

Covariant correlation-disturbance relation and its experimental realization with spin-1/2 particles

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We formulate a precise trade-off relation between correlation and disturbance for sequential n -outcome quantum measurements in Hilbert spaces of arbitrary dimension. This relation highlights key symmetry properties useful for robust estimation and characterization of the measurement parameters against unitary noise, or in scenarios where shared reference frames are unavailable. In addition, we report on the experimental implementation of the proposal for the qubit case, more precisely in a neutron optical experiment, which is particularly valuable for calibrating and optimizing measurement devices, as confirmed by the theoretical results. Finally, we exploit the optimal trade-off relation for direct estimation of the characteristic noise of single-photon detectors, dark counts, and the finite detection efficiency.

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Introduction. Incompatibility between measurements is a fundamental aspect of quantum mechanics, leading to distinctive features such as nontrivial uncertainty [1,2] and complementarity [3] relations. At its core, quantum violations of various classical models—commonly referred to as hidden variable models [4–6]—necessitate this measurement incompatibility, which can be verified solely through observed statistics without relying on specific assumptions about the devices [7]. Identifying scenarios that exhibit the characteristic features of measurement incompatibility and its consequences, such as disturbance, is now a central objective for both fundamental research and practical applications, as it has been recognized as a key resource underlying many quantum information tasks that outperform classical methods [8–12]. Understanding and controlling measurement disturbance has far-reaching implications for developing more precise and reliable quantum technologies [13–15].

One of the most fundamental approaches to quantum complementarity and uncertainty is investigated via a temporal sequence of measurements performed on an initially prepared single system. Many interesting quantum predictions, such as quantum contextuality and the violation of macroscopic realism, have been demonstrated in this scenario [16–20]. The interplay between information gain and disturbance has been formulated in various trade-off relation inequalities [21]. The first information-theoretic—or entropic—uncertainty relation was formulated by Hirschman [22] in 1957 for the position and momentum observables, and later improved in

Refs. [23,24]. The extension to nondegenerate observables on a finite-dimensional Hilbert space was then proposed by Deutsch in 1983 [25] and later improved by Maassen and Uffink [26], yielding the well-known entropic uncertainty relation $H(A) + H(B) \geq -2 \log_2 c$, where H denotes the Shannon entropy and c is the maximal overlap between the eigenvectors $|a_i\rangle$ and $|b_j\rangle$ of the observables A and B . Entropic uncertainty has useful applications in entanglement witnessing [27], complementarity [28], and quantum information theory [29]. More general procedures aiming to quantify error and disturbance in successive (or simultaneous) measurements are Ozawa’s operator-based measures between target observables and measurements [30,31] or distances of the associated probability distributions [32]. Interest has risen in alternative information-theoretic measures, introduced recently by Buscemi [33], and in several subsequent alternative approaches [34–38].

This work proposes a precise trade-off relation between correlation and disturbance within a straightforward, Ramsey-like successive measurement setup. Our scheme effectively captures the inherent incompatibility between two general measurements through a covariant trade-off complementarity inequality, where the characteristic measurement values remain invariant under coordinate choices or unitary transformations of the basis. In contrast to other approaches [2,33], our trade-off relation relies on definitions that are both functionally simple and experimentally practical. This provides a clear and efficient method for robust experimental estimation of measurement devices, enabling tasks such as self-calibration and assessing quantum measurement performance. Additionally, our framework supports various communication protocols, including quantum random access codes and quantum key distribution, making it suitable for (semi) device-independent quantum communication scenarios, as explored in numerous studies. [15,39–46].

Setting. Our proposed scheme consists of three parts: a state preparation ρ followed by two successive observable

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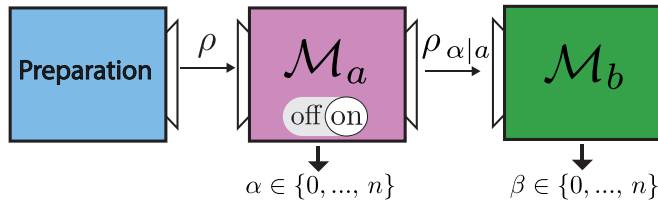


