

Marginally volume-dependent oscillation modes of superfluid pendant droplets in an intervening stability region

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Droplets are ubiquitous in nature and industry, yet their behavior is not yet fully understood, particularly for extreme conditions not typically encountered in viscous classical fluids. In this study, we investigated the oscillations of a superfluid ⁴He pendant droplet formed beneath a cup by superfluid film flow. We controlled the mobility of the droplet's edge by attaching cylinders of varying diameter to the underside of the cup, so that the droplet was suspended from the bottom of the cylinder. For wide cylinders or in the absence of a cylinder, the lowest point of the droplet oscillated vertically, while the edge oscillated along the surface, with only a slight dependence of the oscillation period on the droplet volume, which contradicts the common assumption in classical fluids that larger droplets oscillate with a longer period. For narrow cylinders, the droplet oscillated vertically, while the edge remained fixed at the cylinder's edge, and the oscillation period increased monotonically with increasing droplet volume, displaying the expected behavior. These observations demonstrate that free motion of the droplet's edge contributes to anomalous volume-independent oscillation of pendant droplets. We propose that the marginal volume dependence arises from the stability of superfluid pendant droplets in a region between two instabilities. This behavior is significant only for fluids that completely wet the substrate and possess a mobile edge, a condition rarely observed in classical fluids but naturally occurring in superfluids. Hence, these findings reveal unique properties of droplets in superfluid ⁴He not seen in classical fluids.

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I. INTRODUCTION

Superfluid ⁴He provides a unique means to investigate extreme conditions in fluid dynamics that are difficult to achieve with classical fluids [1,2]. In classical fluids, viscosity and dissipation restrict the phenomena that can be observed, sometimes obscuring intriguing fluid behaviors. A good example is superfluid film flow. When superfluid is transferred into a cup, it spontaneously climbs up the cup surface, forming a nanometer-scale film due to the complete wetting of the wall [3–5]. If it reaches the lip of the cup, it will flow down the outer surface and drip from the undersurface of the cup until the cup is empty. This behavior is prevented in a classical fluid, even if it wets a wall completely, due to the severe dissipation caused by the flow in a thin film. Hence, such film flow is usually thought to be unique to superfluids. However, it is not always easy to distinguish between superfluid flow and classical fluid flow [6–10]. While superfluids possess distinct characteristics such as persistent flow and quantized circulation, these are not apparent under typical conditions.

Exploring extremes with superfluid ⁴He could help uncover previously unexamined properties in classical fluids, leading to significant advances in the general field of fluid physics [11–16]. In this study, we present a comprehensive analysis of the volume dependence of superfluid pendant droplet oscillation. Our findings reveal that the oscillation period shows a marginal volume dependence, challenging the classical understanding that larger droplets should oscillate more slowly.

It is a common belief that larger droplets oscillate with a longer period. A typical example is a free spherical droplet free from a wall, for which Rayleigh calculated the oscillation frequency ω or oscillation period T_0 analytically to be

$$\omega = \frac{2\pi}{T_0} = \sqrt{\frac{\sigma}{\rho R^3} k(k-1)(k+2)} \quad (1)$$

for the modes whose distortion from the sphere is specified by the spherical harmonic function $Y_{k,l}$ [17,18]. Here, k and l are the polar and azimuthal wave numbers, and σ , ρ , and R are the surface energy, density of the liquid, and radius of the droplet. T_0 monotonically increases as the droplet volume V increases, and $T_0(V \rightarrow 0) = 0$. These are also true when the droplets are attached to a wall: Larger droplets, whether they are sessile or pendant, oscillate more slowly [19,20]. The oscillation modes of sessile and pendant droplets can be classified by the same indices (k , l) as for free droplets [21–23]. We have demonstrated a counterintuitive phenomenon in that the oscillation period of superfluid pendant droplets is

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almost volume independent [24–26]. However, the origin of this anomalous oscillation remains unclear. Here, we systematically examined the volume dependence of the oscillations by varying the shape of the wall to restrict the edge motion. We show that the condition for the occurrence of this anomalous oscillation is the free motion of the droplet’s edge along the wall surface. While a mobile droplet edge can occur in principle in classical fluids, it is rarely observed and thus its oscillation has not been examined. In contrast, superfluid ^4He spreads across surfaces by forming a thin film due to complete wetting, with a contact angle of $\alpha = 0$, reducing the effect of surface roughness and facilitating this free motion.

We propose that anomalous oscillations occur in pendant droplets that can stably exist in an intervening volume region between two instabilities. This intervening stability is caused by the Rayleigh-Taylor instability in the small volume, which is significant only for droplets that have a zero-contact angle to a wall and a mobile edge. The influence of the Rayleigh-Taylor instability on droplet dynamics has not been previously understood, as it is usually irrelevant to droplet motion of classical fluids. Intriguingly, superfluidity leads to not just a minor deviation from known behaviors, but a significant and qualitative change in droplet dynamics.

To induce oscillation in a superfluid pendant droplet, superfluid ^4He was dripped from the undersurface of a cup [24–26]. The recoil from the pinch-off caused the remaining pendant droplet to oscillate. This dripping occurred due to a film flow mechanism, as mentioned above. Between each pinch-off event, the pendant droplet continued to oscillate. As it oscillated, superfluid continued to flow into the droplet, leading to steady growth. We measured the oscillation period as a function of the droplet’s volume. To examine the influence of the droplet edge motion, solid cylinders with various diameters were attached to the underside of the cup. The edge of the cylinder effectively confined the droplet motion and pinned its edge.

