

Nonreciprocal multiphoton bundle emission in a spinning resonator

Xue Gao, Jun Xu^{✉,*} and Xiangming Hu^{✉,†}*College of Physical Science and Technology, Central China Normal University, Wuhan 430079, People's Republic of China*

(Received 3 February 2026; accepted 8 July 2026; published 22 July 2026)

We propose a scheme to realize nonreciprocal multiphoton bundle emission in a spinning resonator. The system consists of a rotating two-mode cavity directly coupled to a two-level atom, enabling efficient excitation of the desired multiphoton bundle states. These photon bundles originate from leapfrog transitions between dressed states, established through the coherent atom-cavity coupling under multiphoton resonance conditions. When the rotating resonator is driven in different directions, it generates opposite Sagnac-Fizeau shifts, making the resonant conditions dependent on the direction. As a consequence, distinct multiphoton transition channels can be selectively activated by tuning the driving direction and frequency. The nonreciprocal two-photon or three-photon bundle emission with the same and different modes can be implemented with a high tunability. This scheme provides a feasible route toward direction-controlled multiphoton emitters with potential applications in chiral quantum information processing.

DOI: [10.1103/nlpp-vwbg](https://doi.org/10.1103/nlpp-vwbg)

I. INTRODUCTION

Nonreciprocity originates from the breaking of time-reversal symmetry and constitutes a fundamental physical mechanism for achieving directional selectivity of propagation and one-way information transport. In optical systems, it plays a crucial role in devices such as isolators, circulators, and directional amplifiers [1–3]. To date, nonreciprocal transport has been achieved through the following core physical mechanisms. Breaking of time-reversal symmetry, induced by magnetic biasing or mechanical rotation, provides genuine nonreciprocity and has been widely explored in magneto-optical and spinning systems [4–9]. Time-dependent modulation and synthetic gauge fields enable nonreciprocal transmission and circulation without static magnetic fields [10–20]. Nonlinearity-induced asymmetric responses have been demonstrated in nonlinear photonic and few-photon atom-cavity systems [21–24]. Dissipation engineering and non-Hermitian interference offer an alternative route to nonreciprocity via asymmetric coherent and dissipative couplings [25–30]. Chiral light-matter interactions further enable direction-dependent coupling and nonreciprocal transport at the single-photon level [31–36]. Such directionality is particularly important for quantum information processing, because the emission, absorption, and scattering of single photons can be controlled by their propagation direction. It provides a natural mechanism for realizing nonreciprocal single-photon

devices, deterministic spin-photon interfaces, and cascaded quantum networks, which are relevant to quantum-state transfer, quantum routing, and integrated quantum photonic circuits [3,37–39]. More recently, nonreciprocity has been extended into the quantum regime, giving rise to directional control of quantum noise and amplification [40,41], as well as genuinely quantum phenomena such as nonreciprocal photon (magnon) blockade [42–47] and nonreciprocal entanglement [48–51].

The generation and control of strongly correlated multiquanta states constitute one of the central problems in quantum optics and quantum information science, playing a fundamental role in precision measurements beyond classical limits, the preparation of nonclassical light fields, and the construction of quantum devices [52–55]. Against this backdrop, N -photon bundle emission has attracted increasing attention in recent years as a quantum radiation mechanism capable of producing strongly correlated multiphoton states within a continuous dynamical process [56]. This process typically involves coherent coupling between the ground state and highly excited Fock states and, under appropriate conditions, leads to the formation of periodic multiphoton super-Rabi dynamics. Building on this physical mechanism, multiphoton bundle emission has been investigated in several physical platforms, including cavity and waveguide quantum electrodynamics (cQED and wQED) [57–66], cavity optomechanics [67,68], strongly interacting Rydberg atomic systems [69–72], hybrid magnonic systems [73–77], semiconductor quantum dot system [78], and superconducting microwave resonators [79–83]. Building on these advances, nonreciprocal multiphoton bundle emission has attracted increasing interest due to its potential applications in one-way quantum information processing and chiral quantum networks. Recently, Bin *et al.* proposed a theoretical scheme to realize directional emission of entangled photon-phonon and photon-magnon pairs in a spinning hybrid cavity-atom system via

*Contact author: junxu@ccnu.edu.cn†Contact author: xmhu@ccnu.edu.cn

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International](https://creativecommons.org/licenses/by/4.0/) license. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI.

the Sagnac effect [84]. However, their scheme has several disadvantages. First, its implementation relies on a rather involved hybrid platform combining optical, mechanical, and magnonic modes. Second, the population of the target bundle states remains relatively low due to the indirect atom-phonon (magnon) coupling. Third, the associated super-Rabi dynamics requires stringent multiparameter resonance conditions. These factors posed significant challenges to the experimental implementation.

In this paper, we propose an effective scheme to realize nonreciprocal multiphoton bundle emission in a rotating cavity coupled to a two-level atom. The cavity rotation induces a Fizeau-Sagnac frequency shift between the counterpropagating modes [9], making the resonance conditions dependent on the propagation direction of the driving field. The direction-dependent resonance enables selective excitation of two-photon and three-photon leapfrog transitions, giving rise to directional emission of strongly correlated photon bundles, including both same-mode and hybrid-mode bundles. Our scheme provides several distinct advantages over previous schemes: (1) The system is based on the direct atom-cavity coupling, which enables more efficient preparation of the target bundle states. (2) The nonreciprocal two-photon bundle emission with high purity can be implemented and it can be further extended to the nonreciprocal three-photon bundle emission. (3) The proposal is convenient and highly tunable, allowing flexible control of multiphoton bundle generation via the driving direction and frequency. This scheme offers a practical platform for direction-controlled multiphoton emitters, with potential applications in chiral quantum information processing, directional quantum communication, and nonreciprocal quantum photonic devices.

The remaining part of this paper is organized as follows. In Sec. II, we introduce the model and master equation. In Sec. III, we focus on nonreciprocal two-photon bundle emission, including the analysis of super-Rabi oscillations and dissipation-triggered directional bundle generation. In Sec. IV, we extend the study to nonreciprocal three-photon bundle emission. Finally, a conclusion is given in Sec. V.

II. MODEL AND EQUATIONS

As illustrated in Fig. 1, we consider a rotating optical resonator supporting two independent optical modes, a and b , with frequencies ω_a and ω_b . A two-level atom with states $|g\rangle$ and $|e\rangle$ is coherently coupled to both modes, with interaction strengths g_a and g_b . For a resonator spinning at an angular velocity Ω , the circulating optical modes experience Sagnac-Fizeau frequency shifts Δ_{F_j} ($j = a, b$), i.e., $\omega_j \rightarrow \omega_j + \Delta_{F_j}$, with Ref. [85]

$$\Delta_{F_j} = \pm \frac{nr\Omega\omega_j}{c} \left(1 - \frac{1}{n^2} - \frac{\lambda}{n} \frac{dn}{d\lambda} \right), \quad (1)$$

where n is the refractive index, r is the radius of the cavity, and c and λ denote the speed and wavelength of light in vacuum, respectively. The factor in parentheses corresponds to the Fresnel-Fizeau drag coefficient, including the Lorentz dispersion correction term $dn/d\lambda$, which is typically negligible in common optical materials. For convenience, we scale all frequencies by the atom-cavity coupling strength g , i.e.,

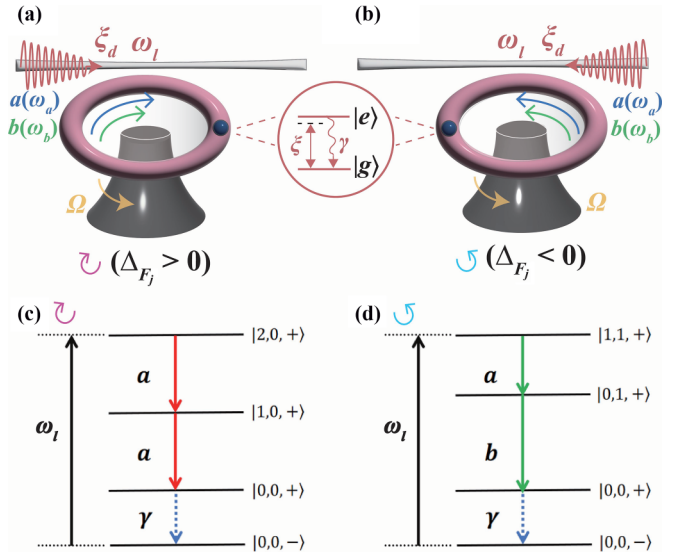


FIG. 1. Schematic of the spinning cQED system and the energy level structure diagram under the two-photon resonance condition. The system states are labeled as $|n_a, n_b, \pm\rangle$, where n_a and n_b denote the photon numbers in cavity modes a and b , respectively, and $|\pm\rangle$ represent the atomic dressed states. (a) Driving from the left side ($\Delta_{F_j} > 0$) and (b) driving from the right side ($\Delta_{F_j} < 0$). (c) Under the left-side driving, the solid red line corresponds to the a^2 two-photon bundle emission (two photons in mode a). (d) Under the right-side driving, the solid green line corresponds to the ab two-photon bundle emission (one photon in each mode a and b). The dotted blue lines indicate a flip of the atomic dressed state from $|-\rangle$ to $|+\rangle$.

$g_a = g_b = g$, which defines the unit of frequency throughout this work. As indicated by Eq. (1), the rotation of the cavity leads to frequency shifts that are linear in Ω and have opposite signs for counterpropagating optical modes. To illustrate the dependence of the Fizeau shifts on the rotation angular velocity, Fig. 2 plots the normalized frequency shifts

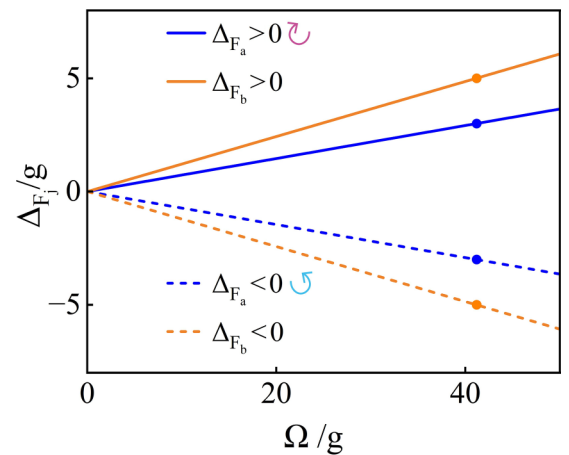


FIG. 2. The frequency shifts Δ_{F_j}/g ($j = a, b$) as functions of the rotation angular velocity Ω/g of the optical microcavity, where all frequencies are normalized to the atom-cavity coupling strength g (with $g_a = g_b = g$). The blue and orange solid lines correspond to $\Delta_{F_j} > 0$, while the dashed lines indicate $\Delta_{F_j} < 0$. The four marked points correspond to $\Delta_{F_a} = \pm 3g$ and $\Delta_{F_b} = \pm 5g$ at $\Omega = 41.2g$.

