

## Kineclinic magnetogenesis in relativistic collisionless plasmas

Modhuchandra Laishram <sup>1</sup>, Suresh Basnet <sup>1</sup>, and Young Dae Yoon <sup>1,2,\*</sup><sup>1</sup>*Asia Pacific Center for Theoretical Physics, Pohang, Gyeongbuk 37673, Republic of Korea*<sup>2</sup>*Department of Physics, Pohang University of Science and Technology, Pohang, Gyeongbuk 37673, Republic of Korea*

(Received 9 January 2026; accepted 9 May 2026; published 26 May 2026)

The relativistic momentum equation of a collisionless plasma is reformulated to describe the time evolution of canonical vorticity. Compared to the nonrelativistic counterpart, an additional source term for canonical vorticity is identified, which embodies the misalignment between the fluid momentum and fluid velocity gradients. This kineclinic term breaks the frozen-in condition of canonical vorticity, thereby enabling the generation or dissipation of magnetic fields and vorticity. We verify the role of this effect through particle-in-cell simulations of a kineclinic flow. Kineclinicity is generically expected in relativistic plasma kinetic plasmas lacking a functional relationship between fluid momentum and fluid velocity.

DOI: [10.1103/mlyq-q8xw](https://doi.org/10.1103/mlyq-q8xw)

## I. INTRODUCTION

The dynamical behavior of astrophysical plasmas spans a broad spectrum of regimes. While jets in planetary nebulae and protostellar systems are typically nonrelativistic, those in active galactic nuclei and gamma ray bursts are relativistic or even ultrarelativistic [1–3]. In addition, many high-energy environments such as pulsar wind nebula and supernova remnants are typically relativistic pair plasmas with Lorentz factors reaching up to 100 [4–7]. Magnetic fields of varying scales and magnitudes are ubiquitous in these extreme plasma systems, and they are responsible for numerous dynamical processes [8–12]. The origin of the magnetic fields and their amplification process—magnetogenesis—in these extreme plasmas remain an interdisciplinary subject of serious debate from both cosmological [12,13] and plasma physics aspects [14,15], and both perspectives are justified in different astrophysical scenarios [16,17].

It is often thought that large-scale magnetic fields originate from the amplification of a microscopic “seed field” via a dynamo process [11,14,18–20]. Popular plasma physics mechanisms that generate these seed fields include the Biermann battery (baroclinicity) effect [21], the Weibel instability [22], and electron-scale Kelvin-Helmholtz instability (ESKHI) [23–25].

Recently, Laishram *et al.* [26] showed that by taking canonical vorticity—the curl of the canonical momentum—as the primary variable, one can reformulate the momentum equation into an induction equation, akin to the plasma induction equation in magnetohydrodynamics (MHD). This formulation

allows for a clear distinction and unification of various magnetogenesis mechanisms. In particular, a nonideal term due to the pressure tensor gives rise to a canonical vorticity source, that is, the “canonical battery” effect [26,27], which generalizes both Biermann battery and Weibel instability and also predicts new mechanisms. The formulation is also useful when analyzing magnetic reconnection and helicity transport [27–31].

One limitation of the previous formulation is that it was derived for subrelativistic velocities. A natural question to ask is whether a similar formulation is possible in relativistic regimes. In fact, many previous works studying Weibel instability and ESKHI consider relativistic (pair) plasmas [4–7,32,33]. There is also significant effort on covariant formulations of canonical vorticity and helicity, where twisting of spacetime can give rise to magnetogenesis [34–37]. This mechanism arises within relativistic ideal magnetohydrodynamics and corresponds to an additional relativistic baroclinic term. As elegant as this covariant formulation is, it makes several important assumptions, such as perfect fluid and local Maxwellian closures, that make it inaccurate for kinetic, collisionless systems. It is also difficult to compare to laboratory-frame experiments or simulations because magnetic field and vorticity are defined in synchronic space: a sentiment shared by some previous work [38–40].

In this article, we formulate the canonical vorticity framework in the relativistic regime. Taking the first moment of the relativistic Vlasov equation and taking the curl, we obtain an equation describing the time evolution of canonical vorticity. Compared to the nonrelativistic case, an additional term arises due to the misalignment of the fluid momentum and velocity gradients, or what we call “kineclinicity.” This term acts as a nonideal term that destroys the conservation of canonical vorticity flux. We perform particle-in-cell (PIC) simulations of a kineclinic flow to demonstrate the breaking of the frozen-in condition of canonical vorticity, generating the seed magnetic field. Various implications are discussed.

\*Contact author: [youngdae.yoon@apctp.org](mailto:youngdae.yoon@apctp.org)

## II. RELATIVISTIC CANONICAL VORTICITY FRAMEWORK

To generalize the canonical vorticity framework to the relativistic regime, we first consider the relativistic Vlasov equation under the Lorentz force,

$$\frac{\partial f_\sigma}{\partial t} + \frac{\mathbf{p}}{\gamma m_\sigma} \cdot \frac{\partial f_\sigma}{\partial \mathbf{x}} + q_\sigma \left( \mathbf{E} + \frac{\mathbf{p}}{\gamma m_\sigma} \times \mathbf{B} \right) \cdot \frac{\partial f_\sigma}{\partial \mathbf{p}} = 0, \quad (1)$$

where  $f_\sigma = f_\sigma(\mathbf{x}, \mathbf{p}, t)$ ,  $q_\sigma$ , and  $m_\sigma$  are the phase-space distribution function, charge, and mass of species  $\sigma$ , and  $\gamma = \sqrt{1 + p^2/m_\sigma^2 c^2}$  is the Lorentz factor, with  $p = |\mathbf{p}|$ .