II. EXPERIMENTS

The experiments were conducted using a glass dewar flask that allowed for visual observation of the motion of the liquid ^4He . A standard procedure for demonstrating superfluid film flow was adapted to observe the dripping of superfluid ^4He [24–26]. ^4He vapor, which coexisted with the liquid, was pumped out to cool the liquid below the superfluid transition temperature of 2.17 K [1,2]. Each experiment was carried out at a base temperature of 1.6 K, at which point the cooling power and heat input were balanced. A rod with a glass cup attached at its lower end was inserted through a flange at the top of the flask, with an O-ring compression seal. The glass cup had an inner diameter of 26 mm, an outer diameter of 30 mm, and a height of 33 mm. In some trials, a brass cylinder with diameters d from 1.0 to 6 mm was glued to the underside of the cup to investigate the impact of surface morphology on the dripping process, as illustrated in Figs. 1(a)–1(c). In this paper, we call a cup without a cylinder the *bare cup* and a cup with a cylinder of d mm in diameter the *d mm cup*. Additionally, the glass surface was coated with a commercial coating agent (Glaco Mirror Coat Zero, Soft99 Co.) to enhance the effective surface area and increase the film

flow rate [5]. This surface treatment improved the effective perimeter for film flow by applying nanometer-scale particles to the surface, making the film flow rate high enough for the experiment.

A portion of superfluid ^4He was scooped into the cup, which was then raised several centimeters above the liquid surface. Dripping was observed as the superfluid spontaneously flowed from the cup. This dripping involved a pendant droplet suspended from the underside of the cup. Oscillation of the pendant droplet was triggered by recoil after the pinch-off, and this oscillation continued until the next pinch-off occurred. During the oscillation, superfluid continued to flow into the droplet, resulting in its steady growth. The relationship between the volume of a droplet and its oscillation period was examined.

To monitor the droplet oscillation, the cup was illuminated from the back of the dewar using parallel light, producing shadowgraph images on the front. Images were captured with a high-speed video camera (Photron, FASTCAM Mini AX50) operating at a frame rate of 125 frames per second or higher. Since the droplets were observed from only one side, the droplet volume was estimated by assuming a cylindrically symmetric shape.

III. RESULTS

The oscillation behaviors of pendant droplets are presented in Fig. 1, showing three typical wall shapes. We begin by presenting the results for the bare cup, as it serves as a baseline without any modifications to the wall shape, highlighting the importance of edge motion. Next, we show the results for the wide cylinder, where edge motion is also a factor, similar to the bare cup case. Finally, we present the results for the narrow cylinder.

Figures 1(a)–1(c) illustrate the bare cup, the 6 mm cup, and the 1.5 mm cup, with the dripping of superfluid from each. The oscillations of the pendant droplet between two pinch-off points are shown in Figs. 1(d)–1(f), with the droplet profiles overlaid. For the bare cup and the 6 mm cup, the lowest point of the droplet oscillated vertically and the droplet edges moved on the substrate surface accordingly. This free motion of the edge was commonly observed for the wide cylinders. In contrast, for the 1.5 mm cup, the droplet edge remained fixed at the edge of the cylinder base. This behavior was typical for narrow cylinders, as the cylinder’s width was much narrower than the natural width of the droplet, hindering its free movement. See also the movie file in the Supplemental Material for Fig. 1 [27].

The length of a droplet measured from the undersurface of the bare cup or cylinder, $L(t)$, is plotted as a function of time between the two pinch-off events in Figs. 1(g)–1(i). The rapid upward motion of $L(t)$ around $t = 0$ s represents the recoil from the pinch-off, which subsequently induces an oscillatory motion of the pendant droplet. The rapid downward motion at the end of the oscillation corresponds to the next pinch-off. The oscillation period of $L(t)$ remains nearly independent of time for the free-edge cases (bare cup and 6 mm cup), while it is time dependent for the pinned-edge case (1.5 mm cup). The positions of the droplet’s right and left edges, $x_R(t)$ and $x_L(t)$, are also plotted in Figs. 1(g)–1(i). For both the bare cup