Δ_{F_j}/g ($j = a, b$) as functions of the normalized rotation angular velocity Ω/g . The parameters are chosen as $r = 30 \mu\text{m}$, $n = 1.4$, and $c = 3.0 \times 10^8 \text{m/s}$. The wavelength of mode b is $\lambda_b = 1064 \text{nm}$, and that of mode a is $\lambda_a = (5/3)\lambda_b$, with $\omega_j = 2\pi c/\lambda_j$ ($j = a, b$). For convenience, we fix the cavity rotation direction as counterclockwise, such that $\Delta_{F_j} < 0$ for corotating modes and $\Delta_{F_j} > 0$ for counterpropagating modes. Under the dipole and rotating-wave approximations, the system Hamiltonian is given by ($\hbar = 1$)

$$H = \omega_{eg}\sigma_{ee} + (\omega_a + \Delta_{F_a})a^\dagger a + (\omega_b + \Delta_{F_b})b^\dagger b + g(a\sigma_{eg} + a^\dagger\sigma_{ge}) + g(b\sigma_{eg} + b^\dagger\sigma_{ge}) + \xi(\sigma_{eg}e^{-i\omega_l t} + \sigma_{ge}e^{i\omega_l t}) + \xi_d(a^\dagger e^{-i\omega_l t} + a e^{i\omega_l t}). \quad (2)$$

Here, a ($a^\dagger$) and b ($b^\dagger$) are the annihilation (creation) operators of the cavity modes. ω_{eg} represents the transition frequency between the excited states $|e\rangle$ and the ground state $|g\rangle$. ξ denotes the Rabi frequency of the classical field driving the two-level atom. $\xi_d = \sqrt{2P\kappa_a}/\omega_l$ is the amplitude of the external cavity driving field, where P is the driving power, ω_l is the driving frequency, and κ_a is the loss rate of mode a . In our model, the external driving field ξ_d is injected into the cavity through a specific input channel that couples to mode a . As a result, only mode a is directly driven, while mode b is populated indirectly through the atom-cavity interaction. $\sigma_{ij} = |i\rangle\langle j|$ ($i, j = g, e$) describe the projection operators when $i = j$ and the spin-flip operators when $i \neq j$.

In the rotating frame $H_r = \omega_l(\sigma_{ee} + a^\dagger a + b^\dagger b)$, the total Hamiltonian of the system can be rewritten as

$$\tilde{H} = \Delta_e\sigma_{ee} + (\Delta_a + \Delta_{F_a})a^\dagger a + (\Delta_b + \Delta_{F_b})b^\dagger b + g(a\sigma_{eg} + a^\dagger\sigma_{ge}) + g(b\sigma_{eg} + b^\dagger\sigma_{ge}) + \xi(\sigma_{eg} + \sigma_{ge}) + \xi_d(a + a^\dagger), \quad (3)$$

where $\Delta_e = \omega_{eg} - \omega_l$ and $\Delta_k = \omega_k - \omega_l$ (for $k = a, b$) denote the detunings of the atomic transition and the cavity modes from the driving field, respectively. In the following sections, we analyze the statistical properties of the two independent optical modes and investigate their nonreciprocal photon bundle emission behavior.

In this rotating frame, the system dynamics are described by a Lindblad master equation [53,54]

$$\dot{\rho} = -i[\tilde{H}, \rho] + \frac{\kappa_a}{2}\mathcal{L}[a]\rho + \frac{\kappa_b}{2}\mathcal{L}[b]\rho + \frac{\gamma}{2}\mathcal{L}[\sigma_{ge}]\rho, \quad (4)$$

with $\mathcal{L}[o]\rho = 2o\rho o^\dagger - o^\dagger o\rho - \rho o^\dagger o$. Here, κ_j ($j = a, b$) and γ denote the decay rates of the cavity modes and the atom, respectively. In general, Eq. (4) can be numerically solved by using the QUTIP package in Python [86].

In the strong driving field limit of $\xi \gg g$, the dressed-state picture can be employed to interpret the possible transition channels. By diagonalizing the Hamiltonian $\tilde{H}_a = \Delta_e\sigma_{ee} + \xi(\sigma_{eg} + \sigma_{ge})$, the two dressed states can be expressed in terms of the bare states as [87]

$$|+\rangle = \cos\theta|e\rangle + \sin\theta|g\rangle, \\ |-\rangle = \cos\theta|g\rangle - \sin\theta|e\rangle, \quad (5)$$

where $\cos\theta = \sqrt{(\bar{\xi} + \Delta_e)/2\bar{\xi}}$, $\sin\theta = \sqrt{(\bar{\xi} - \Delta_e)/2\bar{\xi}}$, and $\bar{\xi} = \sqrt{\Delta_e^2 + 4\xi^2}$. These dressed states $|\pm\rangle$ have their eigenvalues $\varepsilon_\pm = (\Delta_e \pm \sqrt{\Delta_e^2 + 4\xi^2})/2$. Correspondingly, the Hamiltonian H_a is rewritten as $\tilde{H}_a = \varepsilon_+\sigma_{++} + \varepsilon_-\sigma_{--}$. According to the dressed-state picture, the possible spontaneous transition channels and corresponding frequency conditions can be characterized by an effective total detuning Δ . Different multiphoton emission processes generally involve distinct values of Δ , which will be specified in detail in the following discussion. The main channels are given by

$$|+\rangle \rightarrow |-\rangle : \Delta = \bar{\xi}, \\ |-\rangle \rightarrow |+\rangle : \Delta = -\bar{\xi}. \quad (6)$$

As an example, the $|+\rangle \rightarrow |-\rangle$ transition under the two-photon resonance condition is illustrated in Figs. 1(c) and 1(d), showing two-photon emission either entirely in mode a (red solid line) or split between modes a and b (green solid line). The system states are labeled as $|n_a, n_b, \pm\rangle$, where n_a and n_b denote the photon numbers in cavity modes a and b , respectively, and $|\pm\rangle$ represent the atomic dressed states. Subsequently, the cavity mode in the two-photon excited state emits two photons and returns to vacuum, while the atomic dressed state flips from $|-\rangle$ to $|+\rangle$ (dotted blue lines).

III. NONRECIPROCAL TWO-PHOTON BUNDLE EMISSION

In this section, we investigate the generation of nonreciprocal two-photon bundle emission in a rotating optical microcavity. Owing to the Sagnac effect, the cavity resonance frequencies undergo direction-dependent Fizeau shifts, giving rise to optical nonreciprocity. In the following, we fix the cavity rotation direction as counterclockwise. In this case, the Fizeau shift $\Delta_{F_j} > 0$ ($\Delta_{F_j} < 0$) corresponds to driving the resonator from the left (right) side.

A. Super-Rabi oscillations

In the dressed-state picture, the total Hamiltonian of the atom-cavity system can be decomposed as $\tilde{H} = H_0 + V$, where

$$H_0 = \Delta'_a a^\dagger a + \Delta'_b b^\dagger b + \varepsilon_+\sigma_{++} + \varepsilon_-\sigma_{--}, \\ V = g(a + b)(\sin\theta \cos\theta \sigma_{++} + \cos^2\theta \sigma_{+-} - \sin^2\theta \sigma_{-+} - \sin\theta \cos\theta \sigma_{--}) + \text{H.c.} \quad (7)$$

Here, the cavity driving term has been neglected due to the weak cavity driving, i.e., $\xi_d \ll g$, and we have defined $\Delta'_a = \Delta_a + \Delta_{F_a}$ and $\Delta'_b = \Delta_b + \Delta_{F_b}$. In the following, we focus on the dispersive regime $g \ll |\Delta'_{a,b}|$, where the off-resonant single-photon couplings are much smaller than the relevant detunings. Then, an effective Hamiltonian can be obtained by performing a Schrieffer-Wolff transformation [88], $H_{\text{eff}} = e^{-\lambda S}(H_0 + \lambda V)e^{\lambda S}$, where λ is introduced as a formal perturbation parameter by replacing V with λV and is set to $\lambda = 1$ after the perturbative expansion. The non-Hermitian operator

S is given by

$$S = ga \left(\frac{\sin \theta \cos \theta}{\Delta'_a} \sigma_{++} + \frac{\cos^2 \theta}{\Delta'_a - \bar{\xi}} \sigma_{+-} - \frac{\sin^2 \theta}{\Delta'_a + \bar{\xi}} \sigma_{-+} - \frac{\sin \theta \cos \theta}{\Delta'_a} \sigma_{--} \right) + gb \left(\frac{\sin \theta \cos \theta}{\Delta'_b} \sigma_{++} + \frac{\cos^2 \theta}{\Delta'_b - \bar{\xi}} \sigma_{+-} - \frac{\sin^2 \theta}{\Delta'_b + \bar{\xi}} \sigma_{-+} - \frac{\sin \theta \cos \theta}{\Delta'_b} \sigma_{--} \right) - \text{H.c.}, \quad (8)$$

which satisfies the condition $V + [H_0, S] = 0$. In terms of the Baker-Campbell-Hausdorff (BCH) formula, we expand H_{eff} to second order in λ as

$$H_{\text{eff}} = H_0 + \lambda V - \lambda [S, H_0] - \lambda^2 [S, V] + \frac{\lambda^2}{2} [S, [S, H_0]] + O(\lambda^3) = H_0 - \frac{1}{2} [S, V] + O(\lambda^3). \quad (9)$$

Then, we consider a multiphoton resonance condition of the form

$$\Delta = m\Delta'_a + n\Delta'_b = \bar{\xi}, \quad (10)$$

where m and n are non-negative integers. This condition corresponds to an $m + n$ photon resonance, where m and n denote the number of photons emitted into modes a and b , respectively. First, we consider the two-photon resonance case, corresponding to $(m, n) \in \{(2, 0), (1, 1), (0, 2)\}$, corresponding to the emission processes a^2 (two photons in mode a), ab (one photon in each mode a and b), and b^2 (two photons in mode b), respectively. Under these conditions, the system can be described by the following second-order effective interaction Hamiltonians:

$$\begin{aligned} H_{\text{eff}}^{aa} &= g_{\text{eff},1} a^2 \sigma_{+-} + \text{H.c.} = \sum_{p,q=0}^{\infty} g_{\text{eff},1} \sqrt{(p+1)(p+2)} |p, q, +\rangle \langle p+2, q, -| + \text{H.c.}, \\ H_{\text{eff}}^{ab} &= g_{\text{eff},2} ab \sigma_{+-} + \text{H.c.} = \sum_{p,q=0}^{\infty} g_{\text{eff},2} \sqrt{(p+1)(q+1)} |p, q, +\rangle \langle p+1, q+1, -| + \text{H.c.}, \\ H_{\text{eff}}^{bb} &= g_{\text{eff},3} b^2 \sigma_{+-} + \text{H.c.} = \sum_{p,q=0}^{\infty} g_{\text{eff},3} \sqrt{(q+1)(q+2)} |p, q, +\rangle \langle p, q+2, -| + \text{H.c.} \end{aligned} \quad (11)$$

