

## Controlled manipulation of solitons in a recirculating fiber loop using external potentials

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(Received 27 August 2025; accepted 22 October 2025; published 1 December 2025)

Optical solitons are self-sustained wave packets that propagate without distortion due to a balance between dispersion and nonlinearity. Their unique stability underpins key photonic applications while also playing a central role in nonlinear wave physics. However, real-time control over soliton dynamics in nondissipative systems remains a major challenge, limiting their practical applications in photonic systems. Here, we introduce a fiber-based platform for soliton manipulation, by creating programmable external potentials through synchronous arbitrary phase modulation in a recirculating optical fiber loop. We demonstrate precise soliton trapping, parametric excitation, and coupled multisoliton interactions, revealing particle-like behavior in excellent agreement with a Hamiltonian description in which solitons are treated as interacting particles. The strong analogy with matter-wave solitons in Bose-Einstein condensates highlights the broader implications of our approach, which provides a versatile experimental tool for the study of nonlinear wave dynamics and engineered soliton manipulation.

DOI: [10.1103/qcts-whtp](https://doi.org/10.1103/qcts-whtp)

## I. INTRODUCTION

Solitons are localized coherent nonlinear structures involved in the complex dynamical behavior of many nonlinear wave systems [1]. Their ubiquitous nature is reflected in the fact that they occur in very diverse fields, including hydrodynamics, plasmas, quantum gas physics, or nonlinear optics. Their robustness underscores their significance in practical systems such as mode-locked lasers, supercontinuum sources [2], and frequency combs, and they have also been extensively explored for high-speed data transmission [3].

In their simplest—conservative—form, solitons are persisting wave packets that retain their shape and velocity over virtually infinite time owing to the balance between dispersive and nonlinear effects. From a mathematical perspective, these solitons represent exact solutions of the class of integrable partial differential equations that includes the one-dimensional (1D) nonlinear Schrödinger (NLS) equation. This equation that describes, at leading order, the propagation of light fields in optical fibers can be solved using the inverse scattering transform (IST) method [1]. Within this framework, solitons are identified as nonlinear eigenmodes that can be used in practical applications such as fiber telecommunications [4]. A key feature of their dynamics is the ability to interact or *collide* elastically, exhibiting particle-like behavior [1]. Even though the exact dynamics of the optical field during the collision process is complex, the only effect of

the interaction that is visible afterward is a positional shift of the colliding solitons [1,5] that has often been interpreted as resulting from an interaction force [6,7]. In an integrable setting, however, soliton interactions are highly constrained, and external control over their motion is impossible. This implies that the spatiotemporal dynamics of a set of interacting solitons, no matter how complex, is completely determined by the initial wave field and further manipulation during evolution is essentially prohibited. The lack of a general method to dynamically manipulate solitons' trajectories during propagation limits their potential for tunable optical applications as well as more fundamental investigations of complex many soliton interaction.

In contrast, matter-wave solitons in Bose-Einstein condensates (BECs) can be formed and subsequently manipulated using external optical potentials [8,9]. In this context, the degree of freedom introduced by the application of an external potential has proven particularly promising in manipulating the spatiotemporal dynamics of solitons, enabling, for instance, their repeated collision [10] or interaction with barriers that can lead to their controlled splitting [11,12]. Technical constraints in generating stable arbitrary optical traps and physical constraints such as spatial dimensionality or the collapse instability might, however, hinder part of this potential. Achieving an equivalent level of control with optical solitons would significantly benefit the study of nonlinear wave physics and could unlock functionalities for soliton-based applications.

In this work, we present a versatile fiber-optic system for soliton manipulation, based on synchronous phase modulation within a recirculating loop. By applying a configurable spatiotemporal external potential to propagating solitons, we demonstrate soliton trapping, parametric excitation, and controlled multisoliton interactions confirming the particle-like nature of solitons in our experiments. Our approach

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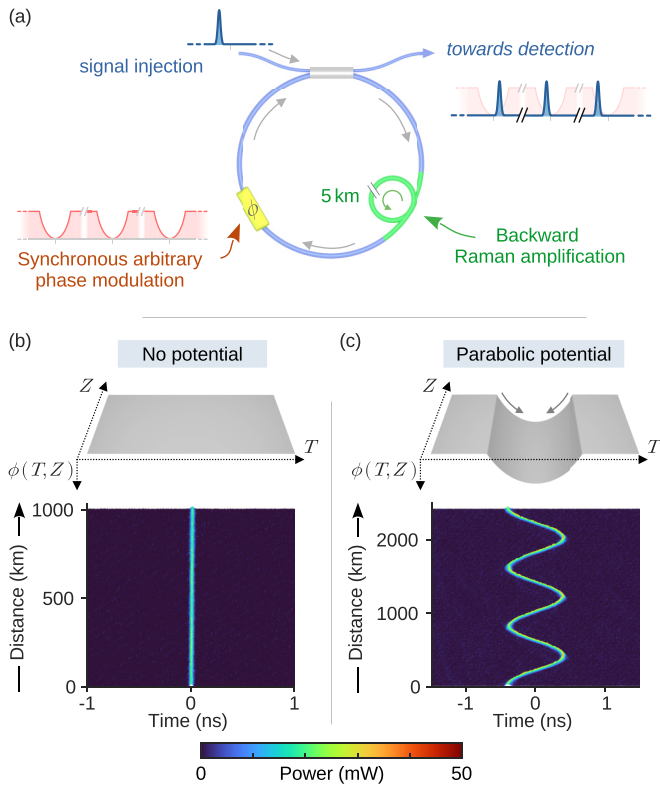


