

Fine particle percolation dynamics in porous media

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The influences of restitution coefficient, e_n , interparticle friction, μ , and size ratio, R , on gravity-driven percolation of fine particles through static beds of larger particles in the free-sifting regime ($R \gtrsim 6.5$) remain largely unexplored. Here, we use discrete element method simulations to study the fine particle percolation velocity, v_p , and velocity fluctuations, v_{rms} , for $7 \leq R \leq 50$ and a range of e_n and μ . Varying e_n modulates the degree of particle excitation and thereby alters the nature of particle trajectories: at low e_n , percolation is dominated by gravity, whereas increasing e_n amplifies velocity fluctuations thereby reducing the mean percolation velocity. Increasing μ decreases v_{rms} but its influence on v_p varies with v_{rms} , decreasing v_p for low v_{rms} and increasing v_p for high v_{rms} . Although the influence of size ratio is weaker, larger values of R increase both v_p and v_{rms} . We also assess the influence of different excitation mechanisms, specifically using static, randomly excited, and sheared beds, finding that an inverse correlation between v_p and v_{rms} persists across all cases and is well-described by the Drude model, where increased scattering reduces mobility, when v_{rms} is large. However, for weakly excited particles with low v_{rms} , the Drude analogy breaks down. In this regime, we introduce a staircase-inspired model that accounts for the gravitationally dominated percolation behavior. These findings provide fundamental insight into the mechanisms governing percolation dynamics in porous media and granular systems.

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I. INTRODUCTION

Granular materials, comprising particles ranging from micrometers to meters in size, constitute the second most widely used material class in industrial and environmental applications after water [1]. They are central to a wide array of systems, including pharmaceutical manufacturing [2], mining and construction [3], chemical and food processing [4,5], and planetary surface evolution [6–8]. In many of these applications, the particles are highly polydisperse, giving rise to size-driven segregation that can alter transport, mixing, and mechanical behavior. A particularly important mode of size-driven segregation involves the downward migration of fine particles through a relatively static bed of larger ones. This mechanism arises in diverse systems, such as powder blending, packed bed reactors, and additive manufacturing, as well as in porous media. In environmental and subsurface flows, the movement of smaller constituents through coarser granular frameworks governs critical processes such as sediment transport in rivers, contaminant dispersion in groundwater, clogging of filtration media, and infiltration in biochar-amended soils [9,10]. Although useful in applications like

particle classification or separation, uncontrolled migration of smaller particles can degrade system performance by disrupting flow pathways or inducing structural instabilities [11].

These dynamics become particularly complex when particle size distributions span several orders of magnitude. For example, fragmented ore in mining ranges from meter-scale blocks to micron-scale fragments [12]; soil and sediment beds include both coarse grains and clay-sized particles [13]; and planetary regolith contains constituents varying in size from nanometers to decimeters [14]. Similar polydispersity is encountered in advanced manufacturing, where smaller particles percolate through coarser powders during powder bed fusion or composite fabrication, affecting layer uniformity and packing fraction [15,16]. In porous geological media, the infiltration of colloids, silt, or microplastics through granular soils and aquifers can significantly influence contaminant mobility, hydraulic conductivity, and long-term material stability [9].

Many studies have investigated segregation of granular materials, examining the effects of particle size, density, and shape on segregation dynamics in dry systems where the effects of the interstitial fluid (typically air) can be ignored. Models to quantify the segregation forces and velocities have also been developed [17–20]. Most of the work has focused on systems where the size ratio between large and small particle diameters is relatively small, $R < 3$. In such systems, segregation is primarily governed by shear in flowing granular materials, which can open interstitial spaces between large particles into which small particles fall and eventually accumulate in lower regions of the flow with large particles accumulating above them. However, as the size ratio increases,

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particularly for $R > 6.5$, the segregation dynamics change significantly. In this regime, smaller particles, often referred to as fine particles or simply fines (rather than small particles), percolate through the voids in a static bed of large particles, even with no flow-related shear. This process, known as free sifting, is a fundamentally different segregation behavior than what occurs in systems with smaller size ratios where flow-related shear is necessary for segregation.

Early experiments investigating free sifting of fine particles were conducted over 50 yr ago by Bridgewater and coworkers [21,22]. These studies focus on measuring the percolation velocity, diffusion, and residence time of fine particles percolating through a static bed of large particles. In static beds, free sifting occurs only when the fine particles are smaller than the smallest pore throats, which represent the narrowest constrictions within the interconnected pore network [23]. While factors such as bed particle polydispersity, particle deformation, and particle shape influence the size of pore throats [24], in the case of monodisperse spherical bed particles of diameter D with negligible deformation, the smallest pore throat is formed between three contacting bed particles and allows fine particles of diameter d with size ratio $R = D/d \gtrsim 6.5$ to percolate freely. Experimental and computational studies find that fine particles rapidly reach a steady-state percolation velocity [21,22,25] that increases with increasing R [23,26,27] and can be related to damping in a spring-dashpot model using an empirical correlation [11]. It is also observed that the dispersion of fine particles is diffusive in all directions [28,29].

Recent investigations examining intermediate size ratios falling between shear-induced percolation and free sifting ($4 \leq R \leq 7.5$) provide correlations for estimating percolation velocity based on particle size and pore throat size distributions [23] for noninteracting fine particles (i.e., in the single fine particle limit). For a fine particle in a sheared bed with size ratio ranging from $2 \leq R \leq 10$, two main factors influence the percolation velocity, the opening and closing of pore throats due to shear, and the fine particle velocity fluctuations [30]. Increased shear rates open interstitial spaces (pores) between large particles more frequently facilitating percolation, but at high enough shear, excite fine particles, which reduces percolation. This behavior, where increased fluctuation leads to reduced percolation, closely parallels electron mobility in metals as described by the Drude model [31], where increased scattering lowers electrical conductivity. Drawing from this analogy, velocity fluctuations can be treated as a granular counterpart to thermal excitation, with percolation velocity playing the role of drift mobility, thereby capturing the inverse relationship between percolation velocity and velocity fluctuations in sheared granular systems, where both excitation and pore-throat evolution are intertwined [30].

To isolate the effect of particle excitation from pore-throat dynamics, we focus on static beds with size ratios $R > 6.464$, where free sifting dominates and pore geometry remains fixed. Unlike sheared beds where particle excitations are driven by shear, in static beds the degree of particle excitation is primarily controlled through the normal restitution coefficient, e_n , which quantifies the fraction of kinetic energy retained after a particle collision. Low values of e_n correspond to highly inelastic collisions, leading to rapid dissipation of kinetic energy and lower excitation. In contrast, high values of e_n

correspond to nearly elastic collisions, which preserve particle kinetic energy and promote more vigorous particle motion. By varying e_n over a wide range ($0.1 \leq e_n \leq 0.99$), we systematically span the spectrum from nearly completely damped to highly excited particle dynamics, allowing us to probe how collisional dissipation influences both the fluctuation velocities and the overall percolation behavior of fine particles. The high e_n values, in particular, enable the exploration of regimes where particle excitations in a static bed become comparable to those observed in sheared systems [30]. In addition to e_n , we vary the interparticle friction coefficient, μ , which governs energy dissipation through sliding interactions between particles. For both parameters, we analyze how they affect velocity fluctuations and how these fluctuations, in turn, modulate the percolation velocity of fine particles. While the findings are relevant to porous media systems such as sediment transport and groundwater contamination, where fine particles migrate through static granular beds, this study focuses on the particle dynamics by excluding the presence of interstitial fluids such as air or water.