The coupling coefficients are defined as

$$\begin{aligned} g_{\text{eff},1} &= -Ag^2 \left(\frac{1}{\Delta'_a} - \frac{1}{\Delta'_a - \bar{\xi}} \right), \\ g_{\text{eff},2} &= -Ag^2 \left(\frac{1}{\Delta'_a} - \frac{1}{\Delta'_a - \bar{\xi}} + \frac{1}{\Delta'_b} - \frac{1}{\Delta'_b - \bar{\xi}} \right), \\ g_{\text{eff},3} &= -Ag^2 \left(\frac{1}{\Delta'_b} - \frac{1}{\Delta'_b - \bar{\xi}} \right), \end{aligned} \quad (12)$$

with $A = \cos^3 \theta \sin \theta$. Here, the direct product state $|p, q, i\rangle = |p\rangle \otimes |q\rangle \otimes |i\rangle$ denotes the state with photon occupancy $|p\rangle$ in mode a , the photon occupancy $|q\rangle$ in mode b when the atom stays in the state $|i\rangle$. Each effective interaction Hamiltonian $H_{\text{eff}}^{o'o'}$ ($o, o' = a, b$) describes a physical process in which the dressed transition from $|+\rangle$ to $|-\rangle$ (or from $|-\rangle$ to $|+\rangle$) is accompanied by the successive emission (or absorption) of two photons.

To characterize the statistical properties of the two-photon emission processes, we introduce the time-delayed second-order correlation function, as defined in Ref. [89]:

$$g_{1,oo'}^{(2)}(\tau) = \frac{\langle o^\dagger(0) o'^\dagger(\tau) o'(\tau) o(0) \rangle}{\langle o^\dagger(0) o(0) \rangle \langle o'^\dagger(\tau) o'(\tau) \rangle}, \quad (o, o' = a, b). \quad (13)$$

Here, the function corresponds to the autocorrelation for $o = o'$ and to the cross-correlation between the two modes for $o \neq o'$. In our numerical calculations, the steady-state density matrix ρ_s is obtained by solving the Lindblad master equation

$\dot{\rho}_s = 0$ based on Eq. (4). In the following, we first focus on the equal-time case $\tau = 0$. Figure 3 shows the logarithm of $g_{1,oo'}^{(2)}(0)$ ($o, o' = a, b$) as a function of the detunings Δ_a/g and Δ_b/g . In these contour plots, regions with warm colors denote strong correlation effects with $g_{1,oo'}^{(2)}(0) \gg 1$, indicating superbunching behavior. Figure 3(a) corresponds to the left-side driving, while Figs. 3(b) and 3(c) show the results for the right-side driving. Here, the two driving directions correspond to opposite signs of the Fizeau shifts Δ_{F_j} in the Hamiltonian, which modify the effective cavity detunings and resonance conditions. It is clearly seen that three prominent transition lines emerge in all panels, along which the correlation function undergoes sharp variations. These lines correspond to the two-photon resonance conditions given by Eq. (10) for the integer pairs $(m, n) \in \{(2, 0), (1, 1), (0, 2)\}$, respectively. In addition to the resonance lines, two characteristic intersection points are highlighted in Fig. 3. The white pentagram marks the common intersection shared by Figs. 3(a) and 3(b), indicating that under the same detuning parameters, reversing the driving direction induces distinct antibunching behaviors. Specifically, the left-side driving produces strong antibunching in the mode a , whereas the right-side driving results in antibunching in mode b . Similarly, the black filled circle denotes the intersection between Figs. 3(a) and 3(c). At this parameter point, the left-side driving yields antibunching in mode a , while the right-side driving activates antibunching in the hybrid-mode correlation, as characterized by $g_{1,ab}^{(2)}(0)$. Without loss of generality, we assume identical

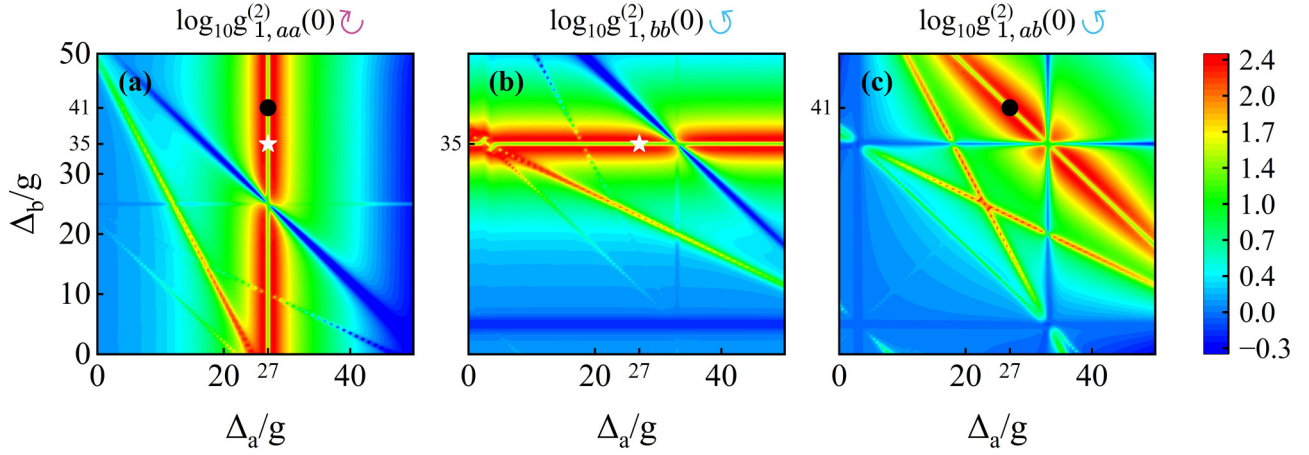


FIG. 3. The logarithm of equal-time second-order correlation function $g_{1,oo'}^{(2)}(0)$ ($o, o' = a, b$) as a function of detunings Δ_a/g and Δ_b/g . The white pentagram and black filled circle denote the intersection points of the resonance lines, corresponding to $(\Delta_a = 27g, \Delta_b = 35g)$ and $(\Delta_a = 27g, \Delta_b = 41g)$, respectively. The other parameters are $\xi = 30g$, $\xi_d = 0.02g$, $\Omega = 41.2g$, $\kappa = 0.1g$, $\gamma = 0.01g$, and $\Delta_e = 0$.

coupling strengths and decay rates for the two cavity modes, i.e., $g_a = g_b = g$ and $\kappa_a = \kappa_b = \kappa$. Other parameters are chosen as $\xi = 30g$, $\xi_d = 0.02g$, $\kappa = 0.1g$, $\gamma = 0.01g$, and $\Delta_e = 0$. In addition, the rotation angular velocity is chosen as $\Omega = 41.2g$, yielding the frequency shifts $\Delta_{F_a} = \pm 3g$ and $\Delta_{F_b} = \pm 5g$, where $\Delta_{F_j} > 0$ ($\Delta_{F_j} < 0$) corresponds to driving the resonator from the left (right) side.

To gain deeper insight into the mechanisms underlying the observed photon correlation patterns, we investigate the super-Rabi oscillations. Here, super-Rabi oscillations refer to coherent population oscillations between a zero-photon state and a resonant multiphoton state under a multiphoton resonance condition [90]. This differs from conventional Rabi oscillations, which usually occur between single-excitation states. In our system, these oscillations are induced by the effective two-photon interactions in Eq. (11). In Fig. 4, the time evolution of the populations $P_{pqi}(t) = |\langle p, q, i | \psi(t) \rangle|^2$ ($p, q = 0, 1, 2$) is shown for five representative cases selected from the resonance lines and their intersection points in Fig. 3 (marked by the black filled circle and the white pentagram). The system is initially prepared in state $|0, 0, +\rangle$; i.e., both cavity modes are in vacuum and the atom is in state $|+\rangle$. Under the exact two-photon resonance condition, the dynamics of each resonant channel can be approximately reduced to a two-state oscillation between $|0, 0, +\rangle$ and the corresponding

final state $|f\rangle$, where $|f\rangle = |2, 0, -\rangle$, $|1, 1, -\rangle$, or $|0, 2, -\rangle$ for the a^2 , ab , or b^2 channel, respectively. The populations are then approximately given by $P_{00+}(t) = \cos^2(\Omega_{\text{eff}}t)$, $P_f(t) = \sin^2(\Omega_{\text{eff}}t)$, with $\Omega_{\text{eff}} = \sqrt{2}g_{\text{eff},1}$, $g_{\text{eff},2}$, and $\sqrt{2}g_{\text{eff},3}$ for the three channels, respectively. In Figs. 4(a)–4(c), successive photon emissions occur only for one driving direction. Specifically, in Fig. 4(a), the left-side driving induces coherent oscillations between the populations P_{00+} and P_{20-} corresponding to a two-photon bundle emission in mode a , whereas the right-side driving suppresses all two-photon bundle channels. In Figs. 4(b) and 4(c), the right-side driving gives rise to the coherent oscillations between the populations P_{00+} and P_{11-} or P_{02-} , corresponding to the two-photon bundle emissions with one photon in each mode [Fig. 4(b)] or only in mode b [Fig. 4(c)], whereas under the left-side driving no two-photon bundle emission is observed. For clarity, only P_{20-} is plotted in Fig. 4(a) under the left-side driving, while P_{11-} and P_{02-} are shown in Figs. 4(b) and 4(c) under the right-side driving, since all other two-photon populations remain zero throughout the evolution. Figures 4(d) and 4(e) illustrate the cases where photon emission occurs under both driving directions. Under the left-side driving, the populations oscillate between P_{00+} and P_{20-} , corresponding to the two-photon emission in mode a . Under the right-side driving, the system exhibits the coherent oscillations between P_{00+} and P_{11-} in