FIG. 1. Experimental setup. (a) Principle of operation of the synchronously phase-modulated recirculating fiber loop. Example of experimentally recorded spatiotemporal dynamics of a single soliton in the case of (b) no external potential and (c) trapping within a truncated parabolic potential [used for describing the setup in panel (a)]. Typical peak power and duration of the solitons are  $\sim 30$  mW and  $\sim 45$  ps FWHM.

establishes an original experimental platform for exploring and engineering nonlinear optical wave dynamics while further strengthening the existing parallel with matter-wave systems.

Our system relies on the recirculating optical fiber loop configuration that gives access to the space-time dynamics of the wave field [13–16] enabling single-shot observation of nonrepetitive dynamics. This has recently been used to observe the nonlinear spatiotemporal dynamics of modulation instability [17–19], soliton collision [20], and soliton gases [21,22]. This approach stands in contrast to most optical methods for accessing space-time dynamics, which typically rely on indirect techniques—such as parameter scanning to simulate different propagation distances [23–27], backscattered signal processing [28], or iterative reconstructions [29]—and are generally limited to deterministic dynamics.

## II. EXPERIMENTAL SETUP AND MODELS

The recirculating loop platform implemented in this work is entirely fiber based and its principle of operation is schematically depicted in Fig. 1(a). A single-frequency, continuous wave laser at 1555 nm is modulated in intensity through a 20 GHz bandwidth electro-optic modulator to generate the initial optical signal, which consists of a custom pattern of

pulses that realizes a specific set of experiments. The electric signal that creates the pulses pattern is obtained using the first channel of a 13.5 GHz bandwidth arbitrary waveform generator (AWG). An acousto-optic modulator is then used to ensure a high extinction of surrounding waves and to prevent the buildup of Brillouin scattering. This field is fed to the recirculating loop through a 90/10 coupler whose ports are arranged such that 90% of the power is recirculating. Consequently, 10% of the circulating field is extracted at each round trip and directed toward the detection system.

The recirculating loop itself measures approximately 5 km (round-trip time  $\approx 24.552$   $\mu$ s) and is mostly comprised of a spool of commercial SMF-28 fiber [group velocity dispersion (GVD) coefficient  $\beta_2 = -22$  ps<sup>2</sup> km<sup>-1</sup> and Kerr nonlinearity coefficient  $\gamma = 1.23$  W<sup>-1</sup> km<sup>-1</sup>] connected at both ends to wavelength division multiplexers (WDMs). Through these, light from a 1455 nm pump laser is coupled in and out of the loop to realize backward Raman amplification. The laser's power is gated to the duration of the experiment and finely tuned to compensate for the total dissipation experienced by the signal during a round trip. In this way, losses are effectively canceled for the signal that can then propagate over extremely long distances, typically several thousands of kilometers. A representative evolution of the optical power during a single experiment is given in the Supplemental Material [30]. We emphasize that in our experiments, even though the optical power is on average almost constant, the amplitude of the optical wave field may undergo significant evolution in the course of one round trip due to the distributed Raman amplification. Upon consecutive circulations, the wave field thus experiences periodic evolution of its power. For the range of parameters used in this work, the typical linear and nonlinear length scales associated with the pulses we consider are  $L_L \approx L_{NL} \approx 30$  km, which is substantially longer than the length of a round trip (see parameters above and in the caption of Fig. 1). In this case, it is known that the nonlinear pulses behave as *effective* solitons also referred to as *guiding-center* solitons [31,32].

We create an effective external potential by using an electro-optic phase modulator (EOM $_{\phi}$ ) embedded in the fiber loop that is driven synchronously with both the intensity pulse pattern and the round-trip time of the loop. This ensures precise and repeatable initial positioning of the pulses relative to the potential and prevents drifting of the latter relative to the circulating field during propagation. Prior to the experiment, a mapping of the targeted potential landscape is designed that consists of a sequence of waveforms (one per round trip). The second channel of the AWG sequentially plays each waveform to generate the electrical signal that drives EOM $_{\phi}$  at each pass of the optical signal. In the general case, a different waveform can be played at each round trip, resulting in a dynamic potential landscape. The AWG can also be set to play the same waveform and thus create a static potential. Note that intraloop phase modulation has already been used to realize experiments in synthetic frequency dimensions [33–37] and generate optical gap solitons [38], typically relying on static harmonic modulations. For the latter, the potential consists of a fast modulation that creates an array of coupled waveguides shorter than the wave packet itself. In contrast, our experiments operate in a different parameter

regime in which the potential timescale is broader than the soliton duration. Application of an external potential is also a powerful means of tailoring the properties of Kerr cavity solitons in resonators [39,40]. In that context, however, phase modulation is constrained by the requirement for the wave-form to match specific cavity resonances, typically limiting it to periodic shapes. These constraints do not apply in our nonresonant loop system, allowing arbitrary and dynamically varying phase profiles. In a different context, it is noteworthy that static periodic potentials have also been proposed as supporting structures to stabilize the propagation of soliton trains for long-haul transmissions, although in that case the potential was generated via cross-phase modulation with an optical clock signal [41–43].