We investigate this problem using discrete element method (DEM) [32,33] simulations. A primary challenge in DEM simulation is the rapid increase in computational cost as the size ratio increases due to the need to detect potential contacts between neighboring particles [34]. However, computational advances over the past decade have facilitated more efficient simulations that make feasible the study of physically relevant systems with size ratios as high as $R = 200$ [35–37], although a revised velocity-Verlet scheme is necessary to avoid unphysical results at large R [38].

In this paper, DEM simulations are used to examine fine particle percolation in the free-sifting regime for size ratios $7 \leq R \leq 50$. We first show how percolation velocity and velocity fluctuations depend on restitution coefficient, size ratio, and friction coefficient in static beds of randomly packed large particles. Based on these results, we develop simple scaling relations that capture the effects of dissipation and geometry (R). We demonstrate that the inverse relationship between percolation velocity and velocity fluctuations holds under different excitation mechanisms, including static, randomly excited, and sheared beds. The Drude model accurately describes this inverse relationship when velocity fluctuations are large, but not when they are small. In the latter case, a staircase-inspired model captures the percolation dynamics in the low-excitation regime.

II. SIMULATION SETUP

The simulation setup is shown in Fig. 1 with the large particles on the right half of the static bed rendered transparent to visualize the fine particles. Monodisperse large particles with diameter $D = 4$ mm are packed in a fully periodic domain of dimensions $20D \times 20D \times 20D$ at a volume packing fraction of 0.60. DEM simulations are conducted using the open-source software LAMMPS [39] utilizing the improved velocity-Verlet integration scheme for large size ratios [38]. The hertz/material contact model [40] coupled with the `coeff_restitution` damping model for inelasticity [38] is used to model the normal forces between particles. Tangential friction is modeled using the `mindlin` model [41], in

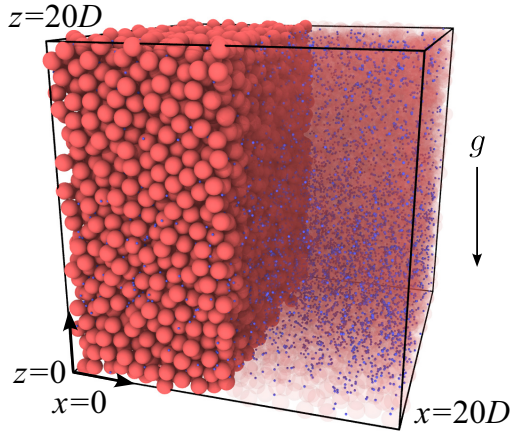


FIG. 1. Simulation setup showing 10^4 fine particles (which only interact with the large particles but not with each other) percolating in a static fully periodic bed of large particles (red) with large particle diameter $D = 4$ mm and $R = 7$. Bed particles for $x > 10D$, are rendered partially transparent to show the fine particles (blue) percolating through the bed.

combination with the marshall twisting model [42]. The material properties, including the Young's modulus and Poisson's ratio, are chosen such that the overlap during contact between large and fine particles remains below 0.1% of the fine particle diameter for size ratios up to $R = 50$ [38]. This requires a Young's modulus of 70 MPa with a Poisson's ratio of 0.3. To ensure adequate temporal resolution for all size ratios, a time step of $0.2 \mu\text{s}$ is used. For efficient neighbor detection at large R , the hierarchical multigrid algorithm in LAMMPS is used [43].

To generate randomly packed large particle beds, the sizes of randomly distributed bed particles are gradually increased from 0.5 to 4 mm within a periodic domain in the absence of gravity [23,44], thereby eliminating the influence of hydrostatic pressure gradients. During this initialization growth phase, a high particle stiffness limits particle overlaps to less than 0.05% of D , ensuring that the pore throat sizes remain unaffected by particle overlap [24]. After the particles reach their target sizes, they are annealed for 2 s and then frozen in place resulting in a large particle packing fraction of 0.60. This configuration allows for the free percolation of fine particles with $R > 6.5$.

To start a simulation, 10^4 fine particles are introduced from approximately 1 mm above the top of the static bed under the influence of gravity ($g = 9.81 \text{ m/s}^2$) acting in the negative z direction. To understand the free-sifting behavior in the single particle limit, fine particles interact only with the large bed particles and not with each other. Once all the fine particles have entered the bed, the top and bottom periodic boundaries of the simulation box are adjusted to match the dimensions of the bed particle domain rather than just above or just below it, which is necessary to initially accommodate the fine particles entering the domain from just above the bed. These adjusted periodic boundary conditions allow fine particles exiting the bottom of the bed to repeatedly re-enter the top of the bed. A total of nearly 2200 simulations are performed for R varying from 7 to 10 in increments of 1, and from 10 to 50

in increments of 5. For each of these cases, the restitution coefficient between large and fine particles is varied from 0.1 to 0.99. While these extreme values of e_n are consistent with those reported in experimental studies [45,46], they are not typical for many granular applications. The primary purpose of spanning this wide range is to use e_n as a tool to modulate the degree of particle excitation, and thereby to probe how this influences the percolation velocity. The friction coefficient, μ , is varied from 0 to 0.3. The nonzero values are consistent with those reported in experiments [47], and we include $\mu = 0$ as an extreme case, again with an aim to explore the influence of frictional dissipation on particle excitation and percolation dynamics.

The distance required for fine particles to attain steady-state velocity varies with the restitution coefficient: at low e_n , steady state occurs after a fine particle travels 2–4 D , whereas at high e_n it requires up to 20 D . In all cases, the steady-state velocity is attained in under 1 s. The mean percolation velocity v_p , defined as the magnitude of the fine particle's z velocity and the root-mean-square (rms) fluctuation velocity v_{rms} , which characterizes the excitation of the fine particles, are both calculated using time and ensemble averaging over 10^4 fine particles. Temporal averages are taken after the particles reaches a steady-state velocity ($t > 1$ s) up to the time that particles are trapped (stop moving downward due to friction [38] as discussed in Sec. III C) or when simulation ends ($t = 5$ s).

III. RESULTS

A. Restitution coefficient effects

During steady-state percolation the gravitational potential energy gained by the fine particle during its descent is balanced by the energy dissipated through both inelastic collisions and friction. To focus initially on dissipation from inelastic collisions alone, we set $\mu = 0$. This allows us to investigate the effects of varying e_n , which modulates the degree of energy loss during inelastic collisions, and R on the mean percolation velocity v_p and the fluctuation velocity v_{rms} . We consider the effect of nonzero friction later in Sec. III C.

The restitution coefficient is varied from 0.1 to 0.85 in increments of 0.05, from 0.85 to 0.9 in increments of 0.025, and from 0.9 to 0.99 in increments of 0.01, for different size ratios. The resulting variation in v_p and v_{rms} with e_n is shown in Figs. 2 and 3, respectively. The velocities are nondimensionalized using the characteristic velocity scale $\sqrt{gD}$ [22,23].