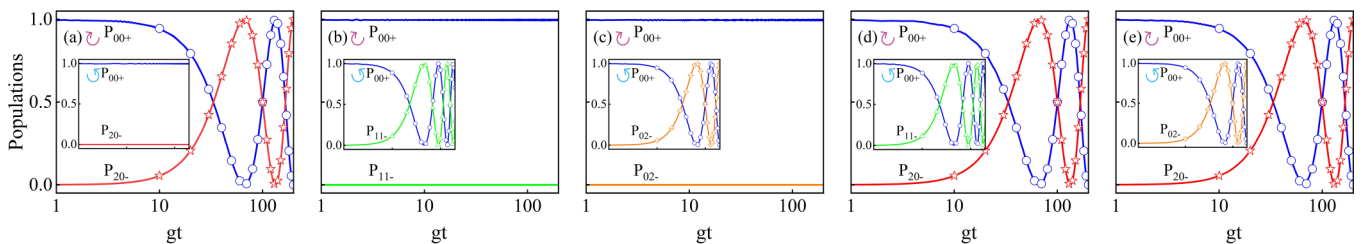


FIG. 4. Time evolution of the dressed-state populations $P_{pqi}(t) = |\langle p, q, i | \psi(t) \rangle|^2$ ($p, q = 0, 1, 2; i = +, -$) without dissipation, at the photon-bundle resonances for five representative cases: (a) $\Delta_a = 27g$ and $\Delta_b = 40g$, (b) $\Delta_a = 28g$ and $\Delta_b = 40g$, (c) $\Delta_a = 40g$ and $\Delta_b = 35g$, (d) $\Delta_a = 27g$ and $\Delta_b = 41g$, (e) $\Delta_a = 27g$ and $\Delta_b = 35g$. The blue, red, green, and orange solid lines correspond to the numerical results obtained from the original Hamiltonian [Eq. (3)], while the circular, star, triangle, and diamond markers denote the results obtained from the effective Hamiltonian [Eq. (11)]. The initial state is $|0, 0, +\rangle$, and the other parameters are the same as in Fig. 3.

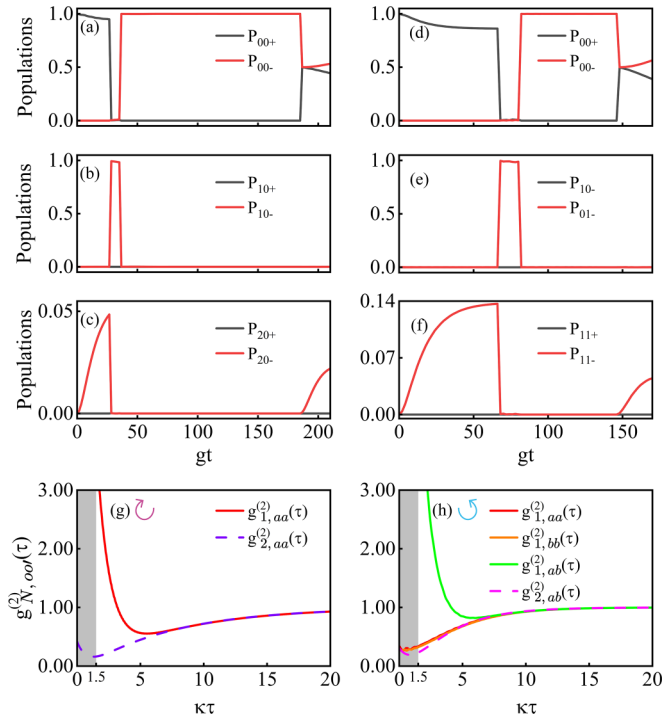


FIG. 5. (a)–(f) Quantum trajectories illustrating nonreciprocal two-photon bundle emission in the same mode and hybrid mode under system dissipation. The lines represent the time evolution of the state populations $P_{pq\pm}$ obtained via Monte Carlo simulations. (g), (h) Time-delayed correlation functions $g^{(2)}_{1,oo'}(\tau)$ and $g^{(2)}_{2,oo'}(\tau)$. The parameters are the same as in Fig. 4(d).

Fig. 4(d), while by adjusting the detuning Δ_b , this changes to oscillations between P_{00+} and P_{02-} in Fig. 4(e). In all these cases, the population oscillates between the initial state $|0, 0, +\rangle$ and the corresponding two-photon dressed states, giving rise to nearly perfect super-Rabi oscillations. The solid lines show the numerical results obtained from the original Hamiltonian in Eq. (3), while the markers represent the corresponding dynamics predicted by the effective Hamiltonian in Eq. (11), demonstrating that the two approaches are fully consistent. To highlight the coherent dynamics, dissipation is temporarily neglected.

B. Nonreciprocal bundle emission triggered by dissipation

The ideal super-Rabi oscillation is demonstrated in Fig. 4 without considering environmental noise. However, dissipation must be taken into account in realistic scenarios, as the system inevitably interacts with its surrounding environments. Therefore, we turn to a more realistic case in which the bundle emission is affected by the cavity losses and the atomic decay.

To characterize the full dynamics of the bundle emission, we use Monte Carlo simulation based on the Lindblad master equation in Eq. (4) to record a single quantum trajectory of the system. Below, we focus on the black filled circle highlighted in Fig. 3, where both driving directions exhibit nonreciprocal bundle emission. First, under the left-side driving with detunings $\Delta_a = 27g$ and $\Delta_b = 41g$ [Figs. 5(a)–5(c)], the system is first driven into a superposition of the initial state $|0, 0, +\rangle$ and the target state $|2, 0, -\rangle$ under the two-photon

resonance condition, with the population of the target state reaching 0.048 at $gt = 27$. During this process, undesired single-photon transitions are largely suppressed, ensuring the selectivity of the target bundle state. When a quantum collapse occurs due to dissipation, a single photon is emitted from mode a , resulting in a transition to the single-photon state $|1, 0, -\rangle$ with a probability close to unity [Fig. 5(b)]. Owing to the strong photon correlation, this is promptly followed by the emission of a second photon, leading to the final state $|0, 0, -\rangle$ and completing the emission of a two-photon bundle in the same mode. Notably, the quantum state $|0, 0, -\rangle$ returns to the initial state when the atomic dressed state flips from $|-\rangle$ to $|+\rangle$. This establishes a cyclic process, enabling successive bundle emissions. In contrast, under the right-side driving [Figs. 5(d)–5(f)], the system evolves into a superposition with the target state $|1, 1, -\rangle$ (one photon in each mode), with the target-state population reaching 0.137. Upon a dissipation-induced collapse, a photon is first emitted from mode b , followed by a correlated photon from mode a , resulting in the final state $|0, 0, -\rangle$. These results demonstrate the nonreciprocal behavior of the bundle emission process. The order of photon emission is random. When the system is driven from the left, it predominantly emits a two-photon bundle in a single mode, whereas the driving from the right leads to the emission of a hybrid photon pair involving two different modes.

Next, we focus on the correlations between photon bundles. To quantify correlations between such bundles, we use the generalized second-order correlation function [84]

$$g^{(2)}_{2,oo'}(\tau) = \frac{\langle (oo')^\dagger(0)(oo')^\dagger(\tau)(oo')(\tau)(oo')(0) \rangle}{\langle (oo')^\dagger(0)(oo')(0) \rangle \langle (oo')^\dagger(\tau)(oo')(\tau) \rangle}, \quad (14)$$

with $o, o' = a, b$. Here, the delay time satisfies $\tau \geq \tau_{\min} = \sum_{m=1}^N 1/m\kappa$, where N denotes the number of photons in a single bundle emission event. For an N -photon bundle, the photons are emitted through a cascade process: The first photon is emitted with the time $1/(N\kappa)$, followed by the subsequent emissions with times $1/[(N-1)\kappa], \dots$, and $1/\kappa$. Thus, the total emission time of the bundle is $\tau_{\min}$. The condition $\tau \geq \tau_{\min}$ is introduced to distinguish the correlation between different photon bundles from the correlation between photons within the same bundle. These correlations can be measured directly thanks to recent advances in two-photon detection techniques [91]. In Figs. 5(g) and 5(h), we plot the time-delayed second-order correlation functions $g^{(2)}_{1,oo'}(\tau)$ and $g^{(2)}_{2,oo'}(\tau)$ as functions of the scaled delay time $\kappa\tau$, representing intrabundle and interbundle correlations, respectively. For the left-side driving, the results in Fig. 5(g) show $g^{(2)}_{1,aa}(\tau) > 1$ and $g^{(2)}_{2,aa}(\tau) < 1$, indicating photon bunching within each two-photon bundle and antibunching between successive bundles in mode a . Correspondingly, for the right-side driving, Fig. 5(h) shows that $g^{(2)}_{1,ab}(\tau) > 1$ and $g^{(2)}_{2,ab}(\tau) < 1$, demonstrating antibunched emission of two-photon bundles across different modes, while both $g^{(2)}_{1,aa}(\tau)$ and $g^{(2)}_{1,bb}(\tau)$ remain below unity, confirming photon antibunching in the individual modes. At large delay times, $g^{(2)}_{1,oo}(\tau)$ and $g^{(2)}_{2,oo}(\tau)$ converge to the same Poissonian limit, indicating that temporally separated emission events become statistically independent. These

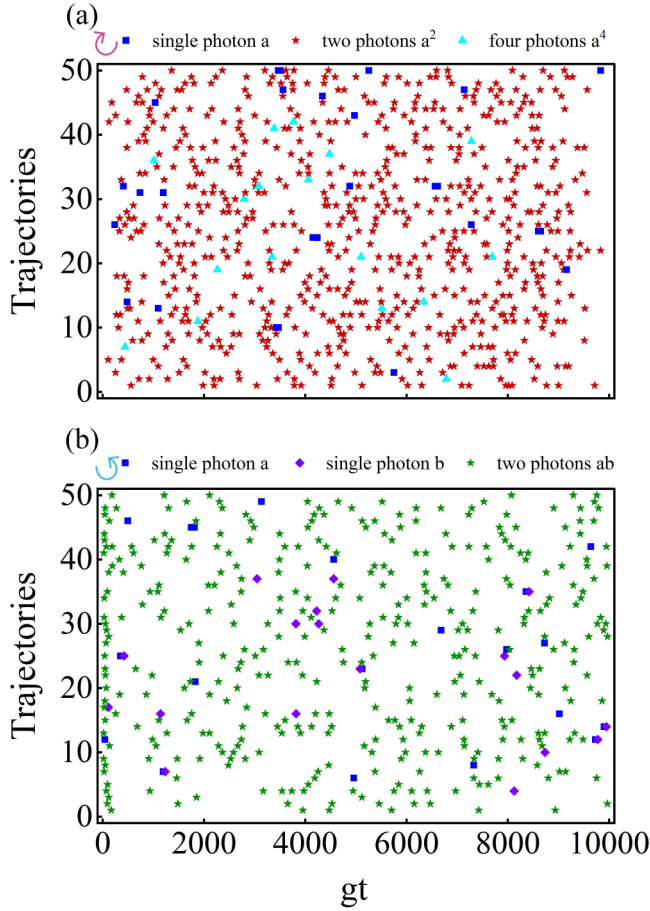


FIG. 6. A series of emission events extracted from the Monte Carlo simulations over 50 quantum trajectories under the two-photon resonance: (a) the system is driven from the left, and (b) the system is driven from the right. The single-, two-, and four-photon emissions in mode a are represented by blue squares, red stars, and cyan triangles, respectively. The single-photon emission in mode b and two-photon emission involving both modes a and b are represented by purple diamonds and green stars, respectively. The other parameters are the same as in Fig. 4(d).

observations collectively verify the nonreciprocal generation of two-photon bundles, specifically in the forms of a^2 and ab .

