

Shortcuts to adiabaticity for loading an ultracold Fermi gas into higher orbital bands of one-dimensional optical lattice

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We present a shortcuts-to-adiabaticity protocol to load a spin-polarized ultracold Fermi gas from the ground (s) band into the first excited (p) band of a one-dimensional optical lattice. In contrast to a Bose-Einstein condensate in a shallow lattice—whose narrow quasimomentum distribution admits uniform lattice modulation—a noninteracting Fermi gas occupies the entire first Brillouin zone, rendering high-efficiency transfer across all momentum classes exceptionally challenging. To address this, we cast the loading process as a global, multiparameter optimization problem: We jointly optimize the timing of lattice-depth pulses and the corresponding phase shifts to maximize the ensemble-averaged transfer fidelity over all quasimomentum states. We demonstrate that fine phase tuning dramatically enhances loading efficiency, though the mean fidelity systematically decreases as more quasimomentum modes are occupied. Finally, we benchmark our shortcuts-to-adiabaticity protocol against established high-band-loading methods, underscoring its minimal hardware requirements and submillisecond timescales. Our work provides a practical path for rapid, high-fidelity preparation of Fermi gases in excited lattice orbitals, opening avenues for exploring orbital dynamics, anisotropic interactions, and many-body phenomena.

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I. INTRODUCTION

One of the most powerful platforms for quantum simulation is ultracold atoms in optical lattices, which affords unrivaled control over interatomic interactions, lattice geometry, and external fields [1–7]. This level of precision enables the emulation of complex condensed-matter systems that remain inaccessible by conventional means [8]. To date, most experiments have been confined to the s band due to technical constraints on loading and coherence [9–23]. Recent breakthroughs, however, have extended this frontier into higher bands—particularly the p and d bands—opening promising pathways to explore orbital physics [24–28], unconventional superfluidity [29–31], rich interaction dynamics [32–34], and even lattice-based superconductivity [35–39].

Several higher-band-loading schemes have been developed for cold atoms, including stimulated Raman transitions [32,40], population-exchange technique [28,31,34], optical dipole trap (ODT) depth modulation [33,41,42], and

shortcuts-to-adiabaticity (STA) approaches [45–52]. While stimulated Raman and population-exchange techniques demand complex multilevel coupling and extensive laser configurations, increasing experimental complexity, ODT modulation typically entails long transfer times and relatively low loading efficiencies. In contrast, the STA methods offer rapid, lattice-only control with minimal hardware overhead, enabling the efficient mapping of s - to p -band Bloch states on submillisecond timescales without requiring additional lasers or external field schemes. Such advantages make STA a powerful tool for investigating high-orbital physics with ultracold atoms.

While STA protocols have proven effective for bosonic gases, efficiently loading Fermi gases into high bands under STA schemes remains challenging. This is primarily due to the broad quasimomentum distribution of Fermi gases in optical lattices; unlike bosons that condense at $q = 0$ in a shallow lattice, fermions populate a wide range of quasimomentum states, which complicates efficient loading with a single, fixed-phase STA trajectory. Therefore, developing an STA-based approach is urgently needed to achieve high-fidelity loading of Fermi gases into higher bands.

Motivated by the STA method in Bose-Einstein condensate (BEC) [46], we here present a phase-optimized STA sequence to load spin-polarized ultracold Fermi gases into the p band of a one-dimensional lattice. Our protocol consists of four key steps: (1) transferring atoms from the ODT to the optical lattice s band; (2) modulating the optical lattice laser intensity to induce energy variations and modify atomic Bloch states;

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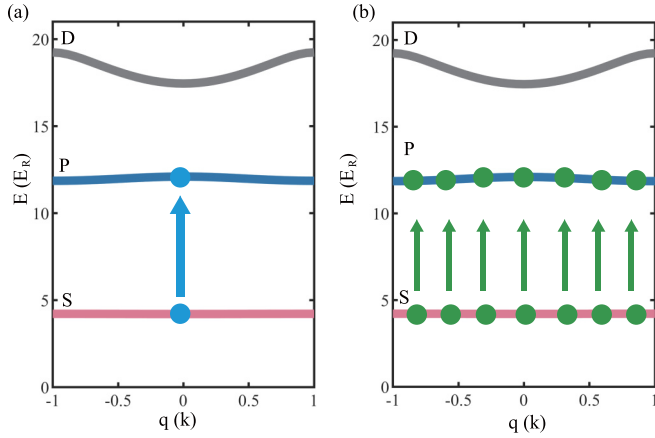


FIG. 1. Loading a BEC in a shallow lattice and a Fermi gas into p band differs significantly, as shown schematically with band diagrams: (a) In a shallow lattice, a BEC's s -band momentum distribution is narrow and centered at $q = 0$, so only that single state is transferred to the p band. (b) For a Fermi gas, the s -band momentum distribution remains broad even in shallow lattices, and each quasimomentum component is independently loaded into the p band without interference. Here, $k = \pi/d$ denotes the lattice wave number, with lattice constant d .

