

Optical quasisymmetry groups for meron lattices

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We report that optical quasisymmetry groups emerge from the commutation between mirror operation and spin-orbit interaction (SOI) of light. Contrary to the principle of symmetry inheritance in free-space optics—where the symmetry of any structured field is strictly constrained by that of its source—we show that strong SOI enables quasi-symmetry-protected formation of topological lattices even when the underlying optical sources violate the nominal rotational symmetry. By analyzing the Hermiticity of the electric-dipole radiation amplitude in a circular polarization basis, we derive an effective mirror operator acting only on a subset of C_3 polarized dipole emitters, forming a quasisymmetry group that commutes with SOI. This quasisymmetry guarantees exact C_3 merons and gives rise to a robust polarization zone within which continuously varying input polarizations generate identical topological textures. Our work establishes quasisymmetry as a fundamental principle in optical physics and opens pathways to engineered topological structures of light beyond conventional symmetry constraints.

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I. INTRODUCTION

Topological quasiparticles such as skyrmions and merons are nontrivial vectorial fields characterized by a robust skyrmion number [1–5]. Originally proposed in nuclear physics [6], skyrmions have been widely investigated in modern magnetic materials, due to their potential in high-density magnetic information storage [7–9], and the emergent singular magnetism that leads to nontrivial transport effects such as topological Hall effect [10,11]. Merons are regarded as half skyrmion with a half-integer topological number. Due to their logarithmically divergent energy, merons and antimerons must come into pairs [12] to form stably as a lattice of periodic structure as meron lattices [13]. These structures can be interpreted as the interference pattern of several chiral spin density waves, manifested as a three-component field vector standing wave pattern [14]. Akin to these magnetic waves,

optical analogs of skyrmions have also been reported in various systems, including surface plasmon polaritons (SPPs) [15–18], photonic crystals [19,20], and the interference of structured light [21,22], with promising applications found in superresolution imaging [23], optical computing [24], and nanoscale metrology [25].

To date, although various optical platforms have been used to demonstrate topological lattices [18,26], none of these cases can violate a symmetry inheritance property of light. The symmetry inheritance from a light source—including all attributes such as amplitude, frequency, momentum, and polarization—is ubiquitously maintained during its free-space propagation, serving as a fundamental property throughout all linear optical domains and even general wave systems [27–30]. This property essentially governs the construction of topological lattices through interference or superposition from several waves [15,31–35]. For instance, it was demonstrated that only optical sources with fourfold or sixfold rotational symmetry are permitted to form skyrmion or meron lattices with identical symmetries [15,31], while topological quasicrystals from higher-rotational-symmetry systems also exhibit strict rotational symmetry inheritance [35–37].

Very recently, the concept of quasisymmetry has been introduced as an extension to the conventional group theory describing the degeneracy of energy bands, enabling the determination of energy splitting effects induced by symmetry-lowering perturbations in condensed matter physics [38–40].

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Unlike the strict crystalline symmetry, quasisymmetry is an approximated symmetry that commutes with the lower-order $k \cdot p$ Hamiltonian in the presence of spin-orbit interaction (SOI), leading to unexpected topology and new classifications of materials beyond the constraints of usual spatial symmetries. For instance, a quasisymmetry emerges when a mirror operation acts on a subset of an object, in contrast to exact symmetry that uniformly acts on the whole object [39]. Although the SOI of light, namely, the interplay between optical spin and orbital angular momentum, has been widely explored [41–43], a quasisymmetry group arising from the nontrivial interplay between symmetry and SOI has not yet been reported in any optical regime.

In this work, we demonstrate the very first quasisymmetry-protected optical meron lattices in free space due to strong SOI of light. The quasisymmetry arises from the Hermitian conjugation of the scattering amplitude of electric-dipole field in a circular polarization basis. Physically, this Hermiticity leads to a particular mirror operation of dipole's polarization upon a subset of C_3 polarized dipole system. In contrast to previous understandings of optical symmetry inheritance, our study shows that quasisymmetry groups—which obviously violate C_3 symmetry—can still generate perfect C_3 merons due to strong SOI of light. Moreover, the probability of polarization states that can generate merons rises abnormally as the SOI enhances, and it ultimately results in a robust polarization zone, within which all continually changed polarization states can lead to C_3 merons without symmetry constraints. We generalized this concept in higher-order rotational symmetries C_{2N+1} and general SOIs beyond dipole radiation, revealing the universal nature of quasisymmetry-protected topology in nonparaxial optical fields.

II. THE DEFINITION OF OPTICAL QUASISYMMETRY GROUP AND ITS IMPACT ON TOPOLOGICAL OPTICAL LATTICES

The main concepts of this work are presented in Fig. 1. We consider a set of $N = 3$ linear-polarized monochromatic dipoles $\mathbf{E}_i$ at the vertices of an equilateral triangle in the xy plane. The distance between a dipole and the origin is r , and the azimuthal angles of the dipoles are $\varphi_i = 2\pi(i-1)/N$, $i = \{1, \dots, N\}$. Each dipole features an in-plane polarization angle $\Phi_i = \varphi_i + \xi_i$, with $\xi_i \in [0, \pi]$ being the angle between a dipole's polarization and its azimuthal vectors $\mathbf{r}_i = r(\cos\varphi_i, \sin\varphi_i)$. This notation allows us to conveniently compare a dipole system to a C_3 symmetric system. For $\xi_i = \xi$, the three-dipole system possesses a threefold rotational symmetry about z axis, since $\hat{\mathcal{R}}_{2\pi/3}\Psi = \Psi$. Here, $\hat{\mathcal{R}}_{2\pi/3}$ is the threefold rotation operator, and $\Psi = \sum_{i=1}^3 \delta(\mathbf{r} - \mathbf{r}_i)\mathbf{E}_i$ denotes the electric field of the three-dipole system on the $z = 0$ plane, as shown in Fig. 1(a). As the dipoles propagate in free space, an ideal Stokes meron is generated at distant xy planes, where the Stokes parameters (S_1, S_2, S_3) are formulated by the superposition of the xy -plane electric field components [Fig. 1(c)]. Here, we define the ideal topological structures as C_N -symmetric periodic or quasiperiodic Stokes textures within the effective interference domain, in which the set of nonparaxial $\mathbf{k}$ vectors approximately retains the corresponding rotational symmetry relative to z axis, thereby

sustaining the long-range order of the interference patterns. This is a natural result of optical symmetry inheritance: Perfect topological textures with N -fold rotational symmetry are produced under interference sources with a global C_N symmetry [15,29,31,36,37]. Crucially, owing to the odd parity of the third Stokes parameter $S_3(x, y)$, nonzero merons can only be formed when N is odd (see Sec. S2 of the Supplemental Material [44] for details); this leaves $N = 3$ the only case to generate periodic meron lattices. Instead of studying this trivial case, we raise a question to seek the possibilities of generating perfect C_3 merons when ξ_i are arbitrarily chosen such that $\hat{\mathcal{R}}_{2\pi/3}\Psi \neq \Psi$.

