

Restitution coefficient dependence of the pressure of a macroscopic active particle

Neta Ben Ari^{*} and Saar Rahav[†]

Schulich Faculty of Chemistry, *Technion—Israel Institute of Technology*, Haifa 3200008, Israel



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Active matter systems serve as a paradigm for diverse nonequilibrium phenomena. Macroscopic realizations of active particles may exhibit inelastic collisions. We study the restitution coefficient dependence of the pressure exerted by a single bristlebot, a type of macroscopic driven particle with hopping dynamics. The finite hopping time allows for low- and high-drag parameter regimes. For low drag, the bristlebot pushes the wall with a momentum transfer rate $\dot{I}$ that satisfies $\dot{I} \simeq F$, where F is the active force, in agreement with results for soft (elastic) walls. In the high-drag regime, we find $\dot{I} \propto (1+r)F$, showing dependence on the coefficient of restitution r . This behavior can be understood analytically by studying the periodic solutions of a simple one-dimensional model. Numerical simulations show that the transition between two regimes also exists for highly active two-dimensional particles.

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I. INTRODUCTION

Active particles use an internal or external source of energy to propel themselves. The properties of active matter are the focus of an intensive research effort [1,2]. The interest in active matter was initiated by Vicsek *et al.* [3], who developed a model that can describe the flocking of birds [4]. Other examples of active systems include the motion of bacteria [5–10] and the dynamics of driven toys [11] or colloids [12].

The explosion of interest in active systems has led researchers to study how their microscopic activity affects their large-scale dynamics [13–15]. Systems of active particles exhibit a variety of motility induced phenomena, such as sticking to walls [16–19], aggregation [11,17,20], and phase separation [21–23]. The variety of different models used to study active particles attests to the breadth of the field [24–26].

Due to their internal propulsion, active particles are inherently out of thermal equilibrium. They exhibit athermal velocity and position distributions [16]. Several works were devoted to the nonequilibrium thermodynamics of active matter [16,27–32]. An important consequence of the out-of-equilibrium nature of active matter is that quantities such as the pressure are no longer state variables [33]. Specifically, the pressure of a system of active particles may depend on the shape of the potential describing their interaction with the container wall.

An interesting type of active matter is bristlebots (bbot), macroscopic toys that are driven by an electrical motor. Their macroscopic size makes them a convenient experimental setup [34–36]. Indeed, bbots were shown to exhibit many of the

dynamical phenomena associated with active matter, such as swarming [11,20,37] or tendency to stick to walls [11,20,38]. Recently, bbots were used as the working substance of an active information engine [39].

Since bbots are macroscopic bodies, their collisions with each other or with walls may be inelastic. It is natural to ask how this inelasticity affects the pressure they exert. Here we explore this question, focusing on the pressure due to a single bbot that makes inelastic collisions with the walls. We find that the effect of inelasticity on the pressure depends on the competition between two mechanisms of momentum relaxation through floor interactions and wall collisions.

Section II presents the need for a kinetic description of the interaction of a bbot with a hard wall. In particular, various issues with alternative descriptions are highlighted. A kicked model of the bbot dynamics is presented in Sec. III. In Sec. IV, we find a family of periodic solutions for a noiseless version of the model and investigate their momentum transfer rate in different regimes. Results for simulations with weak noise are presented in Sec. V. A two-dimensional model is defined and studied analytically in Sec. VI. Its predictions are compared to two-dimensional simulations in Sec. VII. We conclude in Sec. VIII.

II. MOTIVATION

The motivation for this study arose from a recent experimental realization of information engines whose working substance is a collection of bbots [39,40]. Quantifying the work extracted from such engines requires understanding of the interaction between the bbot and the walls. As explained below, commonly used models of active matter may not be adequate for this purpose.

A bbot arriving to a wall typically does not push it gently, but rather executes a series of rapid repeated collisions with it, as depicted in Fig. 1. The figure shows a particle whose active force is directed toward the wall. It collides with the wall (red circles) and is scattered back. During interactions

^{*}Contact author: b.neta@campus.technion.ac.il

[†]Contact author: rahavs@ch.technion.ac.il

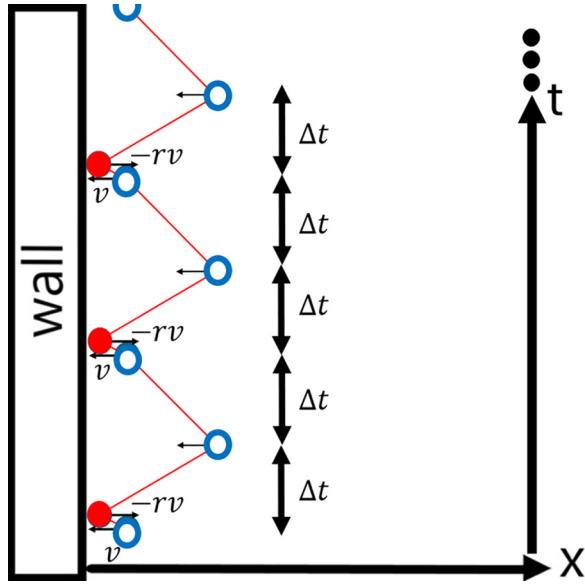


FIG. 1. Schematic depiction of a bbot repeatedly colliding with a wall. The hollow blue disks represent the interactions of the bbot with the floor (with an active force pointing toward the wall), while the red full disks represent the bbot colliding with the wall. This figure shows a trajectory with $j = 2$ interactions with the floor between wall collisions. Note that the figure is not to scale, since the distance the bbot moves between events is typically much smaller than its size.

