

# Enhanced tripartite interaction in a magnon-Andreev-superconducting qubit hybrid system: Collapse-revival phenomena and entanglement redistribution

Sheng Zhao  and Peng-Bo Li 

Ministry of Education Key Laboratory for Nonequilibrium Synthesis and Modulation of Condensed Matter, Shaanxi Province Key Laboratory of Quantum Information and Quantum Optoelectronic Devices, School of Physics, Xi'an Jiaotong University, Xi'an 710049, China



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Tripartite interactions are fundamental for realizing quantum phenomena beyond pairwise physics, yet their native implementation—particularly among distinct quantum systems—remains challenging. Here, we propose a hybrid quantum architecture comprising a magnonic mode [in a yttrium iron garnet (YIG) sphere], an Andreev spin qubit, and a superconducting qubit, to realize a native and strong interaction rather than relying on synthesis from two-body processes, leveraging the coupling between the spin-dependent supercurrent and the magnetic field of YIG sphere. Through analytical and numerical studies, we demonstrate that this interaction induces synchronized collapse and revival in qubit populations when the magnon is initially prepared in a coherent state. Notably, during the collapse region—where populations remain static—the entanglement structure undergoes a dramatic and continuous reorganization. We show that the genuine tripartite entanglement is redistributed into bipartite entanglement between the two qubits, and vice versa, with the total entanglement conserved. These phenomena, unattainable via bipartite couplings, underscore the potential of tripartite interactions for exploring intrinsically new quantum effects and advancing hybrid quantum information platforms.

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## I. INTRODUCTION

Coherent interactions between different degrees of freedom in hybrid quantum systems are essential for quantum science and technology [1]. While most interactions implemented to date are pairwise in nature [2], tripartite interactions [3] play a pivotal role in various physical contexts: the physics of nuclei [4], quantum simulation [5,6], quantum blockade [7], quantum error correction [8,9], and quantum gates [10,11]. Importantly, tripartite interactions can give rise to physical phenomena that are inaccessible with only two-body interactions. However, achieving strong tripartite coupling remains a challenge. Existing approaches largely engineer effective tripartite interactions from underlying two-body couplings via Floquet engineering [12–14] or nonlinear couplers [15], which introduce additional hardware complexity, require precise waveform driving or additional two-local couplers to suppress unwanted two-body terms, and are typically restricted to a single physical platform. So, it is of importance to directly realize a native tripartite interaction without precise driving, extra couplers, and unwanted two-body terms in hybrid quantum architectures.

Recently, Andreev spin qubits (ASQs) have emerged as a promising platform for quantum information processing,

combining the benefits of both superconducting qubit (SCQ) and spin qubit to encode information in the spin of a quasiparticle localized in a semiconductor quantum dot junction [16–21]. Its long lifetimes and small footprint [19,20] make it attractive for large-scale quantum devices [22]. Recent experiments have demonstrated the coherent manipulation of a single ASQ [18,19], the supercurrent-mediated coupling between two distant ASQs [21]. Due to spin-orbit coupling, the quantum dot junction has a spin-dependent supercurrent that depends intrinsically on both spin and phase [17,19], allowing the ASQs to be integrated into superconducting circuits and coherently coupled to SCQs [20]. So, through this spin-dependent supercurrent, it is possible and appealing to realize native tripartite interactions in a hybrid quantum architecture.

Quantum magnonics [23–41] is also a rapidly developing field, which provides a promising platform for studying fundamental quantum phenomena and their applications. Magnon is the quanta of collective spin excitations of magnetic materials [like yttrium iron garnet (YIG)] [42–46]. Owing to their widely tunable frequency and strong coupling to various quantum systems, magnons also serve as a promising carrier for quantum information. Magnon-photon coupling [47–51], magnon-phonon coupling [46,52–55], magnon-solid-state spin coupling [56–64], and magnon-SCQ coupling [65,66] have been proposed and analyzed. This makes magnons a natural building block for hybrid quantum system, which integrate the complementary strengths of distinct physical platforms to overcome the limitations of any single system and hold significant promise for advancing quantum information processing. However, previous studies mostly focus on coherent pairwise interactions between magnons and other completely different physical systems; it seems

\*Contact author: [lipengbo@mail.xjtu.edu.cn](mailto:lipengbo@mail.xjtu.edu.cn)

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to us that a genuine tripartite interaction among magnons, ASQs, and SCQs to construct hybrid quantum setups remains unexplored.

In this work, we propose and investigate native tripartite interactions in a hybrid quantum system consisting of a YIG sphere, an ASQ, and an SCQ. The ASQ possesses spin-supercurrent coupling that enables it to easily couple with the SCQ, giving rise to a spin-dependent supercurrent in the superconducting circuit that depends on both the spin and the phase. Utilizing the coupling between this spin-dependent supercurrent and the stray magnetic field of the YIG sphere, a native and strong tripartite interaction is obtained directly among the magnon, the SCQ, and the ASQ without relying on synthesis from two-body processes. This kind of tripartite interaction, which involves a bosonic mode and two solid-state qubits, has no counterpart in previous studies. It enables the joint excitations of magnons and SCQs upon ASQ de-excitation (or excites two solid-state qubits upon magnon annihilation), allowing for simultaneous manipulation of two solid-state qubits via magnons.

Furthermore, by preparing the magnon in a coherent state, with this tripartite interaction, we show that the population oscillations of the two qubits exhibit synchronized collapse and revival. Most notably, during the collapse region—where the populations remain stationary—the entanglement continues to evolve dynamically. Specifically, we confirm that the entanglement between the magnon and the rest of the system consists solely of residual entanglement, which first increases and then decreases, with no bipartite entanglement ever forming between the magnon and each qubit. Moreover, the entanglement between each qubit and the rest of the system also displays collapse-revival behavior, remaining constant during the collapse region. This composite entanglement comprises both bipartite qubit-qubit entanglement and the minimal residual entanglement. These two components exhibit a trade-off in the collapse region: An increase in one necessarily leads to a decrease in the other. This leads to a remarkable conclusion: During the collapse region, although the energy remains constant, the genuine tripartite entanglement is dynamically redistributed into bipartite entanglement between the two qubits, and vice versa, with the total entanglement conserved. These phenomena have never been uncovered before and cannot be achieved through two-body interactions, highlighting the unique potential of tripartite interactions in revealing genuinely novel quantum effects.

## II. THE MODEL

We investigate a hybrid quantum system consisting of a YIG sphere and a superconducting circuit constructed with a quantum dot junction, a conventional Josephson junction, and a capacitor, as depicted in Fig. 1. The ASQ is implemented via the quantum dot junction. The Hamiltonian of ASQ includes the spin-dependent energy  $E_{SO}$  due to spin-orbit coupling and the Zeeman energy arising from the interaction between spin and an external magnetic field  $B_Z$  applied to the junction [17,19–21], which is given by

$$H_{ASQ} = -E_{SO}\boldsymbol{\sigma} \cdot \mathbf{n} \sin \varphi_1 + \frac{1}{2}E_Z \cdot \boldsymbol{\sigma}, \quad (1)$$

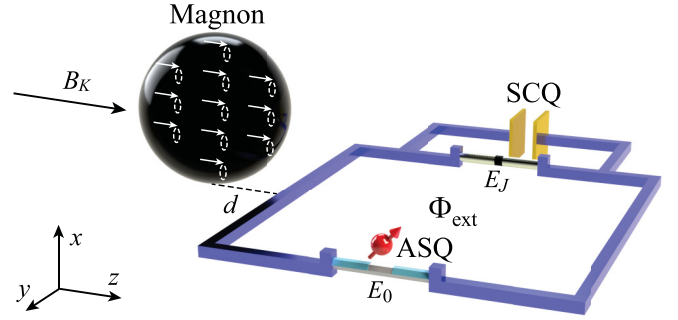


FIG. 1. The hybrid quantum system consisting of the magnon, ASQ from the quantum dot junction, and the SCQ from the SQUID and capacitor.

