

Many-body quantum optics in a Bose-Hubbard waveguide

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Waveguide quantum electrodynamics (QED) studies the interaction between quantum emitters and guided photons in one dimension. When the waveguide hosts interacting photons, it becomes a platform to explore many-body quantum optics. However, the influence of photonic correlations on emitter dynamics remains poorly understood. In this work, we study the collective decay and coherent interactions of quantum emitters coupled to a one-dimensional Bose-Hubbard waveguide, an array of coupled photonic modes with repulsive on-site interactions that supports superfluid and Mott insulating phases. We show that photon-photon interactions alone can trigger a superradiant burst, independent of emitter spacing and transition frequency. In the off-resonant regime, emitters exhibit two distinct types of mediated interactions. Delocalized superfluid excitations yield persistent, long-range couplings, challenging the noninteracting paradigm, where long-range (dissipative) couplings occur in band, whereas short-range and coherent interactions occur in a bandgap. In the Mott limit, quasiparticles generate short-range interactions mediated by doublons and holons. Our work bridges many-body physics and waveguide QED, revealing how photonic many-body states shape emitter dynamics.

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Introduction. Waveguide quantum electrodynamics (QED) [1–12] has emerged as a powerful paradigm for engineering light-matter interactions, where emitters coupled to confined photonic modes exhibit collective phenomena like superradiance [13–16], subradiance [17], and photon-mediated spin models [18–26]. In traditional setups, the photonic bath is assumed to be noninteracting, enabling exact solutions. Recent advances, however, have begun to explore nonlinear photonic environments, where photon-photon interactions introduce correlations that modify emitter dynamics. For instance, attractive interactions can lead to bound photon states [27] or supercorrelated decay [28].

Yet, the role of repulsive photon-photon interactions—ubiquitous in quantum optical platforms [24,29–32]—remains largely unexplored in waveguide QED. In the many-body regime, repulsive interactions drive quantum phases, such as the superfluid-to-Mott insulator transition [33–39], whose quasiparticle excitations (Bogoliubov modes [40] or doublons/holons [41]) could mediate emitter-emitter couplings. Prior works in nonlinear waveguide QED focused on waveguides with attractive photon-photon interactions [27,28,42], nonlinear waveguides with parametric gain [43], or driven-dissipative many-body systems [44], leaving open

how many-body photonic states influence collective emitter phenomena.

Here, we bridge this gap by studying a system made of quantum emitters coupled to a Bose-Hubbard (BH) waveguide—a one-dimensional (1D) lattice hosting repulsive photon-photon interactions. We uncover two key results: (1) The superradiant burst—a hallmark of collective decay—becomes tunable via the photon-photon interaction strength, even for fixed emitter spacing and frequency. (2) Off-resonant emitters develop dipole-dipole interactions mediated by many-body quasiparticles: Delocalized Bogoliubov modes in the superfluid phase induce distance-insensitive couplings (in stark contrast with short-range bandgap photon-mediated interactions), while Mott-insulator excitations recover standard waveguide QED-like behavior, but with doublon/holon dressing. Our results establish an avenue for exploring the interplay between waveguide QED and many-body physics (see Fig. 1).

Bose-Hubbard waveguide. We consider a 1D Bose-Hubbard waveguide, a tight-binding array of N_p photonic modes with on-site repulsion described by

$$\hat{H}_{\text{BH}} = \omega_c \sum_r \hat{a}_r^\dagger \hat{a}_r - J \sum_r (\hat{a}_r^\dagger \hat{a}_{r+1} + \text{H.c.}) + \frac{U}{2} \sum_r \hat{a}_r^\dagger \hat{a}_r^\dagger \hat{a}_r \hat{a}_r, \quad (1)$$

where $\hat{a}_r$ annihilates a photon at site r , the resonator frequency is $\omega_c \gg J, U$ [24,28], $J > 0$ is the hopping rate, and $U > 0$ is the repulsive interaction strength (the lattice spacing d is set to 1). Unlike previous studies focusing on attractive