between its legs and the floor (empty circles), the active force gives the particle a momentum kick, returning the particle to the wall. This results in a series of collisions in which the particle stays near the wall. The force, and the resulting pressure, on the wall should be identified with the mean rate of momentum transfer to the wall. Since collisions involve macroscopic bodies, it is of interest to understand whether inelastic losses affect the pressure. After all, the momentum transfer per collision is $mv_{\perp}(1+r)$. (Here $r = \frac{|v'_{\perp}|}{|v_{\perp}|}$ is the coefficient of restitution, given by the ratio between the orthogonal components of the postcollision velocity $v'_{\perp}$ and precollision velocity $v_{\perp}$.) Thus, kinetic considerations raise the possibility that the pressure may depend on r . While the bbots used in current experiments are stiff (with $r \simeq 1$), investigation of the r dependence of the pressure will reveal if more viscoelastic bbots will exhibit different behavior.

Current approaches used to study active matter are not well suited to describe such repeated collisions between the particle and the wall. Several works have modeled active particles as overdamped Brownian particles, and their interaction with the wall by a soft potential [17,18,33,41,42]. In this description, a particle facing the wall will typically reside near the point where the active force is balanced by the force due to the potential. Clearly, such a description is qualitatively different from the series of collisions that a bbot executes near the wall, and it is of interest to find out under what conditions the two approaches agree. This is emphasized by realizing that the soft wall model is inherently elastic and can never account for energy losses in collisions.

An alternative approach is to describe the bbots as underdamped particles and introduce the momentum transfer

via appropriate boundary conditions in the Fokker-Planck equation. This approach was used to study Brownian particles under gravity [43]. An unexpected and interesting phenomenon, termed inelastic collapse, was found. For restitution coefficients satisfying $r < r^* = e^{-\pi/\sqrt{3}} \approx 0.163$, the particles will make an infinite number of collisions in a finite time [44,45]. The periodic interactions between the bbot and the floor introduce a minimal time between consecutive wall collisions. This has the benefit of regularizing the singular behavior at inelastic collapse. However, one may then wonder whether the resulting pressure will then depend on parameters such as this cutoff time?

The arguments above explain why the interaction between bbots and the wall is not trivial and is already interesting at the single particle level. They furthermore motivate using a kinetic description of the dynamics that explicitly takes into account the repeated collisions. Such a model is introduced below.

III. HEXBUG MODEL

Recently, Horvath *et al.* studied the collisions between a Hexbug nano, a commercially available bbot, and a moving wall [46]. They observed that the Hexbug hops, namely, that its dynamics consists of periods of ballistic motion, interrupted by sudden velocity changes due to interactions between the Hexbug legs and the floor. They proposed a kicked dynamics model and showed that it can be used to describe their experimental observations. The hops introduce a timescale, Δt , namely, the time between successive Hexbug-floor interactions. Here, we modify this kicked dynamics model to account for inelastic collisions. We assume a stationary wall. This is sufficient for the description the work done in quasistatic processes.

The kicked dynamics model includes alternating steps in which the velocity and position are updated. The Hexbug motor retracts and extends its legs periodically, resulting in periodic interactions with the floor at times $t_n = n\Delta t$. In each such interaction, the velocity changes according to

$$v_{n+1} = v_n + \tilde{F} - \tilde{\gamma}v_n + \tilde{\xi}_n. \quad (1)$$

Here, v_n and x_n are the Hexbug velocity and position just before the floor interaction at t_n . $\tilde{\gamma}$ describes the damping in each interaction, $\tilde{F}$ is the impact due to the active force, and $\tilde{\xi}_n$ is a Gaussian noise satisfying $\langle \tilde{\xi}_n \tilde{\xi}_{n'} \rangle = C\delta_{nn'}$. The strength of the noise, C , is given in Eq. (4).

Following each velocity update, the Hexbug travels ballistically for a time. If it does not hit a wall in the next time step, then its position at t_{n+1} is

$$x_{n+1} = x_n + v_{n+1}\Delta t. \quad (2)$$

Alternatively, if the Hexbug collides with a wall (e.g., at $x = 0$), its velocity is updated according to an idealized model of an inelastic collision:

$$v'_{n+1} = -rv_{n+1}. \quad (3)$$

If the collision time was $\tau' = n\Delta t + \Delta t_1$, then its position at the next interaction with the floor will be $x_{n+1} = (\Delta t - \Delta t_1)v'_{n+1}$. r is called the coefficient of restitution. It characterizes the loss of momentum in a collision.

Crucially, a kicked dynamics based on Eqs. (1)–(3) with $r = 1$ was found in Ref. [46] to describe experimental measurements of Hexbug motion. It should be noted that we neglect the dynamical asymmetry that was found in Ref. [46].