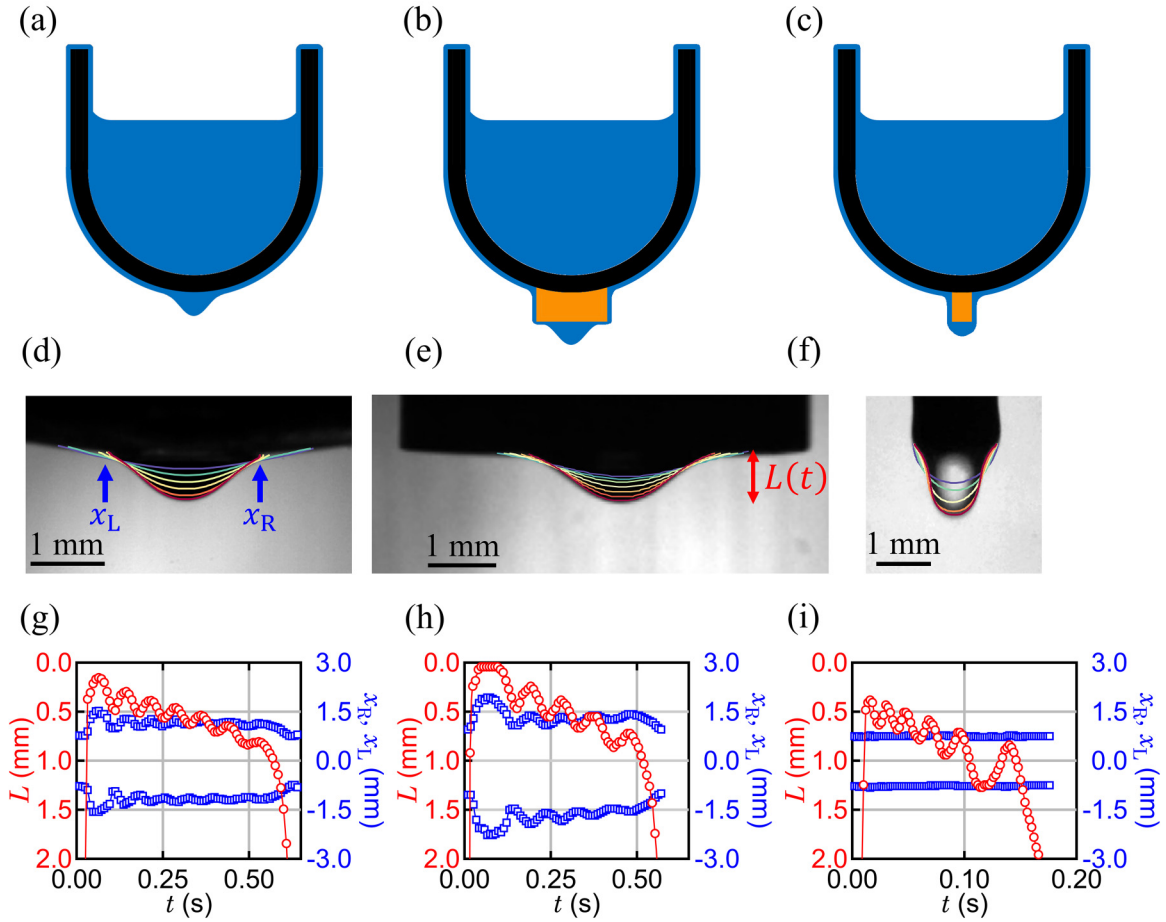


FIG. 1. Oscillations of superfluid pendant droplets on the undersurface of cups with various shapes, induced by dripping via the film flow from the cup. Each column represents a different cup: bare cup, 6 mm cup, and 1.5 mm cup. (a)–(c) Illustrations of the cups, showing pendant droplet. (d)–(f) Images of oscillating pendant droplets on the underside of the cups. Profiles of the droplets at various times are overlaid to illustrate the oscillations. The two-headed arrow in panel (e) represents the length of the droplet $L(t)$ measured from the undersurface. The positions of the left and right edges, $x_R(t)$ and $x_L(t)$, are indicated by the arrows in panel (d). (g)–(i) Droplet shape over time during an oscillation between two pinch-off events. Left axis: $L(t)$, right axis: $x_R(t)$ and $x_L(t)$.

and 6 mm cup, $x_R(t)$ and $x_L(t)$ oscillated in accordance with the changes in $L(t)$. However, for the 1.5 mm cup, $x_R(t)$ and $x_L(t)$ were pinned at the corners of the cylinder base and do not move during the oscillation.

$L(t)$ and the droplet volume $V(t)$ are illustrated at various times between the two pinch-offs in Fig. 2(a) for the bare cup as a typical mobile edge case. The volume increased proportionally to the elapsed time due to the steady superfluid input via the film flow, while the droplet oscillated with nearly

