

How walls shape dissipation intermittency

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(Received 22 June 2025; accepted 19 May 2026; published 30 July 2026)

Intermittency of energy dissipation has long been studied via high-order moments in homogeneous and isotropic turbulence (HIT), but not much where the boundary effects are explicitly included. Here, we derive two fundamental Reynolds number scaling expressions for dissipation moments in wall-bounded flows—one in the outer region where the boundary effects are weak and the other close to the walls where those effects are strong—and support these expressions by direct numerical simulations. Dissipation moments in the outer region follow universal power laws with exponents linked to anomalous scaling of velocity structure functions in HIT. In contrast, moments near the wall follow a bounded defect law, leading to a finite asymptotic limit without intermittency. For very large Reynolds numbers, the outer proposal predicts vanishing dissipation compared to that on the wall, highlighting the need for solid boundaries in generating Onsager-type singularities.

DOI: 10.1103/lsvn-mfrb

Introduction. Turbulence is ubiquitous and constantly challenging, thus inspiring innovative efforts to describe, explore, and understand its complex nature. Far from boundaries and at sufficiently high Reynolds numbers, turbulence is expected to be essentially homogeneous and isotropic [1,2]. In this region, the interscale energy transfer process dominates. In three-dimensional turbulence, the caricature of this process is a cascade of energy from the integral scale L to the dissipation scale η [3], which was succinctly formulated by Kolmogorov in 1941 (K41 henceforth) [4]. In the inertial range ($\eta \ll r \ll L$) where the cascade is thought to be inviscid and self-similar, K41 predicts that the velocity structure functions are determined only by $\langle \epsilon \rangle$ and follow power law scaling $S_p(r) \equiv \langle (\delta_r u)^p \rangle \propto ((\epsilon)r)^{p/3}$. Here, $\delta_r u = u(x+r) - u(x)$, the longitudinal increment of the velocity component u (with x and r in same direction as u), carries information on energy transfer between eddies of scale r , ϵ is the local turbulent dissipation rate, and $\langle \cdot \rangle$ denotes an ensemble average. Underlying K41 is the constancy of mean energy dissipation. In the presence of intermittency for real turbulence, the scaling laws for structure functions differ from K41, require modifications (termed anomalous scaling) of the type $S_p(r) \sim r^{\zeta_p}$, with the exponents ζ_p depending nonlinearly on p , and must be determined separately for each p . Such corrections, together with the understanding of the failure of K41 and the prevalence of

dissipation intermittency, have long been and remain a central topic in modern turbulence research [5–7].

For practical flows, such as channels, pipes, and flow over airfoils, the situation becomes more complex because of the wall. Recently, the universality of small scales in different flow geometries has been observed in terms of the Reynolds number dependence of dissipation [8,9], but these studies focus on the flow center, where a state similar to homogeneous and isotropic turbulence (HIT) develops naturally. In contrast, near the wall, strong mean shear breaks conditions of HIT, and small-scale turbulence feels directly the geometric constraints or large-scale features [10–12], thus violating assumptions of universality. As an extreme example, velocities vanish identically at the wall, while dissipation reaches its maximum and remains highly fluctuating. It is thus essential to examine how dissipation moments vary with wall-normal distance. Nevertheless, the influence of the wall on the intermittency of small-scale turbulence has not been explored much. Rectifying this situation is our goal.

We anticipate the main results by stating here that turbulent dissipation moments in the outer region (including the log layer) follow the universal power law given by

$$\langle \epsilon^p \rangle^{1/p} / \epsilon_\tau = C_p \text{Re}_\tau^{-1+\gamma_p}, \quad (1)$$

and, at the wall, the bounded defect law given by

$$\langle \epsilon^p \rangle^{1/p} / \epsilon_\tau = \alpha_p - \beta_p \text{Re}_\tau^{-1/4}. \quad (2)$$

In the above, ϵ is the streamwise or spanwise dissipation (explicit expressions are given later), $\epsilon_\tau = u_\tau^4/\nu$ is the mean flow dissipation at the wall, $\text{Re}_\tau = u_\tau H/\nu$ is the friction Reynolds number (with H the flow thickness, u_τ the friction velocity, and ν the kinetic viscosity), and γ_p is the intermittency correction in the outer flow ($\gamma_p = 0$ if intermittency is absent); C_p , α_p , and β_p are coefficients independent of Re_τ but may