To answer this question, we must consider the intrinsic SOI of the dipole field during its propagation in free space. For a dipole source $\mathbf{E}_2$ [the purple arrow in Fig. 1(b)], its field distribution in the momentum space is $\mathbf{E}(\mathbf{k}) \cong \mathbf{E}_2 - \mathbf{k}(\mathbf{k} \cdot \mathbf{E}_2)$, where $\mathbf{k}$ is the unit vector of the observation point defined by the spherical coordinates (θ, φ) . Here, θ and φ denote the polar and azimuthal angles of the radiated wave, respectively [Fig. 1(d)]. $\mathbf{E}(\mathbf{k})$ is manifested as the rotation of $\mathbf{E}_2$ on conical $\mathbf{k}$ -distribution waves with a fixed deflection angle θ , as shown in the left panel of Fig. 1(d). This form of spherical projection of the incident polarization $\mathbf{E}_2$ onto the $\mathbf{k}$ sphere, namely, $\mathbf{k}(\mathbf{k} \cdot \mathbf{E}_2)$, naturally sheds light on an analogy of $k \cdot p$ Hamiltonian associated with SOI [39]. To better quantify the SOI in dipole radiation, we rewrite $\mathbf{E}_2 - \mathbf{k}(\mathbf{k} \cdot \mathbf{E}_2)$ as $\hat{\mathcal{H}}(\mathbf{k})\mathbf{E}_2^C$. Here, $\mathbf{E}_2^C = (e^{-i\Phi_2}, e^{i\Phi_2})$ is $\mathbf{E}_2$ formulated in the circular polarization basis, and $\hat{\mathcal{H}}(\mathbf{k})$ is an SOI operator reduced in xy dimension (see Sec. S1 of the Supplemental Material [44] for details):

$$\hat{\mathcal{H}}(\mathbf{k}) = \begin{pmatrix} A & -Be^{-2i\varphi} \\ -Be^{2i\varphi} & A \end{pmatrix}. \quad (1)$$

In Eq. (1), $A = 1 + \cos^2\theta$ and $B = \sin^2\theta$ represent the spin-maintained and spin-flipped components, respectively [45]. Applying Eq. (1) to a linearly polarized dipole source with a polarization angle $\Phi = \varphi + \xi$, the resulting radiation field in the circular basis is

$$\mathbf{E}(\mathbf{k}) \propto \begin{pmatrix} Ae^{-i(\varphi+\xi)} - Be^{-i(\varphi-\xi)} \\ Ae^{+i(\varphi+\xi)} - Be^{+i(\varphi-\xi)} \end{pmatrix} = \begin{pmatrix} \mathcal{S}e^{-i\varphi} \\ \mathcal{S}^*e^{+i\varphi} \end{pmatrix}. \quad (2)$$

Here, we have defined $\mathcal{S}(\xi) = Ae^{-i\xi} - Be^{+i\xi}$. Therefore, the contribution of a dipole's radiation field is separated to a location-originated part “ $e^{\pm i\varphi}$ ” and a polarization-originated part “ $\mathcal{S}$ ”, where $\mathcal{S}$ is mediated by the source's polarization “ ξ ” and the strength of SOI. We have noticed that the $\mathcal{S}$ parameter has a conjugate property: $\mathcal{S}^*(\xi) = -\mathcal{S}(\pi - \xi)$, which bridges two polarization states: $\Phi = \varphi + \xi$ and $\Phi = \varphi + \pi - \xi$. Physically, converting a dipole's polarization from $\varphi + \xi$ to $\varphi + \pi - \xi$ is achieved by introducing an in-plane mirror operator $\hat{\mathcal{M}} = -\hat{V}^\dagger \hat{\mathcal{R}}_\varphi \hat{\sigma}_z \hat{\mathcal{R}}_\varphi \hat{V}$ that acts on the polarization of dipole $\mathbf{E}_2$ along an axis perpendicular to $\mathbf{k}_\varphi$. Here, $\hat{\sigma}_z$ is the third Pauli matrix, $\hat{V}$ denotes the unitary transformation between linear and circular bases (see Sec. S1 of the Supplemental Material [44] for details), and the mirror axis is the dashed line in the right panel of Fig. 1(d). Accordingly, the radiation field $\mathbf{E}(\mathbf{k})$ along $\mathbf{k}_\varphi$ is also mirror flipped [Fig. 1(d), right panel], namely, $\hat{\mathcal{M}}\hat{\mathcal{H}}(\mathbf{k}_\varphi)\mathbf{E}_2 = \hat{\mathcal{H}}(\mathbf{k}_\varphi)\hat{\mathcal{M}}\mathbf{E}_2$. This leads to