FIG. 1. Ramsey-like interferometric sequence. Schematics of the setup involving three parts: state preparation, successive n -outcome measurements $\mathcal{M}_a$ (probe/reference), and measurement $\mathcal{M}_b$ (target). For measuring the measurement disturbance, $\mathcal{M}_a$ is turned on (performed) and off (unperformed).

measurements, denoted by $\mathcal{M}_a = \sum_{\alpha} \alpha E_{\alpha|a}$ and $\mathcal{M}_b = \sum_{\beta} \beta E_{\beta|b}$, respectively. Formally, the general measurements are described by a set of POVMs, where $E_{\alpha|a} \geq 0$ and $\sum_{\alpha} E_{\alpha|a} = \mathbb{1}$ (see Fig. 1). As is clear from the figure, the measurement device $\mathcal{M}_a$ performs a general measurement operation that not only produces outcomes (classical output) but also realizes the corresponding postmeasurement state, according to the transformation rule, described by $\rho \mapsto \mathcal{I}_{\alpha|a}(\rho) = K_{\alpha|a} \rho K_{\alpha|a}^{\dagger}$ with outcome probability $p_{\alpha|a} = \text{tr}[\mathcal{I}_{\alpha|a}(\rho)]$ and $E_{\alpha|a} = K_{\alpha|a}^{\dagger} K_{\alpha|a}$. Summing over the Kraus operators, i.e., $\mathcal{I}_a(\rho) = \sum_{\alpha} \mathcal{I}_{\alpha|a}(\rho)$, acts as a quantum channel corresponding to the case when the measurement is performed but the outcome is ignored or unregistered. This situation, however, affects the outcome statistics of the subsequent measurement. Therefore, quantum instruments facilitate the analysis of measurement disturbance and incompatibility in sequential measurement setups. The important fact is that the Kraus operators ($K_{\alpha|a}$), generally given by $K_{\alpha|a} = U_{\alpha|a} E_{\alpha|a}^{\frac{1}{2}}$, are not unique. We are mainly interested in a particular choice of the instrument associated with the observable measurement effect only and not the unitary afterward, that is, $K_{\alpha|a} = E_{\alpha|a}^{\frac{1}{2}}$, the so-called Lüder instrument, which merely captures the intrinsic effect of the observable's POVM in the state update rule. Therefore, this special case of instrument is more suitable to study the incompatibility and joint measurability of the observables in sequential implementations [47].

Correlation-disturbance trade-off high-dimensional case. First, we want to emphasize that the following derivations are general and apply to n -outcome measurements in a Hilbert space of arbitrary dimension. Central to our formulation/construction is the Born rule for evaluating the joint probabilities of the two measurement outcomes:

$$p(\alpha, \beta) = \text{tr}[\mathcal{I}_{\alpha|a}(\rho) E_{\beta|b}], \quad (1)$$

which encapsulates the essential information required to analyze the interplay between the two measurements. First, the above form fulfills the so-called arrow of time, i.e., $p(\alpha|a) = \sum_{\beta} p(\alpha, \beta|a, b) = \sum_{\beta} p(\alpha, \beta|a, b')$ for all $b \neq b'$, namely, the future measurement will not affect the past measurement.

More characteristically, we have

$$\tilde{p}(\beta|b) = \text{tr}[\rho \mathcal{I}_a^*(E_{\beta|b})] = \sum_{\alpha} p(\alpha, \beta), \quad (2)$$

which is equivalent to the scenario where the $\mathcal{M}_a$ measurement is performed (or turned on), but its outcome is not

recorded. Note that this is different from the case in which measurement $\mathcal{M}_a$ is unperformed (turned off), $p(b|y) = \text{tr}[\rho E_{b|y}]$. This means that the difference in the probabilities of obtaining outcome β when it is preceded by the (unregistered) $\mathcal{M}_a$ measurement, compared to when $\mathcal{M}_b$ is measured alone, reflects the measurement disturbance [8, 48–50]. Therefore, a standard distance measure quantifying the disturbance may be given by

$$\mathcal{D} = \sqrt{\frac{n}{n-1}} \|\tilde{p}(\beta|b) - \tilde{p}(\beta|b)\|_2, \quad (3)$$