Taking the first moment of Eq. (1), i.e., multiplying by  $\mathbf{p}$  and integrating over all  $\mathbf{p}$ , yields the following momentum equation:

$$\frac{\partial \langle \mathbf{p} \rangle_\sigma}{\partial t} + \mathbf{u}_\sigma \cdot \nabla \langle \mathbf{p} \rangle_\sigma = q_\sigma (\mathbf{E} + \mathbf{u}_\sigma \times \mathbf{B}) - \frac{\nabla \cdot \overleftrightarrow{\mathcal{P}}_\sigma}{n_\sigma}, \quad (2)$$

where, denoting  $\mathbf{v} = \mathbf{p}/\gamma m_\sigma$ ,  $n_\sigma = \int f_\sigma d^3\mathbf{p}$  is the density,  $\mathbf{u}_\sigma = n_\sigma^{-1} \int \mathbf{v} f_\sigma d^3\mathbf{p}$  is the fluid velocity,  $\langle \mathbf{p} \rangle_\sigma = n_\sigma^{-1} \int \mathbf{p} f_\sigma d^3\mathbf{p}$  is the fluid momentum, and  $\overleftrightarrow{\mathcal{P}}_\sigma = \int \mathbf{v}' \mathbf{p}' f_\sigma d^3\mathbf{p}$  is the pressure tensor, where  $\mathbf{v}' = \mathbf{v} - \mathbf{u}_\sigma$  and  $\mathbf{p}' = \mathbf{p} - \langle \mathbf{p} \rangle_\sigma$  are the random parts of  $\mathbf{v}_\sigma$  and  $\mathbf{p}_\sigma$ , respectively. For nonrelativistic distributions,  $\langle \mathbf{p} \rangle_\sigma = m_\sigma \mathbf{u}_\sigma$  and so Eq. (2) can be rewritten in terms of  $\mathbf{u}_\sigma$ . On the other hand, for relativistic distributions,  $\langle \mathbf{p} \rangle_\sigma$  is in general not a function of  $\mathbf{u}_\sigma$ , so both variables need to be retained [41].

Taking the curl of Eq. (2), defining the momentum vorticity  $\boldsymbol{\Omega}_\sigma = \nabla \times \langle \mathbf{p} \rangle_\sigma$ , using Faraday's law  $\nabla \times \mathbf{E} = -\partial \mathbf{B}/\partial t$ , and using the vector identity  $\nabla \times (\mathbf{A} \cdot \nabla \mathbf{B} + \mathbf{A} \times \nabla \times \mathbf{B} + \nabla_{\mathbf{A}}[\mathbf{A} \cdot \mathbf{B}]) = 0$  in Feynman subscript notation where  $\nabla_{\mathbf{A}}$  only applies to  $\mathbf{A}$  yields

$$\left[ \frac{\partial \boldsymbol{\Omega}_\sigma}{\partial t} - \nabla \times (\mathbf{u}_\sigma \times \boldsymbol{\Omega}_\sigma) \right] = -q_\sigma \left[ \frac{\partial \mathbf{B}}{\partial t} - \nabla \times (\mathbf{u}_\sigma \times \mathbf{B}) \right] - \nabla \times \left( \frac{\nabla \cdot \overleftrightarrow{\mathcal{P}}_\sigma}{n_\sigma} \right) + \nabla \times \nabla_{\mathbf{u}_\sigma} (\langle \mathbf{p} \rangle_\sigma \cdot \mathbf{u}_\sigma). \quad (3)$$

Now we define the fluid canonical vorticity  $\mathbf{Q}_\sigma = \boldsymbol{\Omega}_\sigma + q_\sigma \mathbf{B}$ , which is the curl of the fluid canonical momentum  $\mathbf{P}_\sigma = \langle \mathbf{p} \rangle_\sigma + q_\sigma \mathbf{A}$ , where  $\mathbf{A}$  is the magnetic vector potential.  $\mathbf{Q}_\sigma$  is now the relativistic generalization of its nonrelativistic counterpart in Ref. [26]. Collecting the temporal and spatial derivatives and simplifying the last term in Eq. (3), we obtain

$$\frac{\partial \mathbf{Q}}{\partial t} = \underbrace{\nabla \times (\mathbf{u} \times \mathbf{Q})}_{\vec{\mathcal{C}}} - \nabla \times \left( \frac{\nabla \cdot \overleftrightarrow{\mathcal{P}}}{n} \right) + \underbrace{\nabla \langle p \rangle_\alpha \times \nabla u_\alpha}_{\vec{\mathcal{R}}}, \quad (4)$$

where  $\sigma$  was dropped to reduce notational clutter, and the index  $\alpha$  in the last term is summed over the three spatial dimensions (i.e.,  $\mathcal{R}_k = \epsilon_{ijk} \partial_i \langle p \rangle_\alpha \partial_j u_\alpha$  in index notation).

Now Eq. (4) is in the form of the canonical induction equation [26,27] that describes the time evolution of  $\mathbf{Q}$  with just three terms on the right-hand side. The first term is the convective term  $\vec{\mathcal{C}}$ , which is isomorphic to the magnetic induction term in ideal MHD and thus signifies that  $\mathbf{Q}$  is frozen

into and evolves with plasma's  $\mathbf{u}$ . Equivalently, it signifies that  $\mathbf{Q}$  flux is ideally conserved in the fluid frame. The second term is the relativistic canonical battery effect due to the pressure tensor  $\vec{\mathcal{B}}$ . The third term is the relativistic kineclinic term  $\vec{\mathcal{R}}$ , which is our principal result. It signifies the misalignment between the fluid momentum and fluid velocity, and reduces to zero in nonrelativistic regimes. Mathematically,  $\mathcal{R}_k$  is the sum of Jacobian determinants (or Poisson brackets)  $\{\langle p \rangle_\alpha, u_\alpha\}_{ij}$ . Equation (4) reveals that  $\vec{\mathcal{R}}$  is an additional source/sink of  $\mathbf{Q}$  affecting its frozen-in property.