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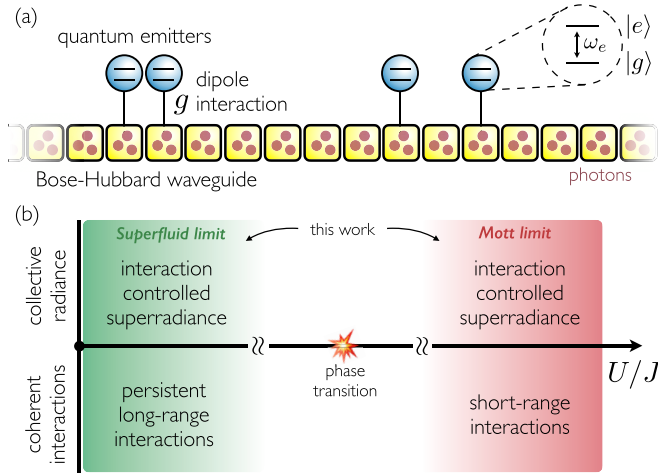


FIG. 1. Setup and summary of main results. (a) Bose-Hubbard waveguide QED setup. Two-level quantum emitters (blue spheres) are coupled via dipole interaction to a Bose-Hubbard waveguide (yellow squares array). The internal transition frequency of the emitters between the excited and ground states $|e\rangle$ and $|g\rangle$ is ω_e . The waveguide array hosts coherent photon hopping at rate J and repulsive local interaction with strength U . In the limit of many photons (red dots) in the waveguide, two regimes can be identified. In the superfluid (Mott) regime, the waveguide can be mapped to Bogoliubov quasiparticles (doublons and holons) living on top of a many-body “superfluid” (Mott insulating) ground state. (b) Phase diagram and main results. In the superfluid and Mott limits, photon-photon interactions can trigger a collective superradiant burst. Coherent interactions between quantum emitters become distance independent in the superfluid limit due to the ground-state delocalized nature, while they are short range in the Mott regime.

interactions [27,28,42], we consider here repulsive photon-photon interactions.

For large photon numbers, the competition between tunneling (J) and repulsion (U) produces two distinct regimes. In the weak interaction limit ($U \ll J$), the many-body ground state is a photon quasicondensate $|\text{SF}\rangle$, which would become a proper superfluid in two dimensions (2D) [45]. Conversely, for strong interactions ($J \ll U$), photons localize into a Mott insulator $|\text{MI}\rangle$. While a true quantum phase transition occurs in 2D [38], the 1D system exhibits clear crossover behavior between these limits [46].

To treat the waveguide as a photonic bath for emitters (see the next section), we employ effective theories in each limit. For the superfluid phase, we use the Bogoliubov theory of weakly interacting Bose gas [40,45,47], while the Mott regime is analyzed via the formalism of Barmettler *et al.* [41]. This maps the interacting Hamiltonian to quadratic quasiparticle models, enabling analytic solutions.

In the superfluid limit ($U \ll J$), we consider the BH waveguide populated by an average number of N photons, with N_0 photons occupying the $k=0$ momentum state such that $N = \sum_k \langle \hat{a}_k^\dagger \hat{a}_k \rangle = \langle \hat{a}_0^\dagger \hat{a}_0 \rangle + \sum_{k \neq 0} \langle \hat{a}_k^\dagger \hat{a}_k \rangle = N_0 + \sum_{k \neq 0} \langle \hat{a}_k^\dagger \hat{a}_k \rangle$. First, we write the Hamiltonian in momentum space, getting $\hat{H}_{\text{BH}} = \sum_k (\omega_c + \varepsilon_k) \hat{a}_k^\dagger \hat{a}_k + (U/2N_p) \sum_{k,k',q,q'} \hat{a}_q^\dagger \hat{a}_q \hat{a}_{k'}^\dagger \hat{a}_{k'} \delta_{q+q',k+k'}$, where $\hat{a}_r = \sum_k \hat{a}_k$