To study the role of the hopping time Δt , we use a parametrization of the dynamics that has the same drag coefficient and terminal velocity for any value of Δt . Comparison to the time-integrated Langevin dynamics of a particle with a fixed force suggests

$$\begin{aligned}\tilde{\gamma} &= 1 - e^{-\gamma\Delta t}, \\ \tilde{F} &= \frac{\tilde{\gamma} F}{\gamma m} = [1 - e^{-\gamma\Delta t}]v_{\text{ter}}, \\ C &= \frac{k_B T_L}{m} [1 - e^{-2\gamma\Delta t}].\end{aligned}\quad (4)$$

$e^{-\gamma\Delta t}$ describes the decay of velocity during a time interval Δt , while $v_{\text{ter}} = \frac{F}{m\gamma}$ is the terminal velocity of a free Hexbug. With this parametrization taking the limit $\Delta t \rightarrow 0$, as well as the overdamped approximation leads to the commonly used active Brownian particle (ABP) model of active matter. In this work, we are interested in highly active particles with $\sqrt{C} \ll \tilde{F}$. Nevertheless, we find in Sec. V that longitudinal noise still plays a role in the dynamics of the particle.

Equations (1) and (2) are one dimensional (1D). The model can be extended to two dimensions using Eqs. (1) and (2) for each component, together with $\tilde{F}_x = \tilde{F} \cos \theta$ and $\tilde{F}_y = \tilde{F} \sin \theta$. The direction of the Hexbug θ evolves diffusively.

IV. NOISELESS PERIODIC SOLUTIONS

Many of the qualitative aspects of the dynamics can be understood by considering a noiseless version of Eq. (1). This simplified dynamics has a family of periodic solutions of a Hexbug that is repeatedly colliding with the wall.

Consider a Hexbug that made j hops without colliding with the wall. Given an initial velocity $v^{(0)}$, the current velocity is

$$v^{(1)} = (1 - e^{-j\gamma\Delta t})v_{\text{ter}} + e^{-j\gamma\Delta t}v^{(0)}. \quad (5)$$

A periodic solution of a Hexbug that collides with the wall every j hops can be found by demanding $v^{(1)} = v$ and $v^{(0)} = -rv$, leading to

$$v = \frac{1 - e^{-j\gamma\Delta t}}{1 + re^{-j\gamma\Delta t}}v_{\text{ter}}. \quad (6)$$

Here, v is the velocity at the collision with the wall. Equation (6) represents a family of periodic solutions, where different solutions are characterized by the number of hops j between wall collisions. One can verify that these solutions are linearly stable. An illustration of a Hexbug exhibiting a periodic solution with $j = 2$ is provided in Fig. 1.

Let us consider the rate of momentum transfer to the wall in one of these solutions. This rate is given by the ratio of the momentum change in a collision, $(1 + r)mv$, and the time between collisions $j\Delta t$. Taking the collision velocity from Eq. (6) and using the definition of the terminal velocity in Eq. (4) gives

$$\dot{I} = \frac{(1 + r)mv}{j\Delta t} = \frac{1 - e^{-j\gamma\Delta t}}{j\Delta t\gamma} \frac{1 + r}{1 + re^{-j\gamma\Delta t}}F. \quad (7)$$

Equations (6) and (7) highlight the important role of the parameter $e^{-\gamma\Delta t}$, which expresses how much of the previous velocity is damped in each interaction of the Hexbug with the floor. Modeling the dynamics as a Langevin equation means taking the limit $\gamma\Delta t \ll 1$, for which one finds

$$\dot{I} \simeq \left(1 - \frac{1}{2} \frac{1 - r}{1 + r} j\Delta t\gamma + O(j^2\Delta t^2\gamma^2)\right)F. \quad (8)$$

To leading order (in $\gamma\Delta t$), the particle simply pushes the wall with force F , in agreement with the soft wall description often used for ABPs.

Interestingly, a different behavior is found for $e^{-\gamma\Delta t} \ll 1$. In this limit, the Hexbug forgets its previous velocity in each hop, and the momentum transfer rate is

$$\dot{I} \simeq \frac{1 + r}{\gamma\Delta t}F, \quad (9)$$

with a nontrivial dependence on r .

This simple analysis neglects two important aspects of the dynamics. First, the periodic solutions we found are not the only ones that exist. As shown in the Appendix, periodic solutions with a more complex pattern of hops between wall collisions are also possible.

Second, the probability of finding the Hexbug in different solutions should be taken into account. For $e^{-\gamma\Delta t} \ll 1$ one must have $j = 1$, since a particle that reaches the wall with $v = v_{\text{ter}}$ is reflected with $v = -rv_{\text{ter}}$. Its maximum distance from the wall is $rv_{\text{ter}}\Delta t$, so it must return to the wall after a single hop. When $\gamma\Delta t \ll 1$ periodic solutions with different j 's are possible. The probability of those is nontrivial, and furthermore may be affected by noise. However, since the leading order is j independent, the conclusion that the Hexbug approximately pushes the wall with force F holds. As a result, the conclusion that there is a crossover from $\dot{I} \simeq F$ to $\frac{1+r}{\gamma\Delta t}F$ as a function of $e^{-\gamma\Delta t}$ is robust.

A physical interpretation of Eqs. (8) and (9) can be gained by considering momentum sources and sinks. The Hexbug gains momentum in its interactions with the floor, since the kicks are always in the same direction. This momentum gain must be balanced by losses due to damping and wall collisions. When $e^{-\gamma\Delta t} \ll 1$, the momentum transfer of a collision with the wall is $m(1 + r)v_{\text{ter}} \approx (1 + r)F/\gamma$, resulting in a transfer rate that depends on r . For $\gamma\Delta t \ll 1$, Eq. (6) gives $v \ll v_{\text{ter}}$ with relatively small momentum changes due to drag, which furthermore have opposing signs when the Hexbug moves away and toward the wall. This means that the momentum gain whose rate is F must be balanced by losses mainly from wall collisions, which explains why the value of r has little effect.