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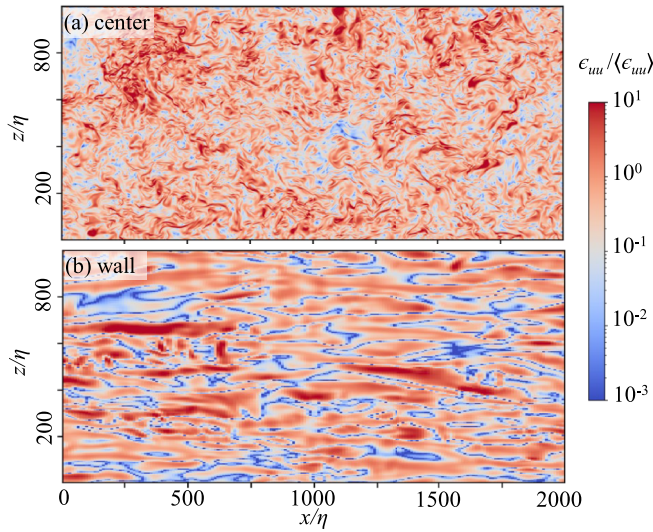


FIG. 1. Typical snapshots of the streamwise component of turbulent dissipation rate on (a) the channel centerplane and (b) the wall, normalized by the ensemble-averaged component turbulent dissipation rate $\langle \epsilon_{uu} \rangle$, obtained from direct numerical simulations (DNS) of channel flow in Ref. [13] at $\text{Re}_\tau = 5200$. Spatial domains are nondimensionalized using the Kolmogorov length η .

depend on the order p . The rest of the Letter deals with the derivation of Eqs. (1) and (2) and their assessment by means of data, followed by a discussion of their significance.

Patterns of dissipation. A direct illustration of the shear effect on dissipation is given in Fig. 1, which compares, for the channel, the patterns of the streamwise component of the dissipation rate ϵ ($\epsilon_{uu} = \nu \partial_j u \partial_j u$, where u is the fluctuation velocity after subtracting the mean) on horizontal planes (x - z) at the channel center ($y/H = 1$) and on the wall ($y/H = 0$). The two domain sizes are specified in Kolmogorov lengths, ensuring comparable information on dissipative scales. On the centerplane, the dissipation field exhibits no preferred direction and shows a nearly isotropic character. In contrast, dissipation at the wall forms a streaky structure in the streamwise direction (similar to other quantities there [13]), with fluctuations reaching more intense extrema.

An important quantitative detail of the patterns is revealed by the probability density functions (PDFs) of ϵ_{uu} in Fig. 2. In particular, the well-collapsing left tails have a distinct slope of $-1/2$ on the wall and $1/2$ at the center (inset). These trends indicate, for zero dissipation, a divergent PDF at the wall and a vanishing PDF for the center. Both slopes have recently been reported in HIT for dissipation defined with different derivative terms [14,15]. The oppositely signed slopes for the wall and the centerplane reflect different dynamics (suggested by Fig. 1): They show that the standard energy cascade and the intermittency characteristic of the centerplane are strongly shaped at the wall by the shear, clearly amplifying the low-dissipation events (possibly also high-amplitude events). They also highlight the role of the wall in shaping dissipation dynamics, leading to different scaling behaviors with changing distance from the wall, as stated toward the end of the Introduction. We now set out to establish these changes.

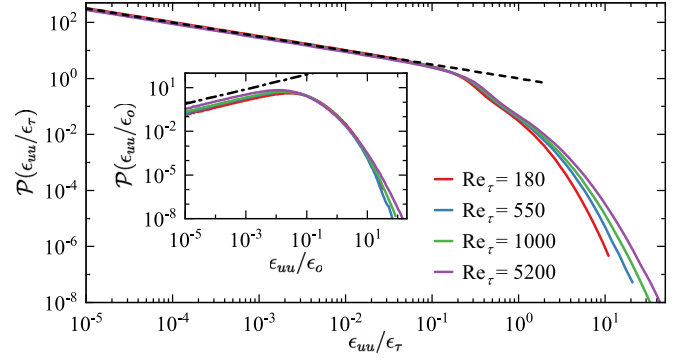


FIG. 2. Probability densities of $\epsilon_{uu}/\epsilon_\tau$ at the wall compared with those of ϵ_{uu}/ϵ_o at the center (inset). Dashed line represents a slope of $-1/2$ and dash-dotted line for slope of $1/2$. Here, ϵ_τ is the wall dissipation [defined below Eq. (2)] and ϵ_o is the scale of outer dissipation [defined just below Eq. (5)]. Data for $\text{Re}_\tau = 180$ and 550 are ours (see the Supplemental Material [16]), and for 1000 and 5200 are from Ref. [13].

