

Vortex refraction and enhanced vortex velocities at tilted superconductor-normal metal interfaces

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(Received 30 July 2025; accepted 30 March 2026; published 27 April 2026)

We derive a refraction law for superconducting vortices at superconductor/normal metal (S/N) interfaces that are exposed to tilted magnetic fields. Simulations of the proximity effect confirm this law and reveal vortex trapping for low effective mass. Under transport currents, we find tilt-angle-dependent core displacements due to vortex viscosities differing across the S/N interface and characteristic voltage oscillations due to the modified vortex dynamics. Our results are important for emerging three-dimensional superconductor nanotechnologies. They offer design principles for high-current S/N devices and their interface orientation in an applied field.

DOI: [10.1103/ld3z-kkmp](https://doi.org/10.1103/PhysRevResearch.8.L022018)

The superconducting proximity effect refers to the leaking of Cooper pairs from a superconducting material (S) into a nonsuperconducting material [1]. This phenomenon has enabled rich and unconventional physics in a variety of systems, including normal metals [2], ferromagnets [3,4], and topological insulators [5]. In a normal metal (N), proximity-induced correlations can be long-ranged, persisting over several hundred nanometers. Of particular interest is the emergence of topological defects of the superconducting order parameter ψ , such as Abrikosov vortices, in coupled S/N systems [6–8]. Until recently, the evolution of a vortex transitioning from S to N has not been well understood. Stolyarov *et al.* [9] studied vortex cores in the mixed state of a superconductor in a perpendicular magnetic field and demonstrated that they expanded when penetrating into the proximity-coupled normal metal layer. The result was confirmed by Panghotra *et al.* [10]. More recently, Vodolazov [11] used time-dependent Ginzburg-Landau simulations and reported the formation of nascent vortices at the S/N interface under an in-plane magnetic field. Earlier theoretical and numerical studies explored vortex interactions with point, tilted line, and plane defects in high-temperature superconductors [12–14]. However, a general understanding of vortex behavior in S/N bilayers for interface normals not parallel to the applied field and their behavior under transport currents remains lacking. Such understanding is key to interpret and optimize vortex-dominated current-voltage characteristics of coated su-

perconducting wedges [15,16] and samples in the emerging field of three-dimensional (3D) superconductivity [17], such as trapezoids, helices, tubes, and bent wires [18–21]. When coated or contacted by metals [18,21], their S/N interface normals and the applied field vectors are generally misaligned.

In this Letter, we investigate how vortices interact with superconductor-normal metal interfaces at arbitrary orientations. We describe the proximity-induced superconductivity in the metal using a modified Ginzburg-Landau framework following Ref. [22]. The time-dependent Ginzburg-Landau (TDGL) equations have been used in Refs. [10,23,24] to reproduce qualitatively proximity effect features. The Ginzburg-Landau results for S/N bilayers were found to be qualitatively consistent with quasiclassical calculations performed on the same system [24]. Recently, a work using a similarly modified TDGL has been used to model composites of type-I and type-II superconductors [25]. Noticeably, the authors reported qualitative agreement with features observed in S/N systems [25]. In this work, we show that vortices crossing a tilted interface undergo a characteristic bending of their core. Bending at tilted insulating interfaces is known [21,26–28], but we find a decisive dependence on the discontinuity in the effective Cooper pair masses. In analogy with the terminology in other fields, such as fluid dynamics [29] and electromagnetism [30], we refer to this phenomenon as refraction. Using analytical arguments, we derive the corresponding vortex refraction law and, through time-dependent Ginzburg-Landau simulations [31,32], we demonstrate that this effect, combined with interface energy barriers, leads to an apparent vortex core displacement in static conditions. We show that applied transport currents modulate the degree of vortex bending. Moreover, we find that tilting the interface introduces directional asymmetry in vortex motion, which manifests as changes in voltage oscillation frequency.