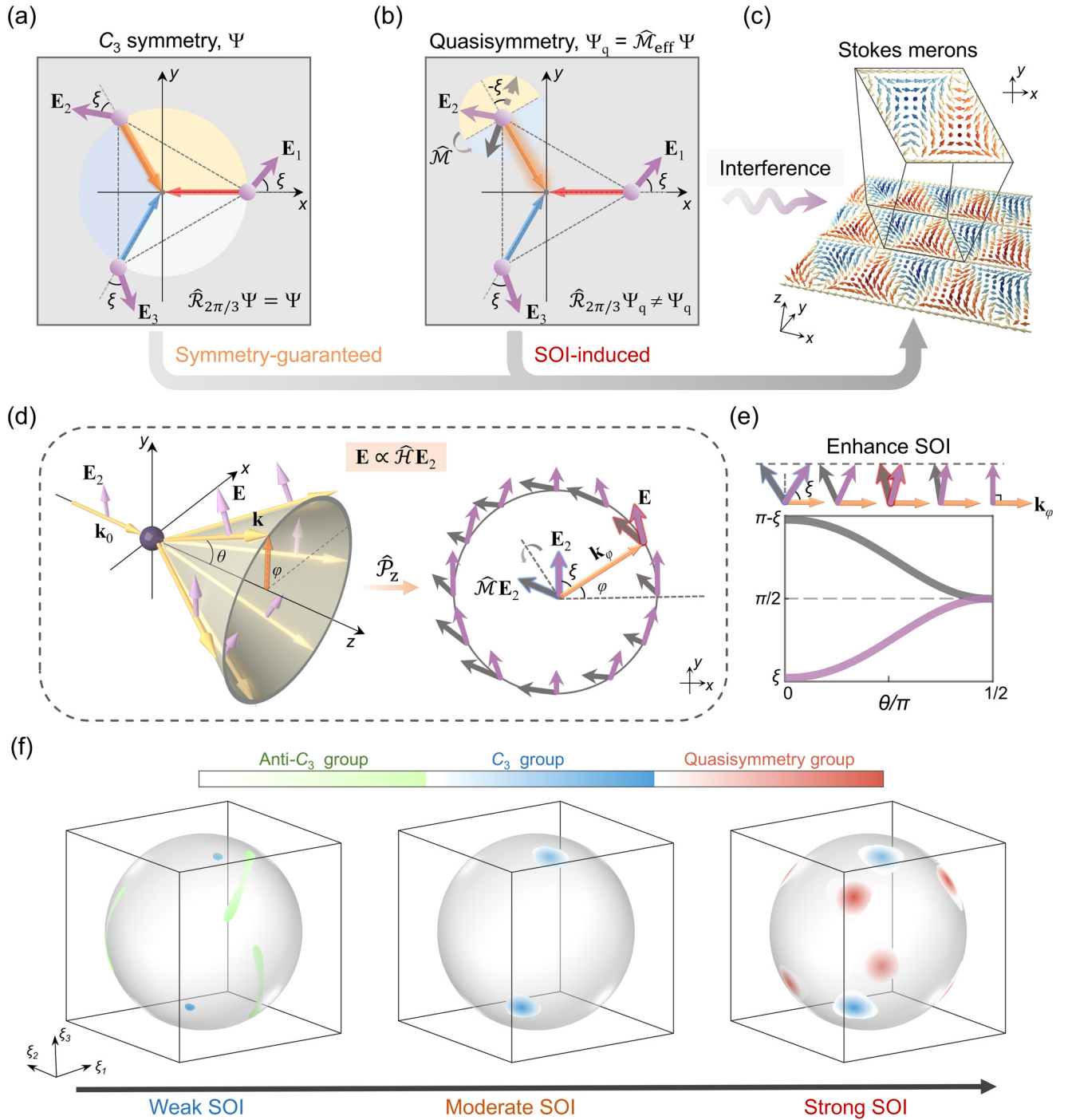


FIG. 1. Concept of the optical quasisymmetry group. (a) A C_3 -symmetric dipole system Ψ in the $z=0$ plane. (b) Definition of a quasisymmetry C_3 system. Ψ_q is formed by applying a partial mirror operator $\hat{M}_{\text{eff}}$ to Ψ in panel (a), which implements a mirror operator $\hat{M}$ on a dipole along the axes perpendicular to $\mathbf{r}$ (the dashed border). (c) Stokes meron lattices (inset shows a unit cell) generated by the interference of the three dipoles at a distant observation plane z . This topological lattice is either symmetry guaranteed by the C_3 system or SOI induced by the q - C_3 system. The three-dimensional (3D) arrows represent the orientation distribution of the Stokes vectors $\mathbf{S}(x, y) = [S_1, S_2, S_3]$ and the color indicates S_3 “up” (red) or “down” (blue). (d) Left panel: Radiation field from a dipole $\mathbf{E}_2$ produces a conical $\mathbf{k}$ distribution (light yellow surface and arrows) and associated adiabatically rotating field $\mathbf{E}(\mathbf{k})$ (purple arrows). Right panel: Projected in-plane field distribution $\mathbf{E}(\mathbf{k})$ at a fixed deflection angle θ originating from $\mathbf{E}_2$ and its mirror image $\hat{M}\mathbf{E}_2$. $\hat{P}_z = \text{diag}(1, 1, 0)$. (e) Evolution of the relative angle between the projected mirror polarization pair and the $\mathbf{k}_\phi$ (orange arrows) direction as θ increases from 0 to $\pi/2$. Initial polarization angle is $\xi \in [0, \pi]$. (f) Typical polarization symmetry groups for ideal merons in the polarization cube (ξ_1, ξ_2, ξ_3) with different SOIs. Distribution of states on the spherical surface corresponding to distinct symmetry groups: anti- C_3 (green), C_3 (blue), and q - C_3 (red). Color gradient from dark to light indicates the deviation degree from exact symmetry operations, quantified by Euclidean distance in parameter space.