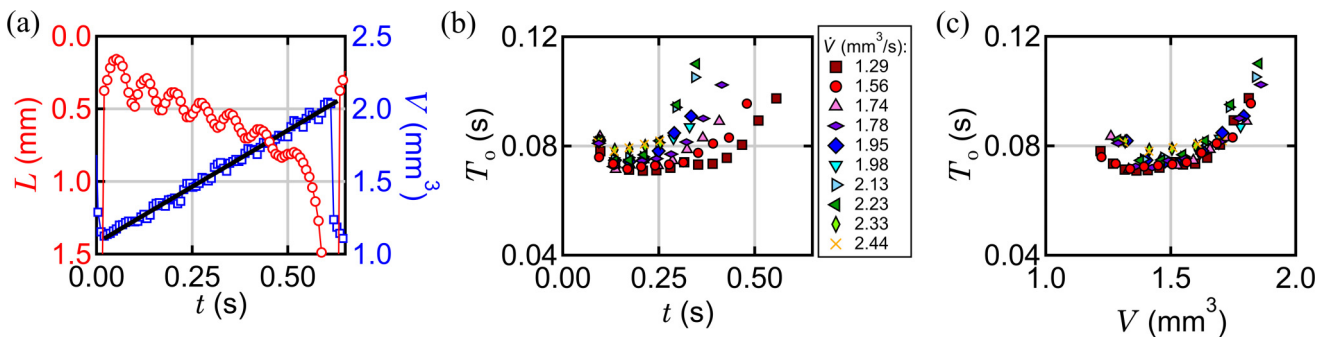


FIG. 2. Oscillation of a superfluid pendant droplet formed on the underside of the bare cup between two pinch-offs. (a) $L(t)$ (circles) and droplet volume $V(t)$ (squares) as functions of time. The black solid line is a linear fit to $V(t)$, which determined $\dot{V} = 1.56 \pm 0.03$ mm³/s. (b) Oscillation period $T_0(t)$ as a function of time measured for various flow rates, represented by different colors. The red circles are derived from panel (a). (c) $T_0(V)$ as a function of $V(t)$. $\dot{V}$ are shown in the legend and are accurate within about $\pm 2\%$.

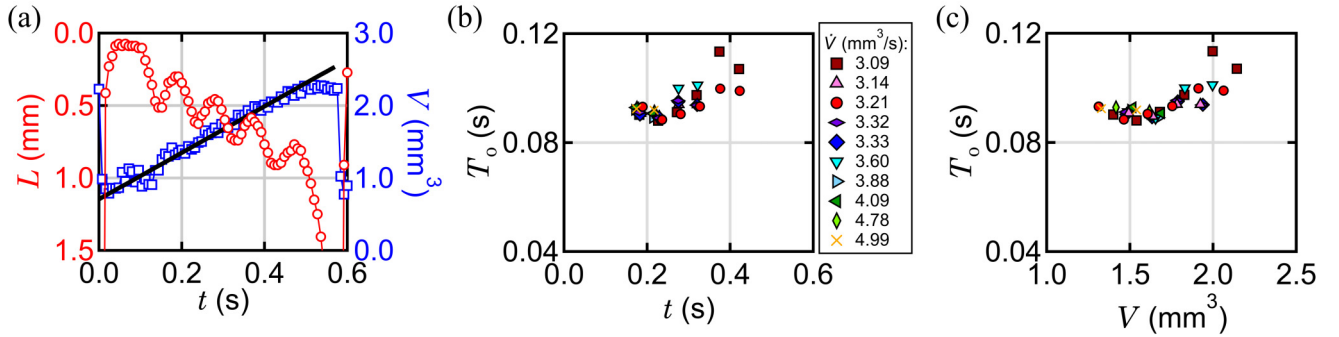


FIG. 3. Oscillation of a superfluid pendant droplet formed on the underside of the 6 mm cup between two pinch-offs. (a) $L(t)$ (circles) and $V(t)$ (squares) as functions of time. The black solid line is the linear fit to $V(t)$, which determined $\dot{V} = 3.21 \pm 0.04$ mm³/s. (b) $T_o(t)$ as a function of time for various flow rates, represented by different colors. The red circles are derived from panel (a). (c) $T_o(V)$ as a function of $V(t)$. $\dot{V}$ are shown in the legend and are accurate within about $\pm 1\%$.

a constant period. This means that the oscillation period was only marginally volume dependent, which we referred to as the anomalous (2, 0) mode in our earlier publication [25]. From the measurement of $L(t)$ and $V(t)$ at various flow rates, detailed characteristics of the oscillation can be obtained and the time evolution of the oscillation period $T_o(t)$ was extracted from the maximum and minimum positions of the oscillations of $L(t)$.

$T_o(t)$ between the two pinch-offs are plotted in Fig. 2(b) for the bare cup at various flow rates. At higher flow rates, the dripping period between two pinch-offs was shorter and there were fewer oscillations. Although $T_o(t)$ had a different evolution depending on the flow rate, each evolution had common features. $T_o(t)$ initially had nearly a constant value of about 0.078 s and increased approaching the pinch-off. Since $V(t)$ increased linearly, the time-independent $T_o(t)$ indicates a volume-independent oscillation period and thus is anomalous. The increase of $T_o(t)$ in the final stage of the oscillation is due to the decrease in the restoring force for the oscillation as it approaches the pinch-off instability, becoming too large to stick to the undersurface at the critical volume. The volume dependence of the oscillation period $T_o(V)$ is shown in Fig. 2(c). The volume of the droplets was estimated by performing a linear fit on the $V(t)$ data at each flow rate. While $T_o(t)$ varied depending on the flow rate, $T_o(V)$ was consistent across the different conditions, showing a collapse to a universal curve. This finding indicates that the oscillation period depicted in Fig. 2(c) accurately represents the functional form of $T_o(V)$, which is primarily determined by the droplet volume and is not significantly affected by factors such as variations in flow rate or evaporation during the oscillations.

Figure 3 presents the same set of analyses for the 6 mm cup as Fig. 2 does for the bare cup. In the case of the 6 mm cup, the edge of the droplet was also mobile. As a result, the key features described in Fig. 2 are also present in Fig. 3, including the volume-independent oscillation periods in $T_o(V)$ and the collapse of these periods. One noticeable difference observed immediately after the pinch-off is that $L(t)$ remained very close to zero for the interval between 0.040 and 0.102 s in Fig. 3(a). During this time, apparent oscillation was not detected, indicating that the period is long. Because there was

no clear maximum in $L(t)$, $T_o(t)$ could not be estimated in this region. The absence of oscillation may serve as a precursor to Rayleigh-Taylor instability, which became evident in the widest cylinder. A discussion of this instability will be provided in the next section.

