

Hardness of measuring magic resource

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Quantum computers promise to solve computational problems significantly faster than classical computers. These “speedups” are achieved by utilizing a resource known as magic resource. Measuring the amount of magic resource used by a device allows us to quantify its potential computational power. Without this property, quantum computers are no faster than classical computers. Whether magic resource can be accurately measured on large-scale quantum computers has remained an open problem. To address this question, we introduce *Pauli instability* as a measure of magic resource and experimentally measure it on the IBM Eagle quantum processor. We show that sufficiently small quantities of magic resource can be measured efficiently. We also prove that measuring large (i.e., extensive) quantities of magic resource with our measure is intractable. Our results lead us to surmise that magic resource may be difficult to measure when a quantum computer provides a speedup. We find that one of the most sought-after properties in quantum computation is only partially accessible to experiments. We support our conclusions with both theoretical and experimental evidence. Our work illustrates the capabilities and limitations encountered when studying the power of quantum computers.

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I. INTRODUCTION

Quantum computers [1–3] are powerful devices with the potential to solve problems in fields as diverse as biology [4,5], chemistry [6–8], physics [1], cryptography [9], machine learning [10], and finance [11]. These devices promise to perform computations significantly faster than classical computers [12], a feat referred to as quantum advantage [9,13–18]. In recent years, claims of significant quantum advantages have been made for sampling problems [19,20].

Quantum computers can only attain such computational speedups by utilizing a property known as magic resource [21,22]. Without this property, quantum computers can perform computations no faster than supercomputers [12]. Informally, the amount of magic resource used by a quantum device quantifies its potential to solve computational problems quickly. Measuring this property is therefore important in assessing the capabilities of real-world quantum computers.

However, previous investigations have suggested that such a measurement may be difficult [23–25]. In this work, we study whether magic resource can be measured experimentally on large-scale quantum computers.

Magic resource is measured by using functions known as *magic monotones*. Examples of well-known monotones include the robustness of magic [22], stabilizer rank [26], mana, and the relative entropy of magic [21], among others [27,28]. These functions have played a central role in finding applications of magic resource in quantum computation. They have been used to prove bounds on the time needed to classically simulate a computation [22,26,27,29–33], which is useful to identify quantum advantages. Monotones have also been used to bound the cost of generating so-called magic states [34–37]; these states are important in realizing an essential feat known as fault-tolerant, universal quantum computation [34,38–40]. Furthermore, monotones have been used to link magic resource to interdisciplinary topics, such as quantum circuit complexity and statistical complexity [41,42].

Our work investigates the hardness of measuring magic monotones in experiments. Monotones are thought to be difficult to measure, as they are typically defined as sums or optimizations over exponentially many variables. This measurement problem has received increasing attention in recent years [23,43–45]. In 2021, Google conducted an experiment detecting signatures of magic resource on their Sycamore quantum processor [43]. A magic monotone was later introduced to explain this measurement [46]. In 2022, Google’s result was followed by a measurement of a magic monotone, known as the stabilizer Rényi entropy, on IBM’s quantum processor [23]. This experiment required a large number of

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physical measurements (exponentially large in the processor size), making it intractable for large systems. An algorithm was later developed to measure this monotone in various regimes [47].

In 2023, a measurement of a magic monotone known as the additive Bell magic was made on IonQ's quantum computer [44] and was speculated to be tractable at the large scale. This was followed by a measurement of the additive Bell magic on a logical quantum processor [48]. These findings motivate the principal question in our work: Can one always measure magic monotones on large-scale quantum computers?

To answer this question, we introduce a magic monotone, which we call *Pauli instability*, and measure it on the IBM Eagle quantum processor. We use Pauli instability to show that larger quantities of magic resource are more difficult to measure (Theorem 1). As the magic resource increases, the number of physical measurements needed grows exponentially. We show that sufficiently small quantities of magic resource can be efficiently measured on large-scale quantum computers. We also find that, when magic resource is extensive, measuring Pauli instability is intractable. We conjecture that measuring an extensive amount of magic resource in typical states or unitaries is intractable for any reliable magic monotone. Our results lead us to posit that magic resource may be difficult to measure when a quantum computer provides a quantum advantage. Furthermore, our results showcase the role that chaos plays in measuring magic resource. Our techniques are compatible with quantum platforms with single-qubit readout [19,49,50].