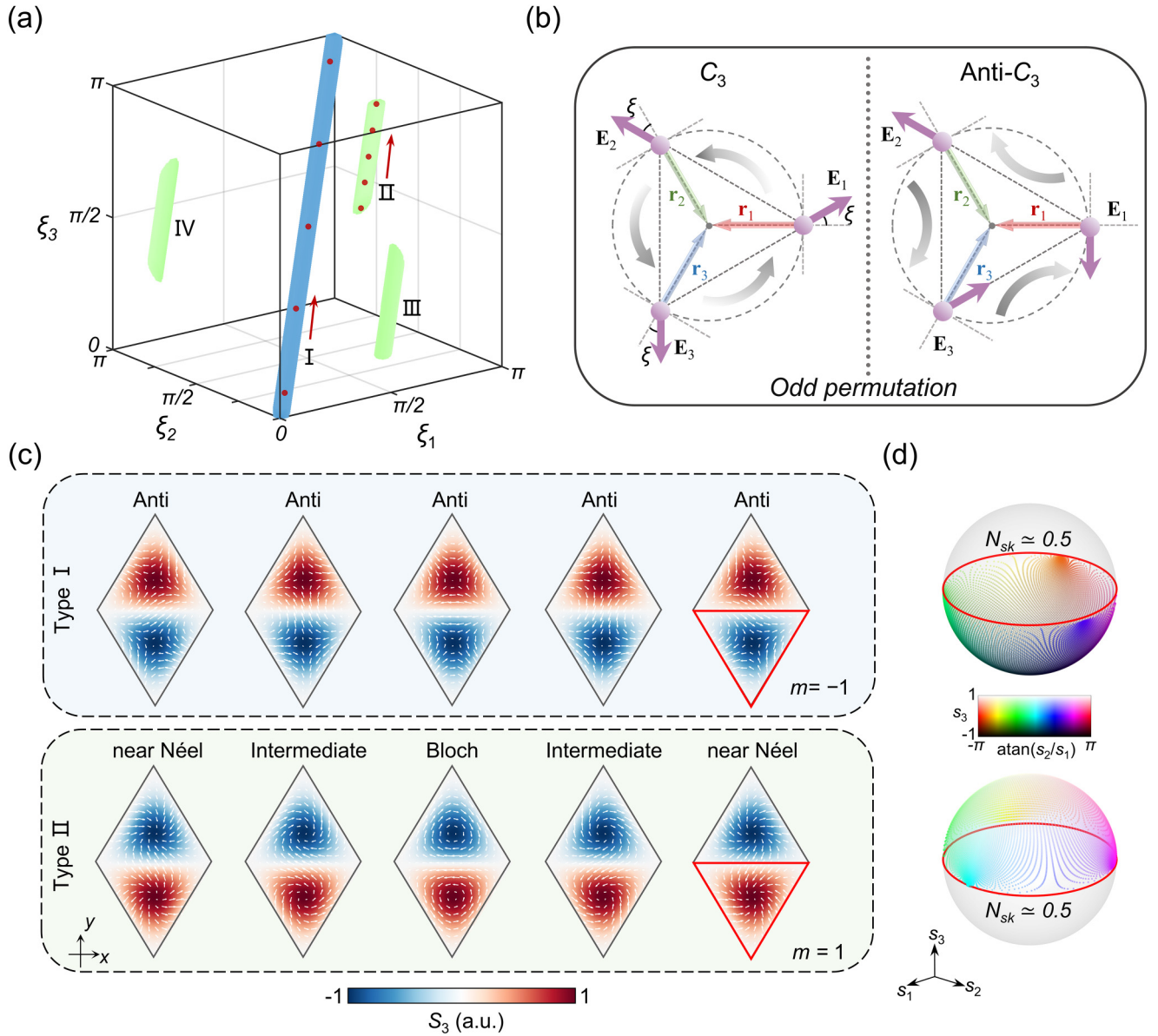


FIG. 2. Polarization symmetric configurations for ideal Stokes meron lattices in the absence of SOI. (a) Polarization states to form C_3 merons in the polarization cube in the paraxial regime ($\theta \rightarrow 0$). Type I (blue solid line) corresponds to C_3 polarization group, while Types II–IV (green solid lines) represent anti- C_3 groups. (b) Schematic of a C_3 (left panel) and anti- C_3 (right panel) polarization state. For the C_3 state, $\hat{R}_{2\pi/3}\mathbf{r}_i = \mathbf{r}_{i+1}$ and $\hat{R}_{2\pi/3}\mathbf{E}_i = \mathbf{E}_{i+1}$ (left panel, curved arrows). For the anti- C_3 state, $\hat{R}_{2\pi/3}\mathbf{r}_i = \mathbf{r}_{i+1}$ and $\hat{R}_{2\pi/3}^{-1}\mathbf{E}_i = \mathbf{E}_{i+1}$ (right panel, curved arrows), $i = 1, 2, 3$ (cyclic order). (c) Stokes vectors $\mathbf{S}(x, y)$ over triangular unit cells corresponding to the selected polarization states (red points) in panel (a). These meron pairs exhibit different vorticity m and helicity γ . The white arrows represent the in-plane vectors (S_1, S_2) and color indicates the S_3 component. (d) The normalized Stokes vectors in the triangular unit cells [the last panels in panel (c)] represented by brightness (s_3) and colormap ($\text{atan}(s_2/s_1)$) are mapped onto the Poincaré spheres with topological number $N_{sk} = \pm 1/2$. The meron boundaries correspond to $S_3 = 0$ (red circles).

a commutation relation between $\hat{\mathcal{M}}$ and $\hat{\mathcal{H}}(\mathbf{k}_\varphi)$:

$$[\hat{\mathcal{M}}, \hat{\mathcal{H}}(\mathbf{k}_\varphi)] = \hat{\mathcal{M}}\hat{\mathcal{H}}(\mathbf{k}_\varphi) - \hat{\mathcal{H}}(\mathbf{k}_\varphi)\hat{\mathcal{M}} = 0. \quad (3)$$

Equation (3) is the main result of this work, which brings forth the definition of a mirror operator $\hat{\mathcal{M}}$ associated with the direction $\mathbf{k}_\varphi$. This commutation relation is analogical to that found in a semimetal, in which a partial mirror operation also commutes with the lower-order $k \cdot p$ Hamiltonian in the presence of SOI [39]. Likewise, Eq. (3) leads to new classifications

of symmetry groups beyond the constraints of usual spatial symmetries. Notably, by observing $\mathbf{E}(\mathbf{k})$ from increased θ , the in-plane components of radiation fields generated by $\mathbf{E}_2$ and $\hat{\mathcal{M}}\mathbf{E}_2$ are gradually aligned with their mirror axis, and they are eventually overlapped under the grazing angle limit ($\theta \rightarrow \pi/2$) with the maximized SOI, as shown in Fig. 1(e). This means that, as SOI intensifies, the in-plane fields from arbitrary polarized dipole sources and their mirror counterparts become indistinguishable, and the angular location of dipole φ solely determines the polarization, and $\mathbf{E}(\mathbf{k}) \perp \mathbf{k}_\varphi$. Note