Figure 2 shows that v_p decreases toward zero at an increasing rate as $e_n \rightarrow 1$. For $e_n \lesssim 0.8$, the decrease is nearly linear, whereas for $e_n \gtrsim 0.8$ the relationship becomes more nonlinear and steeper, consistent with the sharp drop in v_p for $e_n \gtrsim 0.8$. As shown in the log-log inset, the curves are not strictly linear, but their nonlinearity increases markedly for $e_n \gtrsim 0.8$, highlighting the transition in the trend. A different trend is observed in the fluctuation velocity, v_{rms} , in Fig. 3. As e_n increases, v_{rms} increases. This increase is very gradual for $e_n \lesssim 0.8$, as also evident in the log-log inset, but grows sharply once $e_n \gtrsim 0.8$, mirroring the nonlinearity of v_p . These observations suggest the possibility of two different regimes, a low-restitution regime ($e_n \lesssim 0.8$) where the change in v_{rms} is minimal and gravity-driven motion dominates, and

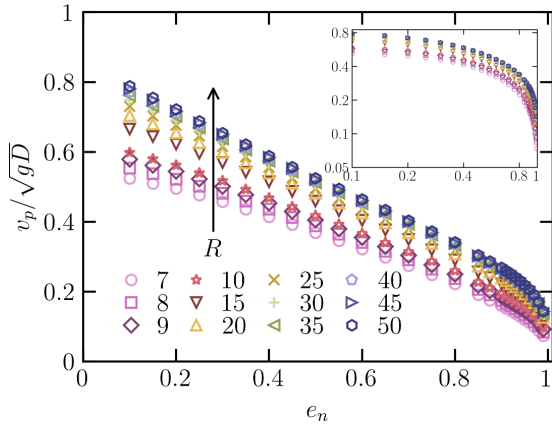


FIG. 2. Scaled percolation velocity, $v_p/\sqrt{gD}$, vs restitution coefficient, e_n , for size ratios $7 < R < 50$, with the inset showing results on a log-log scale.

a high-restitution regime ($e_n \gtrsim 0.8$) where the influence of fluctuations characterized by v_{rms} is more dominant. It is important to note that the value $e_n \approx 0.8$ is not a precise or universal threshold, but rather a rough indicator of where this transition typically occurs.

The trajectory plots in Fig. 4 for ten fine particles with $R = 50$ over 0.5 s visually demonstrate this contrast. In the fluctuation-dominated regime ($e_n = 0.99$), $v_{\text{rms}}/\sqrt{gD} > 1$ and fine particles collide many times with the large bed particles before descending a distance on the order of D [see Fig. 4(a)]. This increased random motion impedes percolation. Consequently, these fines percolate more slowly resulting in a steeper decrease in v_p with e_n in Fig. 2 for $e_n \gtrsim 0.8$.

In the gravity-dominated regime where the particles are inelastic ($e_n = 0.1$), the collisions between the fines and the large bed particles become less frequent and fines descend more directly [see Fig. 4(b)] and v_{rms} remains more or less constant (as seen in Fig. 3 for $e_n \lesssim 0.8$). In this gravity-dominated regime, restitution controls how the inelasticity of fine particle collisions shapes their bouncing behavior during repeated gravitational acceleration and subsequent collisions.

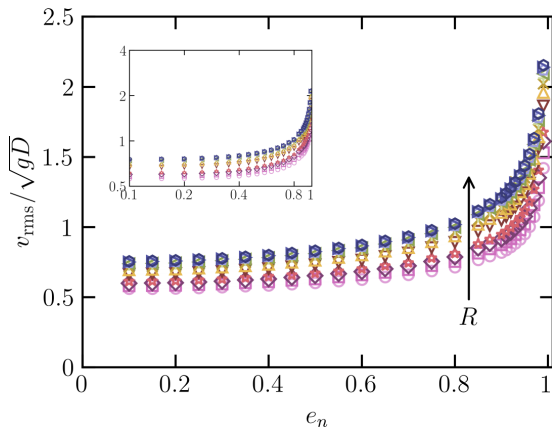


FIG. 3. Scaled fluctuation velocity, $v_{\text{rms}}/\sqrt{gD}$, vs restitution coefficient, e_n , for size ratios, $7 < R < 50$ (symbols as in Fig. 2), with the inset showing results on a log-log scale.

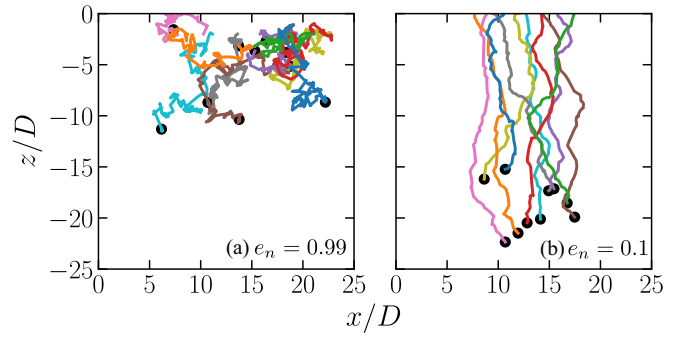


FIG. 4. Sample trajectories of ten fine particles with (a) $e_n = 0.99$ (left) and (b) $e_n = 0.1$ (right) for $\mu = 0$ in a static bed of large particles with $R = 50$.

The details of these two mechanisms are discussed in depth in Secs. III D and III E.

B. Size ratio effects

Even though the system operates in the free-sifting regime and no fine particles are trapped when $\mu = 0$, a size-ratio dependence is also observed in Figs. 2 and 3. In Fig. 2, as R is increased, v_p increases because smaller fine particles pass through pore throats more easily. The rate of increase of v_p is more pronounced for smaller R . For $R > 30$, the fine particles are sufficiently small that further reductions in their size minimally affect their percolation behavior, as indicated by the overlapping data points in Fig. 2. The fluctuation velocity also increases with increasing R (see Fig. 3). Similar to the results for v_p in Fig. 2 the dependence of v_{rms} on R is minimal for $R > 30$. Finally, we examined bed particle polydispersity up to $\pm 10\%$ at all R and observed no significant influence on the results. At larger polydispersities and low size ratios $R \lesssim 10$, however, fine particles experience increased hindrance and can be trapped due to reduced pore throat sizes, as expected [24].

To develop a size-ratio scaling for v_p and v_{rms} , consider how the size ratio affects the ability of a fine particle to pass through a pore throat between large particles. The schematic in Fig. 5(a) shows a general case of a fine particle, with diameter d , passing through a pore throat of diameter d_p formed by a random arrangement of bed particles that may not necessarily be touching one another. The accessible open area of a pore throat for large particles that are in contact with each other depends on the size ratio, as shown in Fig. 5(b). A particle with $R = 50$ has significantly more free space to pass through compared to one with $R = 7$, which is just slightly above the free-sifting limit of $R \approx 6.464$.