where  $\boldsymbol{\sigma}$  is the spin operator,  $\mathbf{n}$  is the unit vector along the spin-polarization direction set by the spin-orbit interaction,  $\varphi_1$  is the phase difference across the quantum dot junction, and  $E_Z$  is the Zeeman field. In the limit of  $|E_Z| \gg E_{SO}$ , the free Hamiltonian of the ASQ is mainly dominated by the second term, while the first term gives rise to interactions with other subsystems. The free Hamiltonian of ASQ can be simplified to  $\hat{H}_{ASQ} = \hbar\omega_a\hat{\sigma}_a^z/2$ , where  $\omega_a \simeq |E_Z|/\hbar$  is the frequency of ASQ [20,22,67] and  $\sigma_a^z = |\uparrow\rangle\langle\uparrow| - |\downarrow\rangle\langle\downarrow|$  is the Pauli  $z$  operator with the eigenstates of ASQ  $\{|\downarrow\rangle, |\uparrow\rangle\}$ .

Furthermore, the quantum dot junction and conventional Josephson junction, with Josephson energies  $E_0$  and  $E_J$ , respectively, form a superconducting quantum interference device (SQUID) with an external flux  $\Phi_{ext}$  through it. The SCQ is obtained by connecting the SQUID loop in parallel with the capacitor that has charging energy  $E_C$ . Its Hamiltonian reads  $\hat{H}_{SCQ} = 4E_C\hat{N}^2 - E_J^{\text{sum}}S(\phi_{ext})\cos(\hat{\phi} - \phi_0)$  with charge number  $\hat{N}$  and average phase difference  $\hat{\phi}$ , where  $E_J^{\text{sum}} = E_J + E_0$ ,  $S(\phi_{ext}) = \sqrt{\cos^2\phi_{ext} + a^2\sin^2\phi_{ext}}$  with junction asymmetry  $a = (E_J - E_0)/E_J^{\text{sum}}$  and phase  $\phi_{ext} = \pi\Phi_{ext}/\Phi_0$ , and  $\phi_0 = \arctan(a \tan \phi_{ext})$ . In the two-level subspace  $\{|g\rangle, |e\rangle\}$  of the SCQ, the Hamiltonian of SCQ is rewritten as  $\hat{H}_{SCQ} = \frac{\hbar\omega_s}{2}\hat{\sigma}_s^z$  in the transmon regime [67,85,86]. Here,  $\hbar\omega_s = \sqrt{8E_CE_J^{\text{sum}}S(\phi_{ext}) - E_C}$  is the qubit excitation energy.

The YIG sphere with radius  $R_K$  is placed at a distance of  $d$  from the circuit in the  $z$  direction and  $R_K$  in the  $x$  direction. Applying a uniform bias magnetic field  $B_K$  to the sphere, long-lived spin wave mode can be excited. In the system, only the Kittel mode is considered, where all spins precess in phase and have the identical amplitude [24]. The free Hamiltonian of the Kittel mode is expressed as  $\hat{H}_M = \hbar\omega_m\hat{m}^\dagger\hat{m}$ , where  $\hat{m}$  ( $\hat{m}^\dagger$ ) represents the annihilation (creation) operator of magnon and  $\omega_m = \gamma_0 B_K$  is the resonance frequency with the gyromagnetic ratio  $\gamma_0$  [87].

The magnetic moment  $\boldsymbol{\mu}$  of the YIG sphere emits a stray field described by  $\mathbf{B}(\boldsymbol{\mu}) = \mu_0/(4\pi r^3)[3\mathbf{r}(\boldsymbol{\mu} \cdot \mathbf{r})/r^2 - \boldsymbol{\mu}]$  [88], where  $\mu_0$  is the permeability of the free space and  $\mathbf{r}$  is the position vector from the center of the YIG sphere to any location on the SQUID loop. By the quantized magnetic moment  $\hat{\mu}_x = \hbar\gamma_0\sqrt{N_S}/2(\hat{m}^\dagger + \hat{m})$ ,  $\hat{\mu}_y = i\hbar\gamma_0\sqrt{N_S}/2(\hat{m} - \hat{m}^\dagger)$ , and  $\hat{\mu}_z = \hbar\gamma_0\hat{m}^\dagger\hat{m}$ , with the total number of spins  $N_S$ , the magnetic field can be quantized as  $\hat{\mathbf{B}} = (\hat{B}_x, \hat{B}_y, \hat{B}_z)$ . The magnetic

field  $\hat{\mathbf{B}}$  induces a flux through the SQUID loop, which reads

$$\Phi(\hat{\boldsymbol{\mu}}) = \iint \mathbf{B}(\hat{\boldsymbol{\mu}}) \cdot d\mathbf{S} = \Phi_{\text{YIG}}(\hat{m} + \hat{m}^\dagger), \quad (2)$$

where  $\Phi_{\text{YIG}}$  is related to the relative positions and relative sizes of the YIG sphere and the SQUID loop [67]. The tripartite interaction originates from the influence of magnetic flux induced by the YIG sphere on the spin-dependent energy of ASQ. Based on the phase relationship  $\hat{\phi}_1 = \hat{\phi} + \pi \Phi_{\text{ext}}/\Phi_0$  [67], the first term in Eq. (1) can be expressed as  $-E_{\text{SO}} \boldsymbol{\sigma} \cdot \mathbf{n} \sin(\hat{\phi} + \phi_{\text{ext}})$ . In the subspace  $\{|\uparrow\rangle, |\downarrow\rangle\}$ , the spin along the polarization direction reads  $\boldsymbol{\sigma} \cdot \mathbf{n} = \cos\theta \hat{\sigma}_a^z + \sin\theta \hat{\sigma}_a^x$ , where  $\theta$  is the angle between the direction of the external Zeeman field and the polarization of the spin [22,67]. Assuming the angle  $\theta = \pi/2$  and considering the influence of the magnetic flux induced by the YIG sphere  $\phi_{\text{ext}} \rightarrow \phi_{\text{ext}} + \phi(\hat{\boldsymbol{\mu}})$ , the interaction term can be rewritten as  $\hat{H}_{\text{int}} = -E_{\text{SO}} \hat{\sigma}_a^x \sin(\phi_{\text{ext}} + \hat{\phi} + \phi(\hat{\boldsymbol{\mu}}))$ , where  $\phi(\hat{\boldsymbol{\mu}}) = \pi \Phi(\hat{\boldsymbol{\mu}})/\Phi_0$ . Expanding the trigonometric function with  $\phi(\hat{\boldsymbol{\mu}}) \ll 1$  for typical parameters, the Hamiltonian becomes

$$\hat{H}_{\text{int}} = -E_{\text{SO}} \phi(\hat{\boldsymbol{\mu}}) \cos(\phi_{\text{ext}} + \hat{\phi}) \hat{\sigma}_a^x - E_{\text{SO}} \sin(\phi_{\text{ext}} + \hat{\phi}) \hat{\sigma}_a^x. \quad (3)$$