The drawback of Eq. (4) is that it is not in a covariant form and thus is frame dependent. However, as mentioned earlier, it provides a better link to laboratory-frame experiments and simulations because the magnetic field and vorticity are both defined in synchronic space at a certain time. Also, Eq. (4) was derived without simplifying approximations such as scalar temperature or enthalpy.

To understand the role of  $\vec{\mathcal{R}}$  quantitatively with respect to magnetogenesis, let us expand the convective term without the magnetic term:

$$\mathcal{C}_k = \epsilon_{ijk} (\partial_i u_\alpha \partial_j \langle p \rangle_\alpha - \partial_i u_\alpha \partial_\alpha \langle p \rangle_j - u_\alpha \partial_\alpha \partial_i \langle p \rangle_j), \quad (5)$$

$$= -\mathcal{R}_k + \epsilon_{ijk} (-\partial_i u_\alpha \partial_\alpha \langle p \rangle_j - u_\alpha \partial_\alpha \partial_i \langle p \rangle_j), \quad (6)$$

which shows that  $\vec{\mathcal{R}}$  in fact partially cancels  $\vec{\mathcal{C}}$ , which embodies the frozen-in property of  $\mathbf{Q}$ . Therefore,  $\vec{\mathcal{R}}$  acts as a relativistic inertial term that prevents perfect convection of  $\mathbf{Q}$  in relativistic regimes.

Aside from the role of  $\vec{\mathcal{R}}$ , let us note the relationship between  $\vec{\mathcal{B}}$  and  $\vec{\mathcal{C}}$  as well. When constructing the relativistic pressure tensor, the symmetry of the momentum flux density  $\mathcal{T}_{ij} = \int p_i p_j f d^3\mathbf{p}/\gamma$  was destroyed, i.e.,  $\mathcal{P}_{ij} = \mathcal{T}_{ij} - n u_i \langle p \rangle_j$ . The asymmetric part of the canonical battery  $\vec{\mathcal{B}}$  is

$$\epsilon_{ijk} \partial_i \left[ \frac{1}{n} \langle p \rangle_j \partial_\alpha (n u_\alpha) + u_\alpha \partial_\alpha \langle p \rangle_j \right], \quad (7)$$

where the second term exactly cancels the terms other than  $\mathcal{R}_k$  in Eq. (6). The first term represents compressibility effects.

Because  $\vec{\mathcal{R}}$  and parts of  $\vec{\mathcal{B}}$  together cancel out  $\vec{\mathcal{C}}$ , the representation in Eq. (4) is not unique and different interpretations may be possible by shifting emphasis to different terms. However, the present framework explicitly decomposes the time evolution of  $\mathbf{Q}$  into three distinct, physically interpretable sources, where the convective term embodies the conservation of canonical vorticity flux, the canonical battery term embodies pressure tensor effects, and the kineclinic term embodies relativistic effects.

One important point to stress is that the fluid momentum in principle has no functional relationship with the fluid velocity and can be drastically different from one another. This is because the former is dominated by the high-momentum population, whereas the latter is dominated by the statistical average velocity [40]. For example, consider the distribution function  $f \sim [a\delta(\mathbf{p} - p_a \hat{y}) + b\delta(\mathbf{p} - p_b \hat{x})]$ , where  $\delta$  is the Dirac delta function,  $a$  represents a dilute, highly relativistic population, and  $b$  represents a dense, nonrelativistic population. The first moments of this distribution function are, in normalized units,  $\langle \mathbf{p} \rangle \sim a p_a \hat{y} + b p_b \hat{x}$  and  $\mathbf{u} \sim$

$ap_a/\sqrt{1+p_a^2}\hat{y} + bp_b/\sqrt{1+p_b^2}\hat{x}$ . It is trivial to see that when  $a \ll b$ ,  $p_a/p_b \gg b/a$ , and  $p_b$  is of order unity, it follows that  $\langle \mathbf{p} \rangle \simeq ap_a\hat{y}$  but  $\mathbf{u} \simeq bp_b/\sqrt{1+p_b^2}\hat{x}$ .

Let us now construct a configuration where  $\vec{\mathcal{R}}$  is the only finite term. We first assume that only  $\partial u_x/\partial y$  and  $\partial \langle p \rangle_x/\partial x$  are finite, in which case  $\vec{\mathcal{C}} = 0$ . Now, requiring the asymmetric part of  $\vec{\mathcal{B}} = 0$  yields

$$\frac{1}{n} \langle p \rangle_x u_x \partial_x n + u_x \partial_x \langle p \rangle_x = 0, \quad (8)$$

which reduces to  $n \langle p \rangle_x = \text{const}$ . These conditions together constitute a situation where purely  $\vec{\mathcal{R}}$  may, at least initially, dominate magnetic field generation. Now we shall demonstrate this prediction with the aid of PIC simulations.

### III. NUMERICAL RESULTS

To validate the model and visualize the role of  $\vec{\mathcal{R}}$  in Eq. (4), several 2D3V PIC simulations were conducted for a pair plasma using the SMILEI code [42]. The fiducial simulation domain was  $(L_x, L_y) = (20, 20) d_e$  divided into 2048 cells in each direction, where  $d_e = c/\omega_{pe}$  is the collisionless electron skin depth, corresponding to cell sizes of  $\Delta x = \Delta y \approx 0.0097 d_e$ . Three hundred particles per cell were placed for both species. The temporal step size was  $\Delta t \approx 0.0066/\omega_{pe}$ , initial  $\mathbf{E} = \mathbf{B} = 0$ , uniform thermal velocity  $u_{th}/c \approx 0.049$ , and periodic boundary conditions were used for both particles and fields.