To quantify how readily a fine particle passes through a pore throat for bed particles that may not be touching [see Fig. 5(a)], we calculate the clearance, $d_p - d$, where d_p is the pore throat diameter and d is the fine particle diameter. The ratio of the clearance to the pore throat diameter is

$$\frac{d_p - d}{d_p} = 1 - \frac{R_p}{R}, \quad (1)$$

where $R_p = D/d_p$ is the pore-throat size ratio and $R = D/d$ is the particle size ratio. We assume without justification that

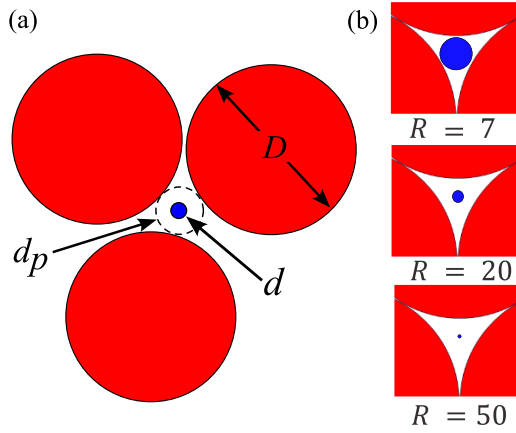


FIG. 5. Sketches showing (a) the clearance for a fine particle of diameter d passing through a pore throat of effective diameter d_p formed by a random arrangement of bed particles of diameter D , and (b) illustration of the increasing free space with increasing R in a pore throat created between three contacting bed particles.

the large particle diameter, D , used in the velocity scaling, $\sqrt{gD}$, can be replaced by $D(1 - R_p/R)$ because of the role that the relative clearance (i.e., the approximate mean free path in the pore throat) certainly plays in determining both v_p and v_{rms} , noting that $\sqrt{gD}$ is recovered for large R , as would be expected. Thus, we scale the percolation and fluctuation velocities as

$$v_p^* = \frac{v_p}{\sqrt{gD(1 - R_p/R)}} \quad \text{and} \quad v_{\text{rms}}^* = \frac{v_{\text{rms}}}{\sqrt{gD(1 - R_p/R)}}. \quad (2)$$

Although the pore throat size ratio, R_p , varies substantially in randomly packed beds, it is possible to estimate a typical value for R_p by considering the probability density function (PDF) of pore throat sizes, shown in Fig. 6, which is obtained following the Delaunay triangulation approach used previously [23]. The PDF in Fig. 6 has a broad peak corresponding to $R_p = 4$, consistent with previous findings for randomly packed static beds [23]. Hence, we use $R_p = 4$ in Eq. (2).

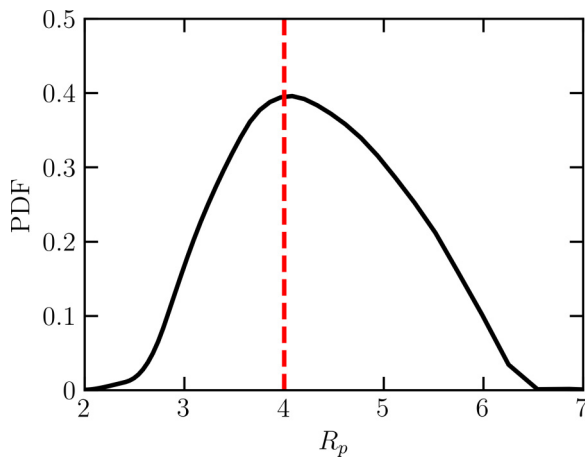


FIG. 6. PDF of the pore throat size ratio, $R_p = D/d_p$, in the randomly packed large particle bed (large particle volume fraction is 0.60).

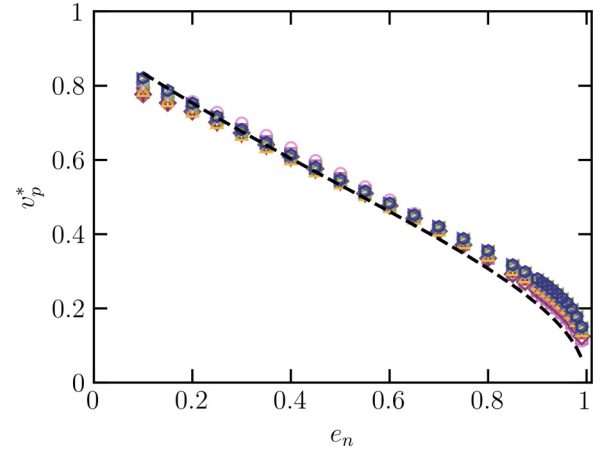


FIG. 7. Size scaled percolation velocity v_p^* vs e_n for $7 \leq R \leq 50$ (symbols as in Fig. 2). Dashed curve corresponds to $v_p/\sqrt{gD}$ from Eq. (15) with $h' = 1.7$.

To demonstrate the effectiveness of the scaling in Eq. (2) in capturing the size ratio dependence, the DEM results in Figs. 2 and 3 are replotted in rescaled form in Figs. 7 and 8, respectively. The collapse across size ratios in Fig. 7 for the scaled percolation velocity has a coefficient of variation below 7%. Similarly using the scaling of Eq. (2) for v_{rms} collapses the data equally well in Fig. 8, with a coefficient of variation that remains below 8%. Note that the $\sqrt{1 - R_p/R}$ correction is relatively small, ranging from 0.65 at $R = 7$ to 0.96 at $R = 50$. While one might anticipate that an area-based scaling would perform better given the geometry of the pore throat, this clearance-based scaling yields a better collapse of the data.

C. Friction effects

In addition to energy losses characterized by the restitution coefficient, friction is the other key energy dissipation mechanism in granular systems. To examine their combined effect on free-sifting particles, we vary the friction coefficient,

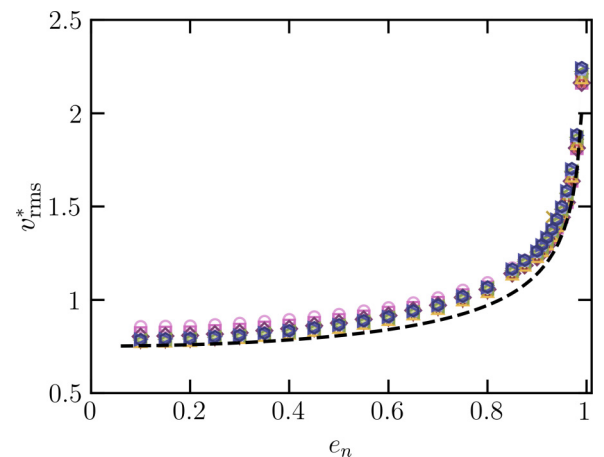


FIG. 8. Size scaled fluctuation velocity v_{rms}^* vs e_n for $7 \leq R \leq 50$ (symbols as in Fig. 2). Dashed curve corresponds to $v_{\text{rms}}/\sqrt{gD}$ from Eq. (10) with $\beta = 0.39$.