Scaling in the outer region. We follow the framework developed for HIT [20–23] and connect dissipation and inertial-range dynamics. The derivation is given in detail in the Supplemental Material [16], but a few steps are summarized here. By applying the point-splitting procedure to local velocity gradients [24], one has $\partial_x u \approx \delta_\eta u / \eta$. Extending the structure function scaling to both ends up to the dissipation and integral scales, together with the simplification of the exact equation for p th order longitudinal structure functions [25,26] in the inertial range, one obtains the dissipation moments via the scaling of structure functions [Eq. (24) in Ref. [20]], as

$$\frac{\langle \epsilon^p \rangle^{1/p}}{\sigma_L^3/L} \sim \text{Re}_L^{\gamma_p}. \quad (3)$$

Here, $\sigma_L = \sqrt{\langle (\delta u_L)^2 \rangle}$ is the velocity difference over the large scale L , $\text{Re}_L = L\sigma_L/\nu$ is the corresponding Reynolds number, and the scaling exponent corrections γ_p are expressed in terms of ζ_p as

$$\gamma_p = 1 + \frac{\zeta_{4p}}{p(\zeta_{4p} - \zeta_{4p+1} - 1)}. \quad (4)$$

To apply the HIT result (3) to channels, two points deserve further clarification. First, according to the definitions, the large-scale L of HIT corresponds naturally to the flow thickness H of the channel. In HIT, σ_L is the fluctuation intensity; for wall flows, since the mean velocity U is not the right variable, σ_L can be chosen as $k = \sqrt{\langle uu \rangle}$, or as u_τ , which is the commonly used metric for the fluctuation intensity in wall flows [19]. Therefore, replacing σ_L by u_τ and L by H , Eq. (3) reads

$$\langle \epsilon^p \rangle^{1/p} / \epsilon_o \sim \text{Re}_\tau^{\gamma_p}, \quad (5)$$

where $\epsilon_o = u_\tau^3/H$. Note that, compared to $\epsilon_\tau = u_\tau^4/\nu$ defined on the wall, ϵ_o is a more reasonable normalization for dissipation moments in the outer flow, due to the vanishing viscous effect. Also, since $\epsilon_o/\epsilon_\tau = \text{Re}_\tau^{-1}$ by definition, Eq. (5) immediately leads to Eq. (1).