II. PRELIMINARIES

We introduce the mathematical definition of magic resource. We define the set of n -qubit Pauli strings as $Q^{\otimes n} = \{\otimes_{i=1}^n P^{(i)} : P^{(i)} \in \{I, X, Y, Z\}\}$, where n is the system size, I is the identity, and X, Y , and Z are Pauli operators. A Clifford unitary, U , is defined as a unitary that maps any Pauli string P to another Pauli string P' (up to a phase ϕ) under conjugation: $U^\dagger P U = e^{-i\phi} P'$. All Clifford unitaries can be generated by the gate set $\{H, S, \text{CNOT}\}$, where $H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$, $S = \text{diag}(1, i)$, and CNOT denotes the controlled not gate [51].

Unitaries that are non-Clifford are defined to contain magic resource. Intuitively, magic resource quantifies the distance between a given unitary and the set of Clifford unitaries. The more magic resource a unitary contains, the more resourceful it is thought to be in completing computational tasks [34]. The best-known example of a non-Clifford gate is the T gate, defined as $T = \text{diag}(1, e^{i\pi/4})$.

III. MAIN RESULTS

We introduce a monotone, which we call Pauli instability, to measure the magic resource of a unitary.

Definition 1. The Pauli instability of a unitary U is

$$\mathbb{I}(U) = -\ln \left[\mathbb{E}_{P_1, P_2 \in Q^{\otimes n}} |\text{OTOC}(U, P_1, P_2)| \right], \quad (1)$$

where $\text{OTOC}(U, P_1, P_2) = \frac{1}{2^n} \text{Tr}\{U^\dagger P_1 U P_2 U^\dagger P_1 U P_2\}$ and $\mathbb{E}$ denotes the uniform expectation over $Q^{\otimes n}$.

Here, $\text{OTOC}(U, P_1, P_2)$ is the out-of-time-ordered correlator. Pauli instability satisfies the following properties:

- (1) (Faithfulness) $\mathbb{I}(U) \geq 0$ for all unitaries U and $\mathbb{I}(U) = 0$ iff U is a Clifford unitary.
- (2) (Invariance) $\mathbb{I}(V_1 U V_2) = \mathbb{I}(U)$, for any Clifford unitaries V_1 and V_2 .
- (3) (Additivity) $\mathbb{I}(U_1 \otimes U_2) = \mathbb{I}(U_1) + \mathbb{I}(U_2)$.
- (4) (Scaling with T gates) $\mathbb{I}(T^{\otimes k} \otimes I^{\otimes n-k}) = k \ln(4/3)$.

Faithfulness guarantees that Clifford unitaries have no magic resource, while non-Clifford unitaries have positive magic resource. Invariance guarantees that appending a Clifford unitary to a circuit cannot increase the measure of magic resource. These two properties make Pauli instability a resource monotone. Furthermore, additivity implies that for any Clifford unitary, V , one has the following invariance relation: $\mathbb{I}(U \otimes V) = \mathbb{I}(U)$. Scaling with T gates makes Pauli instability a reliable monotone, as the number of T gates in a unitary often gives an indication of its classical simulation cost [22]. This property holds regardless of the position of the T gates [52].

Pauli instability can also be interpreted as a measure of chaos, since the OTOC is traditionally used to measure the onset of chaos in chaotic quantum systems [53]. The OTOC notably characterizes a feature of chaos known as scrambling, which describes the delocalization of quantum information. For example, a completely scrambling unitary U can map a Pauli string P to a superposition of many Pauli strings: $U^\dagger P U = \sum_i c_i P_i$; this is interpreted as “delocalization” in Pauli space. This is a property of non-Clifford unitaries (this can be seen from the definition of Clifford unitaries). This property can result in $|\text{OTOC}(U, P_1, P_2)|$ taking on values near 0, leading to a positive value of Pauli instability. By contrast, when U is Clifford, $|\text{OTOC}(U, P_1, P_2)| = 1$ and Pauli instability is 0. In this way, the monotone exploits the chaotic properties of unitaries to measure their magic resource. Furthermore, we name the monotone Pauli instability because it quantifies the extent to which a unitary evolves a Pauli string away from itself.