V. THE ROLE OF LONGITUDINAL NOISE

Neglecting noise is clearly an oversimplification. The Hexbug dynamics exhibit two kinds of noise. Angular noise causes the Hexbug to rotate. Longitudinal noise results in variations in the active force. In 1D only longitudinal noise is present. For macroscopic highly active particles, the longitudinal noise is often assumed to be weak in comparison to the mean active force. For this reason, some models of ABPs include only angular noise [14,16,17,27]. In contrast,

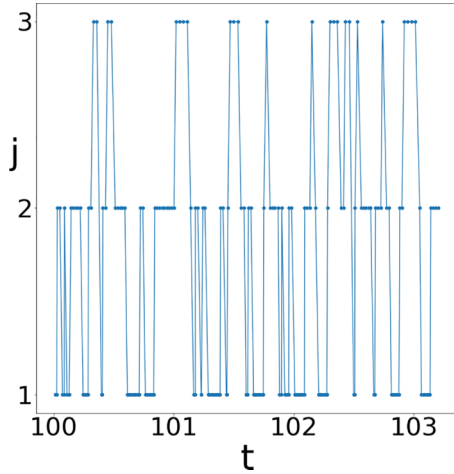


FIG. 2. Three seconds from a trajectory of a one-dimensional Hexbug with noise. $j(t)$ is defined as the number of hops between the previous and next collisions with the wall. Flat regions indicate several or many consecutive segments with the same value of j .

longitudinal noise was found in the only experimental investigation of the motion of a single Hexbug [46]. We therefore study how weak longitudinal noise affects the qualitative behavior found in the previous section.

A qualitative understanding of the dynamics with weak longitudinal noise can be obtained by comparing the difference between the collision velocities of the j 'th and $j + 1$ periodic solutions, Δv , to the noise-induced velocity drift. For $\gamma \Delta t \ll 1$, one has

$$\Delta v \simeq \frac{F \Delta t}{(1+r)m}. \quad (10)$$

The noise-induced velocity drift during a period can be estimated using

$$\Delta v_{\text{drift}}^2 \simeq \frac{k_B T_L}{m} [1 - e^{-2\gamma j \Delta t}]. \quad (11)$$

T_L is a measure of the strength of the noise, expressed as an effective temperature. $\Delta v_{\text{drift}}^2$ is finite for $j \Delta t \rightarrow \infty$, since information on past events decays as $v \rightarrow v_{\text{ter}}$.

When the noise is sufficiently weak, the velocity drift is likely to be smaller than Δv . As a result, the Hexbug is expected to remain in the basin of attraction of its current periodic trajectory. The noisy dynamics then consists of infrequent transitions between metastable periodic trajectories. This happens when

$$\frac{F^2 \Delta t^2}{(1+r)^2 m^2} > \frac{k_B T_L}{m} [1 - e^{-2\gamma j \Delta t}]. \quad (12)$$

When $\Delta t \rightarrow 0$, inequality (12) is violated. Active particles that are modeled using the Langevin equation are in this parameter regime. In this regime, the periodic solutions are no longer a good basis for approximations. Nevertheless, due to small momentum losses in Hexbug floor interactions $\dot{I} \simeq F$, which can be shown from a Fokker-Planck equation [47].

Results from numerical simulations of Eqs. (1)–(3) support these considerations. Figure 2 shows the time evolution

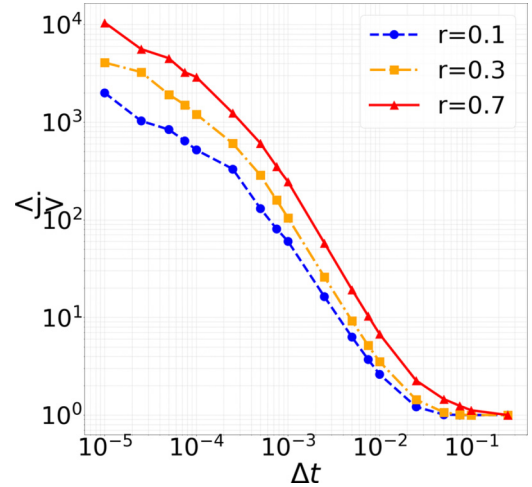


FIG. 3. The average number of Hexbug hops between wall collisions as a function of Δt . Results are for the 1D model of Eqs. (1)–(3).

of the number of hops between wall collisions. Specifically, $j(t)$ is defined as the number of hops between the previous and next wall collisions. The trajectory was simulated using the parameters $m = 0.007$ kg, $F = 0.2$ N, $\Delta t = 0.01$ s, $\gamma = 10$ Hz, $r = 0.7$, and $k_B T_L = 10^{-6}$ J. For these parameters, inequality (12) holds. As expected, the trajectory stays for a while in the vicinity of the same periodic trajectory, before making a transition to another periodic trajectory.

Figure 3 depicts the mean value of j as a function of Δt for several values of r . It shows that as expected $j = 1$ when $\gamma \Delta t \geq 1$. It also shows that $\langle j \rangle$ increases when $\Delta t \rightarrow 0$, but that $\langle j \rangle \Delta t < 1$.

Figure 4 shows the distributions of the momentum transfer rate to the wall for different values of r . These distributions are calculated from values that are the ratio of the momentum transfer in the n th collision and the time between collisions $\tau \equiv \frac{1}{2}(\tau_{n+1} - \tau_{n-1})$. (τ_n denotes the time of the n th collision.) The vertical lines denote the mean value of each distribution.