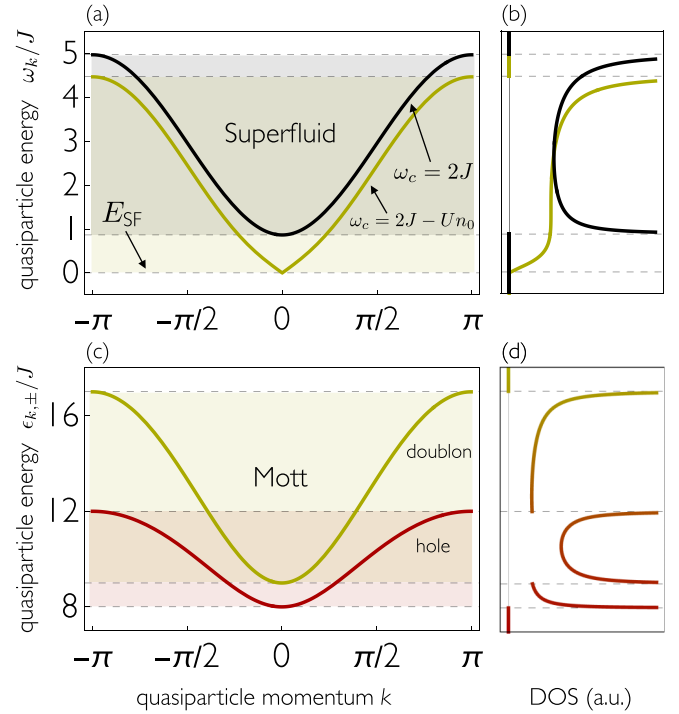


FIG. 2. Quasiparticle dispersion and DOS. (a) Superfluid ($U = J/2$) quasiparticle dispersion for $\omega_c = 2J - Un_0$ (yellow) and $\omega_c = 2J$ (black), with corresponding DOS in panel (b). The DOS, up to a prefactor, is the single-emitter decay rate $\Gamma_{ii}^{\text{SF}}(\omega)$. E_{SF} marks the ground-state energy (zero reference). (c) Mott insulator ($U = 3J$) excitations: doublons (yellow) and holons (red). (d) Corresponding DOS, proportional to $\Gamma_{ii}^{\text{MI}}(\omega)$. The shaded stripes highlight the bandwidth corresponding to each band. Parameters: $n_0 = \bar{n} = 1$, $\omega_c = 10J$ [panels (c) and (d)], and $g = 0.1J$.

$e^{ikr}/\sqrt{N_p}$ and $\varepsilon_k = -2J \cos k$. Then, following Bogoliubov theory, we assume $N_0 \gg 1$ and replace $\hat{a}_0$ with $\sqrt{N_0}$. Keeping terms up to order N_0 (see the Supplemental Material [48]), we get, after a Bogoliubov rotation, the quadratic Hamiltonian

$$\hat{H}_{\text{BH,SF}} = E_{\text{SF}} + \sum_{k \neq 0} \omega_k \hat{b}_k^\dagger \hat{b}_k, \quad (2)$$

with ground-state energy E_{SF} and Bogoliubov excitations $\hat{b}_k$ satisfying $\hat{b}_k |\text{SF}\rangle = 0$ for the quasicondensate state $|\text{SF}\rangle = (\hat{a}_{k=0}^\dagger)^{N_0} |0\rangle$. The quasiparticle dispersion is $\omega_k = \sqrt{f_k^2 - U^2 n_0^2}$, where $f_k = \omega_c + 2Un_0 + \varepsilon_k$ and $n_0 = N_0/N_p$ is the condensate fraction. Setting $\omega_c = 2J - Un_0$ recovers the standard BH dispersion [40]. The Bogoliubov transformation given by $\hat{b}_k = u_k \hat{a}_k + v_k \hat{a}_{-k}^\dagger$, where $u_k = \cosh \alpha_k$, $v_k = \sinh \alpha_k$, and α_k is defined by $\tanh(2\alpha_k) = Un_0/f_k$, connects the quasiparticles to the photonic excitations (see the Supplemental Material [48]). Figures 2(a) and 2(b) show the dispersion and corresponding density of states (DOS).

In the Mott limit ($J \ll U$), we follow the theory developed by Barmettler *et al.* [41] (and previously in Refs. [49,50]), noting that our Hamiltonian (1) differs slightly from the one considered in Refs. [41,51] (see the Supplemental Material [48]). For zero hopping ($J = 0$), the ground state is perfectly localized: $|\text{MI}\rangle = \bigotimes_r |\bar{n}\rangle_r$, where $\bar{n}$ is the integer

filling factor. When small but finite hopping J is introduced, the local density fluctuations around the average filling are limited. We can thus restrict the local Hilbert space to three states: $\{|\bar{n}\rangle_r, |\bar{n} \pm 1\rangle_r\}$. The original bosonic operators are expressed in terms of auxiliary bosonic operators as $\hat{a}_r = \sqrt{\bar{n}+1}\hat{b}_{r,+} + \sqrt{\bar{n}}\hat{b}_{r,-}^\dagger$, where $\hat{b}_{r,+}^\dagger$ creates an excess particle (doublon), $\hat{b}_{r,+}^\dagger|\bar{n}\rangle_r = |\bar{n}+1\rangle_r$, and $\hat{b}_{r,-}^\dagger$ creates a hole, $\hat{b}_{r,-}^\dagger|\bar{n}\rangle_r = |\bar{n}-1\rangle_r$. The Fock state $|\bar{n}\rangle_r$ serves as the vacuum for both auxiliary operators: $\hat{b}_{r,\pm}|\text{MI}\rangle = 0$. While these auxiliary operators obey bosonic commutation relations, they must satisfy the hardcore boson constraints: $\hat{b}_{r,\pm}^2 = 0$ (no double creation on one site) and $\hat{b}_{r,+}^\dagger\hat{b}_{r,+}\hat{b}_{r,-}^\dagger\hat{b}_{r,-} \equiv \hat{n}_{r,+}\hat{n}_{r,-} = 0$ (no simultaneous doublon and holon on one site).