A similar analysis is presented in Fig. 4 for the 1.5 mm cup, which is a representative of the pinned-edge case. Time evolutions of $L(t)$ and $V(t)$ between two pinch-offs for the 1.5 mm cup are shown in Fig. 4(a). As the volume increased proportionally to the elapsed time, the droplet oscillated with a longer period, indicating a volume-dependent oscillation period. We refer to this mode as the normal (2, 0) mode [25]. The time evolutions of $T_o(t)$ extracted from $L(t)$ at various flow rates were different, depending on the flow rates, as shown in Fig. 4(b), but commonly increased over time. The slope became steeper approaching the pinch-off. $T_o(V)$ was determined and exhibited a collapse to a universal curve, shown in Fig. 4(c). The overall dependence is a monotonic increase in $T_o(V)$. In detail, $T_o(V)$ increased almost linearly in the small V region and more steeply approaching the pinch-off. The steep increase approaching the pinch-off indicates a diminishing of the restoring force near the pinch-off instability, as observed in the bare cup.

Additional examples of $L(t)$ for each cup are included in the Appendix to show the reproducibility of the characteristics described above and the extent of data variability.

In Fig. 5(a), $T_o(V)$ is plotted for both the bare cup and eight other cups with cylinders of different diameters. For the bare cup and those with wider cylinders, $T_o(V)$ showed a marginal dependence on volume in the small V regions and exhibited an increase near the pinch-off point, indicating that the oscillations are anomalous. A broad minimum was observed in the intermediate volume. In these cases, the droplets' edges were mobile and oscillated simultaneously with the oscillation of $L(t)$. In contrast, $T_o(V)$ increased monotonically for the cups with narrow cylinders. In these cases, the diameters were significantly smaller than the natural width of the droplet, resulting in the pendant droplet being confined to a narrow region. The edge of these droplets was pinned at the edge of the cylinder and did not move during the droplet oscillation. Thus, the ability for a droplet's edge to move freely appears to

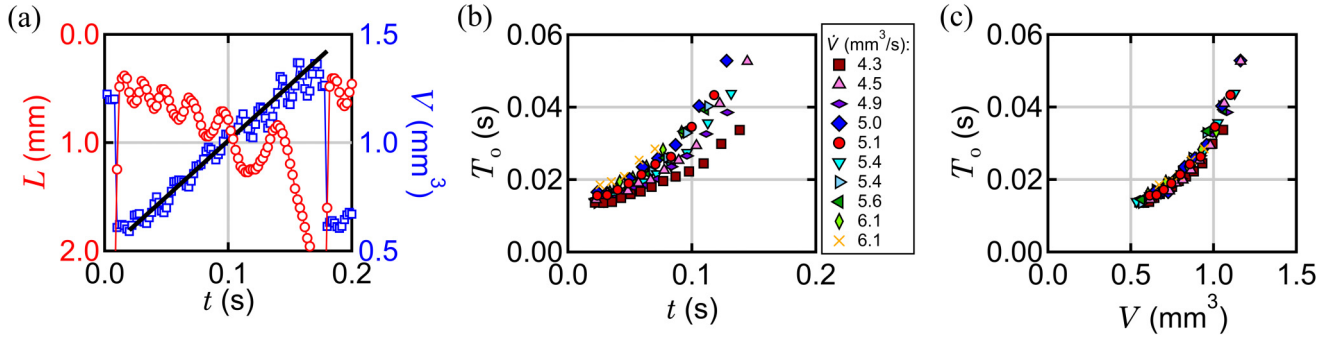


FIG. 4. Oscillation of a superfluid pendant droplet formed on the underside of the 1.5 mm cup between two pinch-offs. (a) $L(t)$ (circles) and $V(t)$ (squares) as functions of time. The black solid line is the linear fit to $V(t)$, which determined $\dot{V} = 5.1 \pm 0.2$ mm³/s. The fitting error was larger than that of the other two cups. (b) $T_o(t)$ as a function of time for various flow rates, represented by different colors. The red circles are derived from panel (a). (c) $T_o(V)$ as a function of $V(t)$. $\dot{V}$ are shown in the legend and are accurate within about $\pm 4\%$.

be a necessary condition for the occurrence of the anomalous oscillation.

To better understand the systematic variation of $T_o(V)$ with changes in d , we fitted $T_o(V)$ in the small V region using a linear function expressed as $T_o(V) = AV + B$, depicted by the dotted lines in Fig. 5(a). Note that the linear fit primarily serves to illustrate the systematic variation qualitatively. The coefficients A and B are plotted in Figs. 5(b) and 5(c) as functions of d for the cups with cylinders and the bare cup. As the bottom of the bare cup, with a curvature radius of 15 mm, is significantly different from those of the cylinder cases, its data are presented separately.