For $\Delta t = 10^{-2}$ s, the results in the top panel of Fig. 4 show distributions with a mean momentum rate that satisfies $\dot{I} \approx F$, as expected from Eq. (8), with only a slight dependence on r . In contrast, the results depicted in the bottom panel of Fig. 4, obtained for $\Delta t = 1$ s, show distinct distributions whose mean values clearly depend on r . Each distribution is centered around the value given by Eq. (9), but is broadened by the noise. This confirms our expectations for Hexbugs with $e^{-\gamma \Delta t} \ll 1$. The results shown in Fig. 4 therefore demonstrate that the qualitative understanding gained from the periodic solutions holds also for weak noise.

VI. TWO-DIMENSIONAL DYNAMICS

So far, we considered Hexbugs with a fixed orientation. Real Hexbugs are able to rotate. A two-dimensional underdamped generalization of Eqs. (1) and (2) is given by

$$\begin{aligned} \vec{v}_{n+1} &= \vec{v}_n + \vec{F}(\theta_n) - \tilde{\gamma} \vec{v}_n + \vec{\xi}_L, \\ \omega_{n+1} &= \omega_n - \tilde{\gamma}_R \omega_n + \tilde{\xi}_R, \end{aligned}$$

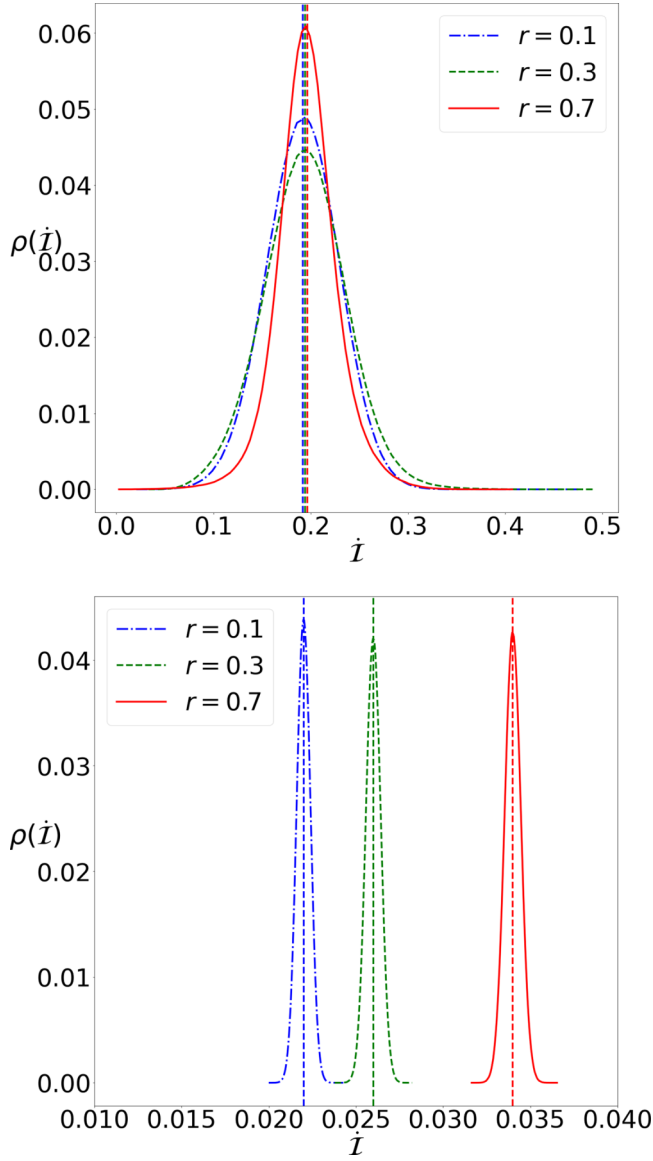


FIG. 4. Distributions of Hexbug momentum transfer rates for $\Delta t = 10^{-2}$ s (top) and $\Delta t = 1$ s (bottom).

$$\begin{aligned}\vec{r}_{n+1} &= \vec{r}_n + \vec{v}_{n+1} \Delta t, \\ \theta_{n+1} &= \theta_n + \omega_{n+1} \Delta t.\end{aligned}\quad (13)$$

Here, $\vec{v} = (v_x, v_y)^T$ and $\vec{r} = (x, y)^T$ are the velocity and coordinates of the Hexbug. The active force is $F(\theta) = F(\cos \theta, \sin \theta)^T$, where θ denotes the Hexbug's direction, with the relation between $|\vec{F}|$ and $|F|$ given by Eq. (4). ω is the rotational velocity, with rotational friction $\tilde{\gamma}_R$ and noise ξ_R , defined similarly to the longitudinal ones in Eq. (4). We note that the longitudinal and rotational parameters typically differ.