The system uses essentially polarization-maintaining (PM) fiber components except for the 5 km fiber spool and the WDMs. A polarization controller is thus placed after this non-PM section to realign the polarization of the signal after each circulation, which mitigates polarization-dependent effects.

The signal extracted from the loop at the 10% port of the coupler is detected with a 65 GHz bandwidth oscilloscope used in a sequential mode: After each round trip, the oscilloscope is triggered to record the signal in a segment. The full recording consists of up to a few thousand segments and thus contains the dynamics of the signal for more than 10 000 km of propagation. As an illustration, two typical examples of fully processed recordings are shown in Figs. 1(b) and 1(c) depicting, respectively, the spatiotemporal dynamics of a single soliton freely propagating and oscillating within a parabolic potential generated via synchronous phase modulation with a truncated quadratic profile. Additional information concerning the experimental setup and details regarding optical power calibration are given in the Supplemental Material [30].

The evolution of the wave field in the recirculating fiber loop over several round trips is well captured by the following 1D NLS equation, also referred to as 1D Gross-Pitaevskii equation in the Bose-Einstein condensate community:

$$i\frac{\partial A}{\partial Z} = \frac{\beta_2}{2}\frac{\partial^2 A}{\partial T^2} - \gamma|A|^2A - i\frac{\alpha_{\text{eff}}}{2}A - \frac{\phi(T, Z)}{L}A, \quad (1)$$

where  $A(T, Z)$  is the envelope of the electromagnetic field,  $Z$  is the propagation distance, and  $T$  is the time defined in the reference frame traveling at the group velocity at the carrier frequency.  $\beta_2$  and  $\gamma$  are the GVD and the Kerr nonlinearity coefficients, respectively.  $\alpha_{\text{eff}}$  is a linear dissipation term that describes the potentially imperfect loss compensation that results in an effective exponential power variation over many round trips.  $L$  is the total length of the recirculating loop.  $\phi(T, Z)$  is the phase modulation applied at distance  $Z = nL$ , where  $n$  is the round-trip number. As explicitly shown in the Supplemental Material [30], Eq. (1) derives from an iterative model accounting for the nonlinear light propagation in the loop and the periodic application of discrete recirculating conditions (local losses and phase modulation). It is valid provided that the round trip-to-round trip evolution of the field envelope  $A_n$  and the phase modulation  $\phi_n$  are small. Consequently, the subscript  $n$  can be dropped, and the term  $\phi(T, Z)/L$  plays the role of an external potential that results from the longitudinal distribution of the phase modulation  $\phi(T, Z)$ . It is noteworthy in this model that, since  $\phi$  can

now be seen as a continuous function of  $T$  and  $Z$ , the phase modulation applied in a discrete fashion (once per round trip) generates a distributed potential landscape. We checked numerically that in the range of parameters of all the experiments reported in this work the mean-field and iterative models yield identical results. The signal measured in our experiments is proportional to the optical power sampled at each round trip, which corresponds to  $|A(T, Z = nL)|^2$ .

It is possible to write Eq. (1) in a normalized form that, considering the anomalous dispersion regime (i.e.,  $\beta_2 < 0$ ), reads

$$i\frac{\partial \psi}{\partial t} = -\frac{1}{2}\frac{\partial^2 \psi}{\partial x^2} - |\psi|^2\psi - i\epsilon\psi + V(x, t)\psi, \quad (2)$$

by introducing the following adimensional variables:  $\psi = A/\sqrt{P_0}$ ;  $x = T\sqrt{\gamma P_0/|\beta_2|}$ ;  $t = Z\gamma P_0$ ;  $\epsilon = \alpha_{\text{eff}}/(2\gamma P_0)$ ; and  $V = -\phi/(\gamma LP_0)$ . Note that within this notation, the physical propagation distance  $Z$  (resp. physical time  $T$ ) is replaced by a normalized time  $t$  (resp. normalized position  $x$ ).

Considering that individual NLS solitons behave like particles, maintaining their shape at all positions in space and time, a simplified Hamiltonian description has been proposed in Refs. [44–46]. In this model, the interaction among solitons is described by an intersoliton potential that has been introduced in order to match the shift experienced by solitons upon pairwise collisions. Considering an initial wave field consisting of  $N$  well-separated single solitons, the initial condition reads

$$\psi(x, t = 0) = \sum_{i=1}^N 2\eta_i \text{sech}[2\eta_i(x - q_{0i})] e^{i(v_{0i}x + \phi_{0i})}. \quad (3)$$