where n is the number of different outcomes. Note that the value of $\mathcal{D}$ depends on the initial state. The maximum value corresponds to the spectral norm, $\mathcal{D} \leq \sup_{\rho} |\mathcal{D}_{\rho}| \leq 1$, realized for the optimal state maximizing the disturbance detection. The effect of the probe measurement on the target measurement is described by the Lüders channel modifying the observable b (target), i.e., $\mathcal{I}_a^*(\mathcal{M}_b) = \sum_{\alpha} E_{\alpha|a}^{\frac{1}{2}} \mathcal{M}_b E_{\alpha|a}^{\frac{1}{2}}$. Here, $\mathcal{I}_a^*$ is the dual of a quantum channel preserving the identity operator, $\mathcal{I}_a^*(\mathbb{1}) = \mathbb{1}$. The maximum disturbance 1 is obtained for $\mathcal{M}_a$ and $\mathcal{M}_b$ being complementary, two different mutually unbiased bases, measurements.

The other measure of interest, defined in terms of Eq. (1), quantifies the correlation between the successive outcomes,

$$\mathcal{C} = \frac{n}{n-1} \left(p(\alpha = \beta) - \frac{1}{n} \right), \quad (4)$$

whose absolute value indicates the predictability of the target measurement given the result of the probe (reference) measurement.

One naturally expects that the measurement disturbance suppresses the predictability and, therefore, expects a mutually complementary role between the defined quantities. In the following, we report fundamental statistical constraints between the outcomes of sequential measurements expressed in the form of a trade-off inequality between correlation and disturbance:

$$\mathcal{C}^2 + \mathcal{D}^2 \leq 1. \quad (5)$$

This inequality, we call $\mathcal{CD}$ trade-off, constituting the starting point of our main result, represents a nontrivial mutual relation: A large correlation implies a small disturbance and vice versa. The sketch of the proof is presented in Sec. I in Supplemental Material [51]. The mathematical form of Eq. (5) appears in a different context in the well-known Englert-Greenberger-Yasin duality relation, also called wave-particle duality relation [52], which has been proposed to illustrate the complementarity between interference visibility and path distinguishability as a quantum feature independent of Heisenberg uncertainty or noncommutativity [53, 54]. Here, in contrast, we want to emphasize that Eq. (5) is explicitly connected to a type of *measurement* uncertainty relation, known from the theoretical frameworks of Ozawa [30, 31], Busch, *et al.* [55, 56], or Buscemi, *et al.* [33], captured via the measurement disturbance in sequential measurement settings. While there is a rather broad consensus on the concept and formulation of *disturbance* within the community, there is an ongoing debate on the appropriate definition of *error* in error-disturbance measurement uncertainty relations

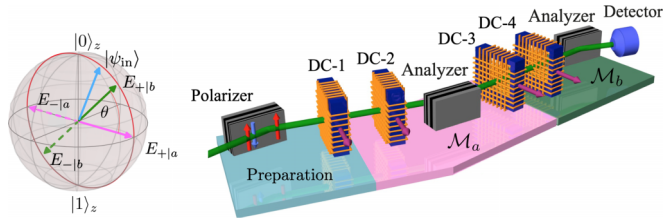


FIG. 2. Neutron optical setup for observing $\mathcal{CD}$ trade-off using supermirrors (polarizers and analyzers) as projection operators and exploiting Larmor recession induced by the magnetic fields and spin turner coils DC-1 to DC-4, any required manipulation of the neutron spin can be performed. Bloch sphere indicates initial (pure) state $|\psi_{\text{in}}\rangle$, and measurement direction of the first $\mathcal{M}_a$ and second measurements $\mathcal{M}_b$, respectively.

[2,57–59]. Therefore, we replaced the *controversial* quantity error by the unambiguous concept of *correlations* in our $\mathcal{CD}$ trade-off relation.

Although the definitions extend to multiple-outcome measurements, in many cases, two-outcome measurements sufficiently capture the essential features. Therefore, we focus on dichotomic measurements, where $\alpha, \beta = \pm 1$. In this case, the expressions for correlation and disturbance are simplified to $\mathcal{C} = 2p(\alpha = \beta) - 1$ and $\mathcal{D} = 2|p(+|b) - \tilde{p}(+|b)|$. To gain a clear intuition about various specifications and applications of Eq. (5), we examine the qubit example and present a precise $\mathcal{CD}$ trade-off holding covariant symmetry, proving to be highly useful for the experimental characterization of measurement parameters.