A schematic depiction of our two-dimensional Hexbug in a channel is given in Fig. 5. We used $L = 3$ m and periodic boundary conditions along the channel. We assume a disklike bbot, thereby neglecting collision-induced alignment. In this case, wall collisions only affect the perpendicular component of the velocity, $v_x \rightarrow v'_x = -rv_x$, and the angular dynamics is decoupled from other degrees of freedom. The angular

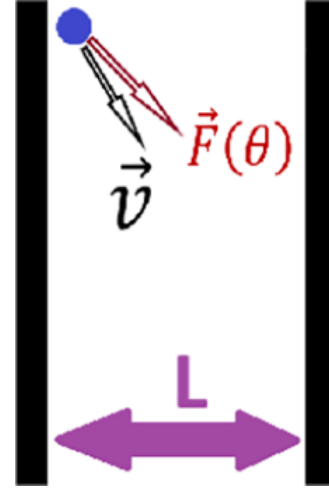


FIG. 5. Schematic depiction of a two-dimensional active particle in a channel of width $L = 3m$. Periodic boundary conditions are assumed along the channel.

dynamics in Eq. (13) is therefore that of a free underdamped Brownian particle. The noise in the angular velocity results in diffusion of the angle, characterized by a diffusion coefficient D_r . Since θ has a period of 2π , the Hexbug forgets its orientation on a timescale of $\tau_r = \frac{\pi^2}{D_r}$. We are interested in highly active Hexbugs, where the rotation time is longer than other timescales, $\tau_r \gg \Delta t, \frac{1}{\gamma}, \frac{L}{v_{\text{ter}}}$. In all our simulations, $\tau_r \approx 13.8$ s. This ensures that the Hexbug spends most of its time near the walls.

Equations (8) and (9) were derived for a one-dimensional Hexbug that always faces toward the same wall. This is no longer the case in two dimensions, and one should account for the probability that the Hexbug is repeatedly colliding with a given wall. This can be done with the help of simple approximations. First, the driving force in Eqs. (8) and (9) is replaced with $F_{\perp} = F \cos \theta$. Then, one accounts for correlations between the Hexbug orientation and position. For slowly rotating Hexbugs, these take a simple form. When $\cos \theta < 0$ ($\cos \theta > 0$), the particle is highly likely to be found near the left (right) wall. The momentum transfer rate to each wall can then be approximated as an average of Eq. (8) or (9) over orientations with $\cos \theta < 0$ or $\cos \theta > 0$. As a result, Eq. (8) is replaced with

$$\dot{i} \simeq \left(1 - \frac{1}{2} \frac{1-r}{1+r} \Delta t \gamma \langle j \rangle\right) F \langle |\cos \theta| \rangle P_w, \quad (14)$$

and Eq. (9) with

$$\dot{i} \simeq \frac{1+r}{\gamma \Delta t} F \langle |\cos \theta| \rangle P_w. \quad (15)$$

P_w is the probability that the Hexbug is near one of the walls.

A naive estimate for $\langle |\cos \theta| \rangle$ will assume a uniform distribution over the orientations that point toward a wall, resulting in $\langle |\cos \theta| \rangle = \frac{1}{\pi} \int_{-\pi/2}^{\pi/2} d\theta \cos \theta = \frac{2}{\pi}$. Similarly, one expects $P_w \simeq \frac{1}{2}$. A more accurate estimate will have to be taken from simulations or experiments.