To enforce these constraints, we employ the generalized Jordan-Wigner transformation [52,53], mapping the auxiliary bosons to fermionic operators $\hat{c}_{r,\pm}$: $\hat{b}_{r,\pm} = \hat{Z}_{r,\pm}\hat{c}_{r,\pm}$, where $\hat{Z}_{r,+} = \exp(i\pi \sum_{\sigma=\pm} \sum_{r'<r} \hat{n}_{r',\sigma})$ and $\hat{Z}_{r,-} = \hat{Z}_{r,+} \exp(i\pi \hat{n}_{r,+})$ are nonlocal string operators. This automatically enforces the hardcore constraint. The remaining constraint against double occupancies of different species is treated by projecting the Hamiltonian with $\mathcal{P} = \prod_r (1 - \hat{n}_{r,+}\hat{n}_{r,-})$ and making the unconstrained fermion approximation $\mathcal{P} \rightarrow 1$ following Ref. [41].

Once the initial Hamiltonian is written in terms of fermionic operators $\hat{c}_{r,\pm}$, transforming in momentum space $\hat{c}_{r,\pm} = \sum_k \hat{c}_{k,\pm} e^{ikr} / \sqrt{N_p}$ and performing the Bogoliubov rotation $\hat{\gamma}_{k,\pm} = (u_k \hat{c}_{k,\pm} - v_k \hat{c}_{-k,\mp}^\dagger)$, where u_k (v_k) is real (imaginary) and even (odd) in k , one can recast the BH waveguide Hamiltonian into

$$\hat{H}_{\text{BH,MI}} = E_{\text{MI}} + \sum_{k,\sigma} \epsilon_{k,\sigma} \hat{\gamma}_{k,\sigma}^\dagger \hat{\gamma}_{k,\sigma}. \quad (3)$$

Here, E_{MI} is the ground-state energy and $\hat{\gamma}_{k,\pm}$'s are the (fermionic) Bogoliubov excitations on top of the ground state, so that, in the limit of vanishing hopping strength, the ground state $|\text{MI}\rangle$ is the vacuum of the Bogoliubov excitations $\hat{\gamma}_{k,\pm}|\text{MI}\rangle = 0$. The doublon and holon dispersions are given by $\epsilon_{k,\pm} = \pm \delta_k + \eta_k$, where $2\delta_k = E_+(k) - E_-(k)$ and $2\eta_k = \sqrt{[E_+(k) + E_-(k)]^2 + 4|\Delta(k)|^2}$. Here, $E_+(k) = \omega_c + U\bar{n} - 2J(\bar{n}+1)\cos k$, $E_-(k) = \omega_c - U(\bar{n}-1) - 2J\bar{n}\cos k$, and $\Delta(k) = -2iJ\sqrt{\bar{n}(\bar{n}+1)}\sin k$. Finally, the coefficients of the Bogoliubov transformation are given in the Mott limit by $u_k = \cos(\theta_k/2)$ and $v_k = i\sin(\theta_k/2)$, where θ_k is defined by $\tan \theta_k = 2i\Delta(k)/[E_+(k) + E_-(k)]$ (see the Supplemental Material [48]). In Figs. 2(c) and 2(d), we show the doublon/holon dispersion and the corresponding DOS.

