) shows that the longitudinal components of right- and left-handed Riemann-Silberstein vectors in the center of the beam's focus are *independently* modulated by the complex functions $a(t)$ and $b(t)$ that determine the beam's paraxial spatial structure (16). It resembles Eq. (13) of the standing-wave scenario, except that the entries of $\vec{\Psi}_f^\parallel$ are not electric and magnetic fields. Choosing

$$a(t) = \cos \frac{\theta_0}{2} e^{i\chi_a}, \quad b(t) = \sin \frac{\theta_0}{2} e^{i\chi_b}, \quad (18)$$

where χ_a and χ_b coincide with χ_x and χ_y in Eq. (14), the EM geometric phase can be obtained over $t \in [0, T]$ from the equivalent of Eq. (11) in the Riemann-Silberstein basis if $|A_R^\perp| = |A_L^\perp|$ (linear transverse electric polarization):

$$\Phi_g = -\frac{1}{2} \int_0^T (1 - \cos \theta_{RL}) d\phi_{RL} = (1 - \cos \theta_0)\pi, \quad (19)$$

where $\theta_{RL} = \theta_0$ is the polar angle defined with respect to the $-W_2$ axis on the EM sphere and $d\phi_{RL} \equiv d\chi_b - d\chi_a = -(2\pi/T)dt$ is the differential of the corresponding azimuthal angle [see Fig. 3(b)]. Equation (19) is equivalent to what would be the modal geometric phase [23–26] in an unfocused paraxial beam, but now it is described by the evolution of the field in a *single point* rather than by a spatially extended modal structure. Figures 3(b) and 3(c) show the $\mathbf{W}$ -sphere representation of the EM geometric phase (19), which is similar to the standing-wave scenario, Figs. 2(b) and (2c), up to a global rotation about the W_1 axis because of the use of the Riemann-Silberstein basis. An additional figure visualizing the evolution of the field distribution and longitudinal components in the focal plane of the beam is provided in the Supplemental Material [40].

IV. CONCLUSIONS

We have presented a generalized theory of optical geometric phase, which incorporates both electric and magnetic fields consistently, and identified an EM geometric phase. Together with the PB and spin-redirection phases, it completes the picture of optical geometric phase. The EM geometric phase emerges only in nonparaxial light and results from cyclic evolutions of the relative amplitude and phase between the electric and magnetic fields. Remarkably, it can be observed in nonparaxial waves where both fields have the same, constant

polarization ellipse, but only vary in relative amplitude and phase. We have presented two examples of this scenario: a standing wave modulated in time and the focal center of a strongly focused first-order beam, whose spatial structure is modulated in time.

Importantly, although the theoretical description of the EM geometric phase involves both electric and magnetic fields, it can be observed experimentally as the phase of the electric field. Moreover, one can analyze the wave phase using an electric-field-only formalism, as in Ref. [22]; in this case, the EM geometric phase will be subsumed into the electric field's dynamical phase analogous to Eq. (4). Geometric phases can be measured in interferometry experiments; however, a standard interferometer setup would not suffice in either of our two case studies since the evolution of the field takes place at a single point in space. In principle, however, a scattering probe particle could be placed at the point of interest in the evolving field, producing a scattered field that could then interfere with the scattered field of a second probe particle placed identically in a reference field.

Optical geometric phases have proved invaluable for the design of advanced optical elements and metasurfaces [10–13]. Much current effort is now devoted to both time-varying materials [53] and time-structured waves [54,55]. The EM geometric phase could aid in the design of new generations of metamaterials. Our formalism also ties into recent studies of the topological features of the six-component electromagnetic field (1) [41,56].

ACKNOWLEDGMENTS

This work was supported in part by the Marie Skłodowska-Curie COFUND Programme of the European Commission (Project No. HORIZON-MSCA-2022-COFUND-101126600-SmartBRAIN3), ENSEMBLE3 Project (MAB/2020/14), which is carried out within the International Research Agendas Programme (IRAP) of the Foundation for Polish Science cofinanced by the European Union under the European Regional Development Fund and Teaming Horizon 2020 program (Agreement No. 857543) of the European Commission, and the project of the Minister of Science and Higher Education “Support for the activities of Centers of Excellence established in Poland under the Horizon 2020 program” (Contract No. MEiN/2023/DIR/3797).

DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

- [1] M. V. Berry, Quantal phase factors accompanying adiabatic changes, *Proc. R. Soc. London A*, **392**, 45 (1984).
- [2] F. Wilczek and A. Shapere, *Geometric Phases in Physics* (World Scientific, Singapore, 1989).

- [3] J. W. Zwanziger, M. Koenig, and A. Pines, Berry's phase, *Annu. Rev. Phys. Chem.* **41**, 601 (1990).
- [4] D. Xiao, M.-C. Chang, and Q. Niu, Berry phase effects on electronic properties, *Rev. Mod. Phys.* **82**, 1959 (2010).