We now introduce an approach to approximate Pauli instability. Due to the average over $Q^{\otimes n}$ in Eq. (1), an exact measurement of Pauli instability necessitates computing 16^n terms. Such a measurement is not feasible for large-scale quantum systems. However, one can efficiently approximate Pauli instability by uniformly sampling N pairs of Pauli strings $\{(P_1^{(i)}, P_2^{(i)})\}_{i=1}^N$ from $Q^{\otimes n}$, where each string in the pair is sampled independently. We refer to N as the *Pauli sample complexity*. By computing the OTOC for each pair of strings, one can construct an approximator of Pauli instability:

$$\mathbb{I}_N(U) = -\ln \left[\frac{1}{N} \sum_{i=1}^N |\text{OTOC}(U, P_1^{(i)}, P_2^{(i)})| \right]. \quad (2)$$

The finite sampling over $Q^{\otimes n}$ introduces an error of approximation. One can use Hoeffding's inequality to compute the Pauli sample complexity needed to approximate Pauli instability up to a given error.

Theorem 1 (Pauli sample complexity). Let $\delta, \eta > 0$. Then, $|\mathbb{I}_N(U) - \mathbb{I}(U)| < \eta$ with probability at least $1 - \delta$ when the

Pauli sample complexity is

$$N = e^{2\mathbb{I}(U)} f(\eta, \delta). \quad (3)$$

Here, $f(\eta, \delta) = \frac{\ln(2/\delta)}{2(1-e^{\eta})^2}$ and $g = \text{sign}(\mathbb{I}(U) - \mathbb{I}_N(U))$.

Intuitively, Theorem 1 shows that measuring more magic resource requires exponentially more samples. A similar result was shown for a monotone known as the stabilizer Rényi entropy [47]. This theorem can help us identify the regime where one can accurately and efficiently measure magic resource. Here, “efficiently” means that the Pauli sample complexity satisfies $N = \text{poly}(n)$. We say that a measurement is intractable when $N = \exp(n)$. “Accurately” means that η satisfies $\eta = \gamma \mathbb{I}(U)$, where $0 < \gamma < 1$ is a constant factor.

Corollary 1. Magic resource, measured via Pauli instability, can be efficiently and accurately approximated when $\mathbb{I}(U) = \ln(n)$. However, an accurate approximation is intractable when $\mathbb{I}(U) = \text{linear}(n)$.

Corollary 1 shows that circuits with small (large) quantities of magic resource can (cannot) be efficiently measured via our monotone. To illustrate, we consider the unitary $U_k = T^{\otimes k} \otimes I^{\otimes n-k}$. Using Theorem 1, we find $N = e^{8k/3} f(\eta, \delta)$. The sample complexity grows exponentially with the number of T gates, k , in the circuit. The measurement is efficient when $k = \ln(n)$ and is intractable when $k = \text{linear}(n)$.

In the case of more complex circuit architectures, one still typically expects the sample complexity to grow exponentially with the number of T gates. This is because magic resource typically scales with the number of T gates. This suggests that one can perhaps only measure the magic resource of circuits that are classically simulable (i.e., those containing $\ln(n)$ T gates). These circuits cannot demonstrate a quantum advantage. This motivates the following conjecture.

Conjecture 1. One cannot efficiently and accurately measure a reliable magic monotone $\mathcal{M}$ for typical states/unitaries with $\mathcal{M} = \text{linear}(n)$.

We say that a monotone is reliable if it grows with the complexity of a unitary. More exactly, such monotones should typically scale linearly with the number of T gates in a circuit or, alternatively, should typically grow linearly under complex Hamiltonian evolution [54]. We further conjecture that measuring $\mathcal{M}$ for a typical circuit with a $\text{linear}(n)$ number of T gates is intractable. Informally, this conjecture captures the idea that, as more T gates are added to a typical circuit, measuring any reliable magic monotone should become harder. This is because the sample complexity is expected to scale exponentially with the amount of magic resource (or alternatively with the number of T gates). This is relevant to random quantum circuits, which typically contain large quantities of magic resource [55–59].