(3) shifting the optical lattice potential to adjust the phase of the Bloch states to match the p -band parity; and (4) continuously adjusting the lattice laser to achieve optimal alignment of the Bloch states with the p band. Given the broad momentum distribution of Fermi gases [53], we adopt a strategy to simultaneously load atoms across all quasimomentum states and perform global optimization throughout the process to achieve the optimal time sequence, as shown in Fig. 1. Under optimal conditions, our method maintains loading efficiencies above 60% across all lattice depths and reaches up to 95% in two specific lattice depths. It offers reduced loading times and simpler implementation compared to existing techniques. Our method is well suited for studying the lifetime of high-orbital atoms and for advancing research in high-orbital physics.

The structure of this paper is as follows: In Sec. II, we present the theoretical model of our optimization scheme. Section III presents the details of the multiparameter optimization method, including optimization using only optical pulse durations, optimization with pulse durations and a single lattice potential phase, and optimization with pulse durations alongside a phase tailored to each pulse. In Sec. IV, we discuss the effects of quasimomentum occupancy on the result of the optimization, and compare our results with other higher-band-loading methods. Finally, we summarize the key findings of this work and offer perspectives for future research in Sec. V.

II. MODEL

In this section, we outline the theoretical framework underlying our loading method. To analyze the Hamiltonian, Bloch states, and time evolution operator, we employ the plane-wave decomposition method, a widely adopted approach in the field. A detailed explanation of this method can be found in Appendix A.

As a starting point, we describe the initial state of a spin-polarized, noninteracting Fermi gas in a one-dimensional optical lattice. Building on our previous work, we have shown that nearly 100% of the atoms can be loaded into the s band by adiabatically raising the one-dimensional optical lattice [41]. Unlike a BEC, which occupies a single quasimomentum state in shallow lattice depth, the Fermi gas exhibits a broad momentum distribution, as shown in Fig. 1. The initial state of the noninteracting Fermi gas is a statistical mixture over s -band quasimomentum, described by the single-particle density operator:

$$\rho_{\text{in}} = \sum_j C_{\text{in}}(q_j) |s, q_j\rangle \langle s, q_j|, \quad (1)$$

where C_{in} represents the equal probability of s -band Bloch states for each q and s means the s band. Here, we have already summed over the transverse harmonic levels $n_{\perp}$ (radial modes) via $C_{\text{in}}(q_j) = \sum_{n_{\perp}} f(q_j, n_{\perp}) / \sum_{q', n_{\perp}} f(q', n_{\perp})$, where $f(q, n_{\perp})$ is the zero-temperature Fermi occupation of the Bloch-harmonic state $|s, q, n_{\perp}\rangle$. This tracing out of radial modes yields the mixed single-particle density operator in the axial quasimomentum basis.

Similarly, we express the target state we aim to achieve:

$$\rho_t = \sum_j C_t(q_j) |p, q_j\rangle \langle p, q_j|, \quad (2)$$

here, we assume that Bloch states in the p band are occupied with equal probability for each quasimomentum, and this probability is identical to the one associated with the s band mentioned above, i.e., $C_t = C_{\text{in}}$. Note that C_t describes only the quasimomentum population, while the full complex structure of the $|p, q\rangle$ states, including the phase, is retained through numerical diagonalization and fully enters the dynamical evolution.

Thus, during the optical lattice-switching process, each quasimomentum state participates in the temporal evolution, which can be expressed as

$$\rho_e(t) = \hat{U}_{\text{load}} \rho_{\text{in}} \hat{U}_{\text{load}}^{\dagger} = \left(\prod_{k=1}^{n_t} \hat{U}_k \right) \rho_{\text{in}} \left(\prod_{k=1}^{n_t} \hat{U}_k \right)^{\dagger}, \quad (3)$$

where $\hat{U}_k = e^{-i\hat{H}_k t_k}$ is the time evolution operator for the k th process and n_t is the total number of processes.

Based on the different time sequences, we divide the Hamiltonian into two scenarios: the shutdown and opening of the lattice potential, here are the Hamiltonians:

$$\hat{H}_s = \frac{p^2}{2m}, \quad \hat{H}_o = \frac{p^2}{2m} + V_o(x), \quad (4)$$

where $\hat{H}_s$ denotes the Hamiltonian with the lattice potential switched off and $\hat{H}_o$ describes the lattice being on at a constant depth V_o throughout the process, where V_o is the lattice depth when the lattice is turned on. The final state is obtained after time evolution.

After the evolution, both the initial and final states are described by density operators. To quantify the overall loading efficiency, we define the fidelity as the expectation value of the projection operator onto the target p -band states,

$\hat{P}_t = \sum_j |p, q_j\rangle\langle p, q_j|$, evaluated on the final evolved state $\rho_e(t)$:

$$F = \text{Tr}[\hat{P}_t \rho_e(t)]. \quad (5)$$

By substituting the expression for the evolved state $\rho_e(t)$ into Eq. (5), the fidelity reduces to the ensemble-averaged transfer probability:

$$F = \sum_j C_{\text{in}}(q_j) |\langle p, q_j | \hat{U}_{\text{load}} | s, q_j \rangle|^2. \quad (6)$$

Because the loading process does not generate coherence between different q_j sectors, this weighted sum is the most direct measure of the fraction of atoms successfully transferred to the target p band. We then employ matlab's global optimization routines to vary the pulse timings and phase shifts, maximizing F as defined in Eq. (6).