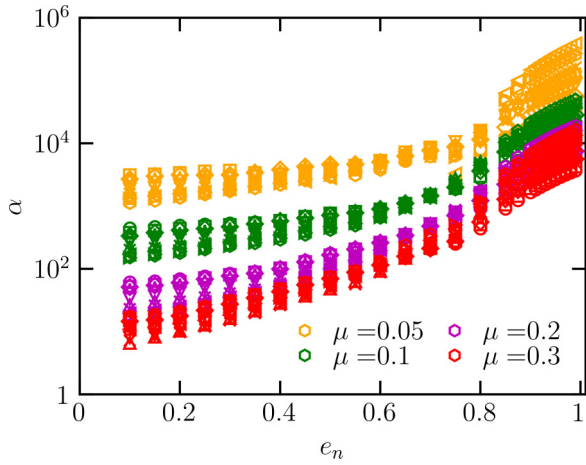


FIG. 9. Trapping depth coefficient, α , vs restitution coefficient, e_n , for different values of friction coefficient, μ (colors) and size ratios, $7 \leq R \leq 50$ (symbol shapes as in Fig. 2).

μ , from 0 to 0.3 and analyze its impact on v_p and v_{rms} . First, however, it is important to note that the introduction of friction can cause fine particles to become trapped between two large particles [38]. To quantify this behavior, we calculate the probability $P_p(\Delta z)$ that a fine particle percolates to a depth Δz without becoming trapped for different values of μ , e_n , and R . As in Ref. [23], this can be expressed as

$$P_p(\Delta z) = e^{-\Delta z/\alpha D}, \quad (3)$$

where αD is the characteristic percolation depth, representing the distance a fine particle travels before being trapped in the large particle bed. For frictionless and low μ systems ($\mu \leq 0.01$), $\alpha \rightarrow \infty$, indicating that fine particles percolate freely. As μ increases, α decreases, signifying more frequent trapping. As shown in Fig. 9, the trapping depth, quantified in terms of α , decreases with increasing μ and decreasing e_n .

In addition to friction, e_n also influences trapping. Higher e_n values lead to greater excitation and bouncing, thereby increasing α since particles only become trapped once they stop bouncing. Although it is difficult to discern because of the overlap of the markers in Fig. 9, α increases with decreasing R , as larger fine particles are less likely to wedge between bed particles. For large friction values, with sufficient time, all particles eventually become trapped, recalling that we consider fine particles that do not interact with one another and cannot dislodge previously trapped particles.

The combined effects of the restitution and friction dissipation mechanisms on v_p and v_{rms} are shown in Figs. 10 and 11, respectively, for $e_n = 0.4$ and $e_n = 0.99$ at $\mu = 0$ and $\mu = 0.3$. To prevent trapping from affecting v_p and v_{rms} measurements, we calculate steady-state values up until the particle becomes trapped [23]. In Fig. 10 for $e_n = 0.4$, v_p decreases with increasing μ , whereas for $e_n = 0.99$, v_p increases. In contrast, in Fig. 11 v_{rms} decreases with increasing μ for both cases, with a more pronounced reduction at $e_n = 0.99$. To explain, at high e_n , the absence of friction minimizes energy loss, leading to greater bouncing and higher v_{rms} , which reduces v_p . Introducing friction enhances energy dissipation, substantially reducing v_{rms} and enabling freer per-

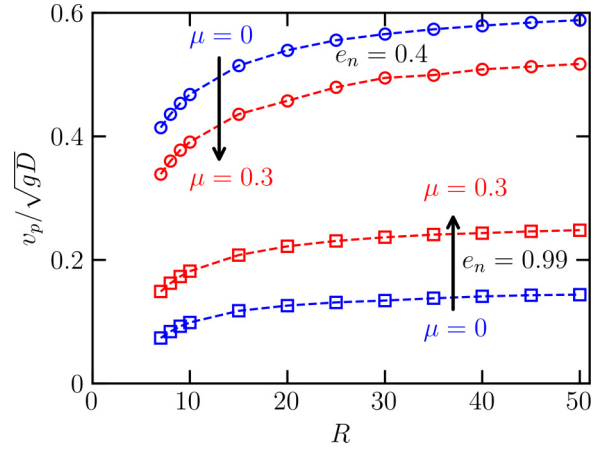


FIG. 10. Scaled percolation velocity, $v_p/\sqrt{gD}$, vs size ratio, R , for friction coefficient, $\mu = 0$ (blue) and 0.3 (red), showing opposite responses for restitution coefficients, $e_n = 0.4$ (circles) and 0.99 (squares).

colation, thereby increasing v_p . Conversely, at low e_n , v_{rms} is already low, so introducing friction only decreases it slightly. However, friction further inhibits sliding during collisions, thereby reducing v_p . Data for other values of e_n and μ fall between the curves shown in Figs. 10 and 11 but are omitted for clarity.

The combined influence of restitution and friction is also evident in the fine particle trajectories in Fig. 12 for $\mu = 0.3$ compared to the frictionless case in Fig. 4. At high restitution ($e_n = 0.99$), the lack of friction results in minimal energy dissipation and strongly scattering trajectories. With friction, however, the additional frictional dissipation reduces the velocity fluctuations v_{rms} and leads to more downwardly directed motion. At low restitution ($e_n = 0.1$), the trajectories remain governed by the gravity-dominated percolation process and are similar to the frictionless case. Yet, as fines percolate through the static bed, they slide against the bed particles, and friction increases the resistance to this sliding, thereby

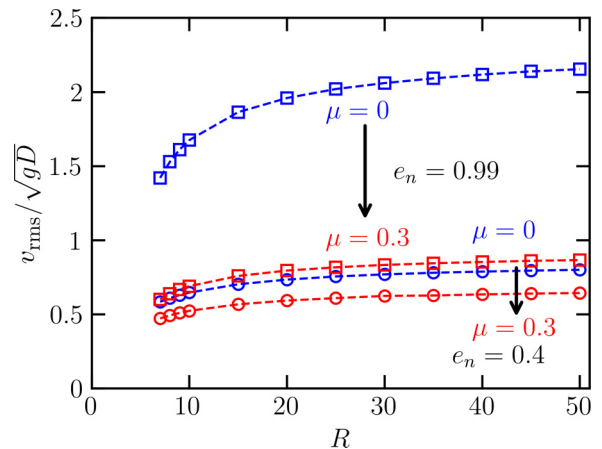


FIG. 11. Scaled fluctuation velocity, $v_{\text{rms}}/\sqrt{gD}$, vs size ratio, R , decreases with increasing friction coefficient [data shown for $\mu = 0$ (blue) and 0.3 (red)] at low- and high-restitution coefficients $e_n = 0.4$ (circles) and 0.99 (squares).