We sketch the following argument to support the conjecture. Many magic monotones that scale linearly with the number of T gates (or T states), N_T , are defined in terms of logarithms. They can, for example, take on the form $\mathcal{M} = -\ln(\exp(-N_T))$. The magic entropy [60], stabilizer Rényi entropy [61], and additive Bell magic [44] are some examples. For typical states, accurately extracting the $\exp(-N_T)$ value often requires a measurement error exponentially small in N_T , leading to a sample complexity exponentially large in

N_T . This makes the measurement intractable for large N_T (i.e., when $N_T = \text{linear}(n)$), consistent with the conjecture. In the Supplemental Material [62], we show that the additive Bell magic satisfies Conjecture 1. Furthermore, it has been shown that, for a variety of monotones, states with magic resource larger than $\ln(n)$ may not be distinguished from random states [24]; similar results hold for other resources [63]. For the special case where a state has an efficient matrix product state representation, it has been shown that the magic resource of these states can be measured, even when large [64,65].

Thus far, we have considered the complexity of Pauli sampling. We must also consider the number of physical measurements needed to measure the OTOC, which we call the *OTOC sample complexity*. Many OTOC measurement protocols have been constructed. Past examples have utilized an interferometric approach [66], a randomized measurement toolbox [67,68], a teleportation-based technique [69,70], and the classical shadows formalism [71,72]. Here, we use the method proposed by Swingle *et al.* [66], given by the circuit in Fig. 2 (we give an alternative circuit in the Supplemental Material [62], which uses only $n + 1$ qubits). By running this circuit a finite number of times, we can approximate the OTOC up to a given error. The following proposition gives the OTOC sample complexity.

Proposition 1 (OTOC sample complexity). With probability at least $1 - \delta$, the number of samples needed to accurately measure $\text{OTOC}(U, P_1, P_2)$ up to an error of $\gamma \text{OTOC}(U, P_1, P_2)$ (where $0 < \gamma < 1$) is

$$M = \frac{2 \ln(2/\delta)}{\gamma^2 \text{OTOC}(U, P_1, P_2)^2}. \quad (4)$$

Proposition 1 shows that measuring smaller OTOC values requires more samples. The following corollary gives the regime where an accurate measurement is possible.

Corollary 2. The OTOC can be efficiently and accurately approximated when $\text{OTOC}(U) = \frac{1}{\text{poly}(n)}$. However, an accurate approximation is intractable when $\text{OTOC}(U) = \exp(-n)$.

In the case of Haar random unitaries, the value of the OTOC is typically $\exp(-n)$ (this can be readily computed through an average over the Haar measure). This renders the OTOC measurement intractable, as the required sample complexity is exponentially large in n . Furthermore, introducing more magic resource into a quantum circuit can decrease the magnitude of the OTOC to near 0, as shown in Ref. [43]. By Proposition 1, a greater number of samples are subsequently required for an accurate measurement, demonstrating the effect of magic resource on measurement precision.

We now turn our attention to the total number of samples needed to measure Pauli instability. The *total sample complexity* is a product of the Pauli sample complexity and the OTOC sample complexity.

Proposition 2 (Total sample complexity). Let $\delta_{\text{tot}}, \eta_{\text{tot}} > 0$. With probability at least $1 - \delta_{\text{tot}}$, the total sample complexity needed to measure $\mathbb{I}(U)$ up to an error of η_{tot} is

$$N_{\text{tot}} = e^{2\mathbb{I}(U)} f'(U, \delta, \delta_{\text{tot}}, \eta, \eta_{\text{tot}}), \quad (5)$$

where δ and η are the parameters given in Theorem 1 and the expression for $f'(U, \delta, \delta_{\text{tot}}, \eta, \eta_{\text{tot}})$ is given in the Supplemental Material [62].