To facilitate efficient loading into p band, it is necessary to adjust the parity of the atoms due to the parity difference between bands. This can be achieved through optical lattice phase modulation, as the parity of Bloch states at $q = 0$ is even in the s band and odd in the p band [46]. Zhou *et al.* demonstrated that a phase modulation of $3\pi/4$ effectively achieves this adjustment, making it essential for successful p -band loading.

As noted earlier, the Fermi gas occupies nearly all quasimomentum states, most of which lack well-defined parity except at $q = 0$. Moreover, the exact phases of the occupied quasimomentum states are unknown due to their broad distribution. To address this challenge, we incorporate a phase φ into the optimization program, where we add φ to the modulated lattice potential $V'(x)$ compared to the lattice potential $V_o(x)$,

$$V'(x) = -sE_R \cos^2\left(\frac{\pi x}{d} + \varphi\right), \quad (7)$$

where s is the lattice depth in units of the recoil energy and E_R is the recoil energy (detailed experimental parameters and setup are provided in Appendix B), and the Hamiltonian is

$$\hat{H}_m = \frac{p^2}{2m} + V'(x). \quad (8)$$

Figure 2(a) shows the time sequence of the scheme, which is divided into five constrained segments. This segmentation represents a balance between achieving high loading efficiency and minimizing optimization time. The five segments correspond to different stages of switching the lattice potential on and off, with phase changes occurring at three yellow points to enhance loading efficiency.

III. METHOD AND RESULTS

In this section, based on the experimental setup outlined in Appendix B, we present the method and results of loading the fermionic atoms into higher orbital band. We model the entire optimization process as a black-box function, with the initial state occupying all quasimomentum states of s band. Then, we use the time evolution operator in Sec. II to describe the time evolution. Finally, the fidelity, representing the loading efficiency, is calculated according to the time sequences. To obtain the maximum fidelity, we optimize the time sequences

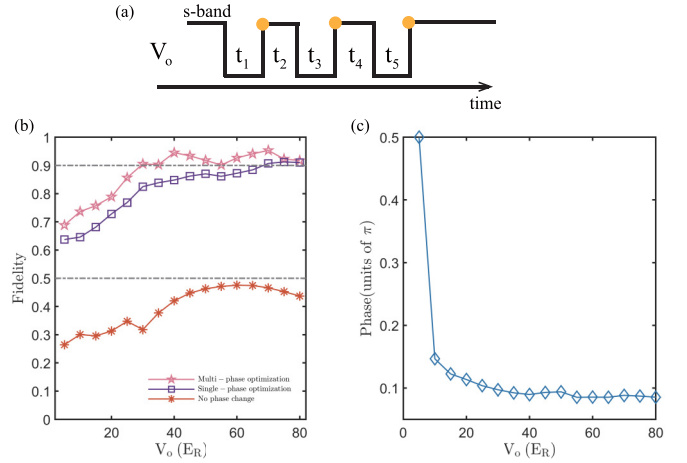


FIG. 2. Loading-efficiency optimization results for 27 sampled quasimomentum states. (a) The yellow dots represent the time points at which we optimized the phase. In the single-phase optimization, the three yellow dots correspond to the same phase, while in the multiphase optimization, each of the yellow dots represents a different phase. (b) The orange line represents the first scenario, with a maximum fidelity of approximately 48%. The purple line represents single-phase optimization, and the pink line represents multiphase optimization, with the latter reaching a peak fidelity of up to 95%. The vertical axis value of 1 corresponds to 100%, and this scale applies to all other cases as well. (c) For single-phase optimization, the optimized phase exhibits a monotonic decrease as a function of the lattice depth.

where the Global Optimization Toolbox in matlab [54] is used.

We prioritize time constraints because excessive off-times during loading can lead to atom loss through ballistic expansion. The loading protocol is divided into five segments with durations t_i ($i = 1, \dots, 5$), and each t_i is treated as an independent parameter in our global optimization. Since the lattice must be turned off during experiments, atoms undergo free ballistic expansion, so we limit the off-time to prevent atoms from escaping. Using the ballistic expansion formula $\sigma_z(t) = b_z(t)\sigma_z(t=0)$, where $\sigma_z(t)$ is the atom cloud width at time t and $b_z(t) = b_z(0)\sqrt{1 + (\omega_z t)^2}$ is the expansion factor, and considering a lattice size of $87 \mu\text{m}$, we determine that the maximum off-time for a lattice depth of $15 E_R$ is $400 \mu\text{s}$, with the maximum optimization time fixed at $226 \mu\text{s}$. This escape time depends on the axial confinement frequency ω_z , which increases with lattice depth and thus leads to shorter allowed off-times at higher depths. In our global optimization, the shortest off-time we obtained is on the order of $10 \mu\text{s}$, which is well within current experimental capabilities.