The first term in Eq. (3) obtained from the influence of the magnetic flux from the YIG sphere on the spin-dependent energy is fully equivalent to the interaction between the spin-dependent supercurrent in the circuit and the magnetic field of the YIG sphere (see the Supplemental Material [67]). In terms of the transmon phase operator  $\hat{\phi} = \hat{\phi} - \varphi_0$ , it  $\hat{H}_{\text{int}1} = E_{\text{SO}} \phi(\hat{\boldsymbol{\mu}}) \hat{\sigma}_a^x / S(\phi_{\text{ext}}) [(1+a) \sin 2\phi_{\text{ext}} \sin \hat{\phi}/2 + (\cos^2 \phi_{\text{ext}} - a \sin^2 \phi_{\text{ext}}) \cos \hat{\phi}]$ . Expanding to order  $\hat{\phi}^2$  and substituting the quantized flux  $\Phi(\hat{\boldsymbol{\mu}})$  and the phase operator  $\hat{\phi}$  with the corresponding magnon bosonic operator  $\hat{m}$  ( $\hat{m}^\dagger$ ) and the SCQ Pauli operator  $\hat{\sigma}_s^+$  ( $\hat{\sigma}_s^-$ ), the tripartite interaction is obtained and can be expressed as

$$\hat{H}_{\text{thr}}/\hbar = G(\hat{\sigma}_s^+ + \hat{\sigma}_s^-)(\hat{m} + \hat{m}^\dagger) \hat{\sigma}_a^x + J \hat{\sigma}_s^+ \hat{\sigma}_s^- (\hat{m} + \hat{m}^\dagger) \hat{\sigma}_a^x. \quad (4)$$

There are also two-body interactions  $\hat{H}_{\text{two}}$  from the second term in Eq. (3) [67]. The total Hamiltonian of the hybrid quantum system is  $\hat{H}_{\text{tot}} = \hat{H}_M + \hat{H}_{\text{ASQ}} + \hat{H}_{\text{SCQ}} + \hat{H}_{\text{thr}} + \hat{H}_{\text{two}}$ . Under the resonance condition  $\omega_a = \omega_m + \omega_s$  (or  $\omega_m = \omega_a + \omega_s$ ), the two-body interaction  $\hat{H}_{\text{two}}$  and the second term of the tripartite interaction in Eq. (4) are far off resonance and can be neglected [67]. Applying the rotating wave approximation, the effective Hamiltonian thus is reduced to

$$\hat{H}_{\text{eff}} = \hat{H}_M + \hat{H}_{\text{ASQ}} + \hat{H}_{\text{SCQ}} + \hbar G(\hat{m}^\dagger \hat{\sigma}_s^+ \hat{\sigma}_a^- + \hat{m} \hat{\sigma}_s^- \hat{\sigma}_a^+), \quad (5)$$

with  $\omega_a = \omega_m + \omega_s$ . For the resonance condition  $\omega_m = \omega_a + \omega_s$ , the interaction becomes  $G(\hat{m} \hat{\sigma}_s^+ \hat{\sigma}_a^+ + \hat{m}^\dagger \hat{\sigma}_s^- \hat{\sigma}_a^-)$ .

In this work, we primarily focus on the tripartite interaction that describes the coherent exchange between the magnon, ASQ, and SCQ. For example, when the resonance condition  $\omega_a = \omega_m + \omega_s$  ( $\omega_m = \omega_a + \omega_s$ ) is selected, the interaction  $\hat{m}^\dagger \hat{\sigma}_s^+ \hat{\sigma}_a^- + \hat{m} \hat{\sigma}_s^- \hat{\sigma}_a^+$  ( $\hat{m} \hat{\sigma}_s^+ \hat{\sigma}_a^+ + \hat{m}^\dagger \hat{\sigma}_s^- \hat{\sigma}_a^-$ ) results in the transmission between  $|n, \uparrow, g\rangle$  and  $|n+1, \downarrow, e\rangle$  ( $|n+1, \downarrow, g\rangle$  and  $|n, \uparrow, e\rangle$ ). This describes magnon and SCQ excitations upon ASQ deexcitation (two qubits excitations upon magnon annihilation) and the inverse process.

Its coupling strength reads

$$G = \frac{E_{\text{SO}} \sin 2\phi_{\text{ext}} (1+a) \pi \Phi_{\text{YIG}}}{2\hbar \Phi_0 S(\phi_{\text{ext}})^{5/4}} \left( \frac{2E_C}{E_j^{\text{sum}}} \right)^{1/4}. \quad (6)$$

In the following, we investigate the influence of relevant parameters on the coupling strength  $G$ . As shown in Fig. 2(a), the coupling strength  $G$  is plotted as functions of the external flux  $\Phi_{\text{ext}}$  with different asymmetry  $a$ . Here, we consider spin-dependent energy  $E_{\text{SO}}/\hbar = 600$  MHz [18–20], typical transmon parameter  $E_j^{\text{sum}}/E_C = 50$  [86], and radius of YIG sphere  $R_K = 30$   $\mu\text{m}$ . The optimal point of the coupling strength  $G$  depends on  $a$  and its maximum value increases as  $a$  decreases.  $G$  varies periodically with  $\Phi_{\text{ext}}/\Phi_0$ , and when  $\Phi_{\text{ext}}/\Phi_0 = 0.5$ ,  $G$  is equal to zero. According to the contour maps shown in Fig. 2(b), the coupling strength  $G$  is proportional to  $E_{\text{SO}}$  and decreases with increasing  $E_j^{\text{sum}}/E_C$ . However, the coupling strength  $G$  is less sensitive to the ratio  $E_j^{\text{sum}}/E_C$  due to  $G \propto (E_C/E_j^{\text{sum}})^{1/4}$ .

Strong coupling is a prerequisite for conducting quantum manipulation. We adopt the decay rates of magnons, ASQs, and SCQs as  $\kappa_m/2\pi = 0.5$  MHz [41,51],  $\kappa_a/2\pi = 0.01$  MHz [20], and  $\kappa_s/2\pi = 0.05$  MHz [89]. The dephasing rates of ASQs and SCQs are taken as  $\gamma_a/2\pi = 1$  MHz [20] and  $\gamma_s/2\pi = 0.05$  MHz [89]. When choosing  $a = 0.3$  and  $\Phi_{\text{ext}}/\Phi_0 = 0.35$ , we have  $G/2\pi \simeq 1.5$  MHz. So the coupling strength  $G$  can reach the strong coupling regime, i.e.,  $G > \kappa_m, \kappa(\gamma)_a, \kappa(\gamma)_s$ . Furthermore, the coupling strength can be enhanced exponentially by the parametric amplification technique [67]. The dephasing rate  $\gamma_a$  of ASQs is expected to decrease by selecting better material for quantum dot junction to reduce the influence of the nuclear environment [20]. The decay rate of magnons  $\kappa_m = \omega_m \alpha_G$  can be reduced to 0.1 MHz with the Gilbert-damping parameter  $\alpha_G = 10^{-5}$  [90]. Therefore, the tripartite coupling strength  $G$  holds promise for being much greater than the dissipations in this hybrid quantum system.