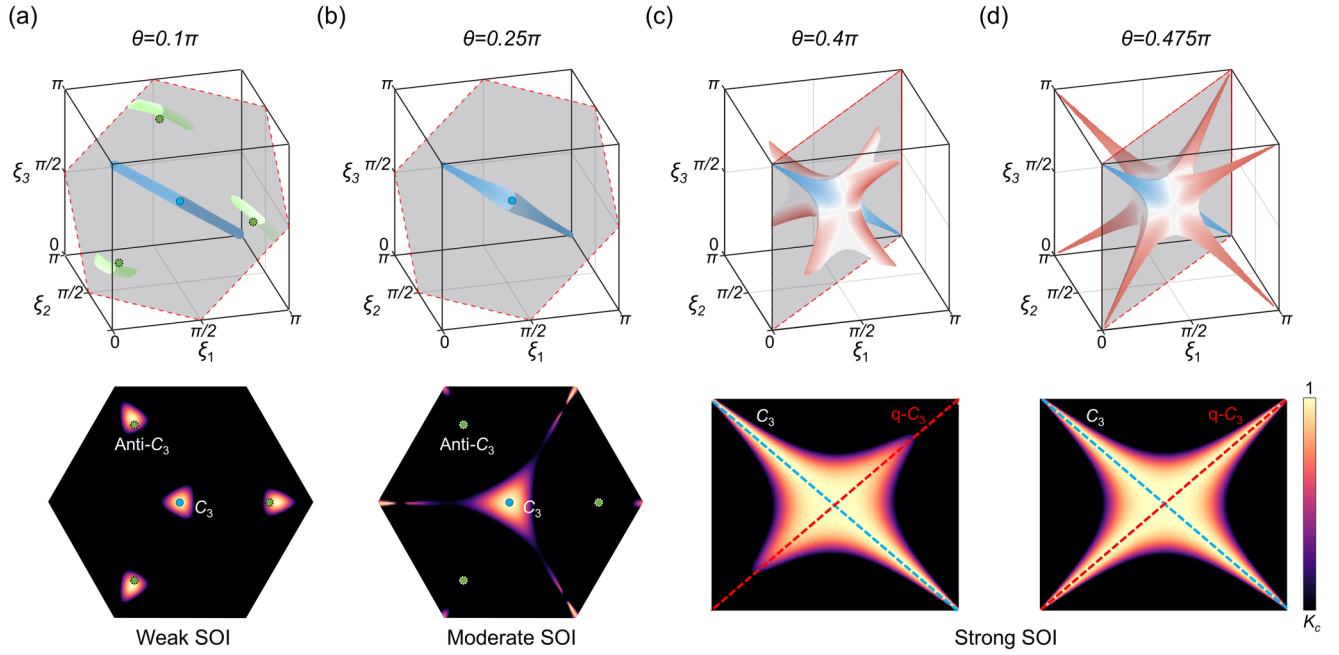


FIG. 3. Evolution of polarization symmetry groups with varying SOIs. (a)–(d) Polarization states in the cube to form perfect C_3 merons across distinct SOI regimes: weak ($\theta = 0.1\pi$), moderate ($\theta = 0.25\pi$), and strong ($\theta = 0.4\pi$ and 0.475π). Polarization groups are represented by various colors using the same scheme as Fig. 1(f). The hexagonal and rectangular cut planes are plotted for better symmetry group characterization, as solutions shown in the bottom panels of this figure. The threshold K_c is set to 0.99.

that the direction of $\mathbf{k}_\varphi$ is fixed for a dipole when N -dipole composes a system with $\varphi_i = 2\pi(i-1)/N$. In this case, by incorporating the mirror operator $\hat{M}$ into a C_N -dipole system and partially acting on any subsets of the dipoles—defined as an $\hat{M}_{\text{eff}}$ operator [Fig. 1(b)]—we find that the corresponding $\Psi_q = \hat{M}_{\text{eff}}\Psi$ belongs to optical *quasisymmetry* C_N groups ($q-C_N$) that are able to produce perfect topological lattices without polarization constraints.

In Fig. 1(f), we summarized all symmetry groups for generating perfect C_3 merons under varying SOIs mediated by the radiation angle θ . The cube is a polarization space determined by the relative polarization angle ξ_i , with $\xi_i \in [0, \pi]$, denoting all possible linear polarization combinations of a three-dipole system depicted in Fig. 1(a). (The sphere is presented for better visualization.) For weak SOI, the generation of merons is symmetry guaranteed, with four symmetry groups: one is C_3 and the others are anti- C_3 (will be discussed in the next section). As the SOI increases, it destroys the anti- C_3 group, but C_3 group is survived. For strong SOI, the $q-C_3$ arises as three additional branches to dominate the production of merons. Notably, the C_3 and $q-C_3$ will merge as a bulky polarization zone, which contains continually changed polarization states to form perfect C_3 merons, indicating a strong polarization robustness to induce topological lattices of light by strong SOI.

III. POLARIZATION SYMMETRY GROUPS FOR C_3 MERONS WITHOUT SOI

We first examine the polarization configurations that produce merons under paraxial approximation in the absence of SOI ($\theta \rightarrow 0$). The cube in Fig. 2(a) contains all linear

polarization states labeled as (ξ_1, ξ_2, ξ_3) , wherein each colored site indicates a polarization state leading to a perfect lattice. These sites are identified by comparing the formed Stokes lattice from $\hat{\mathcal{H}}(\mathbf{k})\Psi$ with a standard C_3 meron, with the normalized overlapped factor $\mathcal{K}(\xi_1, \xi_2, \xi_3) \rightarrow 1$ (see Sec. S4 of the Supplemental Material [44] for details). Figure 2(a) shows four groups in the polarization cube, wherein the diagonal group is the known C_3 group (Type I), as one polarization state illustrated in the left panel of Fig. 2(b). The remaining three groups are labeled as Types II–IV, forming three lines parallel to that of Type I. We find that Types II–IV belong to the same symmetry group merely with a different initial polarization difference. We term to them as an “anti- C_3 ” group, because for each state, the polarization and location of dipoles follow opposite rotation operations: $\hat{\mathcal{R}}_{2\pi/3}\mathbf{r}_i = \mathbf{r}_{i+1}$ and $\hat{\mathcal{R}}_{2\pi/3}^{-1}\mathbf{E}_i = \mathbf{E}_{i+1}$ for $i = 1, 2, 3$. This additional group exhibits a special “anti- C_3 ” vortex configuration characterized by reversed cyclic sequence relative to $\mathbf{r}_i$, distinct from the sequential order of C_3 group (see Table S1 of the Supplemental Material [44] for details).