When d is small, A is large and B is small. Large A indicates that the oscillation period lengthens as the volume increases. Small B indicates that small-volume droplets oscillate with a short period, which is consistent with $T_o(V \rightarrow 0) = 0$. Therefore, normal oscillation occurred when d is small, consistent with Eq. (1). As d increases, A decreases and B increases, indicating that the volume dependence of T_o becomes weaker and that droplets with small volume oscillate with nonzero period as $T_o(V \rightarrow 0) \approx B$. Therefore,

an anomalous oscillation occurred when d is large. The small but negative values of A observed for $d = 4, 5$, and 6 mm indicate the emergence of a shallow minimum in $T_o(V)$, around which T_o has marginal volume dependence.

IV. DISCUSSION

A monotonic increase of $T_o(V)$ is the ordinary behavior observed in many classical systems [19,20]. It is commonly understood that larger droplets oscillate with a longer period. A marginal volume dependence is therefore extraordinary in this sense. At present, it is not known why the anomalous oscillation behavior occurs for superfluid pendant droplets when their edges are mobile. Zhang and Zhou calculated the oscillation frequencies of pendant droplets for arbitrary (k, l) modes for various boundary conditions for the droplet contact angle α and edge mobility [28]. We believe their comprehensive theoretical study of pendant droplets reveals a glimpse of the anomalous mode. They presented calculations of the oscillation frequency scaled by the characteristic time $\lambda = \omega t_c (= 2\pi t_c / T_o)$ as a function of Bond number

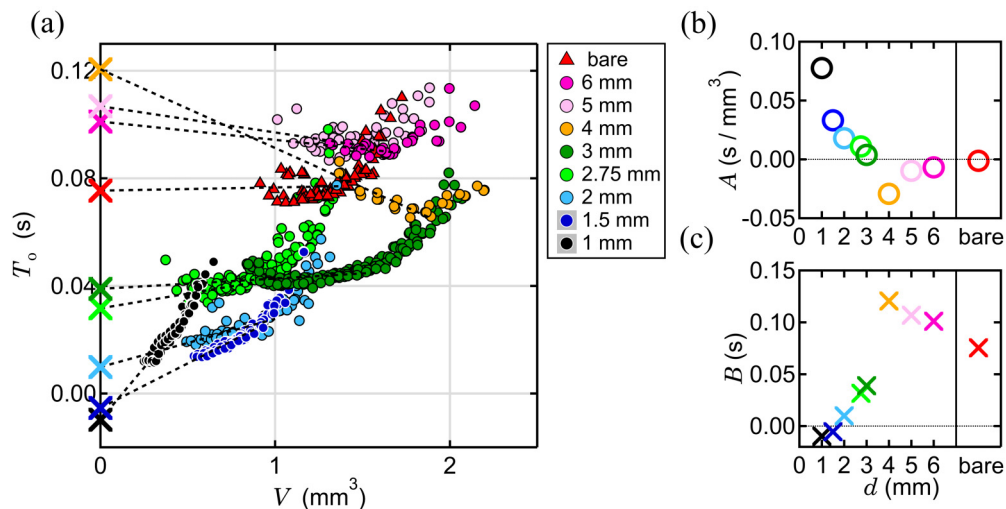


FIG. 5. (a) Oscillation period of pendant droplets formed on the underside of various cups plotted against volume, $T_o(V)$. The diameters of the cylinders d are indicated in the legend. The dotted lines represent linear fits of the data in the small V region and include their extrapolation. (b), (c) Slope A and intercept B of the linear fit as functions of d , with the data for the bare cup presented separately on the right.

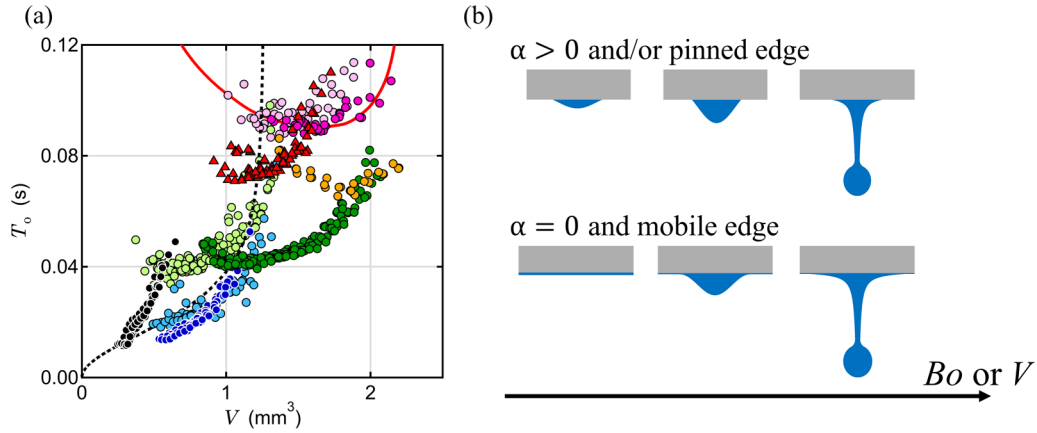


FIG. 6. (a) Comparison of observed $T_0(V)$ with the theoretical calculations, converted for superfluid ^4He from the generic calculations by Zhang and Zhou [28]. The experimental data are the same as those shown in Fig. 5(a). The dotted black line represents the calculation for the pinned-edge droplet with $d = 1.7$ mm. The solid red line is for the free-edge droplet with $\alpha = 0$. (b) Illustrations of the droplet shape as a function of the Bond number or the volume. The upper row depicts the ordinary case ($\alpha > 0$ and/or a pinned edge), while the lower row presents the superfluid case ($\alpha = 0$ and the edge is mobile).