To visualize these nonreciprocal two-photon bundle emissions, we extract photon emission events from Monte Carlo simulations over 50 quantum trajectories at two-photon resonance. In Fig. 6, the horizontal and vertical axes correspond to time and individual trajectories, respectively, with each point representing a photon emission event that may involve one or more photons in the corresponding cavity mode(s). For the left-side driving [Fig. 6(a)], a^2 two-photon events dominate the emission process, occurring with a probability of 94.3% over 50 independent trajectories. Only a small fraction of undesired events, such as single-photon emissions or higher-order events, are observed, indicating the high purity of the two-photon bundle emission in mode a . In contrast, for the right-side driving [Fig. 6(b)], the two-photon bundle emission with one photon in each of the a and b modes prevails, with a probability of 91%. These results confirm the nonreciprocal nature of the two-photon bundle emissions, the system

predominantly producing two-photon bundles in mode a under the left-side driving and with one photon in each of the a and b modes under the right-side driving. The emissions in each case occur stochastically in time and across trajectories, reflecting the probabilistic character of the process. Thus, our scheme enables efficient and high-purity generation of nonreciprocal two-mode two-photon bundles, where different multiphoton emission channels can be selectively activated depending on the driving direction. This direction selectivity originates from the opposite Sagnac-Fizeau shifts for opposite propagation directions, which make the effective detunings and the corresponding multiphoton resonance conditions direction dependent.

IV. NONRECIPROCAL THREE-PHOTON BUNDLE EMISSION

Nonreciprocal two-photon bundles have been demonstrated under appropriate frequency conditions. To further explore the flexibility of the proposed scheme, we extend our analysis to three-photon bundle emission. To derive the effective three-photon interactions, we expand the Hamiltonian using the BCH formula and obtain the effective Hamiltonian up to third order in λ as

$$\begin{aligned} H_{\text{eff}} &= H_0 + \lambda V - \lambda[S, H_0] - \lambda^2[S, V] + \frac{\lambda^2}{2}[S, [S, H_0]] \\ &\quad + \frac{\lambda^3}{2}[S, [S, V]] - \frac{\lambda^3}{6}[S, [S, [S, H_0]]] + O(\lambda^4), \\ &= H_0 - \frac{1}{2}[S, V] + \frac{1}{3}[S, [S, V]] + O(\lambda^4). \end{aligned} \quad (15)$$

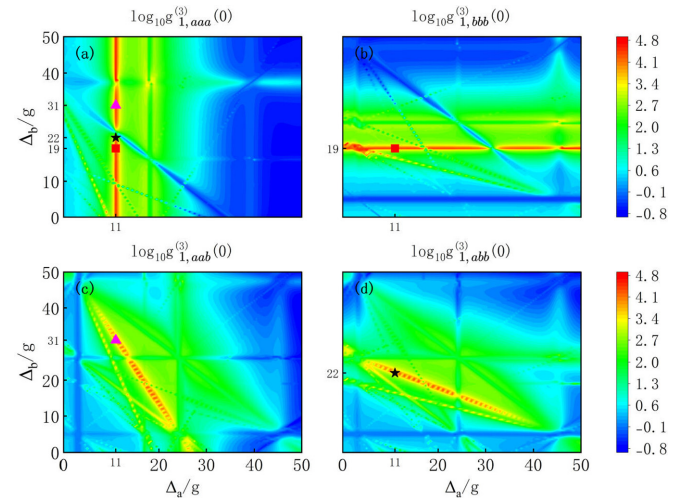


FIG. 7. The logarithm of equal-time third-order correlation function $g_{1,oo'o'}^{(3)}(0)$ ($o, o', o'' = a, b$) as a function of detunings Δ_a/g and Δ_b/g . (a) Driving from the left side and (b)–(d) driving from the right side. The magenta triangle, black pentagram, and red square denote the intersection points of the resonance lines, corresponding to $(\Delta_a = 11g, \Delta_b = 31g)$, $(\Delta_a = 11g, \Delta_b = 22g)$, and $(\Delta_a = 11g, \Delta_b = 19g)$, respectively. We choose $\xi = 20g$, $\xi_d = 0.02g$, $\kappa = 0.01g$, $\gamma = 0.001g$ and $\Delta_e = 14g$. The other parameters are the same as in Fig. 3.

Under the multiphoton resonance condition introduced above Eq. (10), we consider the integer pairs $(m, n) \in (3, 0), (0, 3), (2, 1), (1, 2)$, which correspond to resonant three-photon transitions involving the two cavity modes. Among these channels, we first focus on the single-mode processes, for which the corresponding third-order effective Hamiltonians can be written as

$$\begin{aligned}
 H_{\text{eff}}^{aaa} &= g_{\text{eff},4} a^3 \sigma_{+-} + \text{H.c.} \\
 &= \sum_{p,q=0}^{\infty} g_{\text{eff},4} \sqrt{(p+1)(p+2)(p+3)} |p, q, +\rangle \\
 &\quad \times \langle p+3, q, -| + \text{H.c.}, \\
 H_{\text{eff}}^{bbb} &= g_{\text{eff},5} b^3 \sigma_{+-} + \text{H.c.} \\
 &= \sum_{p,q=0}^{\infty} g_{\text{eff},5} \sqrt{(q+1)(q+2)(q+3)} |p, q, +\rangle \\
 &\quad \times \langle p, q+3, -| + \text{H.c.}, \quad (16)
 \end{aligned}$$

with the effective coupling strengths given by

$$\begin{aligned}
 g_{\text{eff},4} &= \frac{g^3 B}{3} \left[\frac{4}{\Delta'_a} \left(\frac{1}{\Delta'_a} - \frac{1}{\Delta'_a - \xi} \right) \right. \\
 &\quad \left. + \frac{2}{\Delta'_a - \xi} \left(\frac{1}{\Delta'_a - \xi} - \frac{1}{(\Delta'_a + \xi)} \right) \right], \\
 g_{\text{eff},5} &= \frac{g^3 B}{3} \left[\frac{4}{\Delta'_b} \left(\frac{1}{\Delta'_b} - \frac{1}{\Delta'_b - \xi} \right) \right. \\
 &\quad \left. + \frac{2}{\Delta'_b - \xi} \left(\frac{1}{\Delta'_b - \xi} - \frac{1}{(\Delta'_b + \xi)} \right) \right], \quad (17)
 \end{aligned}$$

with $B = \cos^4 \theta \sin^2 \theta$. The hybrid-mode channels (2, 1) and (1, 2) can be derived in an analogous manner but are omitted here for brevity.

To characterize the correlations among photons within a three-photon bundle, we define equal-time third-order correlation functions

$$g_{1,oo'o'}^{(3)}(0) = \frac{\langle o^\dagger o'^\dagger o''^\dagger o' o \rangle}{\langle o^\dagger o \rangle \langle o'^\dagger o' \rangle \langle o''^\dagger o'' \rangle}, \quad (18)$$

with $o, o', o'' = a, b$. Here, the function corresponds to the autocorrelation for $o = o'$ and to the cross-correlation between the two modes for $o \neq o'$. In Fig. 7, we plot $\log_{10} g_{1,oo'o'}^{(3)}(0)$ as functions of the detunings Δ_a/g and Δ_b/g under the three-photon resonance condition. The parameters are chosen as $\xi = 20g$, $\xi_d = 0.02g$, $\kappa = 0.01g$, $\gamma = 0.001g$, and $\Delta_e = 14g$. The warm-colored regions in the contour plots indicate strong correlations ($g_{1,oo'o'}^{(3)}(0) \gg 1$), reflecting pronounced superbunching. Figure 7(a) corresponds to the left-side driving, while Figs. 7(b)–7(d) show the results for the right-side driving. For completeness, all four resonance channels $(m, n) \in \{(3, 0), (0, 3), (2, 1), (1, 2)\}$ are displayed, revealing prominent resonance lines along which the correlation functions are strongly enhanced. Notably, three characteristic intersection points between the resonance lines in Fig. 7(a) and those in Figs. 7(b)–7(d) are highlighted. At identical detuning parameters corresponding to these intersections, reversing the driving direction leads to distinct three-photon correlation behaviors. Specifically, under the left-side driving, strong

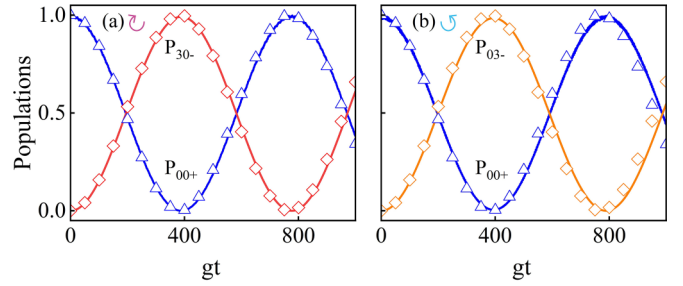


FIG. 8. Time evolution of the dressed-state populations $P_{pqi}(t) = |\langle p, q, i | \psi(t) \rangle|^2$ ($p, q = 0, 3; i = +, -$) without dissipation, at the three-photon resonance conditions marked by the red square in Fig. 7. (a) Driving from the left side and (b) driving from the right side. The initial state is $|0, 0, +\rangle$. The blue, red, and orange solid lines correspond to the numerical results from the original Hamiltonian Eq. (3), while the triangle and square markers denote the results obtained from the effective Hamiltonian Eq. (16).

superbunching appears in the single-mode correlation $g_{1,aaa}^{(3)}(0)$, whereas under the right-side driving the dominant correlations shift to the hybrid-mode correlations $g_{1,aab}^{(3)}(0)$ and $g_{1,abb}^{(3)}(0)$, or to the single-mode correlation $g_{1,bbb}^{(3)}(0)$. These results clearly demonstrate the nonreciprocal characteristics associated with the three-photon bundle correlations.