We now show that the quantum dynamics of the hybrid quantum system is governed by the tripartite interaction  $G(\hat{m}^\dagger \hat{\sigma}_s^+ \hat{\sigma}_a^- + \hat{m} \hat{\sigma}_s^- \hat{\sigma}_a^+)$  under the resonance condition  $\omega_a = \omega_m + \omega_s$ , through numerically solving the quantum master equation. To verify the validity of neglecting the far off-resonant interactions, both the effective Hamiltonian  $\hat{H}_{\text{eff}}$  and the total Hamiltonian  $\hat{H}_{\text{tot}}$  are used to simulate the population evolution of the magnon, ASQ, and SCQ with the initial state  $|0, \uparrow, g\rangle$ . Figure 2(c) shows almost-perfect joint excitation of the magnon mode and SCQ by the ASQ; i.e., if the magnon mode gets excited, the SCQ is also in the excited state. When the dissipation rates of ASQs and magnons are reduced, the populations of the SCQ and the magnon mode approach one after a half Rabi period. We also observe the excellent agreement between the dynamics generated by  $\hat{H}_{\text{eff}}$  and  $\hat{H}_{\text{tot}}$ , confirming that these far off-resonant interactions can be safely neglected. Furthermore, the population evolution under the interaction  $G(\hat{m} \hat{\sigma}_s^+ \hat{\sigma}_a^+ + \hat{m}^\dagger \hat{\sigma}_s^- \hat{\sigma}_a^-)$  shows almost-perfect joint excitation of two qubits by magnons; i.e., if one qubit gets excited, the other one is also in the excited state [67].

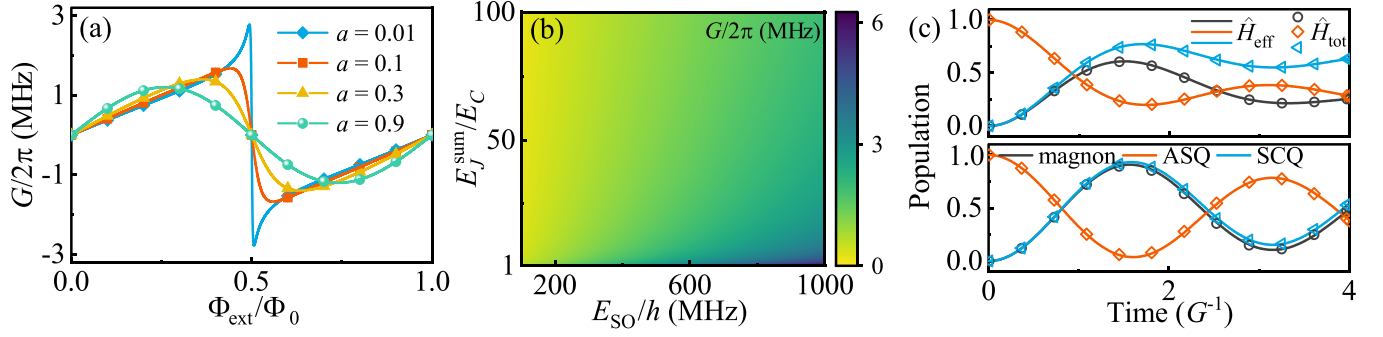


FIG. 2. (a) Variation of the coupling strengths  $G$  with the external flux  $\Phi_{\text{ext}}$  for different asymmetry  $a$ . (b) Contour maps of the coupling strengths  $G$  vs the ratio  $E_J^{\text{sum}}/E_C$  and spin-dependent energy  $E_{\text{SO}}$  with  $a = 0.1$ . (c) The tripartite interaction induces dynamical evolution under the resonance condition  $\omega_m = \omega_a + \omega_s$ . In the upper panel, the dissipation rates are chosen as those in the main text. In the lower panel, the dissipation rates are reduced as  $\kappa_m/2\pi = 0.1$  MHz and  $\gamma_a/2\pi = 0.1$  MHz.

### III. COLLAPSE-REVIVAL PHENOMENA AND ENTANGLEMENT REDISTRIBUTION

Collapse and revival [91–95] is a well-known phenomenon based on the Jaynes-Cummings (JC) interaction, involving the disappearance of oscillations from quantum phase disorder (collapse) and their recovery via phase reordering (revival), which has been observed experimentally [96–98]. In the model of tripartite interaction, the collapse and revival still exist, yet more interesting phenomena have been observed in the dynamic process. We consider the tripartite interaction  $G(\hat{m}^\dagger \hat{\sigma}_s^+ \hat{\sigma}_a^- + \hat{m} \hat{\sigma}_s^- \hat{\sigma}_a^+)$  when  $\omega_a = \omega_m + \omega_s$ . The magnon is prepared in the coherent state  $|\alpha\rangle$  with  $\alpha = 4$ , the ASQ and SCQ are prepared in the excited state  $|\uparrow\rangle$  and ground state  $|g\rangle$ , respectively. The populations of the ASQ  $|\uparrow\rangle$  state and the SCQ  $|e\rangle$  state are analytically calculated to study the collapse-revival phenomena. The results are as follows:  $P_{|\uparrow\rangle} = 0.5 + 0.5X$ ,  $P_{|e\rangle} = 0.5 - 0.5X$ , where  $X = \sum_{n=0}^{\infty} C_n \times \cos(2\Omega_n t)$  with Rabi frequency  $\Omega_n = G\sqrt{n+1}$  and  $C_n = e^{-|\alpha|^2} \frac{\alpha^{2n}}{n!}$ . As shown in Fig. 3(a), when these

components cancel each other out due to random phases, the series summation  $X = 0$ , the oscillations of  $P_{|\uparrow\rangle}$  and  $P_{|e\rangle}$  disappear and the populations remain at 0.5. When these components achieve phase synchronization, their superposition reaches a maximum, manifesting as the revival of oscillations.

Beyond the collapse and revival of populations, an intriguing evolution of entanglement emerges from the dynamics of the tripartite interaction. We now show that during the collapse region, where the populations remain stationary, the entanglement structure undergoes a dramatic and continuous reorganization. For the tripartite quantum system, the entanglement quantified by the logarithmic negativity satisfies the CKW monogamy inequality [99]:  $\mathcal{E}_{i|jk}^2 \geq \mathcal{E}_{i|j}^2 + \mathcal{E}_{i|k}^2$  ( $i, j, k = \text{M, S, A}$ ), where the logarithmic negativity  $\mathcal{E}_{i|j}$  ( $\mathcal{E}_{i|jk}$ ) measures the bipartite entanglement between  $i$  and  $j$  ( $i$  and the single object  $jk$ ). Herein, M, A, and S represent the magnon, ASQ, and SCQ, respectively. The difference between the two sides of the above inequality is the residual entanglement

$$R_{i|jk} = \mathcal{E}_{i|jk}^2 - \mathcal{E}_{i|j}^2 - \mathcal{E}_{i|k}^2, \quad (7)$$

which serves to quantify tripartite entanglement that cannot be accounted for by any bipartite divisions [100–103]. First, we investigate the residual entanglement for the bipartition between the magnon (M) and the remaining two-qubit partition (AS). As shown in Fig. 3(b),  $\mathcal{E}_{\text{M}|A}$  and  $\mathcal{E}_{\text{M}|S}$  remain zero at all times, meaning that there is no bipartite entanglement between the magnon and each qubit, while such bipartite entanglement between the magnon and qubit is present in the JC model [67]. So the residual entanglement for the bipartition M|AS becomes

$$R_{\text{M}|AS} = \mathcal{E}_{\text{M}|AS}^2, \quad (8)$$

which means that the entanglement between the magnon and the rest of the system contains only residual entanglement. The variation of residual entanglement  $R_{\text{M}|AS}$  mirrors that of  $\mathcal{E}_{\text{M}|AS}$ , which gradually decreases to a minimum and then increases, as depicted in Fig. 3(b). Since  $\mathcal{E}_{\text{M}|A}$  ( $\mathcal{E}_{\text{M}|S} = 0$ ), we can also obtain the relation for the bipartition A|MS (S|MA) as

$$R_{\text{A}|MS} (R_{\text{S}|MA}) + \mathcal{E}_{\text{A}|S}^2 = \mathcal{E}_{\text{A}|MS}^2 (\mathcal{E}_{\text{S}|MA}^2), \quad (9)$$

which indicates that the entanglement between any qubit and the rest of the system includes the bipartite entanglement