For the velocity and density profiles of both species, we consider the following setup. To make  $\partial u_x/\partial y$  finite, we set  $u_x(y) = v_0 \sin^2(\pi y/L_y)$ , where  $v_0 = 0.5c$ . To make  $\partial \langle p \rangle_x/\partial x$  finite, we set  $\langle p \rangle_x(x) = p_0 \cos^2(\pi x/L_x) + p_{\text{base}}$ , where  $p_0/m_e c = \sqrt{\gamma - 1}$  for  $\gamma = 100$  and  $p_{\text{base}} = 10m_e c$ . The density profile is then  $n(x) = n_0 p_{\text{base}}/\langle p \rangle_x$ , and so  $p_{\text{base}}$  ensures finite density everywhere. The difference between the fluid velocity and momentum is approximately realized by setting a small fraction (1%) of the particles to follow  $\langle p \rangle_x$  and remaining larger fraction (99%) of the particles to follow  $u_x$ . Because  $p_0 \gg 1$ , the fluid momentum is dominated by the relativistic, dilute species, whereas the fluid velocity is dominated by the nonrelativistic, dense species. This configuration sets  $\vec{\mathcal{C}} = \vec{\mathcal{B}} = 0$  and  $\vec{\mathcal{R}} \sim \hat{z} \sin(2\pi x/L_x) \sin(2\pi y/L_y)$ , i.e., a quadrupole source of canonical vorticity.

This initial condition is rather contrived and intentionally engineered to isolate the kineclinic mechanism, and therefore only acts as a proof-of-principle study. However, the setup physically describes a dilute relativistic stream of particles on top of a dense nonrelativistic plasma flow. Such a situation may be locally realized, for example, when high-energy cosmic rays interact with the denser solar wind or even astrophysical jets. For realistic situations, it may be difficult to have both  $\vec{\mathcal{C}}$  and  $\vec{\mathcal{B}}$  be exactly zero, but here we conceive an ideal situation to illustrate the core mechanics of kineclinic magnetogenesis.

For postprocessing,  $\partial Q_{ez}/\partial t$  was calculated by making three burst saves of the simulation data separated by five time steps at each save point and then using the central difference scheme in time. Vorticity was calculated by a central

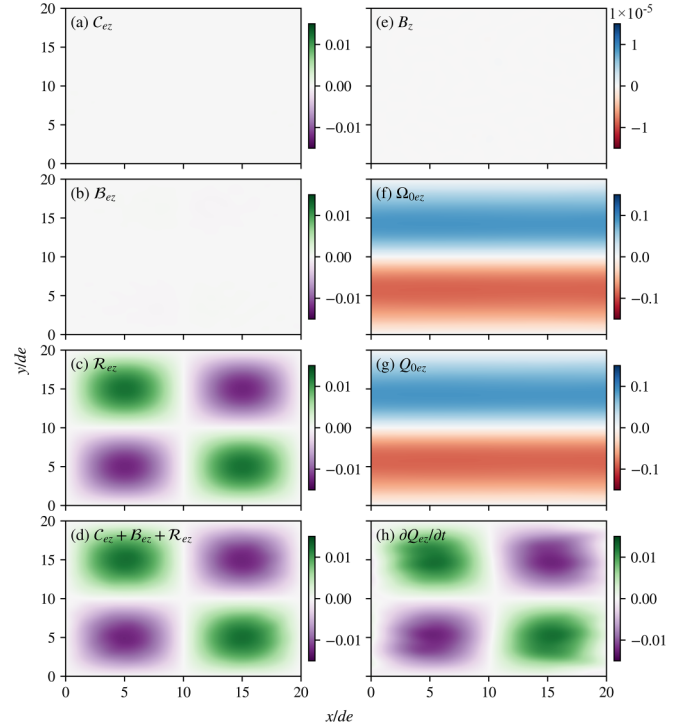


FIG. 1. Slice plots of various quantities (a)  $B_z[m_e\omega_{pe}/e]$ , (b)  $\Omega_{ez}[m_e\omega_{pe}]$ , (c)  $Q_{ez}[m_e\omega_{pe}]$ , (d)  $\partial Q_{ez}/\partial t[m_e\omega_{pe}^2]$ , (e)  $C_{ez}[m_e\omega_{pe}^2]$ , (f)  $B_{ez}[m_e\omega_{pe}^2]$ , (g)  $\mathcal{R}_{ez}[m_e\omega_{pe}^2]$ , and (h)  $(C_{ez} + B_{ez} + \mathcal{R}_{ez})[m_e\omega_{pe}^2]$  from the two-dimensional (2D) PIC simulation at  $t\omega_{pe} = 0$ .

difference scheme in space. All quantities were Gaussian-filtered with 12 grid points to reduce noise arising from taking multiple gradients. Different numbers of filter grid points were tested to ensure that the filter only suppresses high-frequency noise and preserves the larger-scale structure.

Figure 1 presents various quantities at the beginning of the fiducial simulation, focusing on quantities associated with electrons, denoted by a subscript  $e$ . Figures 1(a) and 1(b) show that both  $C_{ez}$  and  $B_{ez}$  are zero at  $t\omega_{pe} \approx 0$  as predicted analytically. Figures 1(c), 1(d), and 1(h) show that kineclinticity effect  $\mathcal{R}_{ez}$  is the only active source term for  $\partial Q_{ez}/\partial t$  at the beginning. The structure is quadrupole as predicted above. There is no initial  $B_z$  but a finite  $\Omega_{ez}$  due to the initial flow profile as shown in Figs. 1(e) and 1(f). Consequently,  $\mathcal{R}_{ez}$  solely drives the initial evolution of  $Q_{ez}$ .