Qubit case and the covariant symmetry. The initial state of a qubit in Bloch representation is $\rho = \frac{1}{2}(\mathbb{1} + \vec{r} \cdot \vec{\sigma})$, where $\vec{r}$ is called Bloch vector. The most general form of the two-outcome POVMs for target measurement is $E_{\pm|b} = ((1 \pm b_0)\mathbb{1} \pm \vec{b} \cdot \vec{\sigma})/2$ giving $\mathcal{M}_b = E_{+|b} - E_{-|b} = b_0\mathbb{1} + \vec{b} \cdot \vec{\sigma}$ in which b_0 and $|\vec{b}|$ are interpreted as the measurement *bias* and *sharpness* (strength), respectively, as characteristic parameters describing the measurement apparatus. The positivity condition imposes $|b_0| + |\vec{b}| \leq 1$. Given the Bloch representation of the POVM elements and the Lüders channel, the disturbance takes the form $\mathcal{D} = (\vec{b} \cdot \vec{\sigma})_\rho - \langle \vec{b} \cdot \vec{\sigma} \rangle_\rho = (\vec{b} - \vec{\tilde{b}}) \cdot \vec{r}$, where $\vec{\tilde{b}} \cdot \vec{\sigma} = \mathcal{I}_a^*(\vec{b} \cdot \vec{\sigma})$. As expected, the amount of disturbance depends on the noncommutativity between the corresponding observables $\mathcal{D} = -s|\vec{b}|\langle [\hat{a} \cdot \vec{\sigma}, [\hat{a} \cdot \vec{\sigma}, \hat{b} \cdot \vec{\sigma}]] \rangle_\rho = -s|\vec{b}|\hat{a} \times (\hat{a} \times \hat{b}) \cdot \vec{r}$. Here, the coefficient $s = 1 - (u_+ + u_-)/2$ where, $u_\alpha = \sqrt{(1 + \alpha a_0)^2 - |\vec{a}|^2}$ accounts for the outcome-dependent *unsharpness* of $\mathcal{M}_a$ measurement. The above form suggests that the optimal input state maximizing $\mathcal{D}$ is obtained by a unit Bloch vector $\vec{r}_{\text{opt}} = (\vec{b} - \vec{\tilde{b}})/|\vec{b} - \vec{\tilde{b}}| = -\hat{a} \times (\hat{a} \times \hat{b})$, which lies in the plane spanned by the measurements $\mathcal{M}_a$ and $\mathcal{M}_b$ and is perpendicular to the measurement direction of $\mathcal{M}_a$, as depicted in Fig. 3(b), resulting to $\langle \mathcal{M}_a \rangle = a_0$. In this case, $\mathcal{D} = s|\vec{b}| \sin \theta$, where $\theta = \arccos(\hat{a} \cdot \hat{b})$ represents the angle between the Bloch vectors of the respective measurements. Correlation for the optimal state is expressed as $\mathcal{C} = a_0 b_0 + |\vec{a}||\vec{b}| \cos \theta + \delta|\vec{b}| \sin \theta$. As an example, consider $\mathcal{M}_x$ being sharp and the second measurement arbitrary; then Eq. (6) reduces to an optimal trade-off