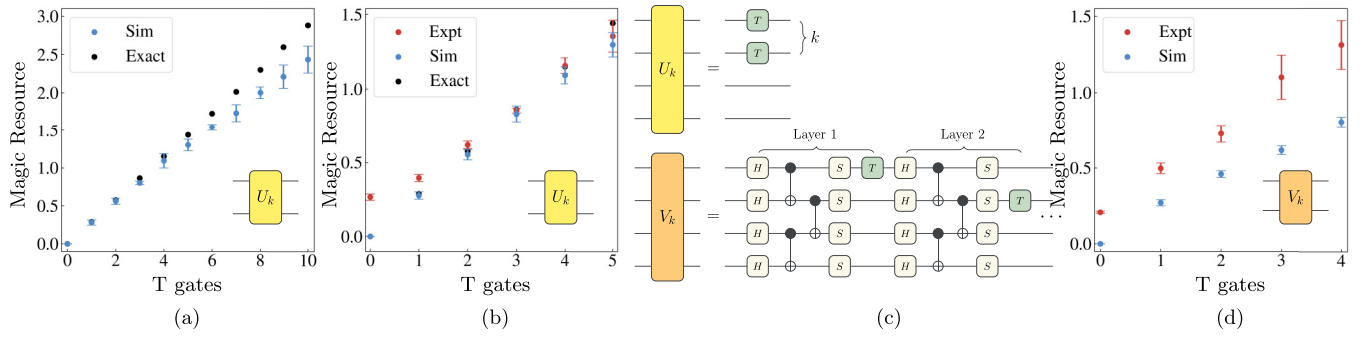


FIG. 1. (a) Numerical simulations (blue points) of $\mathbb{I}_N$ (labeled as “Magic Resource” on the y axis) for a unitary U_k as in panel (c, top), which is a single layer of k T gates. Black points are the exact values of the $\mathbb{I}$: $\mathbb{I}(U_k) = k \ln(4/3)$. The system size is $n = 10$, the Pauli sample complexity is 500, and the OTOC sample complexity is 500. (b) Experimental measurement of $\mathbb{I}_N$ for U_k . The system size is $n = 5$, the Pauli sample complexity is 500, and the OTOC sample complexity is 500. (d) Experimental measurement of $\mathbb{I}_N$ for the unitary V_k in panel (c, bottom), which contains k layers. Layer i is composed of a layer of H gates, two layers of staggered cnot gates, a layer of S gates, and a single T gate applied to the i th qubit. The system size is $n = 4$, the Pauli sample complexity is 500, and the OTOC sample complexity is 500. In all plots, each data point is computed by independently measuring $\mathbb{I}$ (or $\mathbb{I}_N$) five times and averaging. The OTOC is measured using the circuit in Fig. 2.

When the magic resource is sufficiently large, the total number of samples needed for an accurate measurement becomes intractably large, consistent with our previous discussion.

IV. SIMULATIONS AND EXPERIMENTS

We numerically simulate a measurement of magic resource to understand how the measurement accuracy changes with the number of T gates in a circuit. In Fig. 1(a), we simulate $\mathbb{I}_N$ for the unitary U_k in Fig. 1(c, top), which contains k T gates. Each data point is computed with $n = 10$ qubits and a Pauli sample complexity of $N = 500$. This is much smaller than the 16^{10} samples required to compute the exact value of Pauli instability using Eq. (1).

The simulated data initially scales linearly with the number of T gates, agreeing with the exact value. When the number of T gates is on the same order as the system size (10 in this case), the accuracy of the approximation breaks down. This is notably seen after adding five or more T gates. This shows that we require more samples to accurately approximate Pauli instability as more magic resource is introduced into the circuit. This is consistent with Theorem 1.

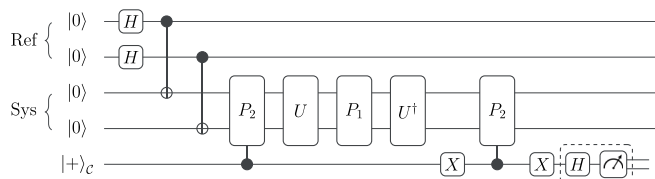


FIG. 2. Quantum circuit to measure the OTOC for a unitary U and Pauli strings P_1 and P_2 . “Ref” denotes n reference qubits and “Sys” denotes n system qubits (the n qubits are denoted as just two qubits in the diagram for notational simplicity). The control qubit is in the state $|+\rangle_C = (|0\rangle_C + |1\rangle_C)/\sqrt{2}$. The dotted box denotes a measurement in the X basis. The circuit outputs the expectation value $\langle X_C \rangle = \text{OTOC}(U, P_1, P_2)$, where C denotes the control qubit.