Figure 2 shows the same plots at  $t\omega_{pe} \approx 7$ . Figures 2(a) and 2(b) show that the contribution of  $C_{ez}$  and  $B_{ez}$  grows, although the kineclinic term [Fig. 2(c)] is still dominant, which maintains the two-dimensional tilted quadrupole pattern. Figure 2(d) shows the total source  $C_{ez} + B_{ez} + \mathcal{R}_{ez}$ , which nearly matches  $\mathcal{R}_{ez}$  because the latter dominates over other sources.

Figure 2(e) shows the self-generated  $B_{ez}$ , along with relatively strong  $\Omega_{ez}$  [Fig. 2(f)], which together yield  $Q_{ez}$  [Fig. 2(g)]. These fields inherit strongly correlated spatiotemporal structure because their linear-phase evolution is driven almost entirely by  $\mathcal{R}_{ez}$ . Finally, Fig. 2(h) shows  $\partial Q_{ez}/\partial t$ , which is the left-hand-side of Eq. (4). The strong resemblance between panels (d) and (h) in Fig. 2 (also in Fig. 1) validates the canonical vorticity formulation given in Eq. (4). Minor differences arise due to numerical errors from the time-difference

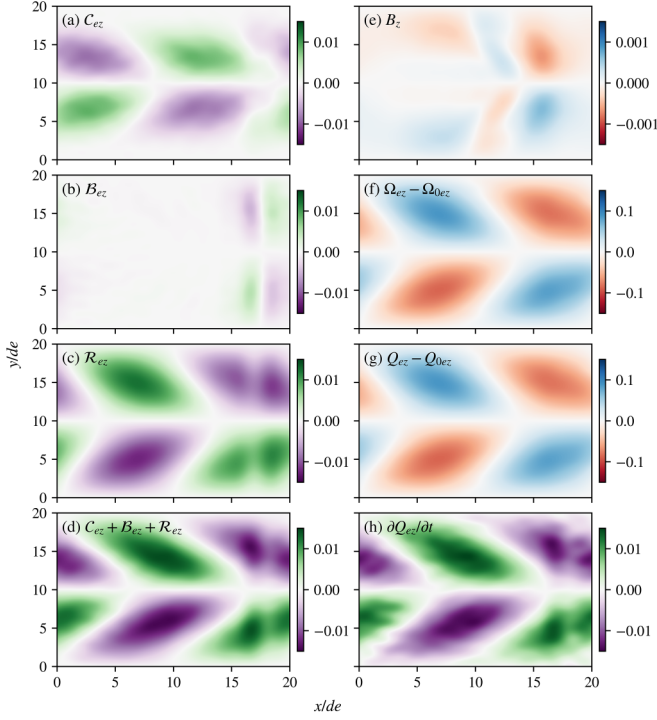


FIG. 2. Slice plots of various quantities (a)  $B_z[m_e\omega_{pe}/e]$ , (b)  $(\Omega_{ez} - \Omega_{0ez})[m_e\omega_{pe}]$ , (c)  $(Q_{ez} - Q_{0ez})[m_e\omega_{pe}]$ , (d)  $\partial Q_{ez}/\partial t[m_e\omega_{pe}^2]$ , (e)  $C_{ez}[m_e\omega_{pe}^2]$ , (f)  $B_{ez}[m_e\omega_{pe}^2]$ , (g)  $R_{ez}[m_e\omega_{pe}^2]$ , and (h)  $(C_{ez} + B_{ez} + R_{ez})[m_e\omega_{pe}^2]$  from the 2D PIC simulation at  $t\omega_{pe} \approx 7$ , where  $\Omega_{0ez}$  and  $Q_{0ez}$  are the vorticity and canonical vorticity at  $t\omega_{pe} \approx 0$ .

scheme used when calculating  $\partial Q_{ez}/\partial t$  and the usage of the noise filter. The contrast between Figs. 2(a) and 2(h) indicates that the frozen-in property of  $Q_{ez}$  is significantly violated, while the close matches between Figs. 2(c) and 2(h) confirm that  $R_{ez}$  governs the early growth of  $Q_{ez}$ , a clear demonstration of  $R_{ez}$  effect as a seed-field generation mechanism.

Figure 3 shows the temporal evolution of the rms value of the various quantities. The exponential growth of quantities such as  $B_z$  indicates the characteristics of the instabilities driving the dynamics. It also demonstrated that the magnitude of  $R_{ez}$  is robust throughout the simulation time. Although later  $C_{ez}$  and  $B_{ez}$  become comparable or dominant over  $R_{ez}$ , the convective term  $C_{ez}$ , in particular, is just trying to “carry” the fields that are being generated by  $R_{ez}$  and so does not participate in magnetogenesis *per se* in the fluid frame. As discussed above, the kineclinic term is zero if and only if

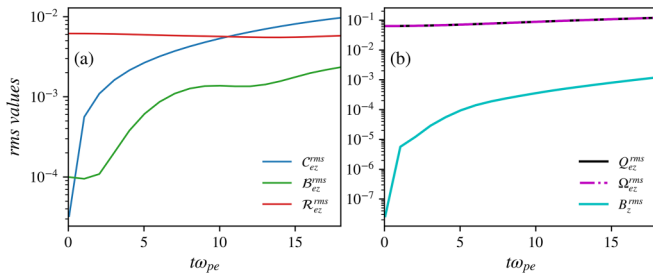


FIG. 3. Temporal variation of rms values of normalized (a)  $C_{ez}$ ,  $B_{ez}$ , and  $R_{ez}$ , and (b)  $Q_{ez}$ ,  $\Omega_{ez}$ , and  $B_z$ .