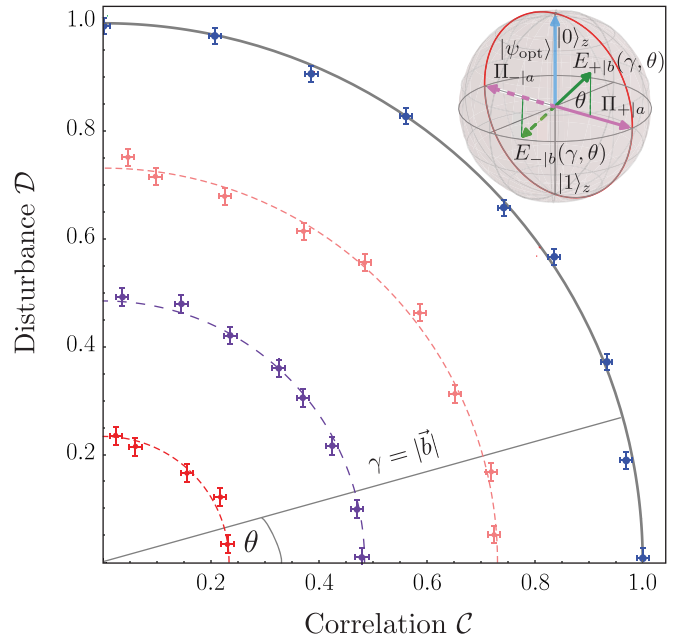


FIG. 3. Disturbance $\mathcal{D}$ vs correlation $\mathcal{C}$ first (fixed) projective $\mathcal{M}_a = \sigma_x$ and second (rotated) unbiased POVM $\mathcal{M}_b(\gamma, \theta) = E_{+|b}(\gamma, \theta) - E_{-|b}(\gamma, \theta)$. The measurement strength γ equals the length of Bloch vector $\vec{b}$ of the target measurement $\mathcal{M}_b$.

$\mathcal{C}^2 + \mathcal{D}^2 = |\vec{b}|^2$. The practical advantage of this finding becomes more significant in high-dimensional measurement in which the readout noise or detector precision as represented by the generalized Bloch length is estimated without resorting to highly demanding process tomography (see Ref. [51] for details in d dimension). Therefore, one can directly measure the strength of the second measurement by estimating the correlation and disturbance. Note that the form of the relation is invariant under the choice of the measurement settings or the observer's frame of reference, hence called covariant correlation disturbance.

Consequently, the entire result for the most general pair of measurements can be summarized into the following parametric equations governing the $\mathcal{CD}$ trade-off:

$$\begin{bmatrix} \mathcal{C} \\ \mathcal{D} \end{bmatrix} = \begin{bmatrix} a_0 b_0 \\ 0 \end{bmatrix} + \begin{bmatrix} |\vec{a}| & \delta \\ 0 & s \end{bmatrix} \begin{bmatrix} |\vec{b}| \cos \theta \\ |\vec{b}| \sin \theta \end{bmatrix}, \quad (6)$$

where $\delta = (u_+ - u_-)/2$ is nonzero for bias of $\mathcal{M}_a$, introducing shear to the original circle. The bias of the target measurement is inferred from the displacement $a_0 b_0$ along the x axis. Thus, the values of $\mathcal{C}$ and $\mathcal{D}$ follow an ellipse equation encapsulating all essential information about the intrinsic properties of the measurement devices in a generic form, regardless of the unitary transformation of the observables, which means that measurement settings (e.g., measurement direction θ) do not have to be known in principle. Therefore, our scheme offers an experimentally robust estimation and characterization of the measurement parameters such as the measurement strength even with limited control over the measurements, or in situations involving uncharacterized local unitary noise. Note that in comparison to the scheme proposed by Buscemi, *et al.* [33], which is also based on correlations

(but between initial states and measurement outcomes), no prior knowledge about the observables—or more precisely—about its eigenstates is required here. This paves the way for self-calibration of quantum measurement devices in (semi) device-independent quantum communication applications.

Experimental results. We experimentally demonstrate various instances of the above relationship. The data offer significant validation and verification of the measurement setup's performance by leveraging the covariance symmetry of the $\mathcal{CD}$ trade-off. The configuration of the experimental setup is depicted in Fig. 2 (see Sec. III A in Supplemental Material for details [51]). In all our experimental configurations, we examine the effects of the first measurement $\mathcal{M}_a$ on the subsequent measurement $\mathcal{M}_b$ for the optimal (pure) input state $|\psi_{\text{opt}}\rangle$, in terms of disturbance $\mathcal{D}$ and correlations $\mathcal{C}$.

For a better understanding of the implementation of a general measurement process in our setup, it is useful to rewrite the POVM in the following form:

$$E_{\pm|b}(\theta, \gamma, b_0) = \gamma \Pi_{\pm}(\theta) + (1 - \gamma) N_{\pm}(b_0) \quad (7)$$

as a convex combination of projective measurement $\Pi_{\pm}(\theta) = (\mathbb{1} \pm \hat{b}(\theta) \cdot \vec{\sigma})/2$, with unit vector $\hat{b}(\theta)$, and biased dummy (noisy) measurement $N_{\pm}(b_0) = \frac{1}{2}(\mathbb{1} \pm \frac{b_0}{1-\gamma})\mathbb{1}$, which is fully characterized by three parameters: rotation angle θ about a specific axis, bias b_0 , and the randomization degree $\gamma = |\vec{b}|$ (see Sec. III D in Supplemental Material [51] for the experimental implementation of POVMs).