Each soliton is characterized by a set of four parameters:  $\eta_i$ ,  $q_{0i}$ ,  $v_{0i}$ , and  $\phi_{0i}$ , encoding its amplitude, initial position, velocity, and phase, respectively. In the particle analogy, the Hamiltonian  $H$  for such system reads [44–46]

$$H = \sum_{i=1}^N \frac{p_i^2}{2\eta_i} + \eta_i V(q_i) - \sum_{1 \leq i < j \leq N} 2\eta_i \eta_j (\eta_i + \eta_j) \text{sech}^2 \left[ \frac{2\eta_i \eta_j}{\eta_i + \eta_j} (q_i - q_j) \right], \quad (4)$$

where  $q_i(t)$  denotes the position of the peak of soliton  $i$  and  $p_i(t) = \eta_i \dot{q}_i$  its momentum. The first two terms represent the kinetic and potential energies, while the last term corresponds to the soliton-soliton interaction potential. This expression is valid for arbitrary potential shapes provided that  $V(q)$  varies slowly compared to the soliton width, ensuring that each soliton experiences the potential through its local value  $V(q_i)$ . Note that the phase of the solitons does not appear in Eq. (4), since it does not affect the positional shift induced by the collisions. The relative phase between solitons, however, influences the exact spatiotemporal dynamics observed during collisions [20]. The simultaneous interaction of more than two solitons is not accounted for by this Hamiltonian, limiting its accuracy only to scenarios involving pairwise collisions occurring at distinct positions in space and time. Note also that this model fails to reproduce the correct shift when solitons interact with vanishing relative velocities [44]. In this work, we

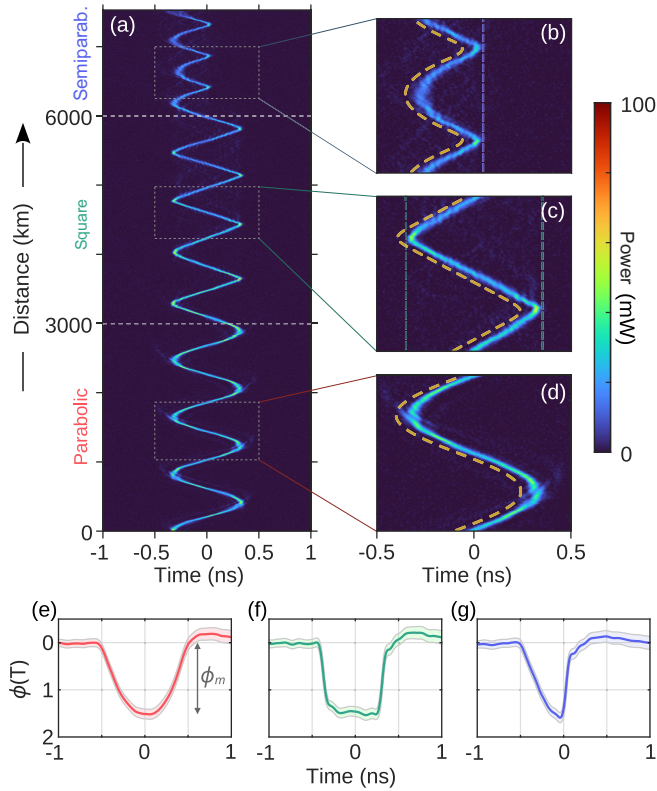


FIG. 2. Dynamics of a single soliton in a trapping potential taking consecutively three different shapes. (a) Space-time dynamics of a single soliton subjected to the potential trap. Close-ups highlighting (d) sinusoidal oscillations, (c) total temporal reflections, and (b) succession of temporal reflections and parabolic trajectories. Dashed lines in panels (b)–(d) are the trajectories estimated by the particle model shifted by  $-80$  ps for clarity. (e)–(g) Shapes of the phase modulations applied in each segment obtained from heterodyne measurement (see the Supplemental Material [30] for details regarding the measurement method). Solid lines are the averaged profiles and colored areas show the standard deviation of the phase measurement.

have solved the Hamilton's equation of motion  $\dot{q}_i = \partial H / \partial p_i$  and  $\dot{p}_i = -\partial H / \partial q_i$  to find the trajectories of solitons  $q_i(t)$ , within the particle analogy, subjected to external potentials  $V(x, t)$  and compared them to our experimental observations.

### III. RESULTS

#### A. Single-soliton dynamics

The first set of experiments that we present focuses on the spatiotemporal dynamics of a single soliton under a trapping potential that sequentially takes three distinct shapes: a quadratic well, a square box, and an asymmetric well. In the spatiotemporal diagram of Fig. 2(a), the horizontal axis is the time in the reference frame that moves at the group velocity of the signal in the absence of external potential. The external potential is set to be stationary in this reference frame. The zero of this axis is the central position of the external potential and the soliton is initially shifted by  $-300$  ps from the center [see Figs. 2(a) and 2(e)]. In the first 3000 km of propagation, a truncated parabolic phase modulation spanning 1 ns is synchronously applied, which creates an effective quadratic

potential. As a result, the soliton's velocity is modulated and its temporal position oscillates sinusoidally [Fig. 2(d)] similar to what is observed in BEC experiments [10,12]. It should be noted that this oscillation of the velocity physically translates into an oscillation of the central frequency of the Fourier spectrum of the soliton. Little distortion of the soliton's shape is observed as a consequence of the interaction with weak radiative waves that are also trapped, but this was not found to alter its trajectory in a significant way. Considering a quadratic phase modulation of the form  $\phi(T)/L = -C/2 \times T^2$ , the trap wave number is determined, by analogy with the quantum harmonic oscillator, to be  $K_0^2 = \beta_2 C$ . When applying a truncated quadratic phase modulation parametrized by its depth in radian  $\phi_m$  and duration (width)  $W$ , as schematically shown in Fig. 3(d), the spatial period of oscillation  $Z_0$  can be expressed as

$$Z_0 = \frac{2\pi}{K_0} = \sqrt{\frac{\pi^2 W^2 L}{2|\beta_2| \phi_m}}. \quad (5)$$