In the following, we conduct global optimization in three scenarios: the first with $\varphi = 0$ for all five time segments, the second with single-phase optimization, and the third with multiphase optimization. In the first scenario, the phase of all the five segments remains zero degree. We observe the loading efficiency increases from 20% to nearly 48% where the lattice depth is raised from $5 E_R$ to $60 E_R$. The loading efficiency slightly decreases at higher depths [Fig. 2(b), orange line]. However, even at its best in this scenario, the efficiency falls

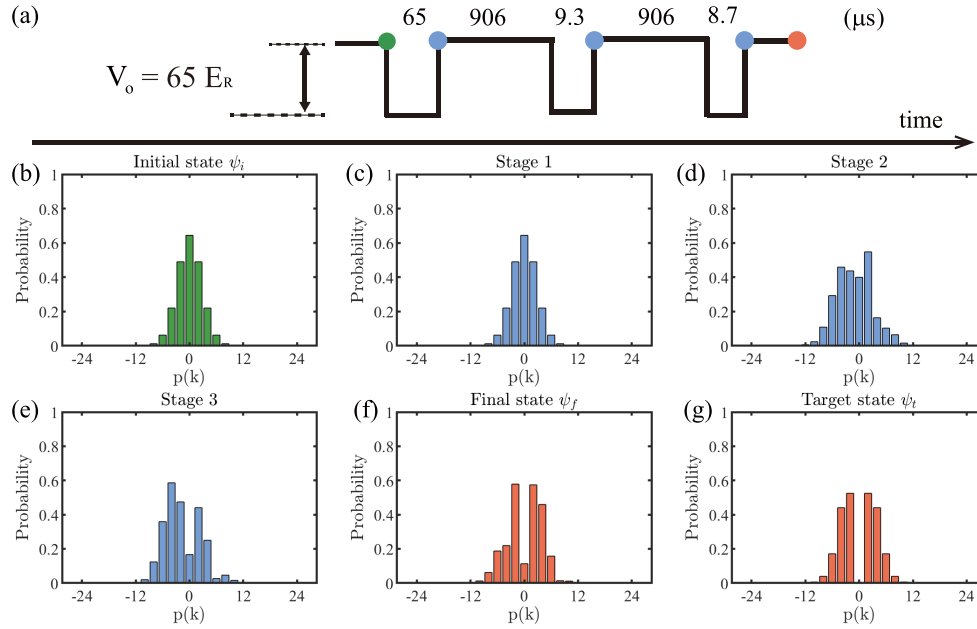


FIG. 3. Time Evolution of the $q = 0$ quasimomentum state at a lattice depth of $65 E_R$, and the fidelity is about 91.33%. (b)–(g) This corresponds to the time points marked with the same color in the above experimental sequence [panel (a)]. It can be observed that the final result is already close to the Bloch states on the p band. Here, $p(k)$ denotes the momentum of the plane-wave basis components used in the Bloch-state decomposition.

short of the best in the BEC case at Ref. [46], motivating us to explore phase optimization in the next scenario.

The second scenario is single-phase optimization, in which we optimize one fixed phase φ applied uniformly across all five time segments of the lattice potential. As shown by the purple line in Fig. 2(b), this yields a clear improvement in loading efficiency, reaching about 90% at its peak. Beyond a lattice depth of $40 E_R$, the efficiency gain saturates. This saturation occurs because, in deep lattices, the plane-wave composition of different Bloch states becomes nearly identical across quasimomentum, so further phase tuning has little effect. Figure 2(c) shows how the optimal phase varies with lattice depth: It is approximately $\pi/2$ at $5 E_R$ and decreases to around $\pi/10$ by $30 E_R$. For comparison, Appendix C presents the result of using a fixed phase of $3\pi/4$ (the value used for BEC p -band loading in Ref. [46]); this yields only $\sim 55\%$ efficiency for the Fermi gas, underscoring the importance of treating φ as a free optimization parameter.

The third scenario is to optimize the phase at each instance when the lattice potential is turned on. This multiphase optimization yields a maximum efficiency of nearly 95%, as shown by the pink line in Fig. 2(b), and outperforms the second scenario. The efficiency generally increases with the lattice depth, reflecting the better performance under the deeper potentials. It is noted that this result represents the best efficiency achievable within the constraints of our system, rather than the absolute theoretical limit. The high-dimensional parameter space and the inherent complexity of the black-box function make achieving the true global optimum challenging. Nevertheless, this result is significant for achieving a high-purity population of atoms in the p -band state.