First, we study the case where the probe (first) measurement is projective and given by $\mathcal{M}_a = \sigma_x$. In contrast, the target (second) measurement is represented by the unbiased POVM $\mathcal{M}_b(\gamma, \theta) = E_{+|b}(\gamma, \theta) - E_{-|b}(\gamma, \theta)$, with POVM elements $\Pi_{\pm|b}(\gamma, \theta)$ from Eq. (7). The resulting tight trade-off relation (Eq. (6)) $\mathcal{C}^2 + \mathcal{D}^2 = |\vec{b}|^2$ can be seen in Fig. 3 (the special case $\mathcal{C}^2 + \mathcal{D}^2 = 1$ with $\gamma = 1$, where both measurements are *projective*, is plotted in blue). This particular configuration allows for a direct determination of the measurement strength γ and in turn for the Bloch length $|\vec{b}|$. This self-calibration features results in the following experimentally determined interaction strengths γ : $\gamma_1 = 0.731(4)$, $\gamma_2 = 0.485(3)$, and $\gamma_3 = 0.233(3)$, which is plotted in Fig. 3 in magenta, violet, and red, respectively. In a configuration where the POVM is the last measurement, the bias b_0 has no effect.

This formulation captures the unsharpness and bias of the probe measurement $\mathcal{M}_a$ through a combined transformation that performs both skewing (shearing) and scaling (squeezing) on the original circle. Here, δ is the shear factor, quantifying the measurement bias. Next, we reverse the order of POVM and projective measurement; now the probe (first) measurement is the POVM $\mathcal{M}_a(\gamma) = E_{+|a}(\gamma) - E_{-|a}(\gamma)$ and the target (second) measurement is projective, given by $\mathcal{M}_b(\theta) = \cos \theta \sigma_x + \sin \theta \sigma_z$. As seen from Fig. 4, the (unbiased) unsharp measurement disturbs the second measurement less compared to a projective measurement, which results in an elliptic shape, as defined in Eq. (6) with $s < |\vec{a}|$, of the correlation-disturbance trade-off, plotted in red and purple for interaction strengths $\gamma = 0.5$ and $\gamma = 0.75$, respectively. Finally, the effect of a biased POVM is studied, where according to Eq. (6) in addition the elliptic shape of the correlation-disturbance

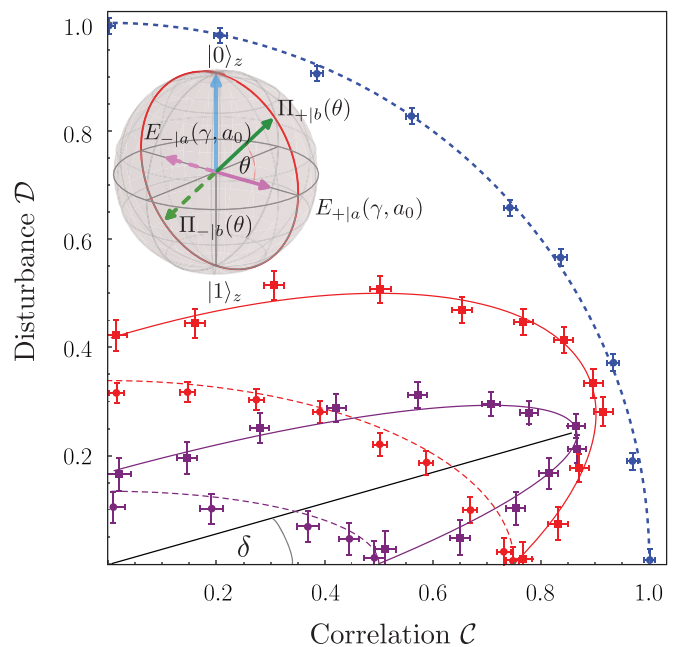


FIG. 4. Disturbance $\mathcal{D}$ vs correlation $\mathcal{C}$ with first unbiased and biased POVM $\mathcal{M}_a(\gamma, a_0) = E_{+|a}(\gamma, a_0) - E_{-|a}(\gamma, a_0)$ and second (varying) projective $\mathcal{M}_b(\theta) = \cos \theta \sigma_x + \sin \theta \sigma_z$. The length of Bloch vector $\vec{a}$ of the first measurement accounts for the maximal possible correlation $\mathcal{C}_{\text{max}}$ in the unbiased case.

relation a tilt and shearing, represented by shear factor δ occur, which is plotted in Fig. 4 for $\gamma = 0, 5$, $a_0 = 0.5$ and $\gamma = 0, 75$, $a_0 = 0.25$ as red and purple squares, respectively.