The experimentally observed period  $Z_0 = 835$  km is consistent with the phase modulation depth obtained via heterodyne measurement  $\phi_m \approx 1.5 \pm 0.1$  rad [Fig. 2(e)]. After 3000 km of propagation, the phase modulation is abruptly switched (from one round trip to the next) to produce a square-well potential. As a result, the spatiotemporal dynamics of the soliton is modified: It experiences total reflections at the edges while crossing the flat bottom of the potential at a constant velocity without changing shape [Fig. 2(c)]. After each reflection, the soliton recovers precisely its shape. The observed dynamics is characteristic of the temporal reflection phenomenon that is known to be the analog of reflection of an optical beam at the interface between two dielectric media [47,48]. At a propagation distance of 6000 km, the phase modulation is switched to generate an asymmetric, semiparabolic well in which the soliton experiences temporal reflection at the sharp right boundary and follows a curved trajectory in the region of quadratic potential [Fig. 2(b)].

The results illustrated in Fig. 2 demonstrate the capacity of our experimental setup to manipulate individual 1D solitons through the application of an arbitrary shaped and reconfigurable external potential. The trajectory of the soliton in response to each potential shape is perfectly reproduced within the particle analogy by integrating the simplified Hamiltonian model [Eq. (4)] for a single particle subjected to corresponding potentials. This is illustrated in Figs. 2(b)–2(d), where the dashed lines (shifted for clarity) represent the trajectories numerically computed and rescaled to physical units.

The recirculating loop configuration allows the phase modulation signal applied to the circulating field to be updated at each round trip. This enables the creation of dynamic potential landscapes (external potentials varying in both  $T$  and  $Z$ ), providing an additional tool for controlling optical waves. As a demonstration, we consider a parabolic trap with  $Z$ -dependent strength, with results presented in Fig. 3. In Figs. 3(a) and 3(b), the parabolic trap strength is longitudinally modulated by varying its temporal width  $W$  while keeping its depth  $\phi_m$  constant [schematized in Fig. 3(d)]. In both cases, the evolution of  $W$  follows a continuous function sampled once per round trip. However, the variation of  $W$

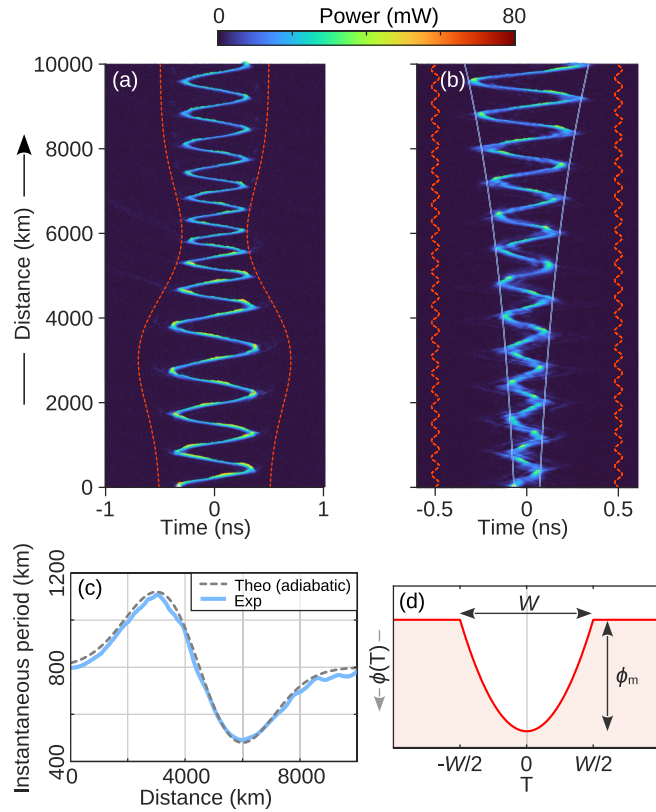


FIG. 3. Manipulation of solitons using spatiotemporally varying harmonic potentials. (a) Soliton in a slowly modulated trap and (b) driven at the parametric resonance. The red dashed lines denote the boundaries of the truncated parabolic traps. Outside of the region enclosed by these lines, the external potential is flat. The blue lines in panel (b) indicate the exponential growth of the amplitude of the oscillations predicted by the particle model. (c) Evolution of the instantaneous period of oscillation of the soliton in panel (a) compared to the prediction of Eq. (5) considering an adiabatic evolution. (d) Sketch of the truncated parabolic phase modulation.