The high fidelity observed in Fig. 2 stems from the robust fidelity of each quasimomentum state. To illustrate this, we

present an example using optimized parameters for the $q = 0$ state at a lattice depth of $65 E_R$, effectively demonstrating the loading process and its precision in reaching the target state. Figures 3(b)–3(g) illustrate this state’s time evolution, plotting the squared modulus of coefficients in the plane-wave basis across various stages, culminating in Fig. 3(f) for the final state and Fig. 3(g) for the target state; their comparison highlights the evolution’s precision and $p(k)$ denotes the momentum of the plane-wave basis components. Subplot colors denote distinct stages, aligned with the time sequence in Fig. 3(a). This visualization intuitively reveals how phase modulation and lattice switching shape the state’s composition. Over time, the initial state aligns closely with the target, achieving an average fidelity of 93.9% and a $q = 0$ state fidelity of 91.33%, as shown in Fig. 4(a).

Figure 4 tracks the time evolution of three quasimomentum states ($q = 0, \pm 26/27k$), with Figs. 4(b)–4(d) illustrating their fidelity trends and Figs. 4(e)–4(g) detailing the corresponding momentum components. Figure 4(a) indicates an average fidelity of 93% across these states, highlighting consistent improvement driven by phase modulation. In Figs. 4(b)–4(d), fidelity surges notably after each lattice-switching event, despite oscillations during lattice-on periods; these follow an upward trajectory, peaking in the final lattice-on segment. A black dashed line marks the time sequence, with interval proportions adjusted for clarity.

Figures 4(e)–4(g) show the corresponding momentum component amplitudes, which exhibit oscillatory behavior during the lattice-on segments and remain unchanged during the lattice-off segments. This steady behavior during the off segments aligns with the fact that plane-wave states are eigenstates of the system in the absence of lattice potential, resulting in no evolution during these intervals. The three selected plane-wave components constitute the dominant

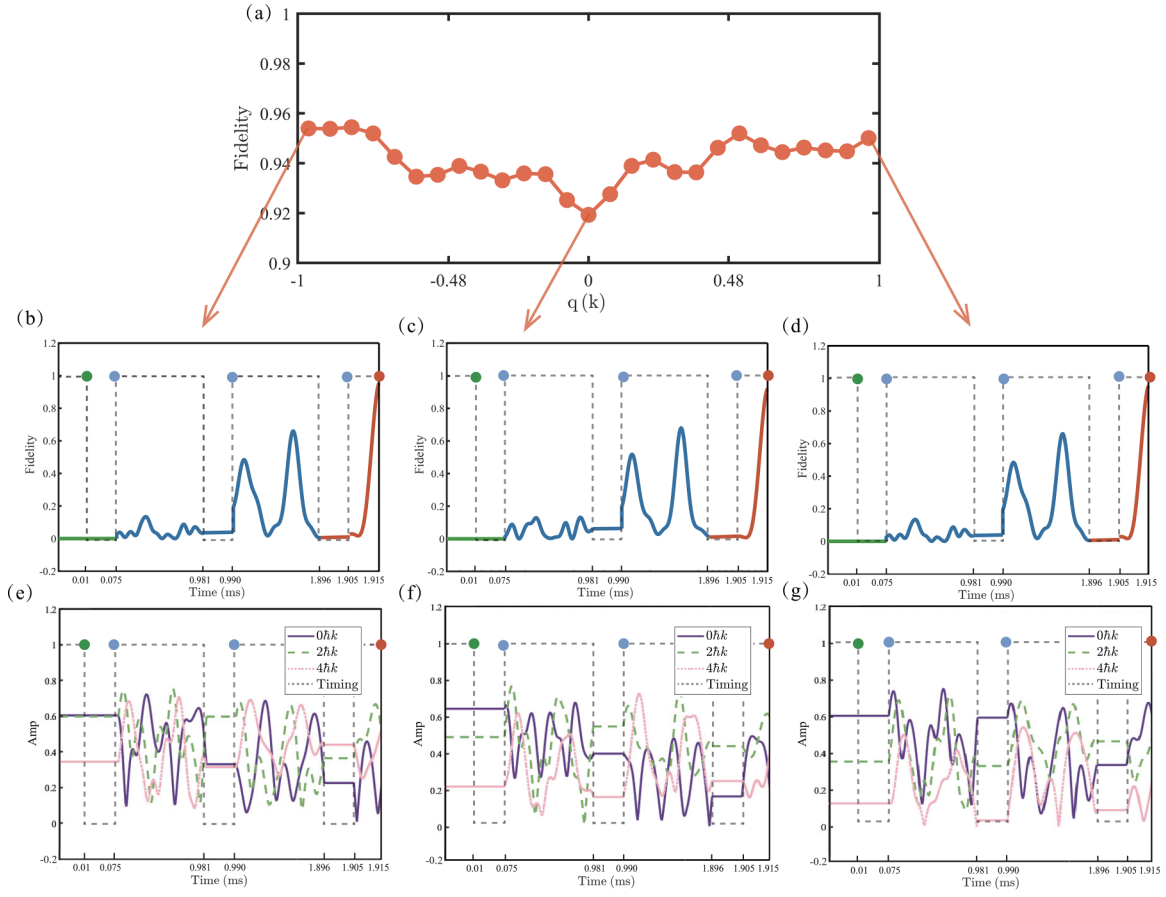


FIG. 4. Evolution of fidelity and momentum components for three quasimomentum states. (a) Fidelity of different quasimomentum states, with final optimized parameters derived as the average across these states. (b)–(g) Time evolution of fidelity and momentum component amplitudes for the three states, with top and bottom subfigures depicting the same state; red arrows in panel (a) point to the corresponding states in panels (b)–(d). The x axis is stretched to visually display both short and long sequence segments; tick marks indicate key cumulative times at the end of each segment, allowing the reader to relate the x axes to actual experimental times.