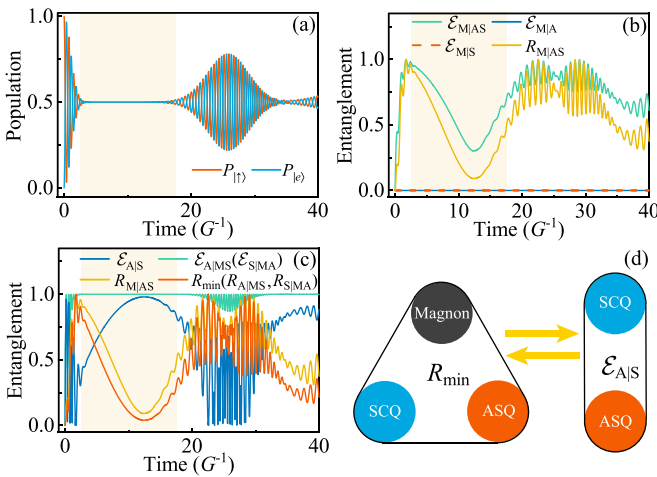


FIG. 3. (a) Simulated evolution of populations without decoherence. The shaded region denotes the collapse region. Panels (b) and (c) show the evolution of the bipartite entanglement and the residual entanglements with time. (d) Schematic diagram of entanglement redistribution.

between two qubits as well as the residual entanglement. Surprisingly, the entanglement between any qubit and the rest of the system exhibits collapse and revival. As shown in Fig. 3(c), in the collapse region,  $\mathcal{E}_{A|MS}$  ( $\mathcal{E}_{S|MA}$ ) is conserved and equal to 1, meaning the entanglement remains maximized. So Eq. (9) becomes  $\mathcal{E}_{A|S}^2 + R_{A|MS}(R_{S|MA}) = 1$  in the collapse region. The conservation of total entanglement forces a dynamic balance: The growth of residual entanglement must be compensated by the consumption of bipartite entanglement between two qubits. As shown in Fig. 3(c), the residual entanglement  $R_{A|MS}$  ( $R_{S|MA}$ ) decreases as  $\mathcal{E}_{A|C}$  increases and vice versa. In particular,  $R_{A|MS}$  ( $R_{S|MA}$ ) coincides with the minimum residual entanglement  $R_{\min} = \min[R_{M|AS}, R_{A|MS}, R_{S|MA}]$  [104,105], since  $R_{A|MS}(R_{S|MA}) \leq R_{M|AS}$  always holds in Fig. 3(c). The nonzero minimum residual entanglement  $R_{\min}$  signifies the existence of genuine tripartite entanglement within the system. Therefore, a conclusion can be drawn regarding the entanglement dynamics in the collapse region: The total entanglement between any qubit and the rest of the system is conserved, yet its form is dynamically transformed from genuine tripartite entanglement into bipartite entanglement between the two qubits and back, as illustrated in Fig. 3(d).

This finding provides unequivocal evidence that energy and entanglement dynamics can become independent. In the collapse region, the energy remains constant, while the entanglement is redistributed from the tripartite form to the bipartite form and vice versa. This highlights the potential of tripartite interaction for exploring complex entanglement dynamics beyond the reach of simple JC interaction, where such entanglement redistribution does not exist [67]. We have also analytically calculated the bipartite entanglement measured by von Neumann entropy or concurrence, and demonstrated

that the redistribution of entanglement still exists even in the presence of dissipation in the Supplemental Material [67].

#### IV. CONCLUSION

We propose a scheme to achieve native and strong tripartite interaction in a hybrid quantum system consisting of a YIG sphere, an Andreev spin qubit, and a superconducting qubit. This intrinsic interaction emerges directly from the coupling between the spin-dependent supercurrent and the magnetic field of YIG sphere, requiring no external drives or couplers. Moreover, by preparing the magnon in a coherent state, we investigate the collapse and revival of the two-qubit populations. Notably, in the collapse region where the populations remain stationary, we find continuous entanglement redistribution from tripartite to bipartite forms and back with the total entanglement conserved. Our work not only provides a mechanism for realizing native tripartite interactions in hybrid quantum architectures that integrate the complementary strengths of distinct physical systems, but also demonstrates the potential of tripartite coupling to generate quantum phenomena—thus holding fundamental significance and offering promising implications for quantum information processing.

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#### DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

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- [1] Z.-L. Xiang, S. Ashhab, J. Q. You, and F. Nori, Hybrid quantum circuits: Superconducting circuits interacting with other quantum systems, *Rev. Mod. Phys.* **85**, 623 (2013).
  - [2] A. Frisk Kockum, A. Miranowicz, S. De Liberato, S. Savasta, and F. Nori, Ultrastrong coupling between light and matter, *Nat. Rev. Phys.* **1**, 19 (2019).
  - [3] C. Fruy, A. Théry, B. Hue, W. Legrand, L. Jarjat, J. Bally, J. Craquelin, M. Delbecq, A. Cottet, and T. Kontos, Observation of strong tripartite coupling in a cavity-quantum circuit-antiferromagnet platform, *Nat. Mater.* (2026).
  - [4] B. Loiseau and Y. Nogami, Three-nucleon force, *Nucl. Phys. B* **2**, 470 (1967).
  - [5] K. Majumdar, D. Furton, and G. S. Uhrig, Effects of ring exchange interaction on the Néel phase of two-dimensional, spatially anisotropic, frustrated Heisenberg quantum antiferromagnet, *Phys. Rev. B* **85**, 144420 (2012).
  - [6] R. Babbush, P. J. Love, and A. Aspuru-Guzik, Adiabatic quantum simulation of quantum chemistry, *Sci. Rep.* **4**, 6603 (2014).
  - [7] S. Chakram, K. He, A. V. Dixit, A. E. Oriani, R. K. Naik, N. Leung, H. Kwon, W.-L. Ma, L. Jiang, and D. I. Schuster, Multimode photon blockade, *Nat. Phys.* **18**, 879 (2022).
  - [8] A. Kitaev, Anyons in an exactly solved model and beyond, *Ann. Phys.* **321**, 2 (2006).
  - [9] A. G. Fowler, M. Mariantoni, J. M. Martinis, and A. N. Cleland, Surface codes: Towards practical large-scale quantum computation, *Phys. Rev. A* **86**, 032324 (2012).
  - [10] H.-T. Liu, B.-J. Chen, J.-C. Zhang, Y.-X. Xiao, T.-M. Li, K. Huang, Z. Wang, H. Li, K. Zhao, Y. Xu, *et al.*, Direct implementation of high-fidelity three-qubit gates for superconducting processor with tunable couplers, *Phys. Rev. Lett.* **135**, 050602 (2025).
  - [11] T. Liu, Implementing a nonpairwise three-body interaction of superconducting qubits and its applications in realizing three-qubit entanglement and quantum gates, *Phys. Rev. Res.* **8**, 013023 (2026).
  - [12] K. S. C. Decker, C. Karrasch, J. Eisert, and D. M. Kennes, Floquet engineering topological many-body localized systems, *Phys. Rev. Lett.* **124**, 190601 (2020).
  - [13] F. Petiziol, M. Sameti, S. Carretta, S. Wimberger, and F. Mintert, Quantum simulation of three-body interactions in weakly driven quantum systems, *Phys. Rev. Lett.* **126**, 250504 (2021).