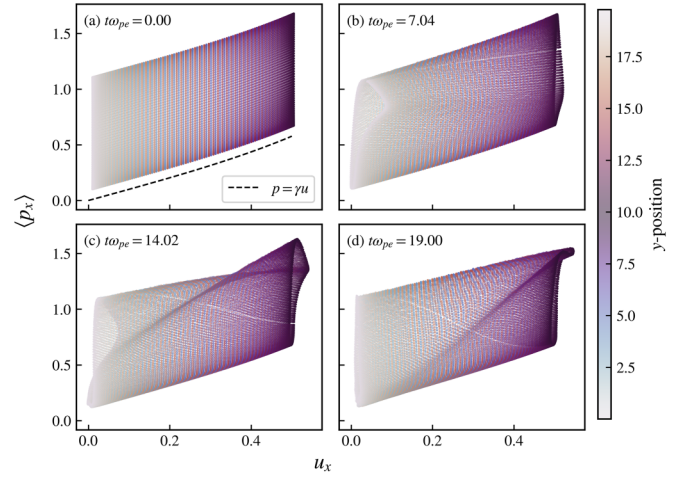


FIG. 4. Scatter plot of normalized  $\langle p \rangle_x$  vs  $u_x$  for every ten grid points in the simulation shown in Fig. 2, displayed at times  $t\omega_{pe} \approx 0, 7, 14$ , and  $19$ . Showing all grid points amounts to a messier plot, hence the sparsity. The black dashed line in panel (a) corresponds to the analytical relation  $\langle p \rangle_x = u_x / \sqrt{1 - u^2}$ . The color indicates the  $y$  location of each grid element.

the fluid momentum  $\langle \mathbf{p} \rangle$  is a function of the fluid velocity  $\mathbf{u}$ , which is generally not true and only approximately true for nonrelativistic regimes [43]. As an explicit example, if  $\langle p \rangle_\alpha = \gamma u_\alpha$  is assumed where  $\gamma = (1 - u_\beta u_\beta)^{-1/2}$  in units of momentum  $mc$  and velocity  $c$ , we have  $\partial_i \gamma = \gamma^3 u_\beta \partial_i u_\beta$  and  $\partial_i \langle p \rangle_\alpha = u_\alpha \partial_i \gamma + \gamma \partial_i u_\alpha = \gamma^3 u_\alpha u_\beta \partial_i u_\beta + \gamma \partial_i u_\alpha$ . Therefore,

$$\begin{aligned} \mathcal{R}_k &= \epsilon_{ijk} \partial_i \langle p \rangle_\alpha \partial_j u_\alpha \\ &= \epsilon_{ijk} [\gamma^3 u_\alpha u_\beta \partial_i u_\beta + \gamma \partial_i u_\alpha] \partial_j u_\alpha \\ &= \gamma^3 \epsilon_{ijk} (u_\beta \partial_i u_\beta) (u_\alpha \partial_j u_\alpha) + \gamma \epsilon_{ijk} (\partial_i u_\alpha) (\partial_j u_\alpha) \\ &= 0, \end{aligned}$$

using the cyclic property of  $\epsilon_{ijk}$ . This is, of course, in general true for any  $\langle p_\alpha \rangle = f(u_\alpha)$  for any arbitrary function  $f$  because  $\partial_i \langle p \rangle_\alpha = f' \partial_i u_\alpha$ . Otherwise,  $\vec{\mathcal{R}}$  is always finite.

Figure 4 explicitly illustrates the origin of finite  $\vec{\mathcal{R}}$  by showing the temporal evolution of normalized  $\langle p \rangle_x$  versus  $u_x$  for the simulation presented in Fig. 2. Each scatter point corresponds to sparse grid points in the simulation. One can see that the single-particle relation  $\langle p \rangle_x = \gamma u_x$  (black dashed line) is not satisfied even from the start, and the nonfunctional relationship between the two quantities is explicit. This nonfunctional relationship persists throughout the whole simulation, thereby maintaining the magnitude of  $\vec{\mathcal{R}}$  in Fig. 3.

#### IV. DISCUSSION

The results presented in Figs. 1–3 clearly demonstrate how  $\vec{\mathcal{R}}$  acts as a dynamically important source of seed magnetic field in relativistic plasmas. The significance of  $\vec{\mathcal{R}}$  compared to other terms, however, depends on several factors. One important requirement is that the system must be at least two dimensional, because kineclincity is exactly zero for one-dimensional systems. Another requirement is that the inverse

of the density ratio of the energetic population has to be at least comparable to its average momentum. For example, if the energetic population has  $p_0/m_e c = 100$  but it constitutes only 0.01% of the bulk density, then its contribution to the average momentum is negligible.

Although the present work used kineclinic flow specifically to isolate the role of  $\mathcal{R}_{ez}$  during magnetogenesis, the generality of this formulation calls for an investigation of the role of kineclinity in various other relativistic situations. In particular, in magnetically dominant situations it may play the opposite role, i.e., dissipating magnetic fields instead of generating them. Our preliminary study shows that the effect may be important in relativistic magnetic reconnection, where the kineclinic term may be an additional source of topology change of  $\mathbf{Q}$  and thus  $\mathbf{B}$  field lines.

For ideal initial setups such as those commonly used in relativistic flows and reconnection with, e.g., Maxwell-Jüttner distribution, kineclinity is initially zero. However, for sufficiently nonlinear phenomena such as collisionless shocks or reconnection, the momentum-carrying population and the bulk population typically become separable very quickly. Therefore, kineclinity is expected to develop in these situations.