Bose-Hubbard waveguide QED. Our main goal is to study how two-level quantum emitters interact with a BH waveguide in its superfluid and Mott insulating phases. The complete system is described by the Hamiltonian $\hat{H} = \hat{H}_{\text{qe}} + \hat{H}_{\text{BH}} + \hat{V}$. The emitters' Hamiltonian $\hat{H}_{\text{qe}} = \omega_e \sum_i \hat{\sigma}_i^\dagger \hat{\sigma}_i$ describes N identical two-level systems with transition frequency ω_e between states $|g\rangle$ and $|e\rangle$, where $\hat{\sigma}_i = |g\rangle_i \langle e|$ is the lowering operator for the i th emitter. The interaction term $\hat{V}$ represents the dipole coupling between emitters and the photonic field,

$$\hat{V} = g \sum_i \hat{\sigma}_i^x \hat{x}_{r_i} = g \sum_i (\hat{\sigma}_i + \hat{\sigma}_i^\dagger)(\hat{a}_{r_i} + \hat{a}_{r_i}^\dagger), \quad (4)$$

where we keep the counterrotating terms—The rotating wave approximation is made in the derivation of the emitters' master equation (see the Supplemental Material [48]). The photonic operators $\hat{a}_r$ take dramatically different forms in each phase of the waveguide in terms of the quasiexcitations:

$$\hat{a}_r^{\text{SF}} = \sqrt{n_0} + \frac{1}{\sqrt{N_p}} \sum_{k \neq 0} (u_k \hat{b}_k - v_k \hat{b}_{-k}^\dagger) e^{ikr}, \quad (5)$$

$$\begin{aligned} \hat{a}_r^{\text{MI}} = & \frac{\sqrt{\bar{n}+1}}{\sqrt{N_p}} \hat{Z}_{r,+} \sum_k e^{ikr} (u_k \hat{\gamma}_{k,+} + v_k \hat{\gamma}_{-k,-}^\dagger) \\ & + \frac{\sqrt{\bar{n}}}{\sqrt{N_p}} \hat{Z}_{r,-} \sum_k e^{-ikr} (u_k \hat{\gamma}_{k,-}^\dagger - v_k \hat{\gamma}_{-k,+}), \end{aligned} \quad (6)$$

with distinct Bogoliubov coefficients u_k, v_k in each phase. We note that previous works studied impurities in BH baths [54,55], though focusing on density-density interactions.

The reduced emitter dynamics may be cast in the following form:

$$\dot{\tilde{\rho}} = -i[\hat{H}_{\text{eff}}(\omega_e), \tilde{\rho}] + \sum_{ij} \Gamma_{ij}(\omega_e) \mathcal{D}[\hat{\sigma}_j, \hat{\sigma}_i^\dagger] \tilde{\rho}, \quad (7)$$

where $\tilde{\rho}$ is the joint state of the quantum emitters in the interaction picture (with respect to $\hat{H}_{\text{qe}} + \hat{H}_{\text{BH}}$) and $\mathcal{D}[\hat{A}, \hat{B}]\rho = \hat{A}\rho\hat{B} - \{\hat{B}\hat{A}, \rho\}/2$. The effective Hamiltonian

$$\hat{H}_{\text{eff}}(\omega_e) = \sum_{ij} [\Delta_{ij}(\omega_e) \hat{\sigma}_i^\dagger \hat{\sigma}_j + \Delta_{ij}(-\omega_e) \hat{\sigma}_i \hat{\sigma}_j^\dagger] \quad (8)$$

describes coherent interactions mediated by the Bogoliubov excitations, while $\Gamma_{ij}(\omega_e)$ gives collective decay rates in the BH waveguide. Both depend on the bath correlation function (see the Supplemental Material [48]):

$$I_{ij}(\omega) = g^2 \int_0^\infty dt e^{i\omega t} \langle \tilde{x}_{r_i}(t) \tilde{x}_{r_j} \rangle, \quad (9)$$

where $\tilde{x}_r(t)$ is $\hat{x}_r$ [cf. Eq. (4)] in the interaction picture and the average is taken over the BH waveguide ground state ($|\text{SF}\rangle$ or $|\text{MI}\rangle$). In fact, we are assuming that at time $t = 0$ the ground state of the BH waveguide is prepared, and that at time $t = 0^+$ the atom light coupling $\hat{V}$ is turned on. The exact form of these quantities reveals how many-body effects modify light-matter interactions. Additionally, the emitters can decay into independent nonguided modes of the surrounding electromagnetic field. We include this effect by adding the local dissipator $\Gamma' \sum_i \mathcal{D}[\hat{\sigma}_i, \hat{\sigma}_i^\dagger]\rho$ to the master equation.