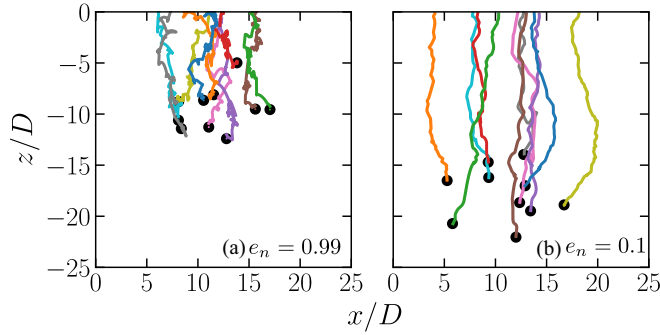


FIG. 12. Sample unwrapped trajectories of 10 fine particles with (a) $e_n = 0.99$ (left) and (b) $e_n = 0.1$ (right) for $\mu = 0.3$ in a static bed of large particles with $R = 50$ showing the difference in motion for low- and high-restitution coefficients.

lowering the percolation velocity. This influence is clearly reflected in Figs. 13 and 14, which present the R -scaled percolation and fluctuation velocities across a range of e_n and μ . For $e_n \lesssim 0.8$, friction enhances sliding resistance, reducing both v_p^* and v_{rms}^* . At higher restitution, however, the introduction of friction produces the opposite effect: v_p^* increases because the stronger reduction in v_{rms}^* suppresses random scattering, resulting in a more gravity-dominated percolation [Fig. 12(a)]. At $e_n \approx 0.8$, these opposing effects nearly balance, and the percolation velocity is largely independent of μ .

An important observation in Figs. 13 and 14 is the contrasting behavior for R -scaled fluctuation velocities depending on whether v_{rms}^* is greater or less than one. For $e_n \gtrsim 0.8$ and $\mu \leq 0.05$, v_{rms}^* rapidly rises above one (Fig. 14) as e_n increases, indicating strongly random motion, consistent with the trajectories in Fig. 4(a) for $e_n = 0.99$. Under the same conditions, Fig. 13 shows a correspondingly steep reduction in percolation velocity. In contrast, for $e_n \gtrsim 0.8$ and $\mu \geq 0.1$, the additional energy dissipation from friction constrains the increase in v_{rms}^* , keeping its value below one (Fig. 14). In this regime, the motion of fines is less random and more directed, as demonstrated by the trajectories in Fig. 12 for $e_n = 0.99$. Consequently, the reduction in percolation velocity is more

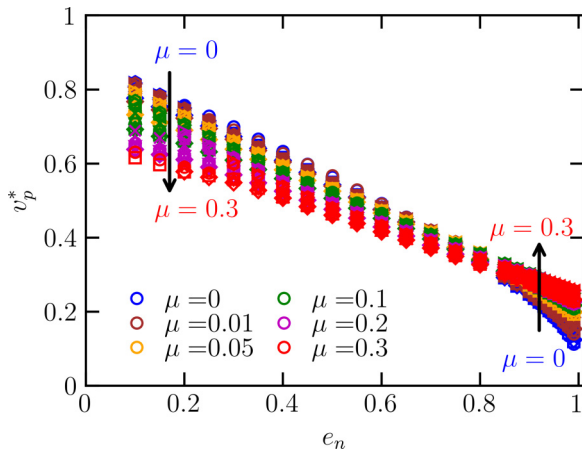


FIG. 13. Scaled percolation velocity v_p^* vs e_n for different μ (colors) with $7 \leq R \leq 50$ as denoted by symbol shapes in Fig. 2.

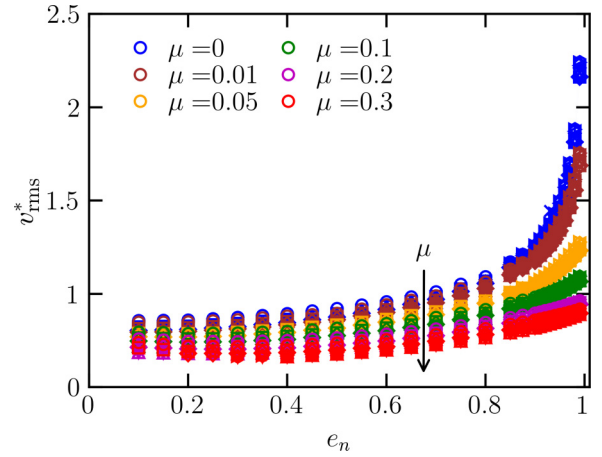


FIG. 14. Scaled fluctuation velocity v_{rms}^* vs e_n for different μ (colors) with $7 \leq R \leq 50$ as denoted by symbol shapes in Fig. 2.

gradual (Fig. 13), resembling the behavior seen at lower restitution ($e_n \lesssim 0.8$). These observations show that the transition from random to directed motion is governed by v_{rms}^* . When fluctuations are small, $v_{rms}^* \lesssim 1$, motion is more directed and percolation velocity is higher. Conversely, when fluctuations are large, $v_{rms}^* \gtrsim 1$, motion becomes more random and the percolation velocity decreases.

In the next two sections, we analyze these two regimes separately and present simple conceptual models that capture their contrasting behaviors.

D. Drude model for fluctuation-dominated percolation

In Figs. 7 and 8 for the fluctuation-dominated regime, $v_{rms}^* \gtrsim 1$, a rapid reduction in percolation velocity is accompanied by a sharp increase in fluctuation velocity for $e_n \gtrsim 0.8$. A similar trend is seen in Figs. 13 and 14 for low values of $\mu \leq 0.05$. Comparable behavior in sheared beds has also been reported by Gao *et al.* [30] at lower size ratios $7 \leq R \leq 10$. They observed that in the presence of shear the percolation velocity decreases not only with increasing e_n but also with increasing shear rate, as higher shear rates generate larger v_{rms} . They further noted that $v_p \propto 1/v_{rms}$ and suggested that this behavior is analogous to that predicted by the Drude model for electron conduction in metals.

To examine whether the Drude model also applies to static beds, we compare our results with those for a sheared bed [30], where the vertical component of the RMS velocity, $v_{z,rms}$, is used to characterize fluctuations. To compare with our results, here we assume that at high shear rates all three components of v_{rms} are equal so that, $v_{rms} = \sqrt{3} v_{z,rms}$.

Since in static beds the value of v_{rms}^* is constrained by e_n , we also consider a third configuration in which fines percolate through a bed of randomly excited large particles. This system retains the same geometry, material properties, size ratios, and packing fraction as the static bed, but introduces random excitation of the large particles using a Langevin thermostat implemented in LAMMPS [39,48] with a damping parameter of 1 s. For this system, we study two restitution coefficients, $e_n = 0.5$ and 0.9 , with $7 \leq R \leq 50$ and $\mu = 0$. The excitation velocity of the large particles is varied from 0.5 to 9 m/s