Moreover, the simulation in the present work involves pair plasmas to avoid the multiscale complexity and computational stiffness associated with the  $m_i/m_e$  ratio. In electron-ion plasmas, one would expect substantially richer structure formation and additional physics of instabilities [6,25]. Because Eq. (4) applies to any species, an ion-driven kineclinic effect should occur in analogy with the electron contribution, but on distinct timescales; determining when and to what extent this ion effect becomes important is an interesting direction for future investigation. Similarly, identifying a more realistic plasma setup where  $\tilde{\mathcal{R}}$  exceeds or competes with the  $\tilde{\mathcal{C}}$  and  $\tilde{\mathcal{B}}$  sources

in, e.g., plasma dynamos, would be an important direction for future work.

## V. SUMMARY

In summary, the curl of the relativistic momentum equation for a collisionless plasma is reformulated in terms of the time evolution of the canonical vorticity. In comparison to the nonrelativistic counterpart, the reformulation yields a term that represents the misalignment between fluid momentum and velocity gradients, which we name “kineclinity.” This term can spontaneously generate vorticity and magnetic fields, and is finite for relativistic plasmas with an absence of a functional relationship between the fluid momentum and velocity. We verify this mechanism in relation to the other two magnetogenesis sources in the formulation with the aid of particle-in-cell simulations of a kineclinic flow.

## ACKNOWLEDGMENTS

This work was supported by an appointment to the JRG Program at the APCTP through the Science and Technology Promotion Fund and Lottery Fund of the Korean government, and also by the Korean Local Governments—Gyeongsangbuk-do Province and Pohang City. This work was also supported by the NRF of Korea under Grants No. NRF-RS-2023-00281272 and No. NRF-RS-2025-00522068. The computations presented here were conducted on the KAIROS supercomputing cluster at the Korean Institute of Fusion Energy and on the APCTP computing server.

## DATA AVAILABILITY

The data that support the findings of this article are openly available [44].

- 
- [1] G. Anglada, L. F. Rodríguez, and C. Carrasco-González, Radio jets from young stellar objects, *Astron. Astrophys. Rev.* **26**, 3 (2018).
  - [2] R. Blandford, D. Meier, and A. Readhead, Relativistic jets from active galactic nuclei, *Annu. Rev. Astron. Astrophys.* **57**, 467 (2019).
  - [3] R. K. Joshi and I. Chattopadhyay, The morphology and dynamics of relativistic jets with relativistic equation of state, *Astrophys. J.* **948**, 13 (2023).
  - [4] E. Liang, M. Boettcher, and I. Smith, Magnetic field generation and particle energization at relativistic shear boundaries in collisionless electron-positron plasmas, *Astrophys. J.* **766**, L19 (2013).
  - [5] K.-I. Nishikawa, P. Hardee, B. Zhang, I. Dušan, M. Medvedev, E. J. Choi, K. W. Min, J. Niemiec, Y. Mizuno, A. Nordlund, J. T. Frederiksen, H. Sol, M. Pohl, and D. H. Hartmann, Magnetic field generation in a jet-sheath plasma via the kinetic Kelvin-Helmholtz instability, *Ann. Geophys.* **31**, 1535 (2013).
  - [6] K.-I. Nishikawa, P. E. Hardee, I. Dutan, J. Niemiec, M. Medvedev, Y. Mizuno, A. Meli, H. Sol, B. Zhang, M. Pohl, and D. H. Hartmann, Magnetic field generation in core-sheath jets via the kinetic Kelvin-Helmholtz instability, *Astrophys. J.* **793**, 60 (2014).
  - [7] H. Chen and F. Fiuza, Perspectives on relativistic electron-positron pair plasma experiments of astrophysical relevance using high-power lasers, *Phys. Plasmas* **30**, 020601 (2023).
  - [8] E. N. Parker, *Cosmical Magnetic Fields. Their Origin and Their Activity* (Oxford University Press, Oxford, England, 1979).
  - [9] A. Bonafede, L. Feretti, M. Murgia, F. Govoni, G. Giovannini, D. Dallacasa, K. Dolag, and G. B. Taylor, The Coma cluster magnetic field from Faraday rotation measures, *Astron. Astrophys.* **513**, A30 (2010).
  - [10] H. Takabe and Y. Kuramitsu, Recent progress of laboratory astrophysics with intense lasers, *High Power Laser Sci. Eng.* **9**, e49 (2021).
  - [11] A. Brandenburg and K. Subramanian, Astrophysical magnetic fields and nonlinear dynamo theory, *Physics Reports* **417**, 1 (2005).
  - [12] T. Vachaspati, Progress on cosmological magnetic fields, *Rep. Prog. Phys.* **84**, 074901 (2021).
  - [13] K. Jedamzik and L. Pogosian, Relieving the Hubble tension with primordial magnetic fields, *Phys. Rev. Lett.* **125**, 181302 (2020).