Superfluid and Mott superradiance. We first study the collective decay of N_e excited emitters in our BH waveguide. Superradiance refers to the cooperative enhancement of spontaneous emission due to coherent coupling between the emitters via the bath. This results in a superradiant burst—a short-lived spike in the total emitted power, scaling faster than linearly with the number of emitters [13,14]. A sufficient condition for the occurrence of a superradiant burst found in Ref. [56] becomes in the waveguide case [16]:

$$\frac{N_e}{2} + \frac{1}{2N_e} \frac{\sin^2(N_e k_{\text{ID}} d)}{\sin^2(k_{\text{ID}} d)} > 2 + \frac{\Gamma'}{\Gamma_{\text{ID}}}, \quad (10)$$

where we have reintroduced explicitly the interatomic distance d (the lattice constant), k_{ID} is the momentum selected

by the atomic frequency (see below), and $\Gamma_{\text{ID}} = \Gamma_{ii}(\omega_e)$ is the single-emitter decay rate in the BH waveguide.

We show that, in the superfluid and Mott limits, the photon-photon interaction strength allows us to control the occurrence of the superradiant burst. In the standard noninteracting waveguide QED scenario, including a non-negligible parasitic decay, condition (10) can be satisfied (or not) by varying the atomic distances for a fixed atomic frequency ω_e (which fixes the momentum $k_{\text{ID}} = \omega_e/v$, v being the speed of light in the waveguide), or vice versa fixing the interatomic distance and varying the momentum [16].

Including photon-photon interactions with strength U , the Bogoliubov momentum becomes dependent on U as well (as the dispersion depends on U). This allows then, for a fixed interatomic distance d and fixed atomic frequency ω_e , to tune the momentum k_{ID} so as to explore regions of parameter space with and without a burst.

In order to show this, we compute the collective decay rates in both limits, which we will label either SF or MI. In the superfluid limit, we get (see the Supplemental Material [48])

$$\Gamma_{ij}^{\text{SF}}(\omega) = \frac{g^2}{J} \mathcal{K}_{\text{SF}}(\omega) \cos[k_{\text{ID}}^{\text{SF}} |i - j| d], \quad (11)$$

where the prefactor reads $\mathcal{K}_{\text{SF}}(\omega) = 2J[\Omega - Un_0]/[\Omega\sqrt{4J^2 - (\omega_c + 2Un_0 - \Omega)^2}]$, $\Omega = \sqrt{\omega^2 + U^2 n_0^2}$, and $k_{\text{ID}}^{\text{SF}}$ is defined by $2J \cos k_{\text{ID}}^{\text{SF}} = \omega_c + 2Un_0 - \Omega$. The collective decay $\Gamma_{ij}^{\text{SF}}(\omega)$ is nonzero only when the atomic frequency is in resonance with the Bogoliubov band, i.e., $\omega_0 < \omega < \omega_\pi$. In the noninteracting limit ($U = 0$), this reduces to the standard waveguide QED results [16] with the typical band-edge divergences. For a finite interaction strength, the collective decay rate can vanish at the lower band edge instead [57]. In the Mott limit, the collective decay rate is given by

$$\Gamma_{ij}^{\text{MI}}(\omega) = \frac{g^2}{J} \sum_{\sigma=\pm} \chi_\sigma(\omega) \mathcal{K}_{\text{MI}}^{(\sigma)}(\omega) \cos[k_{\text{ID},\sigma}^{\text{MI}} |i - j| d], \quad (12)$$

where the explicit form of $\mathcal{K}_{\text{MI}}^{(\sigma)}(\omega)$ is more involved than the superfluid one and provided in Ref. [48]. The term $\chi_\sigma(\omega) = \chi_{[\epsilon_{0,\sigma}, \epsilon_{\pi,\sigma}]}(\omega)$ is the characteristic function $\chi_A(\omega)$ ($\chi_A(\omega) = 1$ if $\omega \in A$ and 0 otherwise), ensuring that the decay rate is zero if the atomic frequency lies in a doublon/holon bandgap. In Eq. (12), the momentum $k_{\text{ID},+}^{\text{MI}}$ is defined by

$$\cos k_{\text{ID},+}^{\text{MI}} = \frac{4J^2 \bar{n}(\bar{n} + 1)}{2J[(2\bar{n} + 1)\omega_c + U + \omega]} + \frac{[\omega_c - U(\bar{n} - 1) + \omega][\omega_c - U\bar{n} - \omega]}{2J[(2\bar{n} + 1)\omega_c + U + \omega]}, \quad (13)$$

and $k_{\text{ID},-}^{\text{MI}}$ is obtained from $k_{\text{ID},+}^{\text{MI}}$ replacing ω with $-\omega$. Using $k_{\text{ID},\pm}^{\text{SF}}$, $k_{\text{ID},\pm}^{\text{MI}}$, and condition (10), in Fig. 3 we show the crossover between burst and no-burst regions, for a fixed interatomic distance, as a function of the number of emitters and interaction strength.