from round trip to round trip is small, ensuring that the wave field experiences a smoothly evolving potential landscape without discernible effects of the discrete nature of the phase modulation. When the trap is slowly modulated [Fig. 3(a)], the soliton's oscillation period and amplitude increase as  $W$  increases, and vice versa. Under adiabatic evolution, Eq. (5) indeed consistently predicts that the soliton oscillation period is proportional to  $W$ . The experimentally measured evolution of the period aligns well with this prediction for adiabatic evolution [Fig. 3(c)], showcasing precise control over the spatiotemporal dynamics of trapped solitons. This concept extends to scenarios involving multiple interacting solitons. Figure 3(b) illustrates another example of soliton control, achieved by periodically modulating the spatial frequency of the harmonic potential at parametric resonance. It has been shown theoretically that the motion of a soliton under such conditions follows the equation of motion of the parametrically driven pendulum [49,50]: A modulation period close to half the oscillator's natural period induces exponential growth in oscillation amplitude. This is realized in our case using a truncated parabolic trap weakly modulated according

to  $W(Z) = W_0(1 - 0.05 \sin(4\pi Z/Z_0))$  with  $W_0 = 1$  ns and  $Z_0 = 800$  km. The oscillations of the soliton initially near the trap's center ( $-75$  ps) exhibit rapid amplitude growth due to parametric excitation. Our experimental results strongly agree with the particle model, as shown by the amplitude of the oscillations closely following the predicted exponential amplification [blue lines in Fig. 3(b)]. It should be noted that we focus here on the positional dynamics of the pulse, but the presence of the trapping potential eventually introduces internal dynamics in the form of width oscillations, which may lead to more complex behavior [51].

The superposition of the predicted trajectories over the experimental observations reported in Figs. 3(a) and 3(b) is provided in the Supplemental Material [30], showing an excellent agreement.

### B. Interacting solitons

The experiments presented thus far all involve a single soliton subjected to an effective external potential whose primary effect is to modulate the velocity of the soliton. We now consider the scenario in which several solitons are confined in a stationary quadratic trapping potential that makes it possible to produce intricate multicollision dynamics. Such scenario is usually precluded in nonlinear fiber optics experiments as solitons with different velocities can only interact once in the absence of potential. We focus on a regime in which the solitons are well separated and distinguishable before and after interaction (i.e., sufficiently broad trapping potential and large initial separation between solitons). Under these conditions, the solitons dynamics can be modeled to a good approximation using the Hamiltonian particle model [Eq. (4)].

We first present experiments realized in the two-soliton configuration for which the particle model is integrable, which has been shown to imply regular (nonchaotic) dynamics of the system [45]. To better emphasize the dynamics of the soliton pair, we designed initial conditions consisting of a soliton placed at the center of the harmonic trap, while a second one is generated shifted from the center [situation depicted in Fig. 4(a)]. In the absence of interaction, the centered soliton would remain pinned to the center of the trap. In contrast, the coherent interaction with the second soliton results in the soliton pair exhibiting coupled dynamics. We varied the strength of the coupling by changing the natural period of the trap (phase modulation depth) and the characteristics of the solitons (amplitude and initial temporal separation) as illustrated schematically in Fig. 4(a).

We show in Figs. 4(b)–4(d) three experiments sorted from weakest to strongest coupling. Over the range of propagation distances plotted, the two solitons collide in each realization at least 20 times without significant alteration of their initial amplitude, duration, and shape. The dashed colored lines serve as visual guides for tracking the dynamics of each soliton. For the weakest coupling [Fig. 4(b)], the central soliton is gradually set into motion and the amplitude of the oscillations of its trajectory increases at the expense of that of the initially shifted soliton. Further increasing the coupling strength [Fig. 4(c)], we observe after  $\sim 9000$  km a point at which the amplitude of oscillation of the shifted soliton is canceled, whereas the initially centered one oscillates strongly.