We experimentally measure magic resource on the IBM Eagle quantum processor. We perform the experiment on a small-scale system of four to five qubits to mitigate noise effects. In Fig. 1(b), we measure $\mathbb{I}_N$ for the unitary U_k on a five-qubit system (red points). The inherent noise in the processor results in the experimental values initially overestimating the true values (black points) and simulated values (blue points). The experimental values gradually approach the exact values, before giving an underestimate at five T gates. The simulated values also underestimate the exact values as the magic resource increases. These results again showcase that when magic resource is large, we require more samples for an accurate estimate.

To explain why noise can lead to an overestimate, we provide a simple example. Assuming that U_k is subject to depolarizing noise with a strength of λ , Pauli instability becomes $\mathbb{I}(U_k) \rightarrow \mathbb{I}(U_k) - 2\ln(1 - \lambda)$. As the noise increases, the monotone increases, giving a false signature of magic resource, consistent with our data [notably, the first two red data points in Fig. 1(b)].

Noise can also increase the number of samples needed for an accurate measurement. When a unitary is subject to depolarizing noise of strength λ , the Pauli sample complexity needed to measure the magic resource of this noisy unitary process is $N_\lambda = \frac{N}{(1-\lambda)^4}$. Here, N is the Pauli sample complexity when the unitary is noiseless. Similarly, the OTOC sample complexity for this noisy unitary process is $M_\lambda = \frac{M}{(1-\lambda)^4}$, where M is the OTOC sample complexity when the unitary is noiseless. Both sample complexities increase as the noise strength increases, implying that more measurements are needed.

In practice, circuits used in quantum computation have more complicated architectures than U_k . Namely, they contain multiple layers of Clifford and non-Clifford gates that generate entanglement [73–76]. We experimentally verify that our monotone can capture an increase in magic resource as T gates are added to more complex circuits. This property verifies the monotone’s reliability. In Fig. 1(d), we experimentally measure the magic resource of the circuit architecture, V_k ,

found in Fig. 1(c, bottom). This circuit is composed of T gates and entangling layers of Clifford gates. The plot displays a roughly linear relation with the number of T gates, similar to Fig. 1(b). This agrees with the intuition that a monotone should typically increase as more T gates are added. Noise effects become more prevalent as the circuit depth increases, leading to larger experimental values of the monotone, compared to the simulated values.

V. CONCLUSION

We have introduced a scalable measure of magic resource for large-scale quantum computers. Our measure allows us to prove that small quantities of magic resource (i.e., in the regime where one does not typically expect a quantum advantage) can be approximated on large quantum devices. When the magic resource is extensive, measurement of our magic measure becomes intractable. While we prove this using one magic measure (which relies on OTOC measurements), we conjecture that measuring large quantities of magic resource for a typical state/unitary is hard for any reliable magic measure. Furthermore, our results suggest that magic resource may become difficult to measure in the regime where one expects a quantum advantage. We leave the proof of these statements as an open problem.

Our result can be interpreted as a precision problem. As more magic resource is introduced into a quantum computer, a greater measurement precision is required, making our task more difficult. This is reminiscent of the barren plateau

problem in quantum machine learning, where ultrafine measurement precision prevents the training of certain learning models [77,78]. As a consequence, only models providing no quantum advantage have been shown to be trainable [79–81]. Our results lead us to pose the following problem: Can one prove that magic resource cannot generally be learned via quantum machine learning? It has already been shown that magic resource can make certain learning problems hard [82].

We have shown, both theoretically and experimentally, that noise can lead to a false signature of magic resource. As quantum processors are inherently noisy, it is useful to develop measurement protocols that are robust to noise. Previous works have successfully measured chaos in the presence of noise [69]. We expect that one can use similar techniques to measure magic resource robustly.

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DATA AVAILABILITY

The data that support the findings of this article are openly available [84].

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