To gain further insight into the mechanisms underlying the observed three-photon bundle correlations, we examine the super-Rabi oscillations induced by the effective three-photon interactions. Starting from the initial state $|0, 0, +\rangle$, we numerically plot the time evolution of the dressed-state populations $P_{pqi}(t)$ for $p, q \in \{0, 3\}$, focusing on the single-mode three-photon resonant transitions. Figure 8 shows a representative case selected from the intersection of the resonance lines in Figs. 7(a) and 7(b) (marked by the red square). In Fig. 8, the population oscillates almost perfectly between the initial state and the corresponding three-photon dressed state, giving rise to nearly ideal super-Rabi oscillations. Specifically, under the left-side driving [Fig. 8(a)], the coherent oscillations occur between P_{00+} and P_{30-} , indicating the three-photon bundle emission in mode a , whereas under the right-side driving [Fig. 8(b)], the oscillations occur instead between P_{00+} and P_{03-} , corresponding to three-photon bundle emission in mode b . The solid lines show the numerical dynamics governed by the full Hamiltonian in Eq. (3), while the markers indicate the corresponding prediction of the effective Hamiltonian in Eq. (16), showing near-perfect consistency between the two approaches. To highlight the coherent dynamics, dissipation is temporarily neglected.

To investigate the impact of dissipation on three-photon bundle emission, we employ Monte Carlo simulations to record the quantum trajectories of the system. First, under the left-side driving with detunings $\Delta_a = 11g$ and $\Delta_b = 19g$ [Figs. 9(a)–9(d)], the system evolves into a superposition of the initial state $|0, 0, +\rangle$ and the target state $|3, 0, -\rangle$, with the three-photon state population reaching 0.081 at $gt = 488$. During this process, undesired transitions are largely suppressed, ensuring the selectivity of the target bundle state. When a quantum collapse occurs due to dissipation, a single photon is emitted from mode a , bringing the system to

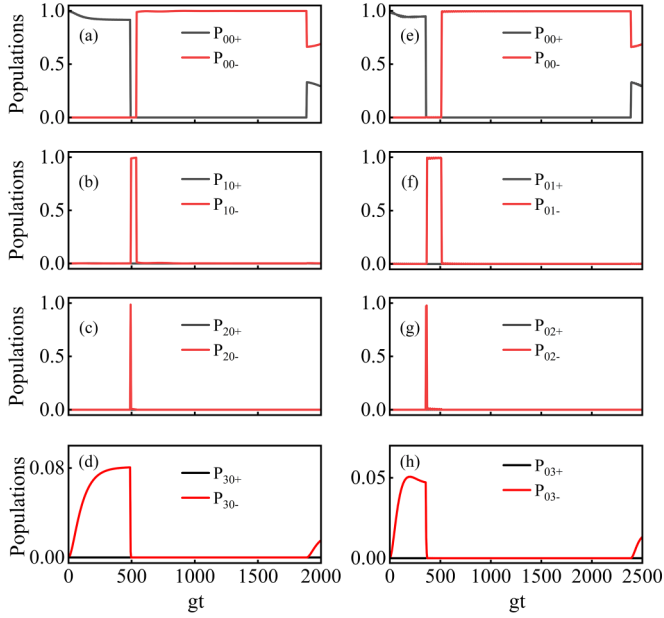


FIG. 9. (a)–(h) Quantum trajectories illustrating nonreciprocal three-photon bundle emission in the same mode under system dissipation. The lines represent the time evolution of the state populations P_{pqi} obtained via Monte Carlo simulations. The parameters are the same as in Fig. 8.

the two-photon state $|2, 0, -\rangle$ with a probability close to unity [Fig. 9(c)]. The strong correlation between the photons ensures that the emission of the second and third photons follows promptly, sequentially reaching $|1, 0, -\rangle$ and then $|0, 0, -\rangle$, thereby completing the three-photon bundle emission from mode a . The atomic dressed state then flips from $|-\rangle$ back to $|+\rangle$, establishing a cyclic process that allows successive bundle emissions. In contrast, under the right-side driving with the same detunings [Figs. 9(e)–9(h)], the system evolves into a superposition with the target state $|0, 3, -\rangle$, with the population of the three-photon state reaching 0.047. Following a dissipation-induced quantum collapse, the three photons are sequentially emitted from mode b , and the system returns to the initial state $|0, 0, -\rangle$. These results clearly illustrate the nonreciprocal character of the emission. The left-side driving predominantly generates a three-photon bundle in mode a , whereas the right-side driving produces a three-photon bundle in mode b .

Next, we analyze the photon correlations within a single bundle and between different bundles. To quantify the correlations between successive three-photon bundles, we introduce the time-delayed second-order correlation functions

$$g_{2,oo'}^{(2)}(\tau) = \frac{\langle (oo'd')^\dagger(0)(oo'd')^\dagger(\tau)(oo'd')(\tau)(oo'd')(0) \rangle}{\langle (oo'd')^\dagger(0)(oo'd')(0) \rangle \langle (oo'd')^\dagger(\tau)(oo'd')(\tau) \rangle}. \quad (19)$$

The correlation functions $g_{1,oo'}^{(2)}(\tau)$ and $g_{2,oo'}^{(2)}(\tau)$ are plotted as functions of the scaled delay time $\kappa\tau$ in Fig. 10. For the left-side driving, we observe $g_{1,aa}^{(2)}(\tau) > 1$ and $g_{2,aaa}^{(2)}(\tau) < 1$, indicating that the three photons in mode a are strongly bunched within each a^3 bundle, while successive bundles are antibunched. Similarly, for the right-side driving,

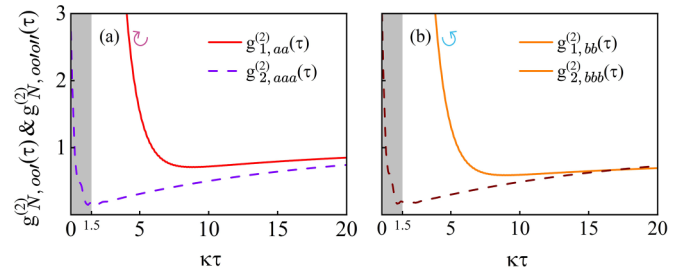


FIG. 10. Time-delayed second-order correlation functions $g_{1,oo'}^{(2)}(\tau)$ and $g_{2,oo'}^{(2)}(\tau)$ ($o, o', o'' = a, b$). (a) Driving from the left side and (b) driving from the right side. The parameters are the same as in Fig. 8.

$g_{1,bb}^{(2)}(\tau) > 1$ and $g_{2,bbb}^{(2)}(\tau) < 1$, demonstrating the formation of antibunched three-photon bundles in mode b . These results demonstrate the nonreciprocal generation of three-photon bundles, with a^3 and b^3 appearing depending on the driving conditions. Thus, our scheme enables efficient generation of nonreciprocal two-mode three-photon bundles, where different three-photon emission channels can also be selectively activated depending on the driving direction through the direction-dependent Sagnac-Fizeau shifts.

The proposed scheme can be realized using a rotating whispering-gallery-mode (WGM) microcavity coupled to a two-level quantum emitter. Strong emitter-cavity coupling via the evanescent field has been routinely achieved in WGM cQED experiments, with coherent coupling rates exceeding both the cavity decay rate and the emitter dissipation rate [92–95]. To induce optical nonreciprocity, we exploit cavity rotation, which generates a propagation-direction-dependent Fizeau-Sagnac frequency shift that can be comparable to the cavity linewidth in micron-scale resonators [9]. By selecting two specific high- Q cavity modes, their effective detunings relative to the emitter can be independently controlled via the rotation. For example, a microcavity with radius $R = 30 \mu\text{m}$ and refractive index $n = 1.4$ can support high- Q resonances at both 1550 and 1064 nm. The statistical properties of multiphoton bundle emission can be characterized using Hanbury Brown-Twiss interferometry combined with high-time-resolution single-photon detectors [96,97]. Therefore, the present scheme provides a realistic and feasible platform for investigating directional multiphoton dynamics in nonreciprocal cQED and offers a promising route toward tunable multiphoton quantum light sources.

V. CONCLUSION

In conclusion, we have proposed a scheme to realize nonreciprocal multiphoton bundle emission in a rotating cavity directly coupled to a two-level atom. Cavity rotation produces opposite Sagnac-Fizeau shifts between counterpropagating modes, making the resonance condition dependent on the driving direction. By tuning the driving direction and frequency, different multiphoton transition channels can be selectively excited, which leads to the nonreciprocal two-photon or three-photon bundle emission with the same and different modes. The proposed scheme combines efficient state preparation, a highly tunable and convenient design, and

a versatile platform for realizing direction-controlled multi-photon sources. This may find applications in chiral quantum information processing, directional quantum communication, and nonreciprocal quantum photonic devices.

ACKNOWLEDGMENTS

This work was supported by the National Natural Science Foundation of China (Grant No. 12274164) and Fundamental

Research Funds for the Central Universities (Grants No. XJ2026002801 and No. CCNU25JCPT030).

DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

- [1] R. J. Potton, Reciprocity in optics, *Rep. Prog. Phys.* **67**, 717 (2004).
- [2] D. L. Sounas and A. Alù, Non-reciprocal photonics based on time modulation, *Nat. Photon.* **11**, 774 (2017).
- [3] P. Lodahl, S. Mahmoodian, S. Stobbe, A. Rauschenbeutel, P. Schneeweiss, J. Volz, H. Pichler, and P. Zoller, Chiral quantum optics, *Nature (London)* **541**, 473 (2017).
- [4] Y.-P. Wang, J. W. Rao, Y. Yang, P.-C. Xu, Y. S. Gui, B. M. Yao, J. Q. You, and C.-M. Hu, Nonreciprocity and unidirectional invisibility in cavity magnonics, *Phys. Rev. Lett.* **123**, 127202 (2019).
- [5] Y. Xu, J.-Y. Liu, W. Liu, and Y.-F. Xiao, Nonreciprocal phonon laser in a spinning microwave magnomechanical system, *Phys. Rev. A* **103**, 053501 (2021).
- [6] Y.-J. Xu and J. Song, Nonreciprocal magnon laser, *Opt. Lett.* **46**, 5276 (2021).
- [7] B. E. Saleh and M. C. Teich, *Fundamentals of Photonics*, 2nd ed. (Wiley Interscience, Hoboken, NJ, 2007).
- [8] D. Jalas, A. Petrov, M. Eich, W. Freude, S. Fan, Z. Yu, R. Baets, M. Popović, A. Melloni, J. D. Joannopoulos, M. Vanwolleghem, C. R. Doerr, and H. Renner, What is—and what is not—an optical isolator, *Nat. Photon.* **7**, 579 (2013).
- [9] S. Maayani, R. Dahan, Y. Kligerman, E. Moses, A. U. Hassan, H. Jing, F. Nori, D. N. Christodoulides, and T. Carmon, Flying couplers above spinning resonators generate irreversible refraction, *Nature (London)* **558**, 569 (2018).
- [10] A. Kamal, J. Clarke, and M. H. Devoret, Noiseless nonreciprocity in a parametric active device, *Nat. Phys.* **7**, 311 (2011).
- [11] J. Kerckhoff, K. Lalumière, B. J. Chapman, A. Blais, and K. W. Lehnert, On-chip superconducting microwave circulator from synthetic rotation, *Phys. Rev. Appl.* **4**, 034002 (2015).
- [12] K. M. Sliwa, M. Hatridge, A. Narla, S. Shankar, L. Frunzio, R. J. Schoelkopf, and M. H. Devoret, Reconfigurable Josephson circulator/directional amplifier, *Phys. Rev. X* **5**, 041020 (2015).
- [13] L. Feng, M. Ayache, J. Huang, Y.-L. Xu, M.-H. Lu, Y.-F. Chen, Y. Fainman, and A. Scherer, Nonreciprocal light propagation in a silicon photonic circuit, *Science* **333**, 729 (2011).
- [14] S. Manipatruni, J. T. Robinson, and M. Lipson, Optical nonreciprocity in optomechanical structures, *Phys. Rev. Lett.* **102**, 213903 (2009).
- [15] Z. Shen, Y.-L. Zhang, Y. Chen, C.-L. Zou, Y.-F. Xiao, X.-B. Zou, F.-W. Sun, G.-C. Guo, and C.-H. Dong, Experimental realization of optomechanically induced nonreciprocity, *Nat. Photon.* **10**, 657 (2016).
- [16] G. A. Peterson, F. Lecocq, K. Cicak, R. W. Simmonds, J. Aumentado, and J. D. Teufel, Demonstration of efficient nonreciprocity in a microwave optomechanical circuit, *Phys. Rev. X* **7**, 031001 (2017).
- [17] N. R. Bernier, L. D. Toth, A. Koottandavida, M. A. Ioannou, D. Malz, A. Nunnenkamp, A. K. Feofanov, and T. J. Kippenberg, Nonreciprocal reconfigurable microwave optomechanical circuit, *Nat. Commun.* **8**, 604 (2017).
- [18] L. M. de Lépinay, E. Damskägg, C. F. Ockeloen-Korppi, and M. A. Sillanpää, Realization of directional amplification in a microwave optomechanical device, *Phys. Rev. Appl.* **11**, 034027 (2019).
- [19] L. M. de Lépinay, C. F. Ockeloen-Korppi, D. Malz, and M. A. Sillanpää, Nonreciprocal transport based on cavity Floquet modes in optomechanics, *Phys. Rev. Lett.* **125**, 023603 (2020).
- [20] Z. Shen, Y.-L. Zhang, Y. Chen, Y.-F. Xiao, C.-L. Zou, G.-C. Guo, and C.-H. Dong, Nonreciprocal frequency conversion and mode routing in a microresonator, *Phys. Rev. Lett.* **130**, 013601 (2023).
- [21] L. Fan, J. Wang, L. T. Varghese, H. Shen, B. Niu, Y. Xuan, A. M. Weiner, and M. Qi, An all-silicon passive optical diode, *Science* **335**, 447 (2012).
- [22] Q.-T. Cao, H. Wang, C.-H. Dong, H. Jing, R.-S. Liu, X. Chen, L. Ge, Q. Gong, and Y.-F. Xiao, Experimental demonstration of spontaneous chirality in a nonlinear microresonator, *Phys. Rev. Lett.* **118**, 033901 (2017).
- [23] P. Yang, X. Xia, H. He, S. Li, X. Han, G. Li, J. Xu, Y. Yang, and T. Zhang, Realization of nonlinear optical nonreciprocity on a few-photon level based on atoms strongly coupled to an asymmetric cavity, *Phys. Rev. Lett.* **123**, 233604 (2019).
- [24] A. Graf, S. D. Rogers, J. Staffa, U. A. Javid, D. H. Griffith, and Q. Lin, Nonreciprocity in photon pair correlations of classically reciprocal systems, *Phys. Rev. Lett.* **128**, 213605 (2022).
- [25] A. Metelmann and A. A. Clerk, Nonreciprocal photon transmission and amplification via reservoir engineering, *Phys. Rev. X* **5**, 021025 (2015).
- [26] M. Liu, C. Zhao, Y. Zeng, Y. Chen, C. Zhao, and C.-W. Qiu, Evolution and nonreciprocity of loss-induced topological phase singularity pairs, *Phys. Rev. Lett.* **127**, 266101 (2021).
- [27] Z. Lin, H. Ramezani, T. Eichelkraut, T. Kottos, H. Cao, and D. N. Christodoulides, Unidirectional invisibility induced by parity-time-symmetric periodic structures, *Phys. Rev. Lett.* **106**, 213901 (2011).
- [28] N. Bender, S. Factor, J. D. Bodyfelt, H. Ramezani, D. N. Christodoulides, F. M. Ellis, and T. Kottos, Observation of asymmetric transport in structures with active nonlinearities, *Phys. Rev. Lett.* **110**, 234101 (2013).
- [29] B. Peng, S. K. Özdemir, F. Lei, F. Monifi, M. Gianfreda, G. L. Long, S. Fan, F. Nori, C. M. Bender, and L. Yang, Parity-time-

- symmetric whispering-gallery microcavities, *Nat. Phys.* **10**, 394 (2014).
- [30] L. Chang, X. Jiang, S. Hua, C. Yang, J. Wen, L. Jiang, G. Li, G. Wang, and M. Xiao, Parity-time symmetry and variable optical isolation in active-passive-coupled microresonators, *Nat. Photon.* **8**, 524 (2014).
- [31] M. Scheucher, A. Hilico, E. Will, J. Volz, and A. Rauschenbeutel, Quantum optical circulator controlled by a single chirally coupled atom, *Science* **354**, 1577 (2016).
- [32] Y. Shen, M. Bradford, and J.-T. Shen, Single-photon diode by exploiting the photon polarization in a waveguide, *Phys. Rev. Lett.* **107**, 173902 (2011).
- [33] K. Xia, F. Nori, and M. Xiao, Cavity-free optical isolators and circulators using a chiral cross-Kerr nonlinearity, *Phys. Rev. Lett.* **121**, 203602 (2018).
- [34] M.-X. Dong, K.-Y. Xia, W.-H. Zhang, Y.-C. Yu, Y.-H. Ye, E.-Z. Li, L. Zeng, D.-S. Ding, B.-S. Shi, G.-C. Guo, and F. Nori, All-optical reversible single-photon isolation at room temperature, *Sci. Adv.* **7**, eabe8924 (2021).
- [35] S.-Y. Ren, W. Yan, L.-T. Feng, Y. Chen, Y.-K. Wu, X.-Z. Qi, X.-J. Liu, Y.-J. Cheng, B.-Y. Xu, L.-J. Deng, G.-C. Guo, L. Bi, and X.-F. Ren, Single-photon nonreciprocity with an integrated magneto-optical isolator, *Laser Photonics Rev.* **16**, 2100595 (2022).
- [36] J.-S. Tang, W. Nie, L. Tang, M. Y. Chen, X. Su, Y. Q. Lu, F. Nori, and K. Y. Xia, Nonreciprocal single-photon band structure, *Phys. Rev. Lett.* **128**, 203602 (2022).
- [37] I. Söllner, S. Mahmoodian, S. L. Hansen, L. Midolo, A. Javadi, G. Kiršanskė, T. Pregolato, H. El-Ella, E. H. Lee, J. D. Song, S. Stobbe, and P. Lodahl, Deterministic photon-emitter coupling in chiral photonic circuits, *Nat. Nanotechnol.* **10**, 775 (2015).
- [38] H. Pichler, T. Ramos, A. J. Daley, and P. Zoller, Quantum optics of chiral spin networks, *Phys. Rev. A* **91**, 042116 (2015).
- [39] K. Stannigel, P. Rabl, and P. Zoller, Driven-dissipative preparation of entangled states in cascaded quantum-optical networks, *New J. Phys.* **14**, 063014 (2012).
- [40] D. Malz, L. D. Tóth, N. R. Bernier, A. K. Feofanov, T. J. Kippenberg, and A. Nunnenkamp, Quantum-limited directional amplifiers with optomechanics, *Phys. Rev. Lett.* **120**, 023601 (2018).
- [41] S. Barzanjeh, M. Aquilina, and A. Xuereb, Manipulating the flow of thermal noise in quantum devices, *Phys. Rev. Lett.* **120**, 060601 (2018).
- [42] R. Huang, A. Miranowicz, J.-Q. Liao, F. Nori, and H. Jing, Nonreciprocal photon blockade, *Phys. Rev. Lett.* **121**, 153601 (2018).
- [43] B. Li, R. Huang, X. Xu, A. Miranowicz, and H. Jing, Nonreciprocal unconventional photon blockade in a spinning optomechanical system, *Photonics Res.* **7**, 630 (2019).
- [44] K. Wang, Q. Wu, Y.-F. Yu, and Z.-M. Zhang, Nonreciprocal photon blockade in a two-mode cavity with a second-order nonlinearity, *Phys. Rev. A* **100**, 053832 (2019).
- [45] H. Z. Shen, Q. Wang, J. Wang, and X. X. Yi, Nonreciprocal unconventional photon blockade in a driven dissipative cavity with parametric amplification, *Phys. Rev. A* **101**, 013826 (2020).
- [46] Y.-W. Jing, H.-Q. Shi, and X.-W. Xu, Nonreciprocal photon blockade and directional amplification in a spinning resonator coupled to a two-level atom, *Phys. Rev. A* **104**, 033707 (2021).
- [47] Y. Wang, W. Xiong, Z. Xu, G.-Q. Zhang, and J.-Q. You, Dissipation-induced nonreciprocal magnon blockade in a magnon-based hybrid system, *Sci. China Phys. Mech. Astron.* **65**, 260314 (2022).
- [48] Y.-F. Jiao, S.-D. Zhang, Y.-L. Zhang, A. Miranowicz, L.-M. Kuang, and H. Jing, Nonreciprocal optomechanical entanglement against backscattering losses, *Phys. Rev. Lett.* **125**, 143605 (2020).
- [49] Y.-L. Ren, Nonreciprocal optical-microwave entanglement in a spinning magnetic resonator, *Opt. Lett.* **47**, 1125 (2022).
- [50] Y.-F. Jiao, J.-X. Liu, Y. Li, R. Yang, L.-M. Kuang, and H. Jing, Nonreciprocal enhancement of remote entanglement between nonidentical mechanical oscillators, *Phys. Rev. Appl.* **18**, 064008 (2022).
- [51] D. Kong, J. Xu, and F. Wang, Nonreciprocal entanglement of ferrimagnetic magnons and nitrogen-vacancy-center ensembles by Kerr nonlinearity, *Phys. Rev. Appl.* **21**, 034061 (2024).
- [52] V. Giovannetti, S. Lloyd, and L. Maccone, Advances in quantum metrology, *Nat. Photon.* **5**, 222 (2011).
- [53] M. O. Scully and M. S. Zubairy, *Quantum Optics* (Cambridge University Press, Cambridge, 1997).
- [54] D. F. Walls and G. J. Milburn, *Quantum Optics* (Springer, Berlin, 2008).
- [55] J. P. Dowling and G. J. Milburn, Quantum technology: The second quantum revolution, *Philos. Trans. R. Soc. London Ser. A* **361**, 1655 (2003).
- [56] C. S. Muñoz, E. del Valle, A. G. Tudela, K. Müller, S. Lichtmannecker, M. Kaniber, C. Tejedor, J. J. Finley, and F. P. Laussy, Emitters of N -photon bundles, *Nat. Photon.* **8**, 550 (2014).
- [57] C. Sánchez Muñoz, F. P. Laussy, E. del Valle, C. Tejedor, and A. González-Tudela, Filtering multiphoton emission from state-of-the-art cavity quantum electrodynamics, *Optica* **5**, 14 (2018).
- [58] Q. Bin, X.-Y. Lü, F. P. Laussy, F. Nori, and Y. Wu, N -phonon bundle emission via the Stokes process, *Phys. Rev. Lett.* **124**, 053601 (2020).
- [59] Q. Bin, Y. Wu, and X.-Y. Lü, Parity-symmetry-protected multiphoton bundle emission, *Phys. Rev. Lett.* **127**, 073602 (2021).
- [60] F. Zou, J.-Q. Liao, and Y. Li, Dynamical emission of phonon pairs in optomechanical systems, *Phys. Rev. A* **105**, 053507 (2022).
- [61] C. Gou, X. Hu, and F. Wang, Antibunched two-mode two-photon bundles via atomic coherence, *Phys. Rev. A* **106**, 063718 (2022).
- [62] S.-Y. Jiang, F. Zou, Y. Wang, J.-F. Huang, X.-W. Xu, and J.-Q. Liao, Multiple-photon bundle emission in the n -photon Jaynes-Cummings model, *Opt. Express* **31**, 15697 (2023).
- [63] C. Liu, J.-F. Huang, and L. Tian, Deterministic generation of multi-photon bundles in a quantum Rabi model, *Sci. China Phys. Mech. Astron.* **66**, 220311 (2023).
- [64] Z. Chen, C. Gou, X. Hu, D. Kong, and F. Wang, Enhanced bundle emission of squeezed photons using parametric amplification, *Phys. Rev. A* **112**, 063703 (2025).
- [65] B. Xiong, Q. Bin, S.-L. Chao, J.-B. Liu, and X.-Y. Lü, Two-photon decay enhanced even photon bundle emission, *Phys. Rev. Res.* **7**, 013238 (2025).
- [66] A. González-Tudela, V. Paulisch, H. J. Kimble, and J. I. Cirac, Efficient multiphoton generation in waveguide quantum electrodynamics, *Phys. Rev. Lett.* **118**, 213601 (2017).
- [67] W. Qin, V. Macrì, A. Miranowicz, S. Savasta, and F. Nori, Emission of photon pairs by mechanical stimulation of the squeezed vacuum, *Phys. Rev. A* **100**, 062501 (2019).