Optical quantum detector noise model. Let us examine the covariant $\mathcal{CD}$ trade-off for optical quantum detectors. Single-photon detectors are an important type of quantum detector [60]. These binary (on-off) detectors register a detection outcome (click or on) upon the arrival of any number of photons, whereas no detection (no click or off) occurs otherwise. Dark count (probability of the click in the absence of a photon, characterized by parameter ν) and detector efficiency (probability of a click upon photon arrival, characterized by parameter η) are critical characteristics of single-photon detectors. The pertinent positive operator-valued measure (POVM) elements, accounting for these primary noise factors, are expressed as $E_{\text{off}} = e^{-\nu} \sum_{n=0}^{\infty} (1 - \eta)^n |n\rangle\langle n|$ and $E_{\text{on}} = \mathbb{1} - E_{\text{off}}$. The aforementioned noise characteristics significantly impact the security and efficiency of numerous quantum communication protocols due to the challenge of differentiating between actual and spurious events, complicating fair sampling. Thus, accurately and reliably assessing intrinsic detector readout noise independent of unitary or transmission channel noises is essential. See Sec. II in Supplemental Material [51] for details and schematic illustration of the optical arrangement. Single photons are prepared using well-established techniques to examine joint measurements, where the initial measurement uses a polarizing beam splitter to evaluate photon polarization encoded in two distinct paths, and the second measurement determines its presence in a specific mode via the on-off detector. We can similarly define correlations as joint probabilities $p(\alpha, \beta)$ with $\alpha \in \{\text{H} \equiv -1, \text{V} \equiv +1\}$ and $\beta \in \{\text{off} \equiv +1,$

on $\equiv -1$ and disturbance. We can directly estimate the noise parameters η and ν by examining two cases: (1) when the reference measurement determining the polarization mode is sharp and (2) when the measurement is fully biased, allowing all photons to pass through. In case (1), according to the $\mathcal{CD}$ trade-off the maximum disturbance is given by $\mathcal{D}^{(1)} = e^{-\nu}\eta$, while $\mathcal{C}^{(1)} = 0$. Conversely, in situation (2), we have $\mathcal{C}^{(2)} = e^{-\nu}(2 - \eta) - 1$ and $\mathcal{D}^{(2)} = 0$. Consequently, the characteristic noise parameters η and ν can be inferred solely from $\mathcal{D}^{(1)}$ and $\mathcal{C}^{(2)}$, eliminating the need for detector tomography [61–65]. Furthermore, a channel may be introduced between measurements to compare outcomes and assess channel loss. It is worth pointing out that this strategy is well tailored for photon detectors where the source of the noise is from the intrinsic part of the detector rather than the random unitaries (polarization rotation).

Conclusion and outlook. The correlation $\mathcal{C}$ and the disturbance $\mathcal{D}$ induced by quantum measurements are not independent quantities; they are mutually related in a nontrivial way. This relationship is expressed as covariant symmetry interplay between the two quantities. Exploiting these symmetries provides a robust tool for estimating characteristic

parameters of measurement devices, such as sharpness and bias, with practical application in high-dimensional settings. Our experimental implementation efficiently demonstrates our theoretical findings, covariant $\mathcal{CD}$ trade-off.

At a fundamental level, we propose that optimal configurations of disturbance, as a way of transmitting information, and correlation, as a measure of predictability, could outperform or even rule out certain classical (hidden variable) models. Associating a classical (noncontextuality) inequality within our scheme offers a perspective on the extent of nonclassicality implied by quantum uncertainties [66,67], which also aligns with schemes of two-time Leggett-Garg macroscopic realism [68,69], sequential random access codes, self-testing of measurement instruments [39–41,44,70,71], and sequential correlation measurements in Bell scenarios [72–74]. Consequently, our scheme could be effectively adapted to optical-based communication tasks where quantum advantage over classical counterparts can be explored.

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