The normalized Stokes vectors $\mathbf{s}(x, y) = [S_1, S_2, S_3]/S_0$ corresponding to Type I and Type II are shown in Fig. 2(c), exhibiting triangular meron textures quantified by the topological number $N_{sk} = (1/4\pi) \iint_A \mathbf{s} \cdot (\partial_x \mathbf{s} \times \partial_y \mathbf{s}) dx dy \simeq \pm 0.5$ (A denotes the region of a unit cell of lattices). As demonstrated in Fig. 2(d), the merons are mapped onto the upper or lower hemisphere of the Poincaré sphere, where the boundary ($S_3 = 0$) corresponds to the equator (red circles), and the centers of triangular lattices with vertically directed $\mathbf{s} = \pm \mathbf{z}$ correspond to the circular points. It is noteworthy that the C_3 group of polarization states exclusively produces anti-type

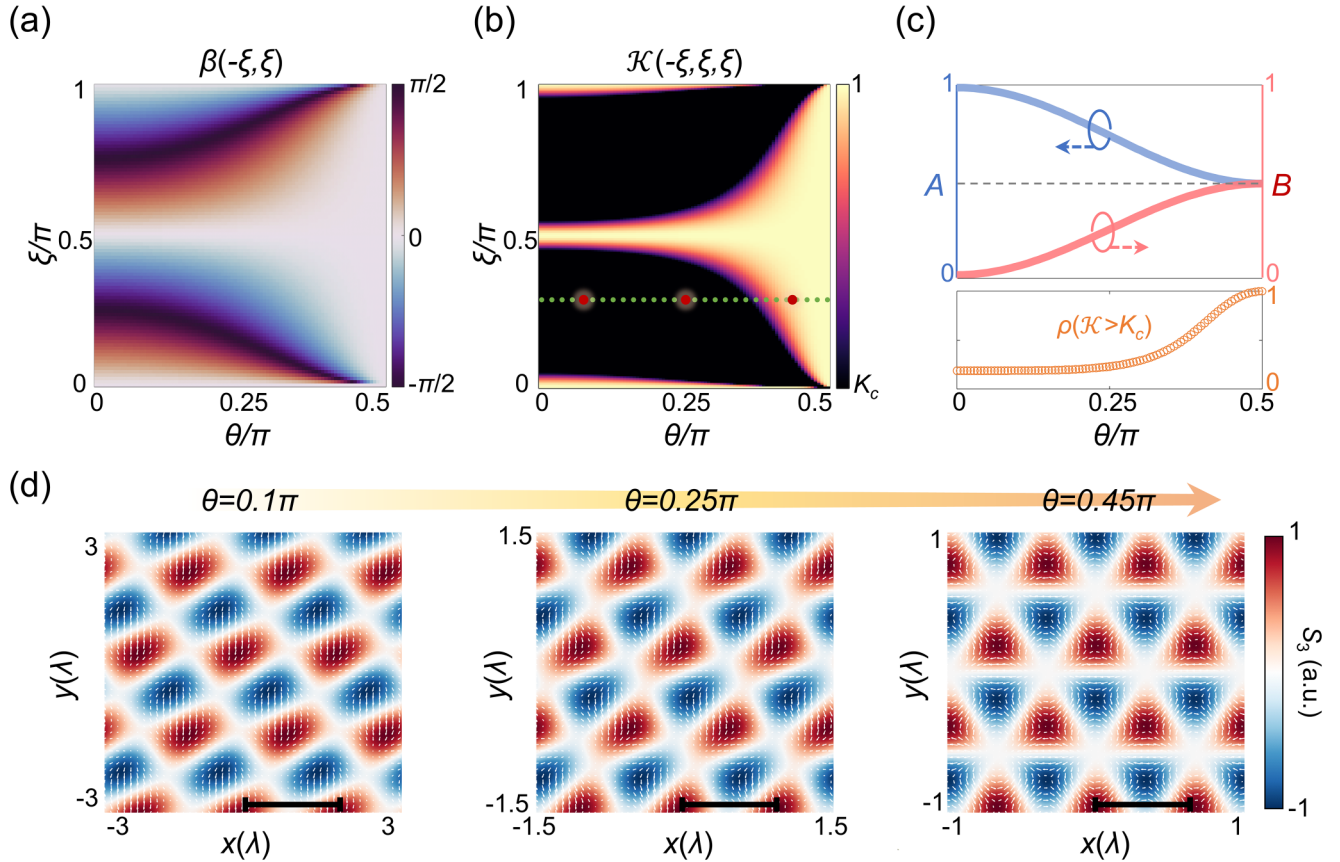


FIG. 4. Details about the proliferation of q- C_3 symmetry group. Calculated β [panel (a)] and $\mathcal{K}$ [panel (b)] factors with different diffraction angles θ . (c) Calculated SOI coefficients A (spin-maintained part, blue curve) and B (spin-flipped part, red curve) as functions of θ , along with the evolution of the normalized density of states $\rho(\mathcal{K} > K_c)$. (d) Stokes vectors constructed from a q- C_3 dipole system $(-\xi, \xi, \xi)$ crossing different θ as marked in panel (b). Here, $K_c = 0.99$ and $\xi = \pi/3$ [green dashed line in panel (b)]. Scale bars: $2\lambda/(3\sin\theta)$.

merons with a vorticity $m = -1$, while the other anti- C_3 group enables the formation of various topological textures with $m = +1$ spanning Néel type, intermediate type (twisted textures), and the Bloch type [18], due to the helicity γ being continuously modulated by ξ varying from $-\pi$ to π (see Sec. S3 of the Supplemental Material [44] for details).