- [68] J.-Q. Liao and C. K. Law, Correlated two-photon scattering in cavity optomechanics, *Phys. Rev. A* **87**, 043809 (2013).
- [69] P. Bienias, S. Choi, O. Firstenberg, M. F. Maghrebi, M. Gullans, M. D. Lukin, A. V. Gorshkov, and H. P. Büchler, Scattering resonances and bound states for strongly interacting Rydberg polaritons, *Phys. Rev. A* **90**, 053804 (2014).
- [70] M. F. Maghrebi, M. J. Gullans, P. Bienias, S. Choi, I. Martin, O. Firstenberg, M. D. Lukin, H. P. Büchler, and A. V. Gorshkov, Coulomb bound states of strongly interacting photons, *Phys. Rev. Lett.* **115**, 123601 (2015).
- [71] O. Firstenberg, C. S. Adams, and S. Hofferberth, Nonlinear quantum optics mediated by Rydberg interactions, *J. Phys. B: At. Mol. Opt. Phys.* **49**, 152003 (2016).
- [72] K. Jachymski, P. Bienias, and H. P. Büchler, Three-body interaction of Rydberg slow-light polaritons, *Phys. Rev. Lett.* **117**, 053601 (2016).
- [73] C. Gou, X. Hu, J. Xu, and F. Wang, Hybrid magnon-photon bundle emission from a ferromagnetic-superconducting system, *Phys. Rev. Res.* **6**, 023052 (2024).
- [74] Q. Bin, Q.-Y. Qiu, Y. Wu, and X.-Y. Lü, Entangled photon-magnon bundle emission, *Laser Photonics Rev.* **18**, 2300977 (2024).
- [75] J. Liu, S. Hu, W. Zhong, G. Cheng, and A. Chen, Controllable antibunching of two-magnon bundle in a hybrid ferromagnet-superconductor system, *Opt. Lett.* **50**, 682 (2025).
- [76] H. Y. Yuan, J. K. Xie, and R. A. Duine, Magnon bundle in a strongly dissipative magnet, *Phys. Rev. Appl.* **19**, 064070 (2023).
- [77] C. Zhao, W. Li, B. Xiong, J.-X. Peng, L. Zhou, and W.-J. Gong, Heralded generation of entangled states based on N -bundle emission in a waveguide magnonics system, *Phys. Rev. A* **112**, 013723 (2025).
- [78] C. Sánchez Muñoz, F. P. Laussy, C. Tejedor, and E. del Valle, Enhanced two-photon emission from a dressed biexciton, *New J. Phys.* **17**, 123021 (2015).
- [79] Y. Chang, A. González-Tudela, C. S. Muñoz, C. Navarrete-Benlloch, and T. Shi, Deterministic down-converter and continuous photon-pair source within the bad-cavity limit, *Phys. Rev. Lett.* **117**, 203602 (2016).
- [80] S.-L. Ma, X.-K. Li, Y.-L. Ren, J.-K. Xie, and F.-L. Li, Antibunched N -photon bundles emitted by a Josephson photonic device, *Phys. Rev. Res.* **3**, 043020 (2021).
- [81] S.-L. Ma, J.-K. Xie, Y.-L. Ren, X.-K. Li, and F.-L. Li, Photon-pair blockade in a Josephson-photonics circuit with two nondegenerate microwave resonators, *New J. Phys.* **24**, 053001 (2022).
- [82] M. Cosacchi, A. Mielnik-Pyszcorski, T. Seidelmann, M. Cygorek, A. Vagov, D. E. Reiter, and V. M. Axt, N -photon bundle statistics in different solid-state platforms, *Phys. Rev. B* **106**, 115304 (2022).
- [83] F. Zou, Y. Li, and J.-Q. Liao, Dynamical N -photon bundle emission, *New J. Phys.* **25**, 043027 (2023).
- [84] Q. Bin, H. Jing, Y. Wu, F. Nori, and X.-Y. Lü, Nonreciprocal bundle emissions of quantum entangled pairs, *Phys. Rev. Lett.* **133**, 043601 (2024).
- [85] G. B. Malykin, The Sagnac effect: Correct and incorrect explanations, *Phys.-Usp.* **43**, 1229 (2000).
- [86] J. R. Johansson, P. D. Nation, and F. Nori, QuTiP: An open-source Python framework for the dynamics of open quantum systems, *Comput. Phys. Commun.* **183**, 1760 (2012).
- [87] C. Cohen-Tannoudji, J. Dupont-Roc, and G. Grynberg, *Atom-Photon Interactions* (Wiley, New York, 1992).
- [88] J. R. Schrieffer and P. A. Wolff, Relation between the Anderson and Kondo Hamiltonians, *Phys. Rev.* **149**, 491 (1966).
- [89] R. J. Glauber, The quantum theory of optical coherence, *Phys. Rev.* **130**, 2529 (1963).
- [90] D. V. Strekalov, A bundle of photons, please, *Nat. Photon.* **8**, 500 (2014).
- [91] F. Boitier, A. Godard, E. Rosencher, and C. Fabre, Measuring photon bunching at ultrashort timescale by two-photon absorption in semiconductors, *Nat. Phys.* **5**, 267 (2009).
- [92] T. Aoki, B. Dayan, E. Wilcut, W. J. Bowen, A. S. Parkins, T. J. Kippenberg, K. J. Vahala, and H. J. Kimble, Observation of strong coupling between one atom and a monolithic microresonator, *Nature (London)* **443**, 671 (2006).
- [93] B. Dayan, A. S. Parkins, T. Aoki, E. P. Ostby, K. J. Vahala, and H. J. Kimble, A photon turnstile dynamically regulated by one atom, *Science* **319**, 1062 (2008).
- [94] D. J. Alton, N. P. Stern, T. Aoki, H. Lee, E. Ostby, K. J. Vahala, and H. J. Kimble, Strong interactions of single atoms and photons near a dielectric boundary, *Nat. Phys.* **7**, 159 (2011).
- [95] E. Will, L. Masters, A. Rauschenbeutel, M. Scheucher, and J. Volz, Coupling a single trapped atom to a whispering-gallery-mode microresonator, *Phys. Rev. Lett.* **126**, 233602 (2021).
- [96] K. M. Birnbaum, A. Boca, R. Miller, A. D. Boozer, T. E. Northup, and H. J. Kimble, Photon blockade in an optical cavity with one trapped atom, *Nature (London)* **436**, 87 (2005).
- [97] C. Hamsen, K. N. Tolazzi, T. Wilk, and G. Rempe, Two-photon blockade in an atom-driven cavity QED system, *Phys. Rev. Lett.* **118**, 133604 (2017).