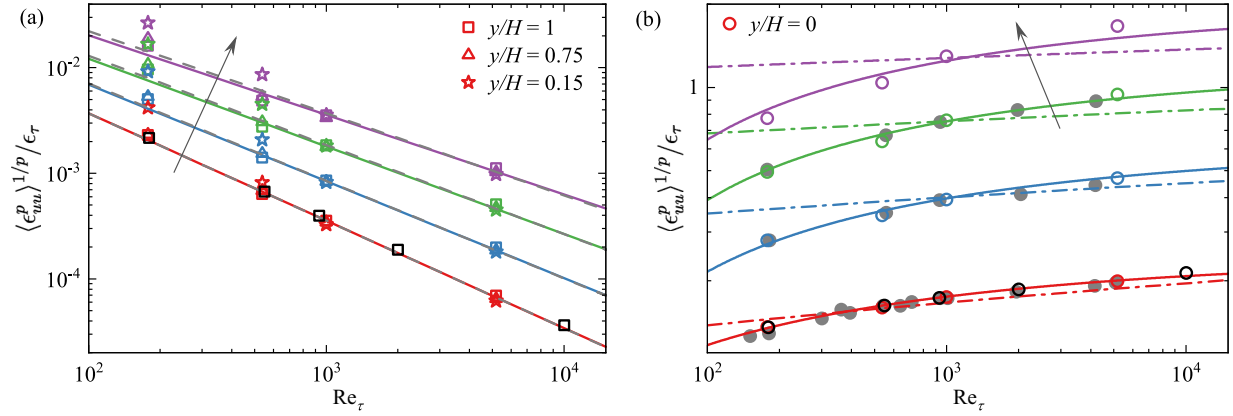


FIG. 3. Streamwise dissipation moments. (a) Re_τ scaling of dissipation moments in the outer flow. Predictions of Eq. (1) are represented by lines—solid lines from Eq. (6) and dashed lines from Eq. (7). For clarity, data at different wall-normal positions are vertically shifted to align with the center values, without affecting the validation of scaling slopes. (b) Re_τ scaling of dissipation moments at the wall. Dash-dotted lines indicate Eq. (8), while solid lines indicate Eq. (2). In both panels, colors denote different orders with arrows depicting increasing order for integer $p = 1 - 4$. Gray symbols are data from Ref. [17], black symbols are from Refs. [18,19], while others are the same as in Fig. 2. The parameter values for the theoretical lines are listed in Table I. Note that the parameters for the log scaling Eq. (8) are adopted directly from Ref. [17].

It should be said that the outer k profile is usually viewed as Re independent when normalized by u_τ [27–29]. This is reminiscent of the Townsend’s outer similarity [30,31] established for mean velocity. However, Eq. (1) could change somewhat if there exists a weak Re dependence of k/u_τ . To clarify this, in the Supplemental Material [16], we collect the DNS and experimental data for Re_τ reaching 5×10^4 . It turns out that k/u_τ saturates at a constant for Re_τ above about 1000, thus implying that the choices of $\sigma_L = u_\tau$ or $\sigma_L = k$ are equivalent for large Re_τ , with no change of Eq. (1). For moderate Re_τ ($\lesssim 1000$), however, a weak dependence $k/u_\tau \sim Re_\tau^{0.028}$ does indeed occur. Yet, compared to the scaling exponent $-1 + \gamma_p$ in Eq. (1), the additional correction is minor (at most about 9%). Hence, the influence of choosing $\sigma_L = k$ is not essential, and the conclusion here remains unchanged.

Second, the exponent γ_p in Eq. (3) or (5) corresponds to an intermittent correction to the zeroth law [32,33] and its generalization to higher order p ’s. To see this, first note that K41’s ζ_p of $p/3$ yields $\gamma_p = 0$ in Eq. (4). Consequently, $\langle \epsilon^p \rangle^{1/p} \sim \epsilon_o$, or $\langle \epsilon^p \rangle^{1/p} / \epsilon_\tau \propto Re_\tau^{-1}$. The Re_τ^{-1} scaling applies quite well for $p = 1$, as shown in Fig. 3(a). For higher p , the data differ increasingly from Re_τ^{-1} because of the increasing

deviation of ζ_p from $p/3$. Thus, a specific anomalous scaling model of ζ_p is required to characterize the dissipation intermittency.

Despite the close agreement among most intermittency models [1,20,34–36] for low-order exponents, recent findings based on very high Reynolds number HIT simulations suggest the saturation of ζ_p when $p \rightarrow \infty$ [20,37,38]. We thus adopt the particular model of Ref. [20] that captures this saturation. That is,

$$\zeta_p = 7.3/(19/p + 1). \quad (6)$$

Taking Eqs. (4) and (6) as valid, the prediction of Eq. (1) is shown in Fig. 3(a), in satisfactory agreement with the data for orders $p = 1-4$. Note that three wall-normal positions are collected here from the log layer to the centerline (see Fig. 3), all obeying prediction (1). This universality indicates a minor effect of the mean shear on dissipation intermittency for the outer flow. Deviations in the data at $y/H = 0.15$ at $Re_\tau = 180$, also observed in earlier work [8,39], are believed to be due to the low Re effects with undeveloped inertial range [12]. The parameters used for the verification of Eq. (1) are summarized in Table I; data sources and convergence of

TABLE I. Parameter values for theoretical lines used in Figs. 3 and 4. Note that values of C_p for Eq. (1) are not shown here because Fig. 3(a) verifies only the scaling exponent γ_p . Also note that γ_p values in Figs. 3(a) and 4(a) are the same.