$\text{Bo} = \rho g / \sigma \times (3V/2\pi)^{2/3}$. Here, $t_c = \sqrt{3\rho V/2\pi\sigma}$. Note that λ is inversely proportional to T_0 . Bo is larger when the gravitational energy is larger than that of the surface energy. Therefore, Bo is larger for the larger volume droplets generally. While in most cases λ monotonically decreased as a function of Bo , as expected from the established understanding of droplet oscillations that larger droplets oscillate at lower frequency or over a longer period, a nonmonotonic dependence on Bo was calculated for $(k, l) = (2, 0)$ mode only under the boundary conditions of $\alpha = 0$ and a freely mobile edge. Such a specific condition is rarely realized in classical fluids, but naturally met in superfluid ^4He .

While they showed the results for λ , they did not present the oscillation period itself or explicitly address the marginal volume dependence [28]. To directly compare these results with our own, we converted their calculated results of $\lambda(\text{Bo})$ presented in Fig. 9 of their paper to $T_0(V)$ using the physical parameters of superfluid ^4He , $\rho = 0.145$ g/cm³ and $\sigma = 0.345$ dyne/cm [1,10]. The converted results of $T_0(V)$ with $\alpha = 0$ for free and pinned edges are shown in Fig. 6(a), as the solid red and dotted black lines, respectively. For the pinned case, we used $d = 1.7$ mm. The $T_0(V)$ data from Fig. 5(a) are included in the figure for easier comparison. Note that their calculations are for a flat and infinitely large surface, not the slightly curved, finite-diameter surface of a cup, and thus perfect agreement with our results is not expected. The result for the pinned edge has a monotonic increase with V and semiquantitatively reproduces the experimental results for the 1.5 mm cup. Furthermore, the results for the mobile edge have a nonmonotonic dependence on V . $T_0(V)$ exhibits a minimum and is marginally volume dependent around the minimum. This again reasonably reproduces the experimental results for the mobile edge condition.

A divergent increase in the large V region is common in their calculations for both the free and pinned edges and is due to the reduction in the restoring force of the droplet oscillation as it approaches the pinch-off instability. However, in the small V region, $T_0(V)$ decreases as V decreases for the pinned case and increases for the mobile case. It was not

explicitly explained in their paper why the anomalous oscillation occurs for the specific condition of $\alpha = 0$ and a free edge. It is noteworthy that only under this condition another type of instability occurs in the small V region known as the Rayleigh-Taylor instability [29,30]. This instability involves a uniform thin film that forms beneath a wall becoming unstable, which results in the development of undulations in the film's thickness and finally the formation of droplets. Provided that the restoring force diminishes near the Rayleigh-Taylor instability, the increase in $T_0(V)$ as V decreases in the small V region can be accounted for.

The Rayleigh-Taylor instability is a well-known phenomenon that occurs at the interface between two immiscible fluids when the heavier fluid is positioned above the lighter fluid. This instability spans a wide range of fields in physics, including classical fluid dynamics, atomic Bose condensates, and planetary or astrophysical studies, and has been extensively researched across many disciplines [29–33]. However, its influence on droplet dynamics has not been previously understood, particularly because it does not apply to droplet motion when the droplet edge is pinned. As a result, the phenomenon of anomalous oscillation had gone unrecognized until our observations in superfluid droplets, in which the heavier fluid is the superfluid ^4He and the lighter fluid is its vapor.

The stability of pendant droplets is illustrated in Fig. 6(b). At large volumes, a phenomenon known as pinch-off instability occurs, in which the droplet volume is too large to be suspended from the undersurface due to the gravitational energy. The droplet can no longer attach to the undersurface and drips. Under ordinary conditions, shown in the upper row, where $\alpha > 0$ and/or the edge is pinned—conditions typically met in classical fluids—a droplet of any smaller volume V than that at the pinch-off instability can exist stably. However, under the specific condition of $\alpha = 0$ and a free edge, as presented in the lower row, which is rarely satisfied in classical fluids but naturally met in superfluid ^4He , the Rayleigh-Taylor instability exists at small volume, in which a uniform thin film is more stable than the droplet due to the surface energy

[29,30]. The surface energy makes the liquid surface flat, and the pendant droplet can no longer stably exist on the undersurface. Therefore, superfluid pendant droplets can exist stably only in the intervening volume region between two instabilities. Both instabilities lead to a diminishing restoring force for oscillations near the instability points in the large- and small-volume regions, causing the oscillation period to diverge as it approaches these limits. In the intervening volume region where the oscillation period exhibits its minimum, the superfluid pendant droplets are stable and oscillate with periods that are marginally dependent on volume. However, this anomaly is suppressed when the edge is pinned by a narrow cylinder, resulting in behavior that aligns more closely with that of classical fluids. This explains why superfluid pendant droplets display anomalous oscillatory behaviors not seen in classical fluids: the Rayleigh-Taylor instability is usually irrelevant to droplet motion in classical fluids.