- [14] A. Brandenburg, D. Sokoloff, and K. Subramanian, Current status of turbulent dynamo theory: From large-scale to small-scale dynamos, *Space Sci. Rev.* **169**, 123 (2012).
- [15] M. Zhou, V. Zhdankin, M. W. Kunz, N. F. Loureiro, and D. A. Uzdensky, Spontaneous magnetization of collisionless plasma, *Proc. Natl. Acad. Sci. USA* **119**, e2119831119 (2022).
- [16] L. M. Widrow, Origin of galactic and extragalactic magnetic fields, *Rev. Mod. Phys.* **74**, 775 (2002).
- [17] E. Garaldi, R. Pakmor, and V. Springel, Magnetogenesis around the first galaxies: The impact of different field seeding processes on galaxy formation, *Mon. Not. R. Astron. Soc.* **502**, 5726 (2021).
- [18] J. Mason, L. Malyskin, S. Boldyrev, and F. Cattaneo, Magnetic dynamo action in random flows with zero and finite correlation times, *Astrophys. J.* **730**, 86 (2011).
- [19] M. Zhou, V. Zhdankin, M. W. Kunz, N. F. Loureiro, and D. A. Uzdensky, Magnetogenesis in a collisionless plasma: From Weibel instability to turbulent dynamo, *Astrophys. J.* **960**, 12 (2024).
- [20] A. Grassi, M. Grech, F. Amiranoff, F. Pegoraro, A. Macchi, and C. Riconda, Electron Weibel instability in relativistic counterstreaming plasmas with flow-aligned external magnetic fields, *Phys. Rev. E* **95**, 023203 (2017).
- [21] L. Biermann, Über den Ursprung der Magnetfelder auf Sternen und im interstellaren Raum, *Z. Naturforsch. A* **5**, 65 (1950).
- [22] E. S. Weibel, Spontaneously growing transverse waves in a plasma due to an anisotropic velocity distribution, *Phys. Rev. Lett.* **2**, 83 (1959).
- [23] A. Gruzinov, GRB: Magnetic fields, cosmic rays, and emission from first principles? [arXiv:0803.1182](https://arxiv.org/abs/0803.1182).
- [24] E. P. Alves, T. Grismayer, S. F. Martins, F. Fiúza, R. A. Fonseca, and L. O. Silva, Large-scale magnetic field generation via the kinetic Kelvin-Helmholtz instability in unmagnetized scenarios, *Astrophys. J.* **746**, L14 (2012).
- [25] T. Grismayer, E. P. Alves, R. A. Fonseca, and L. O. Silva, Theory of multidimensional electron-scale instabilities in unmagnetized shear flows, *Plasma Phys. Control. Fusion* **55**, 124031 (2013).
- [26] M. Laishram, G. S. Yun, and Y. D. Yoon, Magnetogenesis via the canonical battery effect, *Phys. Rev. Res.* **6**, L032052 (2024).
- [27] Y. D. Yoon and P. M. Bellan, The electron canonical battery effect in magnetic reconnection: Completion of the electron canonical vorticity framework, *Phys. Plasmas* **26**, 100702 (2019).
- [28] Y. D. Yoon and P. M. Bellan, A generalized two-fluid picture of non-driven collisionless reconnection and its relation to whistler waves, *Phys. Plasmas* **24**, 052114 (2017).
- [29] Y. D. Yoon and P. M. Bellan, An intuitive two-fluid picture of spontaneous 2D collisionless magnetic reconnection and whistler wave generation, *Phys. Plasmas* **25**, 055704 (2018).
- [30] S. You, A field theory approach to the evolution of canonical helicity and energy, *Phys. Plasmas* **23**, 072108 (2016).
- [31] J. V. D. Linden, J. Sears, T. Intrator, and S. You, Measurements of the canonical helicity of a gyrating kink, *Phys. Rev. Lett.* **121**, 035001 (2018).
- [32] E. N. Nerush, D. A. Serebryakov, and I. Y. Kostyukov, Weibel instability in hot plasma flows with the production of gamma-rays and electron-positron pairs, *Astrophys. J.* **851**, 129 (2017).
- [33] D. Kagan, L. Sironi, B. Cerutti, and D. Giannios, Relativistic magnetic reconnection in pair plasmas and its astrophysical applications, *Space Sci. Rev.* **191**, 545 (2015).
- [34] S. M. Mahajan, Temperature-transformed “minimal coupling”: Magnetofluid unification, *Phys. Rev. Lett.* **90**, 035001 (2003).
- [35] S. M. Mahajan and Z. Yoshida, Twisting space-time: Relativistic origin of seed magnetic field and vorticity, *Phys. Rev. Lett.* **105**, 095005 (2010).
- [36] Y. Kawazura, Z. Yoshida, and Y. Fukumoto, Relabeling symmetry in relativistic fluids and plasmas, *J. Phys. A: Math. Theor.* **47**, 465501 (2014).
- [37] J.-H. Gao, B. Qi, and S.-Y. Wang, Vorticity and magnetic field production in relativistic ideal fluids, *Phys. Rev. D* **90**, 083001 (2014).
- [38] A. H. Taub, Relativistic Rankine-Hugoniot equations, *Phys. Rev.* **74**, 328 (1948).
- [39] T. P. Wright and G. R. Hadley, Relativistic distribution functions and applications to electron beams, *Phys. Rev. A* **12**, 686 (1975).
- [40] M. Hesse and S. Zenitani, Dissipation in relativistic pair-plasma reconnection, *Phys. Plasmas* **14**, 112102 (2007).
- [41] M. Melzani, C. Winisdoerffer, R. Walder, D. Folini, J. M. Favre, S. Krastanov, and P. Messmer, Apar-t: Code, validation, and physical interpretation of particle-in-cell results, *Astron. Astrophys.* **558**, A133 (2013).
- [42] J. Derouillat, A. Beck, F. Pérez, T. Vinci, M. Chieramello, A. Grassi, M. Flé, G. Bouchard, I. Plotnikov, N. Aunai, J. Dargent, C. Riconda, and M. Grech, Smilei: A collaborative, open-source, multi-purpose particle-in-cell code for plasma simulation, *Comput. Phys. Commun.* **222**, 351 (2018).
- [43] A. M. Anile, *Relativistic Fluids and Magneto-Fluids* (Cambridge University Press, Cambridge, England, 1989).
- [44] M. Laishram, S. Basnet, and Y. D. Yoon, Kineclinic magnetogenesis PIC data, Version 1.0, [Dataset], Zenodo (2026), doi: [10.5281/zenodo.20304345](https://doi.org/10.5281/zenodo.20304345).