- [5] M. Baggio, V. Niarchos, and K. Papadodimas, Aspects of Berry phase in QFT, *J. High Energy Phys.* **04** (2017) 062.
- [6] E. Cohen, H. Larocque, F. Bouchard, F. Nejdassattari, Y. Gefen, and E. Karimi, Geometric phase from Aharonov–Bohm to Pancharatnam–Berry and beyond, *Nat. Rev. Phys.* **1**, 437 (2019).
- [7] S. I. Vinitskii, V. L. Derbov, V. M. Dubovik, B. L. Markovski, and Y. P. Stepanovskii, Topological phases in quantum mechanics and polarization optics, *Sov. Phys. Usp.* **33**, 403 (1990).
- [8] R. Bhandari, Polarization of light and topological phases, *Phys. Rep.* **281**, 1 (1997).
- [9] C. Cisowski, J. B. Götze, and S. Franke-Arnold, *Colloquium: Geometric phases of light: Insights from fiber bundle theory*, *Rev. Mod. Phys.* **94**, 031001 (2022).
- [10] E. Hasman, G. Biener, A. Niv, and V. Kleiner, Space-variant polarization manipulation, *Prog. Opt.* **47**, 215 (2005).
- [11] L. Marrucci, E. Karimi, S. Slussarenko, B. Piccirillo, E. Santamato, E. Nagali, and F. Sciarrino, Spin-to-orbital conversion of the angular momentum of light and its classical and quantum applications, *J. Opt.* **13**, 064001 (2011).
- [12] K. Y. Bliokh, F. J. Rodríguez-Fortuño, F. Nori, and A. V. Zayats, Spin-orbit interactions of light, *Nat. Photon.* **9**, 796 (2015).
- [13] C. P. Jisha, S. Nolte, and A. Alberucci, Geometric phase in optics: From wavefront manipulation to waveguiding, *Laser Photon. Rev.* **15**, 2100003 (2021).
- [14] S. Pancharatnam, Generalized theory of interference, and its applications, *Proc. Indian Acad. Sci.* **44**, 247 (1956).
- [15] M. V. Berry, The adiabatic phase and Pancharatnam’s phase for polarized light, *J. Mod. Opt.* **34**, 1401 (1987).
- [16] R. Bhandari and J. Samuel, Observation of topological phase by use of a laser interferometer, *Phys. Rev. Lett.* **60**, 1211 (1988).
- [17] S. M. Rytov, On transition from wave to geometrical optics, *Dokl. Akad. Nauk SSSR* **18**, 263 (1938).
- [18] V. V. Vladimirskiy, The rotation of a polarization plane for curved light ray, *Dokl. Akad. Nauk SSSR* **31**, 222 (1941).
- [19] J. N. Ross, The rotation of the polarization in low birefringence monomode optical fibres due to geometric effects, *Opt. Quantum Electron.* **16**, 455 (1984).
- [20] A. Tomita and R. Y. Chiao, Observation of Berry’s topological phase by use of an optical fiber, *Phys. Rev. Lett.* **57**, 937 (1986).
- [21] J. H. Hannay, The Majorana representation of polarization, and the Berry phase of light, *J. Mod. Opt.* **45**, 1001 (1998).
- [22] K. Y. Bliokh, M. A. Alonso, and M. R. Dennis, Geometric phases in 2D and 3D polarized fields: Geometrical, dynamical, and topological aspects, *Rep. Prog. Phys.* **82**, 122401 (2019).
- [23] S. J. van Enk, Geometric phase, transformations of Gaussian light beams and angular momentum transfer, *Opt. Commun.* **102**, 59 (1993).
- [24] E. J. Galvez, P. R. Crawford, H. I. Sztul, M. J. Pysher, P. J. Haglin, and R. E. Williams, Geometric phase associated with mode transformations of optical beams bearing orbital angular momentum, *Phys. Rev. Lett.* **90**, 203901 (2003).
- [25] G. F. Calvo, Wigner representation and geometric transformations of optical orbital angular momentum spatial modes, *Opt. Lett.* **30**, 1207 (2005).
- [26] M. J. Padgett and J. Courtial, Poincaré-sphere equivalent for light beams containing orbital angular momentum, *Opt. Lett.* **24**, 430 (1999).
- [27] M. R. Dennis and M. A. Alonso, Swings and roundabouts: Optical Poincaré spheres for polarization and Gaussian beams, *Philos. Trans. R. Soc. A* **375**, 20150441 (2017).
- [28] K. Y. Bliokh, Geometrical optics of beams with vortices: berry phase and orbital angular momentum Hall effect, *Phys. Rev. Lett.* **97**, 043901 (2006).
- [29] C. N. Alexeyev and M. A. Yavorsky, Topological phase evolving from the orbital angular momentum of ‘coiled’ quantum vortices, *J. Opt. A: Pure Appl. Opt.* **8**, 752 (2006).
- [30] I. V. Kataevskaya and N. D. Kundikova, Influence of the helical shape of a fibre waveguide on the propagation of light, *Quantum Electron.* **25**, 927 (1995).
- [31] S. Wang, G. Ma, and C. T. Chan, Topological transport of sound mediated by spin-redirection geometric phase, *Sci. Adv.* **4**, eaq1475 (2018).
- [32] Y. Aharonov and J. Anandan, Phase change during a cyclic quantum evolution, *Phys. Rev. Lett.* **58**, 1593 (1987).
- [33] I. Bialynicki-Birula, Photon wave function, *Prog. Opt.* **36**, 245 (1996).
- [34] M. V. Berry, Optical currents, *J. Opt. A: Pure Appl. Opt.* **11**, 094001 (2009).
- [35] K. Y. Bliokh, Y. S. Kivshar, and F. Nori, Magnetoelectric effects in local light-matter interactions, *Phys. Rev. Lett.* **113**, 033601 (2014).
- [36] S. Golat, A. J. Vernon, and F. J. Rodríguez-Fortuño, The electromagnetic symmetry sphere: A framework for energy, momentum, spin and other electromagnetic quantities, *Phys. Scr.* **100**, 105518 (2025).
- [37] M. V. Berry and P. Shukla, Geometry of 3D monochromatic light: Local wavevectors, phases, curl forces, and superoscillations, *J. Opt.* **21**, 064002 (2019).
- [38] M. V. Berry, A geometric-phase timeline, *Opt. Photon. News* **35**, 42 (2024).
- [39] M. V. Berry and M. R. Dennis, Polarization singularities in isotropic random vector waves, *Proc. R. Soc. London A* **457**, 141 (2001).
- [40] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/qs1r-6g64> for geometric phase in paraxial light, a derivation of Eq. (11), derivation of Eq. (18), and a visualization of the transverse field distribution of the evolving focused beam of Fig. 3, which includes Ref. [57].
- [41] A. J. Vernon, Topologies of light in electric-magnetic space, *arXiv:2507.16721*.
- [42] I. Freund, Optical Möbius strips in three-dimensional ellipse fields: I. Lines of circular polarization, *Opt. Commun.* **283**, 1 (2010).
- [43] T. Bauer, P. Banzer, E. Karimi, S. Orlov, A. Rubano, L. Marrucci, E. Santamato, R. W. Boyd, and G. Leuchs, Observation of optical polarization Möbius strips, *Science* **347**, 964 (2015).
- [44] K. Y. Bliokh, M. A. Alonso, D. Sugic, M. Perrin, F. Nori, and E. Brasselet, Polarization singularities and Möbius strips in sound and water-surface waves, *Phys. Fluids* **33**, 077122 (2021).
- [45] R. D. Muelas-Hurtado, K. Volke-Sepúlveda, J. L. Ealo, F. Nori, M. A. Alonso, K. Y. Bliokh, and E. Brasselet, Observation of polarization singularities and topological textures in sound waves, *Phys. Rev. Lett.* **129**, 204301 (2022).