IV. SOI-DRIVEN DYNAMICS OF THE POLARIZATION SYMMETRY GROUPS

A clear evolution of polarization configurations for C_3 merons is observed in the (3D) parameter space with varying SOI. Figures 3(a)–3(d) present several typical states across different SOI strengths, as quantified by the radiation angle θ . It can be seen that as θ increases from 0 to 0.1π , corresponding to a weak SOI, the anti- C_3 groups in the paraxial regime are distorted, deviating from their original spatial regions [Fig. 3(a)]. This means that the anti- C_3 group is fragile from the SOI due to its lack of global C_3 symmetry (see Sec. S8 of the Supplemental Material [44] for details). Meanwhile, the diagonal line corresponding to the C_3 group remains unchanged, as it was guaranteed by the global symmetry. In the moderate SOI regime ($\theta = 0.25\pi$), the anti- C_3 groups are completely eliminated, while the C_3 region expands slightly around its center [Fig. 3(b)]. When the SOI is further enhanced [Figs. 3(c) and 3(d)], the allowed

number of polarization states increases dramatically, and it eventually forms a bulky starlike region, which is dominated by four diagonal branches, as shown in Fig. 3(d). In the bottom panel of Fig. 3, we selected several cut planes to explicitly present the dynamics of different symmetric groups with varying SOI. These cut planes clearly show the preservation of C_3 , the elimination of anti- C_3 , and the proliferation of q- C_3 groups with enhanced SOI. Particularly, for strong SOI, the possibility to observe meron lattices is significantly increased due to q- C_3 groups. The continuous evolution of polarization combinations for perfect lattices is shown in the movies of the Supplemental Material [44].

V. THE PROLIFERATION OF QUASISYMMETRY GROUP UNDER STRONG SOI

We focus on the evolution of the q- C_3 group under different SOIs. Based on the formalism of Eq. (2), we give the expression of the $\mathcal{K}$ factor associated with a q- C_3 polarization state $(-\xi, \xi, \xi)$ (see Sec. S4 of the Supplemental Material [44] for details):

$$\mathcal{K}(-\xi, \xi, \xi) = \frac{\sum_{i \neq j}^3 |\sin(\varphi_{ji} + \beta_{ij})|}{\sqrt{3} \sqrt{\sum_{i \neq j}^3 |\sin(\varphi_{ji} + \beta_{ij})|^2}}.$$

Here, $\varphi_{ji} = 2\pi(j-i)/3$ and $\beta_{ij} = \arg(S_i S_j^*)$ characterizes the phase difference of the radiation fields between any two dipoles. For the two dipoles with the same polarization ξ , $\beta_{ij} = \beta(\xi, \xi) = \arg(|S(\xi)|^2) = 0$. Therefore, a C_3 dipole system naturally has $\mathcal{K}(\xi, \xi, \xi) = 1$ to produce perfect merons. In the q- C_3 system, $\beta_{ij} = \beta(-\xi, \xi; \theta)$, which possesses an odd parity with respect to $\xi = \pi/2$, as shown in the (θ, ξ)

space [Fig. 4(a)]. For small θ , perfect merons are only generated around $\xi = \pi/2$. In this case, q- C_3 and C_3 are degenerate and indistinguishable. As θ increases, $\beta(-\xi, \xi; \theta)$ rapidly converges to 0 for any $\xi \in [0, \pi)$, consequently driving the $\mathcal{K}$ factor to approach 1 [Fig. 4(b)]. As shown in Fig. 4(d), we selected several Stokes vectors $\mathbf{S}(x, y)$ from the same $(-\xi, \xi, \xi)$ dipole system but with different θ . These lattices