Region	Model	Parameter	$p = 1$	$p = 2$	$p = 3$	$p = 4$
Outer	Equations (1), (4), and (6)	γ_p	−0.017	0.084	0.172	0.247
	Equations (1), (4), and (7)	γ_p	−0.017	0.077	0.156	0.222
		$\alpha_{p,uu}$	0.25	0.63	1.22	2.02
Wall	Equation (2)	$\beta_{p,uu}$	0.42	1.31	2.62	4.34
		$\alpha_{p,ww}$	0.13	0.48	1.15	2.20
		$\beta_{p,ww}$	0.31	1.23	2.93	5.25
	Equation (8)	A_p	0.08	0.25	0.54	1.00
		B_p	0.01	0.02	0.03	0.04

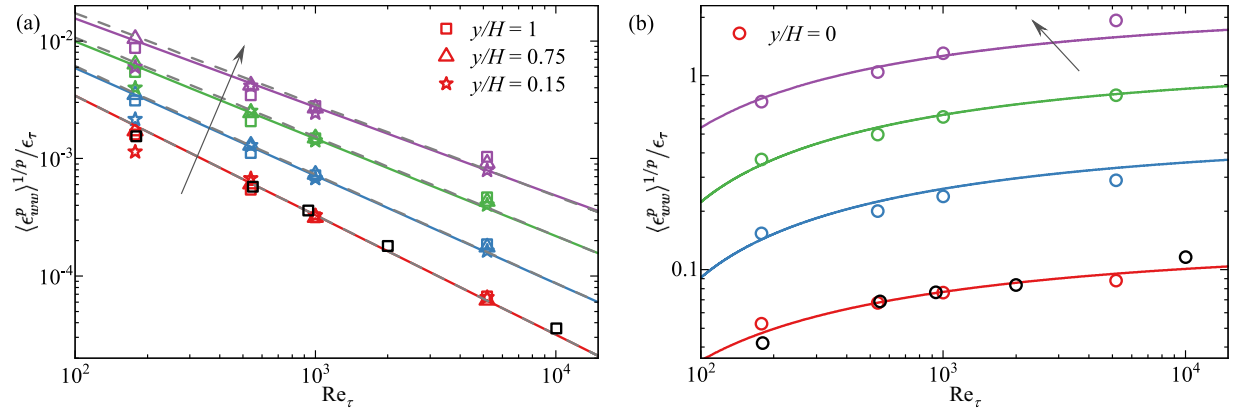


FIG. 4. Spanwise dissipation moments. Symbols and lines are the same as in Fig. 3. Only fewer data points compared to the streamwise components are available.

moments are presented in detail in the Supplemental Material [16].

To show the robustness of Eq. (1), the log-Poisson model [35] is now considered as an alternative to Eq. (6). That is,

$$\zeta_p = p/9 + 2 - 2(2/3)^{p/3}. \quad (7)$$

This model also adheres closely to the data [Fig. 3(a)]. However, as $p \rightarrow \infty$, it yields $\gamma_\infty = 3/5$ in contrast to $\gamma_\infty = 1$ arising from Eq. (6). These two models yield different results for large p but not for the orders considered here. We note that Benzi and Biferale [40] have provided another multifractal model for the scaling of dissipation moments, which show similar agreement with data for low-order moments. Since our purpose is not to judge the superiority of different intermittency models, the performance of other choices of ζ_p is not shown here.

Scaling at the wall. Dissipation exhibits fundamentally different behaviors on the wall. As shown in Fig. 3(b), wall dissipation increases with Re_τ , a trend that is in excellent agreement with the bounded defect law given by Eq. (2). For the particular case of $p = 1$, the basis of Eq. (2) has been discussed in Ref. [41]: The main point is that the energy production limits the dissipation near the wall, which itself is bounded by a finite value that is approached according to a $-1/4$ power in Re_τ . Several consequences of this result have been deduced, e.g., the bounded behavior of the intensity of pressure fluctuation [42], the wall-normal profiles of velocity variance [43], and its higher-order moments [44]. Independent evidence for this behavior can be found in Refs. [27,29,45–47].

Equation (2) is an extension to higher orders of p of this known behavior for $p = 1$, via a linear p -norm Gaussian process. We have performed this extension for moments of velocity fluctuations in Ref. [44]. The novelty here is that we confirm its applicability to wall-dissipation moments. The order-dependent coefficients α_p are found to increase with p , which is consistent with a Gaussian-related random process (see the Supplemental Material [16] for more discussion).