- [14] W. Liu, W. Feng, W. Ren, D.-W. Wang, and H. Wang, Synthesizing three-body interaction of spin chirality with superconducting qubits, *Appl. Phys. Lett.* **116**, 114001 (2020).
- [15] T. Menke, W. P. Banner, T. R. Bergamaschi, A. Di Paolo, A. Vepsäläinen, S. J. Weber, R. Winik, A. Melville, B. M. Niedzielski, D. Rosenberg, K. Serniak, M. E. Schwartz, J. L. Yoder, A. Aspuru-Guzik, S. Gustavsson, J. A. Grover, C. F. Hirjibehedin, A. J. Kerman, and W. D. Oliver, Demonstration of tunable three-body interactions between superconducting qubits, *Phys. Rev. Lett.* **129**, 220501 (2022).
- [16] N. M. Chtchelkatchev and Y. V. Nazarov, Andreev quantum dots for spin manipulation, *Phys. Rev. Lett.* **90**, 226806 (2003).
- [17] C. Padurariu and Y. V. Nazarov, Theoretical proposal for superconducting spin qubits, *Phys. Rev. B* **81**, 144519 (2010).
- [18] M. Hays, V. Fatemi, D. Bouman, J. Cerrillo, S. Diamond, K. Serniak, T. Connolly, P. Krogstrup, J. Nygård, A. L. Yeyati, A. Geresdi, and M. H. Devoret, Coherent manipulation of an Andreev spin qubit, *Science* **373**, 430 (2021).
- [19] A. Bargerbos, M. Pita-Vidal, R. Žitko, L. J. Splitthoff, L. Grünhaupt, J. J. Wesdorp, Y. Liu, L. P. Kouwenhoven, R. Aguado, C. K. Andersen, A. Kou, and B. van Heck, Spectroscopy of spin-split Andreev levels in a quantum dot with superconducting leads, *Phys. Rev. Lett.* **131**, 097001 (2023).
- [20] M. Pita-Vidal, A. Bargerbos, R. Žitko, L. J. Splitthoff, L. Grünhaupt, J. J. Wesdorp, Y. Liu, L. P. Kouwenhoven, R. Aguado, B. van Heck, *et al.*, Direct manipulation of a superconducting spin qubit strongly coupled to a transmon qubit, *Nat. Phys.* **19**, 1110 (2023).
- [21] M. Pita-Vidal, J. J. Wesdorp, L. J. Splitthoff, A. Bargerbos, Y. Liu, L. P. Kouwenhoven, and C. K. Andersen, Strong tunable coupling between two distant superconducting spin qubits, *Nat. Phys.* **20**, 1158 (2024).
- [22] M. Pita-Vidal, J. J. Wesdorp, and C. K. Andersen, Blueprint for all-to-all-connected superconducting spin qubits, *PRX Quantum* **6**, 010308 (2025).
- [23] A. V. Chumak, V. I. Vasyuchka, A. A. Serga, and B. Hillebrands, Magnon spintronics, *Nat. Phys.* **11**, 453 (2015).
- [24] Y. Tabuchi, S. Ishino, A. Noguchi, T. Ishikawa, R. Yamazaki, K. Usami, and Y. Nakamura, Quantum magnonics: The magnon meets the superconducting qubit, *C. R. Phys.* **17**, 729 (2016).
- [25] D. Lachance-Quirion, Y. Tabuchi, A. Gloppe, K. Usami, and Y. Nakamura, Hybrid quantum systems based on magnonics, *Appl. Phys. Express* **12**, 070101 (2019).
- [26] Z.-X. Li, Y. Cao, and P. Yan, Topological insulators and semimetals in classical magnetic systems, *Phys. Rep.* **915**, 1 (2021).
- [27] H. Yuan, Y. Cao, A. Kamra, R. A. Duine, and P. Yan, Quantum magnonics: When magnon spintronics meets quantum information science, *Phys. Rep.* **965**, 1 (2022).
- [28] P. Pirro, V. I. Vasyuchka, A. A. Serga, and B. Hillebrands, Advances in coherent magnonics, *Nat. Rev. Mater.* **6**, 1114 (2021).
- [29] X. Zhang, C.-L. Zou, N. Zhu, F. Marquardt, L. Jiang, and H. X. Tang, Magnon dark modes and gradient memory, *Nat. Commun.* **6**, 8914 (2015).
- [30] Y. Iguchi, S. Uemura, K. Ueno, and Y. Onose, Nonreciprocal magnon propagation in a noncentrosymmetric ferromagnet  $\text{Fe}_5\text{O}_8$ , *Phys. Rev. B* **92**, 184419 (2015).
- [31] S. Sharma, Y. M. Blanter, and Gerrit E. W. Bauer, Optical cooling of magnons, *Phys. Rev. Lett.* **121**, 087205 (2018).
- [32] J. Li, S.-Y. Zhu, and G. S. Agarwal, Squeezed states of magnons and phonons in cavity magnomechanics, *Phys. Rev. A* **99**, 021801(R) (2019).
- [33] C. Kong, H. Xiong, and Y. Wu, Magnon-induced nonreciprocity based on the magnon Kerr effect, *Phys. Rev. Appl.* **12**, 034001 (2019).
- [34] H. Yu, J. Xiao, and H. Schultheiss, Magnetic texture based magnonics, *Phys. Rep.* **905**, 1 (2021).
- [35] F.-Z. Ji and J.-H. An, Kerr nonlinearity induced strong spin-magnon coupling, *Phys. Rev. B* **108**, L180409 (2023).
- [36] C.-J. Yang, Q. Tong, and J.-H. An, Quantum-state engineering in cavity magnomechanics formed by two-dimensional magnetic materials, *New J. Phys.* **26**, 023032 (2024).
- [37] X.-F. Pan, P.-B. Li, X.-L. Hei, X. Zhang, M. Mochizuki, F.-L. Li, and F. Nori, Magnon-skyrmion hybrid quantum systems: Tailoring interactions via magnons, *Phys. Rev. Lett.* **132**, 193601 (2024).
- [38] Q. Bin, H. Jing, Y. Wu, F. Nori, and X.-Y. Lü, Nonreciprocal bundle emissions of quantum entangled pairs, *Phys. Rev. Lett.* **133**, 043601 (2024).
- [39] F. Wang, J. Liu, D. Kong, and X.-Y. Lü, Multipartite magnon entanglement and collective steering in cavity magnonics, *Phys. Rev. A* **111**, 033712 (2025).
- [40] S. Zhao, Y.-L. Ren, X.-L. Hei, X.-F. Pan, and P.-B. Li, Magnon blockade in spin-magnon systems with frequency detuning, *Phys. Rev. A* **112**, 013712 (2025).
- [41] R.-C. Shen, J. Li, Y.-M. Sun, W.-J. Wu, X. Zuo, Y.-P. Wang, S.-Y. Zhu, and J. You, Cavity-magnon polaritons strongly coupled to phonons, *Nat. Commun.* **16**, 5652 (2025).
- [42] J. Barker and Gerrit E. W. Bauer, Thermal spin dynamics of yttrium iron garnet, *Phys. Rev. Lett.* **117**, 217201 (2016).
- [43] M. Collet, X. De Milly, O. d'Allivy Kelly, V. V. Naletov, R. Bernard, P. Bortolotti, J. Ben Youssef, V. Demidov, S. Demokritov, J. L. Prieto, *et al.*, Generation of coherent spin-wave modes in yttrium iron garnet microdiscs by spin-orbit torque, *Nat. Commun.* **7**, 10377 (2016).
- [44] A. J. Princep, R. A. Ewings, S. Ward, S. Tóth, C. Dubs, D. Prabhakaran, and A. T. Boothroyd, The full magnon spectrum of yttrium iron garnet, *npj Quantum Mater.* **2**, 63 (2017).
- [45] X.-Y. Wei, O. A. Santos, C. S. Lusero, G. Bauer, J. Ben Youssef, and B. van Wees, Giant magnon spin conductivity in ultrathin yttrium iron garnet films, *Nat. Mater.* **21**, 1352 (2022).
- [46] X. Zhang, C.-L. Zou, L. Jiang, and H. X. Tang, Cavity magnomechanics, *Sci. Adv.* **2**, e1501286 (2016).
- [47] O. O. Soykal and M. E. Flatté, Strong field interactions between a nanomagnet and a photonic cavity, *Phys. Rev. Lett.* **104**, 077202 (2010).
- [48] D. Zhang, X.-M. Wang, T.-F. Li, X.-Q. Luo, W. Wu, F. Nori, and J. You, Cavity quantum electrodynamics with ferromagnetic magnons in a small yttrium-iron-garnet sphere, *npj Quantum Inf.* **1**, 15014 (2015).
- [49] S. Sharma, Y. M. Blanter, and Gerrit E. W. Bauer, Light scattering by magnons in whispering gallery mode cavities, *Phys. Rev. B* **96**, 094412 (2017).
- [50] B. Bhoi, B. Kim, S.-H. Jang, J. Kim, J. Yang, Y.-J. Cho, and S.-K. Kim, Abnormal anticrossing effect in photon-magnon coupling, *Phys. Rev. B* **99**, 134426 (2019).