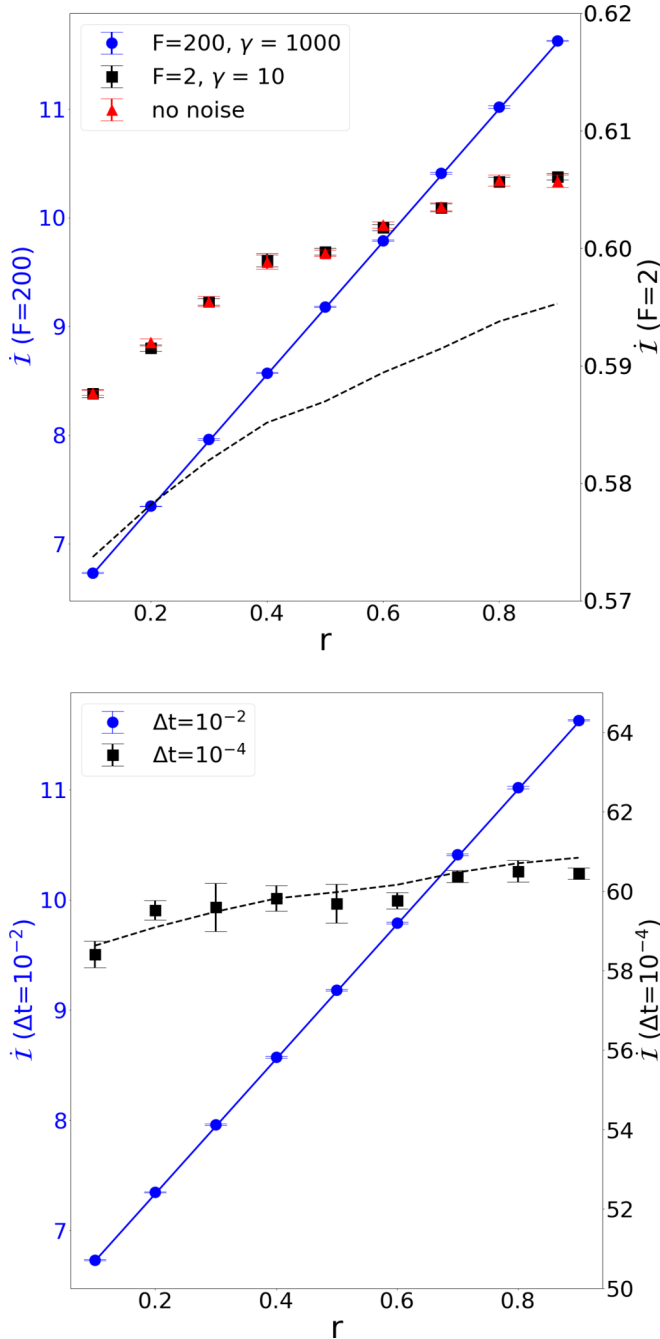


FIG. 6. Two-dimensional simulations of a Hexbug in a channel. Top panel: Momentum transfer rate as a function of r for different active forces. The circles are for a Hexbug in the high-drag regime, while the squares are for low drag. Red triangles are for simulations with low drag and no longitudinal noise. Lines are analytical estimates based on Eqs. (14) and (15); see main text. Bottom panel: Similar to the top panel, but with $F = 200$ N and $\gamma = 1000$ Hz. Different regimes are sampled by changing Δt .

VII. TWO-DIMENSIONAL SIMULATIONS

Figure 6 shows results from simulations based on Eq. (13). The top panel in Fig. 6 compares systems with different F, γ , but with the same v_{ter} . In this panel, $\Delta t = 10^{-2}$ s. The black squares correspond to simulations with $\gamma = 10$ Hz,

and $F = 2$ N, which are in the low-drag regime. The blue circles in both panels show results for $\gamma = 10^3$ Hz, $F = 200$ N, with $e^{-\gamma\Delta t} \ll 1$. The black squares in the bottom panel are obtained, for $\Delta t = 10^{-4}$ s, $\gamma = 10^3$ Hz, and $F = 200$ N. The bottom panel thus examines how Δt causes the transition between the two regimes. Other parameters used were $m = 0.007$ kg, and $k_b T_L = 10^{-6}$ J. Each symbol in the panels was calculated using six simulations with 10^8 hopping events.

We compare the simulation results to the predictions of Eqs. (8) and (9). For the purpose of the comparison, we used the simulation results to estimate P_w and $\langle |\cos \theta| \rangle$. The angular average is done by recording $|\cos \theta|$ at each collision and dividing the sum of these values by the number of collisions. We obtain $\langle \cos \theta \rangle \simeq 0.702$ for the black squares in the top panel and $\langle \cos \theta \rangle \simeq 0.695$ otherwise. These angular averages are somewhat larger than the naive estimate of $2/\pi$, since particles with $|\cos \theta| \ll 1$ only need to rotate a little to escape from the wall.

We also used simulations to obtain a more accurate value of P_w . This probability is essentially the fraction of time that a particle spends near the wall while repeatedly colliding with it. This time was calculated by summing over times between consecutive collisions with $j \leq 10$. This cutoff was introduced to remove time segments where the particle moves between walls or wanders in the bulk of the system. For $r = 0.5$, we found $P_w \simeq 0.440$ for the black squares in the top panel and $P_w \simeq 0.426$ otherwise.

Finally, $\langle j \rangle$ was estimated from one-dimensional simulations with the appropriate parameters, by summing the number of hops between consecutive collisions along a trajectory and dividing by the number of collisions.

The lines in the two panels of Fig. 6 show Eqs. (14) and (15) with the parameters given above. The agreement with the simulation results (symbols) is excellent. There is a systematic deviation in the results for $F = 2$ N and $\gamma = 10$ Hz (black squares, top panel). However, the deviation is small as can be seen from the scale of the relevant y axis. This deviation is attributed to transient dynamics. A Hexbug that reaches the wall will take some time to converge to the periodic solution. During this transient time, its momentum transfer rate will deviate from Eq. (7). Such deviations are neglected in Eqs. (14) and (15), since they are obtained from weighted averages of steady states. A comparison with Langevin dynamics suggests that the relaxation time is roughly $\frac{\ln(1+r)}{\gamma}$. It is therefore not surprising to find deviations in the results with the smallest value of γ . Indeed, we have found an even larger deviation in results for $\gamma = 1$ Hz (not shown).

Longitudinal noise may also be a source of similar deviations, since it tends to increase the collision velocities. To test for this possibility, we ran a set of simulations with $F = 2$ and $\gamma = 10$, but without longitudinal noise. The results are shown in the top panel of Fig. 6 (red triangles). They show the same deviations as the results that included longitudinal noise. This reinforces our conclusion that the deviations are due to transient behavior.

In summary, the results shown in Fig. 6 demonstrate that the two-dimensional Hexbug undergoes the same qualitative transition between low- and high-drag regimes as its one-dimensional counterpart.

VIII. CONCLUSIONS

We have studied how the pressure of an inelastic macroscopic active particle depends on the restitution coefficient. The existence of a finite hopping time allows for a richer than anticipated dependence on parameters. We found a transition from a low-drag regime in which the particle effectively pushes the wall to a regime with r -dependent momentum transfer, $\dot{I} \propto (1+r)F$. This is supported by simple analytical approximations and numerical simulations. Our results suggest that commonly used models, which use potentials for particle-wall interactions and/or infinitesimal time steps, may be inadequate for macroscopic particles with inelastic collisions. In particular, continuous-time Brownian dynamics [43], which is recovered when $\Delta t \rightarrow 0$, is constrained to always be in the low-drag regime. The results presented above then explain why in this regime the force on the wall is r independent despite the r dependence of the momentum transfer in a single collision.