In our previous publication, we reported not only anomalous oscillations but also robustly discretized dripping periods for superfluid dripping from a bare cup [24]. This discretization represents another anomaly in superfluid dripping that is not observed in classical fluids; it is well known that the dripping periods of water from a faucet display a broad distribution due to chaotic behavior [34–37]. These two anomalies in superfluids are closely connected, and the effect of the wall shape on the dripping periods was reported in a separate publication [38].

Since we used spontaneous film flow to induce droplet oscillation, the experimental volume range was limited. We were unable to obtain $T_o(V)$ in both the small and large volume regions very close to the instabilities. Future experiments using an isolated cell will enable investigation over a broader volume range and at higher temperatures where spontaneous film flow is nonreproducible.

V. CONCLUSIONS

The oscillations of superfluid pendant droplets were studied as a function of droplet volume by altering the mobility of the droplet edge. The droplets formed on the undersurface of a cylinder glued to the bottom of a cup, with cylinders of varying diameters used. The droplets grew due to mass input via superfluid film flow from the cup and eventually pinched off, triggering an oscillation. This allowed the oscillation period to be examined as a function of the volume between two pinch-off events. In the case of wide cylinder cups and cups without a cylinder, the entire droplet body oscillated, including its edge. During oscillation, the lowest point of the droplet moved vertically while the droplet edge oscillated cooperatively along the undersurface. The oscillation periods were nearly constant in the small-volume region and increased slightly near the pinch-off. This marginal volume dependence contradicts the commonly accepted belief that larger droplets oscillate with longer periods, making it anomalous. Superfluidity caused not just a minor deviation from known behaviors, but a qualitative difference in the dynamics of droplets. Conversely, droplets formed under a narrow cylinder, whose edge was pinned at the edge of the cylinder base due to

confinement, exhibited volume-dependent oscillation periods; larger droplets oscillated with longer periods, consistent with the conventional understanding. Thus, the mobility of the droplet edge is a key factor contributing to the anomalous oscillation.

Experimental results were compared with recent theoretical calculations of the oscillation frequency of pendant droplets, showing reasonable agreement. We propose that the anomalous oscillation arises because stable superfluid pendant droplets can only exist in an intervening stability region between pinch-off instabilities observed in large volumes and the Rayleigh-Taylor instability in small volumes. Both instabilities lead to an increase in the oscillation period in both large and small volumes due to a weakening restoring force. In the intervening volume region, away from the instabilities, the oscillation period decreases and exhibits a broad minimum as a function of the volume; around this minimum, the oscillation period is only marginally dependent on the volume. Complete wetting of the superfluid on the solid wall, along with the resulting free-edge motion of the droplet, is a necessary condition for both the intervening stability and the anomalous oscillation. This is the first acknowledgment that Rayleigh-Taylor instability, which is usually irrelevant to droplet motion in classical fluids, does impact droplet oscillations. These observations demonstrate that superfluid droplets exhibit a peculiar oscillation feature under the extreme conditions of complete wetting and free-edge movement, which are naturally fulfilled in superfluid ^4He but rarely realized in classical fluids.

ACKNOWLEDGMENT

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

APPENDIX

$L(t)$ and $V(t)$ for each cup are illustrated in Fig. 7 over an extended period to demonstrate the reproducibility of the characteristics discussed in Sec. III and the extent of data variability. The data for the bare, 6 mm, and 1.5 mm cups are presented in panels (a), (b), and (c), respectively. Immediately after $t = 0$ s, the same data shown in Figs. 2(a), 3(a), and 4(a) are included, along with data during two additional successive drippings.

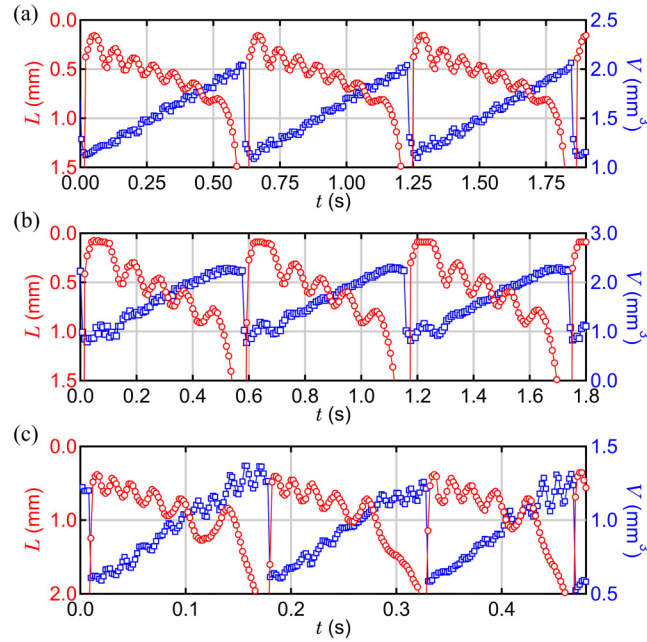


FIG. 7. $L(t)$ and $V(t)$ for the bare, 6 mm, and 1.5 mm cups are shown over an extended period in panels (a), (b), and (c), respectively. The data immediately following $t = 0$ s, which is the same as that shown in Figs. 2(a), 3(a), and 4(a), are supplemented with data from two additional successive drippings. In panels (a) and (b), the number of oscillations was 6 and 4 after each dripping, respectively. In panel (c), there were five oscillations following the initial dripping, which decreased to four for the subsequent drippings.

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