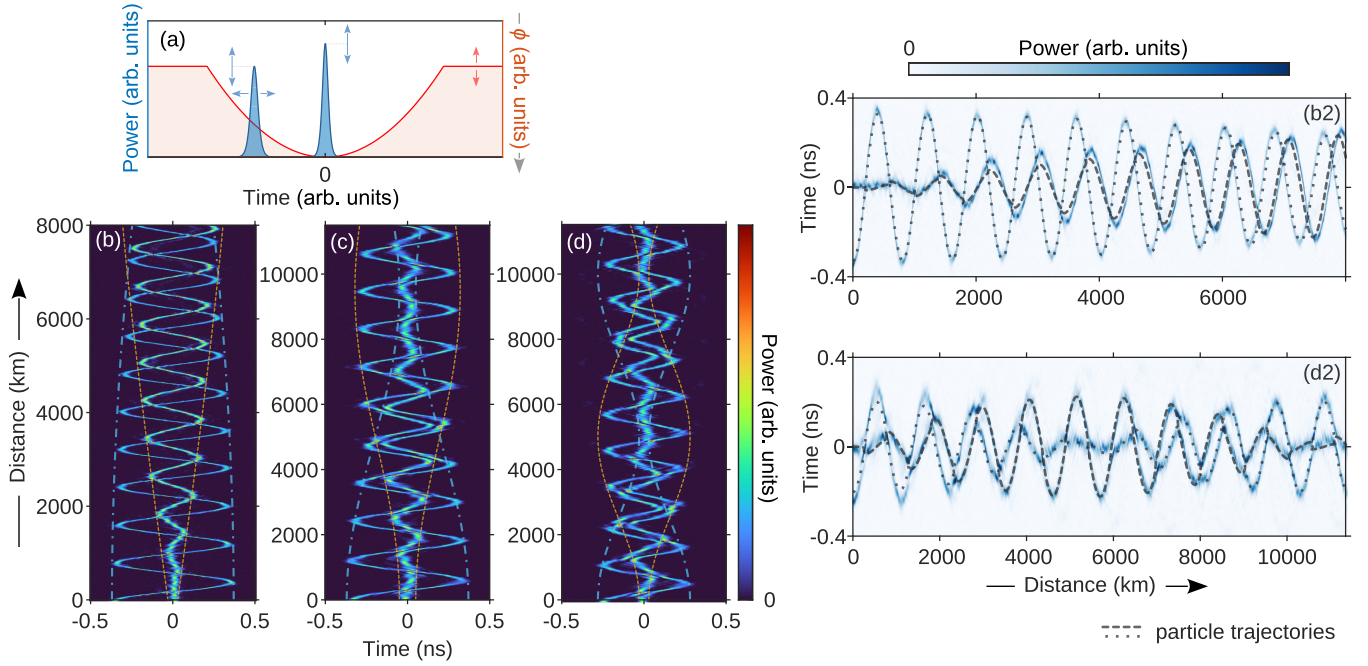


FIG. 4. Coherent dynamics of soliton pairs. (a) Schematic view of the initial condition of the experiments: One soliton is initially centered in the quadratic trap, while the other is detuned. (b)–(d) Spatiotemporal dynamics recorded for three increasing coupling strengths:  $\Delta Z/Z_0 = -1.6\%$ ,  $-6.9\%$ , and  $-11.2\%$ , respectively. Dashed blue and orange lines denote the amplitude of oscillation of each soliton. Panels (b2) and (d2) are same as panels (b) and (d) rotated by  $90^\circ$ . Simulated particle trajectories obtained from Eq. (4) are superimposed as dashed and dotted lines.

Figure 4(d) shows the most strongly coupled dynamics where the solitons apparently exchange their initial position twice, reaching back almost perfectly the initial configuration after 10 000 km. Although the trajectory of each soliton is less distinguishable because of their smaller separation, their coupled motion is still remarkably regular.

A convenient way to quantify the coupling strength that has been reported in Ref. [10] is to measure the relative shift of the period of oscillation of the solitons with respect to the natural period  $Z_0$  of the trap (i.e., the period of oscillation experienced by a single soliton within the same trap)  $\Delta Z_0/Z_0$ . The collision process shifts solitons forward in their propagation direction such that the coupling between solitons effectively decreases their period of oscillation [52]. Using reference single-soliton experiments performed simultaneously to those reported in Figs. 4(b)–4(d) but not shown for clarity, we were able to estimate these shifts as  $\Delta Z_0/Z_0 = -1.6\%$ ,  $-6.9\%$ , and  $-11.2\%$ , respectively. The latter is roughly 4 times larger than the matter-wave soliton experiments reported in Ref. [10].

Our experiments can be compared to the predictions of the Hamiltonian model that treats solitons as interacting classical particles. By integrating Hamiltonian dynamical equations with the Hamiltonian given by Eq. (4), we obtain the positional dynamics  $q_1(t)$  and  $q_2(t)$  of the two equivalent particles associated with the solitons of amplitude  $\eta_1$  and  $\eta_2$ , respectively. The computed trajectories, rescaled to physical units, corresponding to the cases illustrated in Figs. 4(b) and 4(d) are superimposed on the experiments in Figs. 4(b2) and 4(d2) and reproduce quantitatively very well the coupled dynamics of the soliton pairs. For better visualization, the spatiotemporal representations are rotated by  $90^\circ$  and the

colormap has been changed. It should be noted that the initial temporal detuning between the solitons in the particle model was lowered slightly relative to the experiments to account for a transient initial regime during which the external potential stabilizes.

Finally, we realized the same type of experiment with three solitons interacting in a harmonic trap. In practice, the number of solitons and their initial temporal position are easily set by tailoring the electric signal sent to the intensity EOM. The three-particle system is not integrable and supports both chaotic and regular regimes [45,46]. We demonstrate here [see Fig. 5(a)] that we are able to specifically generate soliton triplets with a regular dynamics by carefully shaping the initial state, i.e., the position and relative velocity of each soliton. To do so, three equally spaced solitons are first generated and propagate freely for 500 km before a specifically shaped phase modulation is applied for a few consecutive round trips. The profile of the phase modulation [dashed green line in Fig. 5(b)] is specifically designed such that the central soliton acquires a positive velocity, while the two surrounding ones get a velocity of opposite sign and half amplitude. This, in turn, produces a triplet of solitons characterized by regular pairwise collisions when the trapping harmonic potential is turned on.