Finally, to quantify the impact of photon-photon interactions on the dynamics of correlated emission, we compute from Eq. (7) the time-resolved emission rate $R(t) = \sum_{ij} \Gamma_{ij}(\omega_e) \langle \hat{\sigma}_i^\dagger \hat{\sigma}_j \rangle_t$ [see Figs. 3(e) and 3(f)]. We consider as a minimal example two identical emitters initially in

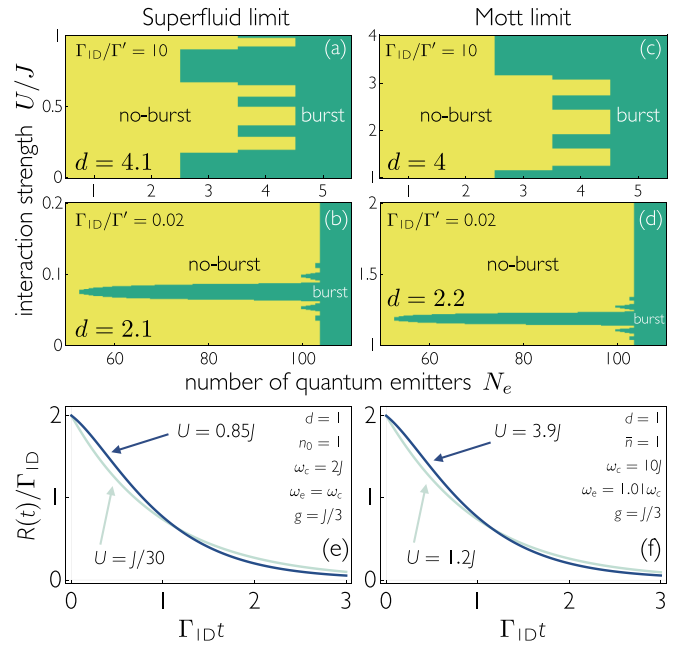


FIG. 3. Superfluid and Mott superradiance. Prediction for a superradiant burst in a superfluid (left) or Mott insulating (right) waveguide. The key difference here is that the interatomic distance d and the atomic frequency ω_e are fixed; still the occurrence of the burst is controlled by the interaction strength U , which modulates the momentum k_{ID} . Superfluid parameters [panels (a) and (b)]: $n_0 = 1$, $\omega_c = 2J$, and $\omega_e = 2J$. Mott parameters [panels (c) and (d)]: $\bar{n} = 1$, $\omega_c = 10J$, and $\omega_e = 11J$. In all panels, $g = 0.1J$. For comparison, see Fig. 2 in Ref. [16]. (e), (f) Emission rate $R(t)$, scaled with the single-emitter decay rate Γ_{ID} , for different values of U/J in the superfluid and Mott limits, respectively. All other parameters are specified within the plot frame.

$|\psi_{\text{init}}\rangle = |ee\rangle$. With d and ω_e fixed, tuning only the photon-photon interaction strength U qualitatively reshapes $R(t)$ in both the superfluid and Mott regimes, demonstrating that U alone controls correlated emission.

Bogoliubov-mediated interactions. We now study the coherent part of the master equation (7). For both the superfluid and Mott insulating cases, we will assume that the atomic frequency is off-resonant with the quasiparticle spectrum, namely, above it. Therefore, the collective decay rates vanish and the dynamics is fully coherent and driven by $\hat{H}_{\text{eff}}$.

The methods we use are formally equivalent to those employed in the studies of photon-mediated interactions. The key difference here is the physical picture. The quantum emitters are coupled to an interacting environment that is in a many-body ground state, and the dipole-dipole interactions are now mediated by the Bogoliubov excitations on top of the ground state.

Contrary to the collective decay rates, we approximate the dispersion relations (for weak U and J in the superfluid and Mott limits, respectively). This allows for the analytical computation of the coherent couplings $\Delta_{ij}(\omega_e)$ from the autocorrelation function using the residue theorem (see the Supplemental Material [48]).