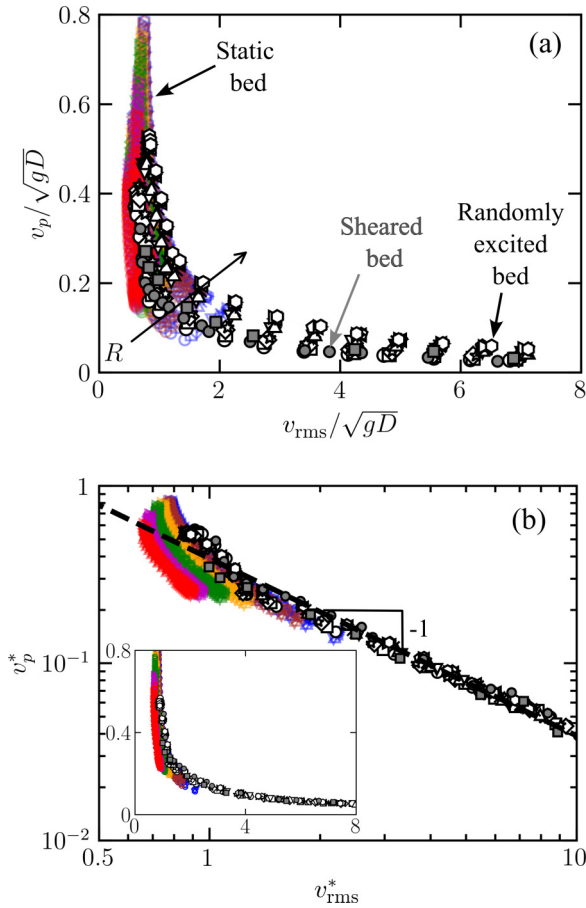


FIG. 15. (a) Inverse relationship between v_p and v_{rms} for static (colored symbols, varying μ as in Fig. 13), sheared (gray symbols), and randomly excited (open black symbols) beds, with $7 \leq R \leq 50$ and $0.1 \leq e_n \leq 0.99$. Symbols represent R values as in Fig. 2. (b) Same data scaled by R [see Eq. (2)] on a logarithmic scale, with inset on a linear scale. The dashed curve corresponds to Eq. (5) with $\beta = 0.39$.

to obtain fine particle fluctuation velocities between 0.2 and 1.4 m/s, comparable to those reported in the sheared bed [30]. Similar to the static configuration, the velocities of percolating fines reach steady state within 1 s, after which both v_p and v_{rms} are obtained as time- and ensemble-averaged quantities.

The inverse relationship between v_p and v_{rms} for static, sheared, and randomly excited beds is shown in Fig. 15. Figure 15(a) demonstrates that the inverse correlation between percolation velocity and fluctuation velocity holds across all excitation mechanisms at large v_{rms} , indicating that the relationship is largely independent of how fine particle velocity fluctuations are generated. Figure 15(b) shows that R scaling collapses the data for $v_{rms}^* \gtrsim 1$, independent of the fine particle excitation mechanisms. The log-log representation in Fig. 15(b) emphasizes the robustness of this trend at high v_{rms}^* but also shows clear contrasting behavior at low fluctuation velocities, where the inverse correlation does not hold. The inset shows the collapse of the data on a linear scale.

The Drude model for electron conduction in metals mirrors this behavior, as described previously for sheared beds [30]. The inverse relationship, $v_p \propto 1/v_{rms}$, for fine particle

mobility parallels how increased lattice scattering reduces electron drift velocity in the Drude framework under an applied electric field E . In the Drude model, the mean momentum of the electron is given by $p = mv = qE\tau$, where m and q are the electron mass and charge, respectively, and τ is the mean time between collisions with atoms in the lattice. Drawing an analogy to fine particle percolation in the limit of a point particle ($R \rightarrow \infty$), we map the terms in the Drude model as follows: the electron drift velocity v is replaced by the percolation velocity v_p , the driving acceleration qE/m is replaced by gravitational acceleration g , and the mean free time τ is represented as $\beta D/v_{rms}$, where βD represents the mean free path of a point particle, which must be of order D . This analogy then gives the relation

$$v_p = \frac{\beta g D}{v_{rms}}, \quad (4)$$

which captures the inverse dependence of the percolation (drift) velocity on the fine particle's fluctuation (excitation) velocity for a point particle. Incorporating R scaling by substituting $\beta D(1 - R_p/R)$ for βD to account for finite fine particle size effects on the mean free path as discussed in regard to Eqs. (1) and (2) gives

$$v_p = \frac{\beta g D}{v_{rms}} \left(1 - \frac{R_p}{R}\right) \text{ or } v_p^* = \frac{\beta}{v_{rms}^*}. \quad (5)$$

In Fig. 15(b), we compare the DEM results with predictions from the Drude model. When $v_{rms}^* \gtrsim 1$, the Drude model agrees well with the DEM data using a fitted value of $\beta = 0.39$. Since βD represents the mean free path of a point particle, this corresponds to the characteristic pore size encountered by a fine particle traversing a static bed, which lies between the typical void diameters of tetrahedral ($0.225D$) and octahedral ($0.414D$) bed particle configurations, indicating that the model reflects realistic geometrical constraints of granular packings.

In the Drude model, the mean free time is governed by the temperature, which determines the frequency of electron-lattice collisions. In contrast, for granular systems, the mean free time, represented as the ratio of mean free path to fluctuation velocity, $\beta D/v_{rms}$, is governed by the particle dynamics and the geometry of the bed. By combining this granular analog of the Drude model with a power balance argument, we estimate the dependence of v_{rms} on e_n , capturing how energy dissipation due to inelastic collisions influences fine particle excitation. In a frictionless system, where energy losses stem solely from inelastic collisions, a simple power balance can be expressed as

$$mgv_p = \frac{1}{2}mv_{rms}^2(1 - e_n^2)n, \quad (6)$$

where n is the collision frequency. Solving for v_{rms} , the percolation velocity can be approximated as

$$v_p = nv_{rms}^2 \frac{(1 - e_n^2)}{2g}. \quad (7)$$

The collision frequency for high v_{rms} systems can be estimated as $n = v_{rms}/\beta D$, giving

$$v_p = \frac{v_{rms}^3(1 - e_n^2)}{2\beta g D}. \quad (8)$$

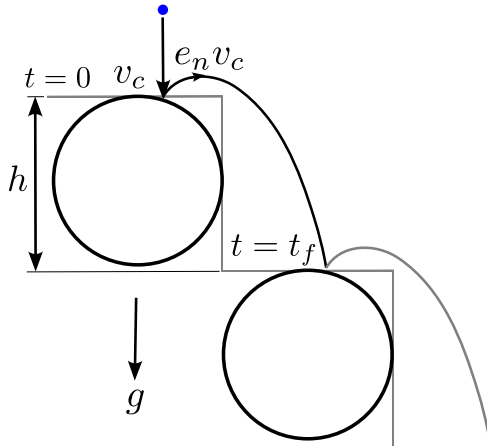


FIG. 16. Schematic showing a simple model of a fine particle bouncing down a staircase of bed particles.

Combining this with Eq. (4) gives an expression for the fluctuation velocity as

$$\frac{v_{\text{rms}}}{\sqrt{gD}} = \sqrt{\beta} \left(\frac{2}{1 - e_n^2} \right)^{1/4}. \quad (9)$$

To account for the size ratio, we apply the $\sqrt{1 - R_p/R}$ correction to the left hand side so that

$$v_{\text{rms}}^* = \sqrt{\beta} \left(\frac{2}{1 - e_n^2} \right)^{1/4}. \quad (10)$$

As shown in Fig. 8, this scaling captures the dependence of v_{rms}^* on e_n in a static bed. While the agreement in Fig. 8 is expected for $v_{\text{rms}}^* \gtrsim 1$, it is surprising that the model also fits the data reasonably well for $v_{\text{rms}}^* \lesssim 1$.