contributions to the final state, providing a clear demonstration of how phase modulation affects the state evolution. Together, these results emphasize the effectiveness of phase modulation in guiding the initial states toward the target state, significantly enhancing fidelity and ensuring a closer match with the desired high-orbital population.

IV. DISCUSSION

A. Fidelity depends on the number of quasimomentum states

With the multiphase optimization, we achieve a maximum loading efficiency of approximately 95% in two specific lattice depth, which is still a little bit lower than the theoretical result of the BEC case [46]. This is likely due to the occupation of multiple quasimomentum states for Fermi gas. To investigate this, we vary the number of occupied quasimomentum states Q_0 and perform separate optimizations for each case to obtain the corresponding best performance. In all of the data plotted in Fig. 2, we use $Q_0 = 27$, corresponding to one q point per lattice site in our finite-size model. When $Q_0 = 1$, the system occupies only the $q = 0$ state, similar to the BEC case. As shown in Fig. 5(a), for higher Q_0 , the efficiency initially decreases, then increases with lattice depth,

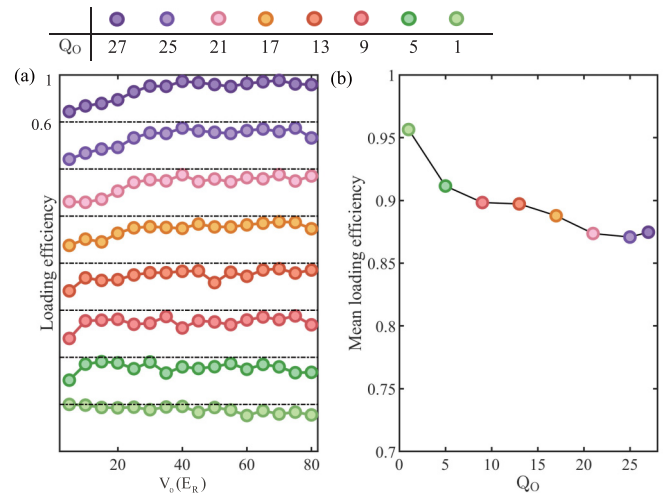


FIG. 5. The relationship between loading efficiency and the number of quasimomentum states Q_0 . (a) Illustrates the correlation between loading efficiency and lattice depth across various quasimomentum occupancies. (b) Plots average loading efficiency against Q_0 , revealing an inverse dependence on the number of occupied states, aligning with expected physical behavior. All subplots in panel (a) use a common y -axis scale.

suggesting that deeper lattices may improve efficiency for Fermi gases.

Figure 5(b) plots the average loading efficiency across all lattice depths, F_m , as a function of Q_O . At $Q_O = 1$, F_m peaks at 95%, but declines with increasing Q_O , suggesting that occupying multiple quasimomentum states hampers efficiency. Beyond $Q_O = 10$, F_m drops below 90%, reflecting the reduced loading efficiency of a Fermi gas compared to a BEC in shallow lattice, attributable to its broader momentum distribution.

B. Comparison with other loading schemes

To better illustrate the advantages of our approach, we compare our results with those from other research groups. We compare our results with those reported in Zhou *et al.*'s work [46]. Excluding the a_2 sequence in their study, our findings align well with other three cases, confirming the reliability of our simulation program.

We also compare our method with those used by other Fermi gas research groups. For three-dimensional optical lattices, Ref. [55] achieved an 85% loading efficiency into the p band using Raman transitions, completing the process in approximately 20 μ s. While their approach offers faster loading, it requires additional laser systems and results in lower efficiency compared to our approach.

For two-dimensional optical lattices, Ref. [33] used chemical potential adjustments to load atoms into high bands, achieving 15% efficiency in 300 ms, which is slower and less efficient than our method. In a similar context, Ref. [28] achieved 60% loading efficiency with population exchange and a superlattice, but this approach is more complex and less efficient than ours.

In a recent study, Ref. [42] extended their method to one-dimensional lattices, achieving 35% loading efficiency into the p band. While the loading time was not specified, it is likely to be on the order of hundreds of ms, further demonstrating the higher efficiency of our method.

We summarize the comparison results in Fig. 6, including the BEC case. Our loading method consistently outperforms alternatives in terms of efficiency—achieving up to 95% under optimal conditions (see Fig. 2, where all efficiencies exceed 60%, and a peak of 95% is reached at two specific lattice depths)—while also excelling in loading time, second only to the Raman transition technique. This is particularly advantageous for experiments studying the lifetime of atoms in high-orbital states.