- [51] J. Xu, C. Zhong, S. Zhuang, C. Qian, Y. Jiang, A. Pishehvar, X. Han, D. Jin, J. M. Jornet, B. Zhen, J. Hu, L. Jiang, and X. Zhang, Slow-wave hybrid magnonics, *Phys. Rev. Lett.* **132**, 116701 (2024).
- [52] A. Kani, B. Sarma, and J. Twamley, Intensive cavity-magnomechanical cooling of a levitated macromagnet, *Phys. Rev. Lett.* **128**, 013602 (2022).
- [53] Z. Shen, G.-T. Xu, M. Zhang, Y.-L. Zhang, Y. Wang, C.-Z. Chai, C.-L. Zou, G.-C. Guo, and C.-H. Dong, Coherent coupling between phonons, magnons, and photons, *Phys. Rev. Lett.* **129**, 243601 (2022).
- [54] M. F. Colombano, G. Arregui, F. Bonell, N. E. Capuj, E. Chavez-Angel, A. Pitanti, S. O. Valenzuela, C. M. Sotomayor-Torres, D. Navarro-Urrios, and M. V. Costache, Ferromagnetic resonance assisted optomechanical magnetometer, *Phys. Rev. Lett.* **125**, 147201 (2020).
- [55] Y. Li, C. Zhao, W. Zhang, A. Hoffmann, and V. Novosad, Advances in coherent coupling between magnons and acoustic phonons, *APL Mater.* **9**, 060902 (2021).
- [56] X.-L. Hei, P.-B. Li, X.-F. Pan, and F. Nori, Enhanced tripartite interactions in spin-magnon-mechanical hybrid systems, *Phys. Rev. Lett.* **130**, 073602 (2023).
- [57] T. Neuman, D. S. Wang, and P. Narang, Nanomagnonic cavities for strong spin-magnon coupling and magnon-mediated spin-spin interactions, *Phys. Rev. Lett.* **125**, 247702 (2020).
- [58] X.-L. Hei, X.-L. Dong, J.-Q. Chen, C.-P. Shen, Y.-F. Qiao, and P.-B. Li, Enhancing spin-photon coupling with a micromagnet, *Phys. Rev. A* **103**, 043706 (2021).
- [59] W. Xiong, M. Tian, G.-Q. Zhang, and J. Q. You, Strong long-range spin-spin coupling via a Kerr magnon interface, *Phys. Rev. B* **105**, 245310 (2022).
- [60] P. Andrich, C. F. De Las Casas, X. Liu, H. L. Bretscher, J. R. Berman, F. J. Heremans, P. F. Nealey, and D. D. Awschalom, Long-range spin wave mediated control of defect qubits in nanodiamonds, *npj Quantum Inf.* **3**, 28 (2017).
- [61] M. Fukami, D. R. Candido, D. D. Awschalom, and M. E. Flatté, Opportunities for long-range magnon-mediated entanglement of spin qubits via on- and off-resonant coupling, *PRX Quantum* **2**, 040314 (2021).
- [62] I. C. Skogvoll, J. Lidal, J. Danon, and A. Kamra, Tunable anisotropic quantum Rabi model via a magnon–spin-qubit ensemble, *Phys. Rev. Appl.* **16**, 064008 (2021).
- [63] C. Gonzalez-Ballester, T. van der Sar, and O. Romero-Isart, Towards a quantum interface between spin waves and paramagnetic spin baths, *Phys. Rev. B* **105**, 075410 (2022).
- [64] M. Bejarano, F. J. T. Goncalves, T. Hache, M. Hollenbach, C. Heins, T. Hula, L. Körber, J. Heinze, Y. Berencén, M. Helm, J. Fassbender, G. V. Astakhov, and H. Schultheiss, Parametric magnon transduction to spin qubits, *Sci. Adv.* **10**, eadi2042 (2024).
- [65] M. Kounalakis, Gerrit E. W. Bauer, and Y. M. Blanter, Analog quantum control of magnonic cat states on a chip by a superconducting qubit, *Phys. Rev. Lett.* **129**, 037205 (2022).
- [66] S. P. Wolski, D. Lachance-Quirion, Y. Tabuchi, S. Kono, A. Noguchi, K. Usami, and Y. Nakamura, Dissipation-based quantum sensing of magnons with a superconducting qubit, *Phys. Rev. Lett.* **125**, 117701 (2020).
- [67] See supplemental material at <http://link.aps.org/supplemental/10.1103/mqz1-99xc> for detailed derivations of the Hamiltonians, enhanced coupling strength, and analytical solutions for the collapse-revival dynamics, which includes Refs. [68–84].
- [68] J. Clarke, A. N. Cleland, M. H. Devoret, D. Esteve, and J. M. Martinis, Quantum mechanics of a macroscopic variable: The phase difference of a Josephson junction, *Science* **239**, 992 (1988).
- [69] J. M. Martinis, M. H. Devoret, and J. Clarke, Quantum Josephson junction circuits and the dawn of artificial atoms, *Nat. Phys.* **16**, 234 (2020).
- [70] Y. Makhlin, G. Schön, and A. Shnirman, Quantum-state engineering with Josephson-junction devices, *Rev. Mod. Phys.* **73**, 357 (2001).
- [71] J. Van Kranendonk and J. H. Van Vleck, Spin waves, *Rev. Mod. Phys.* **30**, 1 (1958).
- [72] J. Clarke and F. K. Wilhelm, Superconducting quantum bits, *Nature (London)* **453**, 1031 (2008).
- [73] D. Lachance-Quirion, S. P. Wolski, Y. Tabuchi, S. Kono, K. Usami, and Y. Nakamura, Entanglement-based single-shot detection of a single magnon with a superconducting qubit, *Science* **367**, 425 (2020).
- [74] Y.-P. Wang, J. W. Rao, Y. Yang, P.-C. Xu, Y. S. Gui, B. M. Yao, J. Q. You, and C.-M. Hu, Nonreciprocity and unidirectional invisibility in cavity magnonics, *Phys. Rev. Lett.* **123**, 127202 (2019).
- [75] C. C. Rusconi, M. J. A. Schuetz, J. Gieseler, M. D. Lukin, and O. Romero-Isart, Hybrid architecture for engineering magnonic quantum networks, *Phys. Rev. A* **100**, 022343 (2019).
- [76] Y. Tabuchi, S. Ishino, A. Noguchi, T. Ishikawa, R. Yamazaki, K. Usami, and Y. Nakamura, Coherent coupling between a ferromagnetic magnon and a superconducting qubit, *Science* **349**, 405 (2015).
- [77] J. R. Johansson, P. D. Nation, and F. Nori, Qutip: An open-source Python framework for the dynamics of open quantum systems, *Comput. Phys. Commun.* **183**, 1760 (2012).
- [78] T. Holstein and H. Primakoff, Field dependence of the intrinsic domain magnetization of a ferromagnet, *Phys. Rev.* **58**, 1098 (1940).
- [79] Y.-P. Wang, G.-Q. Zhang, D. Zhang, X.-Q. Luo, W. Xiong, S.-P. Wang, T.-F. Li, C.-M. Hu, and J. Q. You, Magnon Kerr effect in a strongly coupled cavity-magnon system, *Phys. Rev. B* **94**, 224410 (2016).
- [80] M.-A. Lemonde, N. Didier, and A. A. Clerk, Enhanced nonlinear interactions in quantum optomechanics via mechanical amplification, *Nat. Commun.* **7**, 11338 (2016).
- [81] S. C. Burd, R. Srinivas, J. J. Bollinger, A. C. Wilson, D. J. Wineland, D. Leibfried, D. H. Slichter, and D. T. C. Allcock, Quantum amplification of mechanical oscillator motion, *Science* **364**, 1163 (2019).
- [82] R. Horodecki and M. Horodecki, Information-theoretic aspects of inseparability of mixed states, *Phys. Rev. A* **54**, 1838 (1996).
- [83] G. Vidal and R. F. Werner, Computable measure of entanglement, *Phys. Rev. A* **65**, 032314 (2002).
- [84] S. A. Hill and W. K. Wootters, Entanglement of a pair of quantum bits, *Phys. Rev. Lett.* **78**, 5022 (1997).
- [85] J. Koch, T. M. Yu, J. Gambetta, A. A. Houck, D. I. Schuster, J. Majer, A. Blais, M. H. Devoret, S. M. Girvin, and R. J. Schoelkopf, Charge-insensitive qubit design derived from the Cooper pair box, *Phys. Rev. A* **76**, 042319 (2007).