Returning to Fig. 15(b), the spread of the data for v_p^* broadens for $v_{\text{rms}}^* \lesssim 1$. This is consistent with the physical picture: under these conditions, fine particle motion is gravity dominated and violates the Drude model assumption of random motion, as evident from the trajectory plots in Figs. 4 and 12, which show a more vertically biased descent. Thus, for typical values of e_n and μ in physical systems, Drude-like behavior will not occur in static bed percolation. This motivates the development of a new model, introduced in the following section, that captures percolation dynamics in the low- v_{rms} regime by explicitly accounting for gravity-driven motion and energy dissipation during successive collisions of a fine particle with bed particles.

E. Staircase model for gravity-dominated percolation

Knowing that low restitution minimizes excitation of a fine particle as it percolates through the bed, we propose a simplified representation of a single fine particle percolating through a static bed in the gravity dominated regime where $v_{\text{rms}}^* \lesssim 1$. This description, referred to as the “staircase model,” is shown schematically in Fig. 16.

At steady state, a fine particle with mass m and typical vertical collision velocity $v_z = v_c$ bounces off the surface of a large particle with a reduced rebound velocity of $e_n v_c$. The fine particle then follows a parabolic trajectory before colliding again at a vertical distance $h \sim D$ below the previous

collision location. The steady-state collision velocity v_c can be estimated by balancing the gravitational potential energy gained between impacts, mgh , with the energy dissipated by inelastic loss, characterized by the restitution coefficient e_n , during each collision.

$$\frac{1}{2} m v_c^2 (1 - e_n^2) = mgh, \quad (11)$$

which gives

$$v_c = \sqrt{\frac{2gh}{1 - e_n^2}}. \quad (12)$$

Since the fine particle follows a parabolic trajectory with time of flight t_f between consecutive impacts (steps), the associated average percolation velocity through the bed is $v_p = h/t_f$. By solving the quadratic kinematic equation for the time of flight for the parabolic trajectory after each collision, the percolation velocity v_p can be expressed as

$$v_p = \sqrt{\frac{gh}{2}} \sqrt{\frac{1 - e_n}{1 + e_n}}. \quad (13)$$

Note that the staircase model ignores the additional dissipative effects associated with friction. When e_n approaches zero, the percolation velocity approaches $\sqrt{gh/2}$, which corresponds to the particle falling from step to step without bouncing. Conversely as e_n approaches one, the percolation velocity approaches zero, as most of the kinetic energy is retained and the particle bounces higher and with a longer t_f , resulting in minimal downward percolation through the bed. In contrast to the Drude model, where the fine particle undergoes multiple collisions as it bounces in a bed of large particles and v_p and τ are strongly influenced by v_{rms} , the staircase model assumes a single collision between steps and is governed solely by the parabolic motion under gravity between successive impacts, which depends on e_n .

The step height h represents the vertical distance between collisions and should be proportional to the bed particle diameter as well as the mean free path. Consequently, we express h as

$$h = h'D \left(1 - \frac{R_p}{R} \right), \quad (14)$$

where $h'D$ represents the characteristic distance a point particle ($R \rightarrow \infty$) falls between collisions. Substituting this expression into Eq. (13) and rearranging gives

$$v_p^* = \sqrt{\frac{h'}{2}} \sqrt{\frac{1 - e_n}{1 + e_n}}. \quad (15)$$

Equation (15) matches the data in Fig. 7 well using $h' = 1.7$. The agreement is best at higher R for $e_n \lesssim 0.8$ ($v_{\text{rms}}^* \lesssim 1$), whereas for $e_n \gtrsim 0.8$ the predictions align more closely with the DEM results at lower R , where v_{rms}^* is also relatively small as seen in Fig. 3. Hence, Eq. (15) can be used to estimate the percolation velocity for a given size ratio and restitution coefficient. While the staircase model could be further refined by incorporating effects like large particle curvature, energy dissipation due to friction during interparticle collisions, and rolling and sliding motions of fine particles, it is striking

how well this simple model captures the observed percolation behavior in the regime where $v_{\text{rms}}^* \lesssim 1$.

IV. CONCLUSIONS

In this study, we investigate the percolation of noncohesive spherical fine particles in randomly packed granular beds in the absence of interstitial fluids. We examine the influence of different interaction parameters, the coefficient of restitution (e_n), the friction coefficient (μ), and the large-to-fine diameter ratio (R), on fine particle percolation and fluctuation velocities. In static beds, e_n strongly influences percolation: as e_n increases, percolation velocity v_p decreases while fluctuation velocity, v_{rms} , increases. Two regimes emerge based on the fluctuation velocity, v_{rms} . When $v_{\text{rms}}/\sqrt{gD} \lesssim 1$ percolation is gravity-dominated and steplike, whereas when $v_{\text{rms}}/\sqrt{gD} \gtrsim 1$, random collisional dynamics dominate, leading to an inverse v_p - v_{rms} relationship. This inverse scaling applies not only to static beds but equally well to sheared, and randomly excited beds. However, even in these externally excited system when the excitation is weak, the percolation will fall into the gravity dominated regime. While the Drude model captures the high-fluctuation regime, it fails at capturing behavior for $v_{\text{rms}} \lesssim 1$, where a staircase model better describes the gravitational descent of fines.

The size ratio R also influences v_p and v_{rms} , although not as strongly as e_n . As R increases, both v_p and v_{rms} initially rise but then plateau. This behavior is attributed to smaller particles navigating pore throats more readily, enhancing percolation and thereby reducing their collision frequency with the bed particles, which in turn decreases energy dissipation and results in increased fluctuation velocities. Friction affects v_p and v_{rms} differently depending on e_n . For restitution coefficients $e_n \lesssim 0.8$, increasing friction reduces v_p due to inhibited sliding of fine particles over larger ones during collisions. At higher restitution coefficients ($e_n \gtrsim 0.8$), friction decreases v_{rms} , which, in turn, increases v_p as particles lose energy and percolate more quickly.

While the present study focuses exclusively on dry systems without interstitial fluids, simple force-balance estimates

allow us to identify particle sizes above which fluid and cohesive effects are negligible. In air, a fine glass or sand particle larger than roughly 200 μm can be considered to percolate freely under gravity, as fluid drag is negligible. In water, this threshold increases to approximately 1 mm, due to buoyancy and viscous resistance reducing the effective gravitational acceleration. Cohesive forces, particularly van der Waals interactions, are another important consideration for very fine particles. For dry glass beads, such effects become significant below about 150 μm , meaning that for particles larger than 200 μm both cohesion and fluid effects can be safely ignored in air, but for particles smaller than 200 μm these effects must be considered.

Although this study has untangled the intertwined effects of restitution, friction, and size ratio over a wide parameter space, $0.1 \leq e_n \leq 0.99$, $0 \leq \mu \leq 0.3$, and $7 \leq R \leq 50$, many questions remain. First and foremost is to explore percolation of cohesive and noncohesive interacting fine particles (rather than the individual fine particles considered here), which would reflect more typical segregation conditions. Another critical gap in understanding, particularly for industrial and geophysical applications, is that of interacting fine particles in sheared and randomly excited granular beds for both low concentrations of fine particles, where fine particles mostly interact with large particles collisionally, and for high enough concentrations of fine particles that the fine particles act as a “continuous phase” with a “dispersed phase” of large particles suspended within them.

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DATA AVAILABILITY

The data that support the findings of this article are openly available [49].

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