Our approach is especially effective for one-dimensional optical lattices and can be adapted to other lattice configurations with minimal modifications. Our method efficiently loads Bloch states even with large momentum spreads, offering a more comprehensive solution for creating quantum states across a range of systems.

V. SUMMARY

In this work, we introduce a protocol for loading spin-polarized ultracold Fermi gases into the p band of a one-dimensional optical lattice. By formulating the sequence as a global optimization problem and leveraging matlab's optimization toolbox to jointly tune pulse timings and

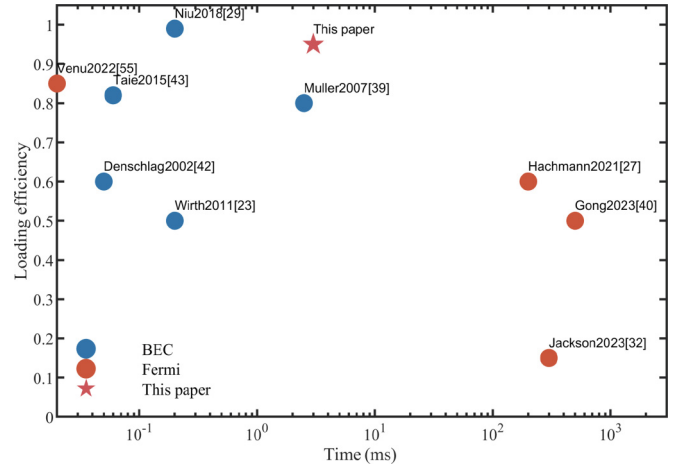


FIG. 6. We have summarized the work of several groups, the red dots represent the Fermi systems [28,33,41,55], while the blue dots represent the BEC systems [24,30,40,43,44]. The red pentagram represents this work, and it is evident that our approach achieves the highest loading efficiency within the Fermi system, with relatively short loading times. Our loading efficiency exceeds 60% at every lattice depth investigated and peaks at 95% for two specific depths. In contrast, while the BEC system achieves even higher loading efficiency, it also benefits from shorter loading times. Some loading-time values were not specified in the original references and are shown here as estimates.

lattice phase shifts, we surmount the challenges posed by the gas's broad quasimomentum distribution. Treating the phase as an independent optimization parameter yields a pronounced boost in loading efficiency that grows with lattice depth. Finally, we explore the role of quasimomentum occupancy, demonstrating that reducing the number of populated modes—approaching the narrow-momentum limit of a BEC—further enhances transfer fidelity.

Our theoretical analysis shows that lattice phase influences atom loading, even with broad momentum distributions, due to the distinct parities of Bloch states. These findings contribute to understanding loading techniques for ultracold Fermi gases in optical lattices. Although we illustrate our phase-optimized loading strategy in a one-dimensional, non-interacting lattice, the approach readily generalizes to higher dimensions whenever the dynamics factorize. Moreover, since many experiments first load atoms into noninteracting bands before introducing interactions, our protocol is immediately applicable to a wide range of realistic settings.

ACKNOWLEDGMENTS

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DATA AVAILABILITY

The data analyzed in this study include results from previous works, as cited in Refs. [24,28,30,33,40,41,43,44,55]. Additional datasets generated during this study are available from the corresponding author upon reasonable request.

APPENDIX A: PLANE-WAVE DECOMPOSITION METHOD

In our setup, the one-dimensional optical lattice is created using two laser beams that form an interference pattern with a specific angle between them, resulting in a simplified optical lattice potential

$$V(x) = -sE_R \cos^2\left(\frac{\pi x}{d}\right) = \sum_G V_G e^{iGx}, \quad (\text{A1})$$

where $E_R = \hbar^2(2\pi \sin \phi)^2/(2m\lambda^2)$ is the recoil energy, ϕ is angular separation of two laser beams, λ is the laser wavelength, and d is the lattice constant. The latter part of this expression can be understood as decomposing the periodic structure into a series of superimposed plane waves, $G = 2l\pi/d$ is reciprocal lattice vectors, $l = -\frac{N_s-1}{2}, \dots, \frac{N_s-1}{2}$ with N_s (odd) the number of lattice sites, and V_G is the intensity coefficient for different plane wave.

Although Bloch states originate from a single-particle model, we model noninteracting Fermi gases as independent single particles. Thus, the Hamiltonian can be expressed as follows, with $\hbar = 1$:

$$\left(\frac{1}{2m}p^2 + \sum_G V_G e^{iGx}\right)\psi(x) = \varepsilon\psi(x). \quad (\text{A2})$$

The solutions to this Schrödinger equation are known as Bloch states, which can also be expressed as a superposition of plane waves:

$$\begin{aligned} \psi(x) &= |n, q\rangle = u_{n,q}(x)e^{iqx} \\ &= \sum_G C_{q+G}^n \frac{e^{i(q+G)x}}{\sqrt{N_s d}} = \sum_l c_{n,q,l} |2lk + q\rangle, \end{aligned} \quad (\text{A3})$$

where n is the band index, $|2lk + q\rangle$ presents plane-wave basis for different reciprocal lattice vectors, $k = \pi/d$ is the wave number of lattice light, $d = \lambda/(2 \sin \theta)$ is the lattice constant, λ is the wavelength of laser, and q is the quasimomentum, restricted to the values of $q = 2m\pi/(N_s d)$, $m = -\frac{N_s-1}{2}, \dots, \frac{N_s-1}{2}$. Each plane wave has a wave vector corresponding to the quasimomentum and reciprocal lattice vector, i.e., $p = 2l\pi/d + q$, subsequently, we describe the entire physical process in this basis.