- [86] A. Blais, A. L. Grimsmo, S. M. Girvin, and A. Wallraff, Circuit quantum electrodynamics, *Rev. Mod. Phys.* **93**, 025005 (2021).
- [87] B. Zare Rameshti, S. Viola Kusminskiy, J. A. Haigh, K. Usami, D. Lachance-Quirion, Y. Nakamura, C.-M. Hu, H. X. Tang, G. E. Bauer, and Y. M. Blanter, Cavity magnonics, *Phys. Rep.* **979**, 1 (2022).
- [88] J. Gieseler, A. Kabcenell, E. Rosenfeld, J. D. Schaefer, A. Safira, M. J. A. Schuetz, C. Gonzalez-Ballester, C. C. Rusconi, O. Romero-Isart, and M. D. Lukin, Single-spin magnetomechanics with levitated micromagnets, *Phys. Rev. Lett.* **124**, 163604 (2020).
- [89] M. Kjaergaard, M. E. Schwartz, J. Braumüller, P. Krantz, J. I.-J. Wang, S. Gustavsson, and W. D. Oliver, Superconducting qubits: Current state of play, *Annu. Rev. Condens. Matter Phys.* **11**, 369 (2020).
- [90] Y. Tabuchi, S. Ishino, T. Ishikawa, R. Yamazaki, K. Usami, and Y. Nakamura, Hybridizing ferromagnetic magnons and microwave photons in the quantum limit, *Phys. Rev. Lett.* **113**, 083603 (2014).
- [91] J. H. Eberly, N. B. Narozhny, and J. J. Sanchez-Mondragon, Periodic spontaneous collapse and revival in a simple quantum model, *Phys. Rev. Lett.* **44**, 1323 (1980).
- [92] J. Gea-Banacloche, Collapse and revival of the state vector in the Jaynes-Cummings model: An example of state preparation by a quantum apparatus, *Phys. Rev. Lett.* **65**, 3385 (1990).
- [93] V. Bužek, H. Moya-Cessa, P. L. Knight, and S. J. D. Phoenix, Schrödinger-cat states in the resonant Jaynes-Cummings model: Collapse and revival of oscillations of the photon-number distribution, *Phys. Rev. A* **45**, 8190 (1992).
- [94] V. S. Malinovsky and I. R. Sola, Phase-controlled collapse and revival of entanglement of two interacting qubits, *Phys. Rev. Lett.* **96**, 050502 (2006).
- [95] J. Casanova, G. Romero, I. Lizuain, J. J. García-Ripoll, and E. Solano, Deep strong coupling regime of the Jaynes-Cummings model, *Phys. Rev. Lett.* **105**, 263603 (2010).
- [96] M. Bild, M. Fadel, Y. Yang, U. von Lüpke, P. Martin, A. Bruno, and Y. Chu, Schrödinger cat states of a 16-microgram mechanical oscillator, *Science* **380**, 274 (2023).
- [97] G. Rempe, H. Walther, and N. Klein, Observation of quantum collapse and revival in a one-atom maser, *Phys. Rev. Lett.* **58**, 353 (1987).
- [98] M. Brune, F. Schmidt-Kaler, A. Maali, J. Dreyer, E. Hagley, J. M. Raimond, and S. Haroche, Quantum Rabi oscillation: A direct test of field quantization in a cavity, *Phys. Rev. Lett.* **76**, 1800 (1996).
- [99] V. Coffman, J. Kundu, and W. K. Wootters, Distributed entanglement, *Phys. Rev. A* **61**, 052306 (2000).
- [100] Y.-C. Ou and H. Fan, Monogamy inequality in terms of negativity for three-qubit states, *Phys. Rev. A* **75**, 062308 (2007).
- [101] G. H. Aguilar, A. Valdés-Hernández, L. Davidovich, S. P. Walborn, and P. H. Souto Ribeiro, Experimental entanglement redistribution under decoherence channels, *Phys. Rev. Lett.* **113**, 240501 (2014).
- [102] B. Jungnitsch, T. Moroder, and O. Gühne, Taming multiparticle entanglement, *Phys. Rev. Lett.* **106**, 190502 (2011).
- [103] J. Li, S.-Y. Zhu, and G. S. Agarwal, Magnon-photon-phonon entanglement in cavity magnomechanics, *Phys. Rev. Lett.* **121**, 203601 (2018).
- [104] G. Adesso and F. Illuminati, Entanglement in continuous-variable systems: Recent advances and current perspectives, *J. Phys. A: Math. Theor.* **40**, 7821 (2007).
- [105] G. Adesso and F. Illuminati, Continuous variable tangle, monogamy inequality, and entanglement sharing in Gaussian states of continuous variable systems, *New J. Phys.* **8**, 15 (2006).