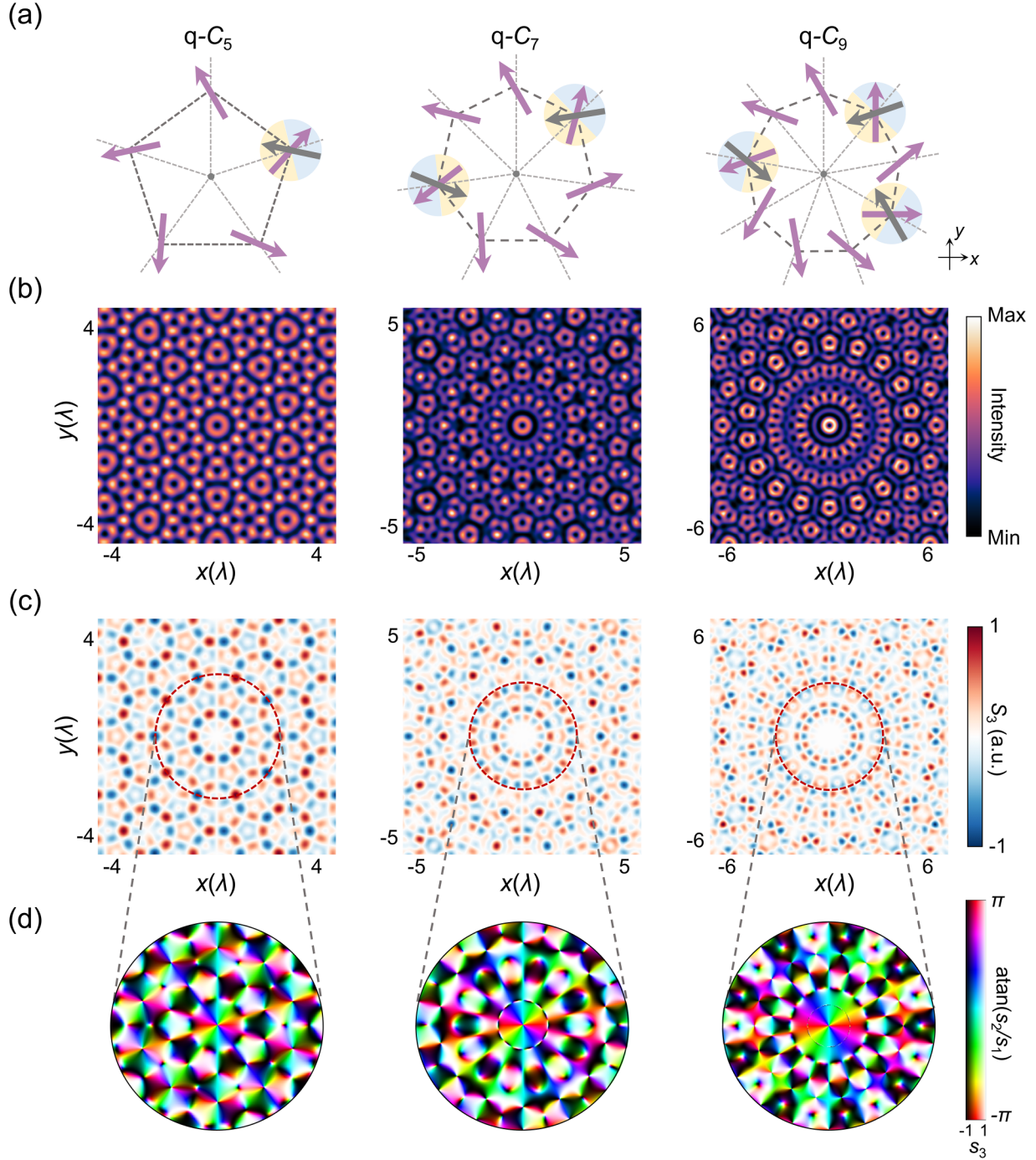


FIG. 5. (a) Schematic of N -fold rotational dipole sources with $N = 5, 7$, and 9 . Random partial mirror operations (gray arrows) are embedded into these C_N sources (purple arrows) where the mirror axes are strictly perpendicular to the corresponding $\mathbf{r}_i$ of that dipole. Initial angles of these dipoles are set as $\xi = \pi/6$. (b) Quasicrystallike C_N light intensities and (c) S_3 distributions on the observation plane with a strong SOI at $\theta = 0.45\pi$. (d) Selected Stokes vector textures marked by the red circle in panel (c) exhibiting mixed topological quasiparticles including vortices, skyrmions, and meron pairs.

present a progression of merons from broken symmetry to “perfect” C_3 symmetry as the SOI enhances. Besides, we calculated the density of polarization states $\rho(\mathcal{K} > K_c)$ for generating merons. Here, $\rho(\mathcal{K} > K_c)$ stands for the percentage of polarization states among all $q-C_3$ configurations to generate perfect merons with a criterion $\mathcal{K} > K_c$. As shown in Fig. 4(c), $\rho(\mathcal{K} > K_c)$ grows rapidly as θ enters the strong SOI region, with the peak occurring at $\theta = \pi/2$, which corresponds to a critical SOI condition: $A = B$. Mathematically, this critical SOI condition is guaranteed by the Hermitian conjugation of dipole’s radiation amplitude $\mathcal{S}(\xi) = \mathcal{S}^*(-\xi)$, consistent with the commutation relation [Eq. (3)]. The fact that the critical SOI condition occurs at the grazing limit ($\theta \rightarrow \pi/2$) is also originated from the radiation nature of electric dipole. For general multipole scattering systems such as electric quadrupoles, magnetic dipoles, etc., A and B can even be complex values, and the critical SOI condition will shift to $\theta < \pi/2$. In general, a critical SOI condition to reach maximal $\rho(\mathcal{K} > K_c)$ from the quasisymmetry groups is

$$\begin{cases} |A| = |B| \neq 0, \\ \phi_{BA} = \arg(B) - \arg(A) = m\pi, \quad m \in \mathbb{Z}, \end{cases} \quad \exists \theta \in (0, \pi/2].$$

This condition has been verified with customized Mie scattering and focused beam systems to obtain optimized θ and different $\hat{\mathcal{H}}(\mathbf{k})$ (see Sec. S6 of the Supplemental Material [44] for details).

VI. DISCUSSION

In conclusion, we have discovered optical quasisymmetry groups in a C_N system composed of N optical dipole sources. This quasisymmetry is defined by one or a few mirror operations on a subset of the system, which originated from the commutation relation of our Eq. (3). Moreover, we further extend this concept to arbitrary elliptically polarized dipole systems (see Sec. S5 of the Supplemental Material [44] for details). Due to the strong SOI of light in free space, quasisymmetry groups can produce perfect meron lattices, deepening our understanding of previously symmetry-guaranteed conditions to generate topological lattices of light. Owing to the ubiquitous nature of SOI, quasisymmetry may be discovered in diverse nontrivial vector fields, including spin angular momentum [16], energy flux [46,47], and pseudospin [19] in real or momentum space, even spatiotemporal fields [23]. Moreover, analogous phenomena about approximate symmetry may also be revealed in other

wave systems including water wave [32,33], acoustic wave [48], etc., where effective SOI can be constructed similarly. Our work leads to extended optical symmetry groups for the observation and manipulation of topological structures of light, possibly useful for applications in exotic light-matter interactions [49] such as constructing topological optical lattices to manipulate atoms [50] and nanoparticles [51] or generating higher-order harmonics [52,53].

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DATA AVAILABILITY

The data that support the findings of this article are openly available [54].

APPENDIX: QUASICRYSTALLIKE TOPOLOGICAL TEXTURES IN QUASISYMMETRY POLARIZATION GROUPS UNDER STRONG SOI

Since an odd number N is required to form the Stokes nontrivial textures, this quasisymmetry property does not exist in other crystalline symmetric groups for even N (see Sec. S2 of the Supplemental Material [44] for details). Therefore, we extended the dipole systems to $N = 5, 7$, and 9 , forming distinct quasicrystalline Stokes textures. Akin to the operation in C_3 , we introduced similar operators $\hat{\mathcal{M}}_{\text{eff}}$ to these highly rotational symmetry C_N dipole sources to construct $q-C_N$ groups and presented C_N Stokes textures under a strong SOI, as shown in Fig. 5(a). Ideal quasicrystalline intensity and spin textures strictly possessing N -fold rotational symmetry are generated due to the introduction of quasisymmetry mirror operation, demonstrating the generality of our quasisymmetry mechanism [Figs. 5(b) and 5(c)]. Particularly, the formation of quasiperiodic textures reveals unprecedented topological structures of light [Fig. 5(d)], which exist not only in SPP systems [35–37] but also in nonparaxial light in free space.

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