We find that in the superfluid limit, the coherent interactions have a component that decays exponentially with

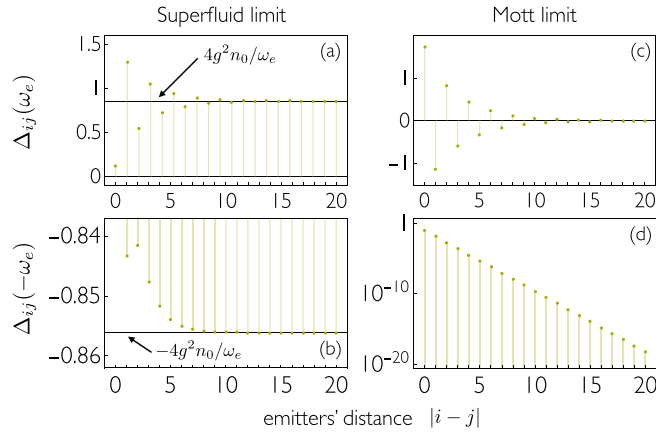


FIG. 4. Bogoliubov-mediated interactions. Coherent couplings $\Delta_{ij}(\pm\omega_e)$, as in Eq. (8), in units of g^2/J . In the superfluid limit [panels (a) and (b)], residual dipole-dipole interactions persist even at large emitters' distance as a result of the underlying many-body ground state driving the emitters. Parameter values in panels (a) and (b): $U = J/2$, $n_0 = 1$, $\omega_c = 2J - Un_0$, and $\omega_e = \omega_\pi + 0.2J$. In the Mott limit [panels (c) and (d)], the emitters interact with the doublon/holon quasiparticles as in the standard waveguide QED case, leading to an exponentially decreasing coupling strength with the emitters' distance. Parameter values in panels (c) and (d): $U = 2J$, $\bar{n} = 1$, $\omega_c = 10J$, and $\omega_e = E_+(\pi) + 0.2J$. In all panels, $g = 0.1J$.

emitters' distance and one that is independent of it, namely,

$$\Delta_{ij}^{\text{SF}}(\omega_e) = \frac{4g^2n_0}{\omega_e} + \frac{2g^2}{\omega_e}F_1(U)e^{-|i-j|/\Lambda_1(U)} + \frac{2g^2}{\omega_e}F_2(U, \omega_e)e^{-|i-j|/\Lambda_2(U, \omega_e)}, \quad (14)$$

where the explicit form of $F_1(U)$ and $F_2(U, \omega)$ is not essential here and given in the Supplemental Material [48]. Equation (14) is formally valid both for $\omega_e > \omega_\pi$ (upper bandgap) and for $\omega_e < 0$ [needed for Eq. (8)].

Figures 4(a) and 4(b), showing the coherent interactions' strength as a function of emitters' distance [Eq. (14)], highlight a central message of this work. In standard (non-interacting) waveguide QED ($U = 0$), resonant emitters (in-band) feature all-to-all, long-range dissipative couplings,

whereas emitters inside a photonic bandgap interact coherently but only over a short range dictated by the exponential tails of atom-photon bound states. In the superfluid case, we find a stark difference: When the emitters lie in a (Bogoliubov) bandgap, the interactions remain coherent and persist at long range.

In the Mott limit, the Bogoliubov-mediated interactions behave similarly to bandgap photon-mediated interactions in waveguide QED setups, having only the exponential dependence on the emitters' distance. Their explicit expression and derivation are detailed in the Supplemental Material [48]. In Fig. 4, we show the coupling strengths as a function of emitters' distance in the superfluid and Mott limits.

Conclusion. We have shown that many-body photonic environments enable access to previously unexplored regimes of light-matter interaction. By coupling quantum emitters to a Bose-Hubbard waveguide, we demonstrated two key phenomena. First, photon-photon interactions can trigger superradiant bursts without changing emitter spacing or frequency. Second, by breaking the noninteracting short-range/coherence correspondence, the superfluid case furnishes a practical pathway to long-range coherent gates via bandgap engineering. The interaction strength and range are controllable through detuning and interactions, making the effect compatible with existing waveguide QED platforms and amenable to scalable gate design.

Our theoretical framework, combining quantum optical and many-body techniques, provides a powerful tool for studying correlated photonic environments. This work opens several research directions, including exploration of topological interacting waveguides, higher-dimensional nonlinear lattices, and nonequilibrium quantum phases. More broadly, it establishes a pathway toward harnessing many-body quantum optical phenomena for quantum simulation and information processing.

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Data availability. The data supporting this study's findings are available within the article.

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