An interesting challenge will be to observe both regimes experimentally. Such an experiment will require the design of a softer bbot with a tunable hopping time Δt . On the theoretical side, it is of interest to extend our results to systems with several or many active particles. This introduces additional aspects. For instance, will inelastic collisions between particles change their tendency to align, and how such collective effects affect the resulting pressure. (See [48] for treatment of elastic collisions between active rods.) Even when such many-particle effects come into play, one still expects that the Hexbugs closest to the wall will collide with it repeatedly, and that the results presented above will be relevant for the description of this part of the dynamics.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

APPENDIX: ADDITIONAL PERIODIC SOLUTIONS AND THEIR PROPERTIES

The simple periodic solutions (6) were instrumental in demonstrating the different limiting behaviors of a Hexbug

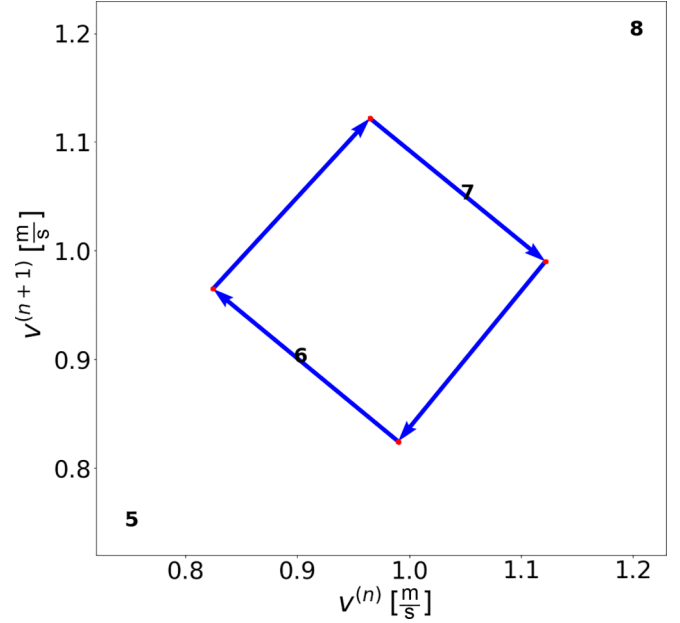


FIG. 7. Depiction of a nontrivial periodic trajectory. The trajectory makes $j_1 = 6$, $j_2 = 6$, $j_3 = 7$, and $j_4 = 7$ hops between wall collisions before repeating itself. The collision velocities are much smaller than the terminal velocity $v_{\text{ter}} \approx 28.57$ m/s.

colliding with a wall. However, the hopping dynamics of Eqs. (1)–(3) allows for more complicated periodic solutions. One such orbit is shown in Fig. 7. Each point in Fig. 7 records a pair of consecutive collision velocities $(v^{(n+1)}, v^{(n)})$. In this representation, a simple periodic trajectory is depicted as a single point, whereas a general periodic trajectory corresponds to repeatedly moving between a finite set of points. The places where the simple different periodic solutions of Eq. (6) would have appeared are denoted by a single number, corresponding to the number of hops between collisions.

The trajectory shown in Fig. 7, which was found in simulations, makes $j_1 = j_2 = 6$ and then $j_3 = j_4 = 7$ hops between collisions with the wall. It repeats itself only after making four collisions with the wall. Its collision velocities are around those of the simple trajectories with $j = 6$ and 7. Its net contribution to the momentum transfer is therefore similar to that of a simple solution with $j = 6.5$.

One can demonstrate that such periodic solutions are qualitatively similar to their simpler counterparts that were discussed in the main text. Consider a Hexbug that reaches the wall, with velocity of $v^{(0)}$, and then makes a series of n wall collisions with $j_1, j_2, \dots, j_n$ hops between them. Its velocity just before the n th collision is

$$v^{(n)} = v_{\text{ter}} \left[1 - e^{-j_n \gamma \Delta t} + \sum_{l=1}^{n-1} (-r)^l (e^{-\gamma \Delta t \sum_{k=n-l+1}^n j_k} - e^{-\gamma \Delta t \sum_{k=n-l}^n j_k}) \right] + (-r)^n e^{-\gamma \Delta t \sum_{k=1}^n j_k} v^{(0)}. \quad (\text{A1})$$

Demanding periodicity, namely, that $v^{(n)} = v^{(0)}$, we find

$$v^{(n)} = \frac{1 - e^{-j_n \gamma \Delta t}}{1 - (-r)^n e^{-\gamma \Delta t \sum_{k=1}^n j_k}} v_{\text{ter}} + \frac{\sum_{l=1}^{n-1} (-r)^l [e^{-\gamma \Delta t \sum_{k=n-l+1}^n j_k} - e^{-\gamma \Delta t \sum_{k=n-l}^n j_k}] v_{\text{ter}}}{1 - (-r)^n e^{-\gamma \Delta t \sum_{k=1}^n j_k}}. \quad (\text{A2})$$

When $e^{-jk\gamma\Delta t} \ll 1$, only the simple solution with $j = 1$ exists. Therefore, complicated solutions of this type may be relevant when $\gamma\Delta t \ll 1$. However, the momentum transfer of such a complex periodic solution in this limit satisfies

$$j = \frac{(1+r)m \sum_k v^{(k)}}{\Delta t \sum_k j_k} \approx \frac{(1+r)\gamma\Delta t m \sum_k j_k}{(1+r)\Delta t \sum_k j_k} v_{\text{ter}} = F. \quad (\text{A3})$$

The existence of these additional periodic solutions does not affect the limiting contribution to the force on the wall that was described in the main text.

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