Also shown in Fig. 3(b) as dash-dotted lines are the logarithmic growth prediction according to the attached-eddy model [30,48], which states that

$$\langle \epsilon^p \rangle^{1/p} / \epsilon_\tau = A_p + B_p \ln Re_\tau. \quad (8)$$

Here, A_p and B_p are constants independent of Re_τ but dependent on p . They are listed in Table I along with γ_p of Eq. (1) and α_p and β_p of Eq. (2). The comparison of Eq. (8) with the data, given in Ref. [17] and reproduced here in Fig. 3(b) for the same parameters, shows larger deviations than Eq. (2). The parameters of log scaling in Eq. (8) could be fine-tuned for better agreement, but this would then not correspond to the case of the attached-eddy model in Ref. [17] where the Gaussian assumption is adopted.

In addition to the evidence from ϵ_{uu} , the spanwise component of the dissipation, $\epsilon_{ww} = v \partial_j w \partial_j w$, is shown Fig. 4 to exhibit the same Reynolds number scaling as the streamwise component, both in the outer region and at the wall. This consistency reaffirms the scaling of Eqs. (1) and (2), as one might indeed have expected. We also note that no prediction of spanwise wall-dissipation moments has been discussed before. The dissipation of wall-normal velocity $\epsilon_{vv} = v \partial_j v \partial_j v$ also satisfies the same law in the outer region, but it is zero on the wall [which means that α_p and β_p in Eq. (2) are zero because of the no-slip condition]. They are therefore not shown here. Note: The coefficients for the streamwise and spanwise components do differ due to the anisotropy introduced by the wall; however, this does not affect the universality of the scaling exponents.

Discussion. We have examined here the boundary effects on dissipation intermittency and obtained two fundamentally distinct Reynolds number scalings in turbulent channel flows. For the outer region where the mean shear is weak, we have identified a universal power law that extends the inertial-range intermittency to the dissipative range. Close to the wall where the mean shear is strong, the bounded defect law discussed here implies a nonintermittent character of dissipative events on the wall. Note that the mean dissipation at the wall remains to be $O(Re_\tau)$ larger than that in the bulk. We point to three of several consequences of the present findings.

(1) It is frequently argued (see, e.g., Refs. [5,6,14]) that intermittency requires increasingly finer grid resolution of DNS for resolving high-order moments of ϵ at larger Reynolds numbers. It may be thought that this consideration might apply for the dissipation near the channel walls as well [49]. However, the present result [see Eq. (2)] shows that all finite high-order moments of ϵ are bounded, so that the DNS resolution could remain $O(v/u_\tau)$ even at infinite Reynolds

numbers. In contrast, Eq. (8) requires a continually increasing grid resolution to resolve high-order wall-dissipation events as Reynolds number grows.

(2) The existence of outer flow intermittency, reflected by the nonzero exponent γ_p , and the absence of intermittency at the wall, as seen by a common exponent for all p in Eq. (2), are consistent with our recent findings on wall-parallel circulation [12]: In the region where Eq. (1) applies, the intermittency of circulation (obtained by the integration of u and w along closed Eulerian loops) takes the form of universal bifractality as in HIT [50], whereas, close to the wall, the bifractality simplifies to unifractality suggestive of the nonintermittent behavior. Equations (1) and (2) support this picture.

(3) Finally, due to the concavity of ζ_p [1], one has $\gamma_p < 1$. Thus, for finite order p , Eq. (1) implies a negative scaling, hence vanishingly small dissipation in the bulk relative to the wall at very large Reynolds numbers. This means that away from the wall, the flow does not generate sufficiently strong

turbulence. However, the extreme nature of dissipative events on the wall (compared to those in the bulk of the channel) suggests that walls may generate the necessary strength of singularities. This brings us to the broader possibility that the Onsager-type singularities [51,52] might be generated only near the wall. To the extent that such singular structures are essential for characterizing turbulence, we can ask whether there can be turbulence without walls somewhere in the flow domain.

Acknowledgments. We gratefully acknowledge the authors of Ref. [13] and the Johns Hopkins Turbulence Database [53] for access to DNS data at $Re_\tau = 1000$ and 5200. X.C. appreciates the support by the National Natural Science Foundation of China, Grant No. 92252201, and the “Fundamental Research Funds for the Central Universities.” K.R.S.’s research was supported by New York University.

Data availability. The data that support the findings of this article are openly available [11].

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