- [46] K. Y. Bliokh, A. Y. Bekshaev, and F. Nori, Dual electromagnetism: Helicity, spin, momentum and angular momentum, *New J. Phys.* **15**, 033026 (2013).
- [47] R. P. Cameron, S. M. Barnett, and A. M. Yao, Optical helicity, optical spin and related quantities in electromagnetic theory, *New J. Phys.* **14**, 053050 (2012).
- [48] Y. Cheng, Y.-S. Zeng, W. Xiao, T. Fu, J. Wu, G.-B. Wu, D. P. Tsai, and S. Wang, Riemann-Silberstein geometric phase for high-dimensional light manipulation, [arXiv:2510.09112](https://arxiv.org/abs/2510.09112).
- [49] M. V. Berry, Quantum phase corrections from adiabatic iteration, *Proc. R. Soc. London A* **414**, 31 (1987).
- [50] K. Y. Bliokh, Generalization of Berry's geometric phase, equivalence of the Hamiltonian nature, quantizability and strong stability of linear oscillatory systems, and conservation of adiabatic invariants, *J. Math. Phys.* **43**, 5624 (2002).
- [51] D. Green and K. A. Forbes, Optical chirality of vortex beams at the nanoscale, *Nanoscale* **15**, 540 (2023).
- [52] K. A. Forbes, Vortex light at the nanoscale: Twists, spins, and surprises, *Rep. Prog. Phys.* **89**, 016401 (2026).
- [53] E. Galiffi, R. Tirole, S. Yin, H. Li, S. Vezzoli, P. A. Huidobro, M. G. Silveirinha, R. Sapienza, A. Alù, and J. B. Pendry, Photonics of time-varying media, *Adv. Photon.* **4**, 014002 (2022).
- [54] M. Yessenov, L. A. Hall, K. L. Schepler, and A. F. Abouraddy, Space-time wave packets, *Adv. Opt. Photon.* **14**, 455 (2022).
- [55] Y. Shen, Q. Zhan, L. G. Wright, D. N. Christodoulides, F. W. Wise, A. E. Willner, K.-H. Zou, Z. Zhao, M. A. Porras, A. Chong, *et al.*, Roadmap on spatiotemporal light fields, *J. Opt.* **25**, 093001 (2023).
- [56] A. J. Vernon, S. Golat, and F. J. Rodríguez-Fortuño, Electromagnetic symmetry dislocations, *Phys. Rev. A* **112**, L021504 (2025).
- [57] K. A. Forbes and G. A. Jones, Measures of helicity and chirality of optical vortex beams, *J. Opt.* **23**, 115401 (2021).