Figure 5(c) shows an example of chaotic motion produced following a similar procedure. The central soliton is here slightly detuned and no relative velocity is imprinted before the trap is turned on. As a result, the coupled dynamics becomes very intricate, with in particular the three solitons overlapping at their first encounter. In this configuration especially, the particle model is not able to describe properly

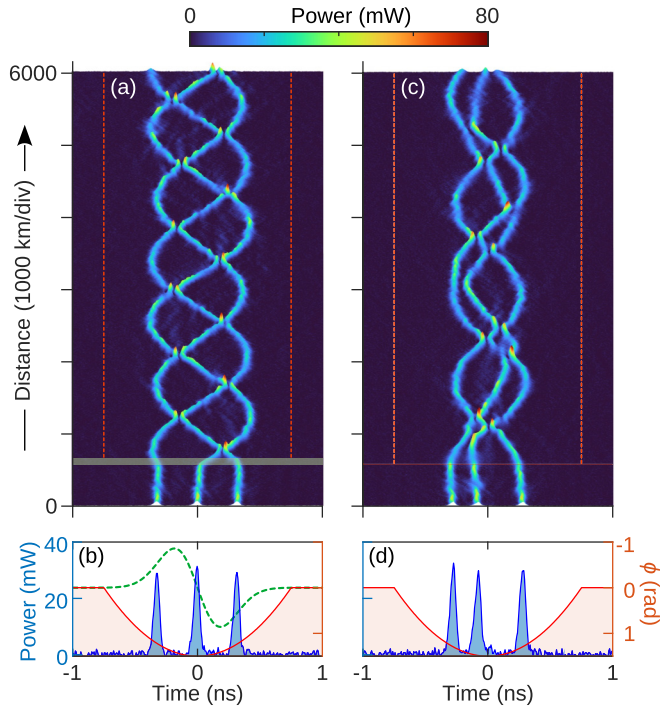


FIG. 5. Regular and chaotic dynamics of soliton triplets. Spatiotemporal dynamics of (a) a regular and (c) a chaotic soliton triplet. The green horizontal line in panel (a) shows the range of distance over which velocities of the solitons are initiated. Dashed vertical lines indicate the boundaries of the truncated parabolic trap. (b), (d) Respective intensity profiles recorded at  $z = 500$  km in blue. Solid red lines and shaded areas illustrate the trapping potential applied after 500 km, while the dashed green line in panel (b) indicates the phase modulation applied for  $500 \text{ km} \leq z \leq 585 \text{ km}$  to initiate velocities of the solitons.

the observed dynamics and exhibits an increased sensitivity to the initial setting (see the Supplemental Material [30] for a related discussion). These experiments provide a clear demonstration of the capabilities offered by this experimental system in designing arbitrary multisoliton optical wave packets. In particular, the results presented in Fig. 5 show how our system can be a powerful experimental platform for exploration of chaotic dynamics in wave mechanics.

#### IV. CONCLUSION

In summary, we have experimentally realized the spatiotemporal manipulation of NLS solitons within a recirculating optical fiber loop. The implementation of intraloop synchronous phase modulation enables the generation of an effective external potential acting on the trajectories of solitons. We take advantage of the capacity to arbitrary shape the potential landscape in both space and time to demon-

strate various control scenarios, including periodic oscillation, temporal reflection, and parametric excitation. In all cases, the solitons showcase particle-like behaviors with trajectories akin to that of classical particles. In the situation where two solitons collide repeatedly within a trapping potential, the collective motion of the system is also accurately described by a simplified Hamiltonian model treating solitons as interacting particles. Precise design of the initial wave field and external potential reveals the predicted onset of chaotic motion when more than two solitons are interacting.

The photonic platform presented here benefits from a high degree of configurability in addition to truly single-shot operation, which makes it readily compatible for more advanced manipulation of optical wave fields. In particular, it constitutes a promising approach toward shaping of the IST “nonlinear” spectrum of complex multisoliton waveforms with possible implications for nonlinear “eigenvalue” telecommunications [4,53].

The weak breaking of integrability provided by an external potential offers a possible route toward the observation of thermalization of soliton gases, i.e., large random ensembles of solitons [54]. In this sense, our system proves highly valuable for testing and refining the predictions of the recent theoretical framework of generalized hydrodynamics (hydrodynamics of many-body integrable systems), which, to date, has been almost exclusively focused on quantum systems ([55] and references therein, [56]). Localization and branched flow phenomena are also expected to occur within periodic [57] or random [58,59] potential landscapes, respectively. The study of the scattering of nonlinear waves from potential barriers is one fundamental topic that could be addressed and that can greatly benefit from the strong analogy between nonlinear optics and matter waves. Although there is a wealth of literature on this topic, practical implementations in matter-wave contexts are limited [12,60–63]. Many of these may be more easily realized using photonic platforms.

#### ACKNOWLEDGMENTS

The authors are grateful to H. Damart, G. Dekyndt, and the Centre d’Etudes et de Recherche Lasers et Application (CERLA) for technical support. This work was partially supported by the Agence Nationale de la Recherche through the StormWave (ANR-21-CE30-0009) and SO-GOOD (ANR-21-CE30-0061) projects, the LABEX CEMPI project (ANR-11-LABX-0007), the Ministry of Higher Education and Research, Hauts de France council, and European Regional Development Fund (ERDF) through the Contrat de Projets Etat-Région (CPER Photonics for Society P4S).

#### DATA AVAILABILITY

The data that support the findings of this article are openly available [64].

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