APPENDIX B: EXPERIMENT SETUP

Our experimental scheme aims to investigate high-orbital loading in an optical lattice, focusing on precise control of the lattice phase to alter the parity of Bloch states. The current experimental setup consists of ultracold lithium-6 atoms [41,56], with the optical lattice constructed using lasers with a wavelength $\lambda = 1064$ nm. The two laser beams forming the optical lattice intersect at an angle of $2\theta = 20^\circ$, resulting in a lattice constant $d = 3.06 \mu\text{m}$ and the

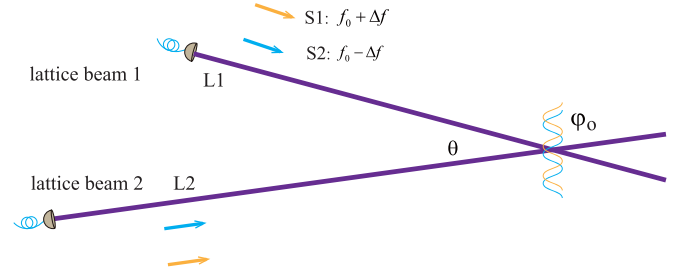


FIG. 7. The modified optical setup, creating a fixed phase difference to meet the requirements for phase adjustment.

recoil energy $E_R = (2\pi \hbar \sin \theta)^2/2m\lambda^2 \approx 883$ Hz, the maximum lattice-depth value is $127.3 E_R$. In this optical lattice, each lattice site forms a pancake-type trap, where the trap frequencies in the x , y , and z directions satisfy $\omega_x:\omega_y:\omega_z \approx 5.8:1:514$.

To modulate the optical lattice phase, we introduce an optical path difference between the two laser beams by horizontally displacing them along the lattice axis. The phase shift is given by the equations

$$\phi_1 = k_1(L_1 - L_2), \quad \phi_2 = k_2(L_1 - L_2), \quad (\text{B1})$$

where k_1 and k_2 are the wave vectors, and L_1 and L_2 are the optical paths of the two lattice beams. The phase of the lattice potential can be optimized by adjusting the laser frequency using an acousto-optic modulator [57], as shown in Fig. 7.

APPENDIX C: CASE OF FIXED PHASE CHANGE TO $3\pi/4$

Based on the theoretical model, we modulated the optical lattice phase to $3\pi/4$ and performed the simulation. As shown by the yellow line in Fig. 8(b), the maximum loading efficiency reaches 53% at shallow lattice depths and gradually decreases, stabilizing around 40%. This contrasts with the first scenario, indicating that phase modulation significantly impacts loading efficiency across the full range of lattice depths.

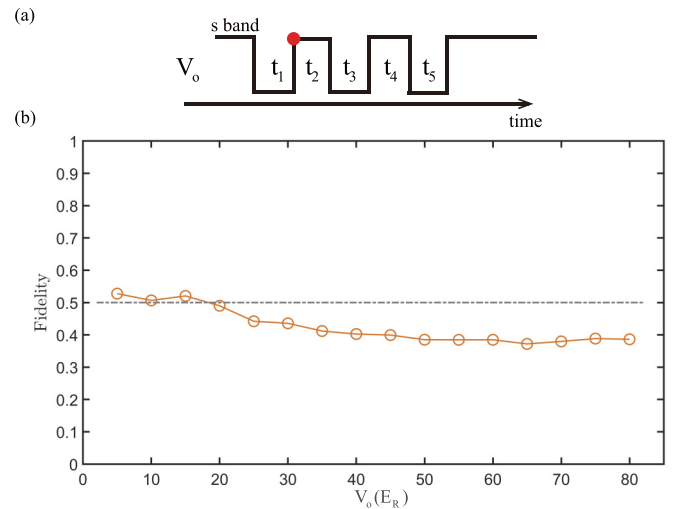


FIG. 8. The optimization results for the fixed phase of $3\pi/4$ are shown. (a) Time sequence with phase transitions marked by red dots. (b) Optimized fidelity plotted against lattice depth.

Comparing this with the first case, fixing the phase at $3\pi/4$ enhances loading efficiency at lower lattice depths but reduces it at higher depths, with both scenarios intersecting at $V_0 \approx 37 E_R$. This suggests that different loading

strategies depend on lattice depth, and phase alterations play a crucial role. However, simply adjusting a fixed phase does not yield optimal results, highlighting the need for phase optimization.

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