In the following, subscripts S and N designate the parameter values taken in the superconductor and normal metal,

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respectively. We use a modified version of the TDGL equation valid for gapless superconductors in the dirty limit close to their critical temperatures to describe the evolution of the order parameter ψ and account for the proximity effect. We assume that the temperature T is close to the critical temperature of both the normal metal T_{cN} and the superconductor T_{cS} , and fulfilling $T_{cN} < T < T_{cS}$. Under these conditions, we consider the following dimensionless TDGL equation [23–25,33]:

$$\frac{1}{D_{S/N}} \left(\frac{\partial \psi}{\partial t} + i\kappa \phi \psi \right) + \left(\frac{i}{\kappa} \nabla + \mathbf{A} \right)^2 \psi + (a_{S/N} + b_{S/N} |\psi|^2) \psi = 0. \quad (1)$$

Here, ϕ is the scalar potential, $\mathbf{A}$ is the vector potential, and $D_{S/N}$ denotes the electronic diffusion constant normalized to $D_S = 1$. The parameters $a_{S/N}$ and $b_{S/N}$ are normalized such that $a_S = -1$, $b_S = 1$, $a_N > 0$, and $b_N = 0$. The parameter, a_N is taken as phenomenological and adjusted accordingly. We denote by $\kappa = \lambda/\xi$ the Ginzburg-Landau parameter with λ the London penetration depth and ξ the coherence length. We use the following units: $[r] = \lambda$, $[t] = \tau = \xi^2/D_S$, $[\mathbf{A}] = \Phi_0/(2\pi\xi)$, and $[\phi] = V_0 = \Phi_0 D_S \lambda / (2\pi\xi^3)$, with Φ_0 the magnetic flux quantum. The order parameter ψ is scaled to its bulk value at zero magnetic field. Further information can be found in Supplemental Material [34].

Our simulations showed that for the geometrical parameters considered here, the screening field did not significantly alter the vortex shape in either the superconductor or the metal, as shown in Supplemental Material [34]. In the following, we therefore assume a time-independent vector potential $\mathbf{A}$ under the Coulomb gauge $\nabla \cdot \mathbf{A} = 0$, and only solve for Eq. (1) coupled with the Poisson equation for the scalar potential [24]

$$\Delta \phi = \frac{1}{m_{S/N} \sigma_{S/N}} \nabla \cdot \left(-\frac{i}{2\kappa} (\psi^* \nabla \psi - \psi \nabla \psi^*) - |\psi|^2 \mathbf{A} \right), \quad (2)$$

with $\sigma_{S/N}$ the normal-state conductivity and $m_{S/N}$ the effective mass of Cooper pairs, taking $m_S = 1$ by normalization. Boundary conditions at the interface with the insulator surrounding the system are $\nabla \psi \cdot \mathbf{n} = 0$, $\mathbf{A} \cdot \mathbf{n} = 0$, $\nabla \phi \cdot \mathbf{n} = 0$, and $(\nabla \times \mathbf{A}) \times \mathbf{n} = \mathbf{H} \times \mathbf{n}$, with $\mathbf{n}$ the normal to the interface and $\mathbf{H}$ the applied magnetic field. At the leads where current is injected, we apply $\sigma_{S/N} \nabla \phi \cdot \mathbf{n} = -\mathbf{J} \cdot \mathbf{n}$, with $\mathbf{J}$ the applied transport current, and set $\psi = 0$. At the interface between the superconductor and normal metal, the boundary conditions are [22,23,25,34,35] $[\psi] = 0$, $[\mathbf{A}] = \mathbf{0}$, $[\phi] = 0$, $[(\nabla \times \mathbf{A}) \times \mathbf{n} / \mu_{S/N}] = \mathbf{0}$, $[\sigma_{S/N} (\partial_t \mathbf{A} + \nabla \phi) \cdot \mathbf{n}] = 0$, and

$$\left[\mathbf{n} \cdot \frac{1}{m_{S/N}} (i\nabla / \kappa + \mathbf{A}) \psi \right] = 0, \quad (3)$$

where $\mu_{S/N}$ is the magnetic permeability and $[\cdot]$ denotes the jump of the enclosed quantity across the interface.

To investigate the orientation and shape of vortices at a tilted boundary, we performed numerical simulations of an S/N bilayer structure (Fig. 1) using the finite element method (FEM) [36] implemented in COMSOL [37–39]. The system dimensions are $h = 35\xi$ and $L = W = 15\xi$, with the interface

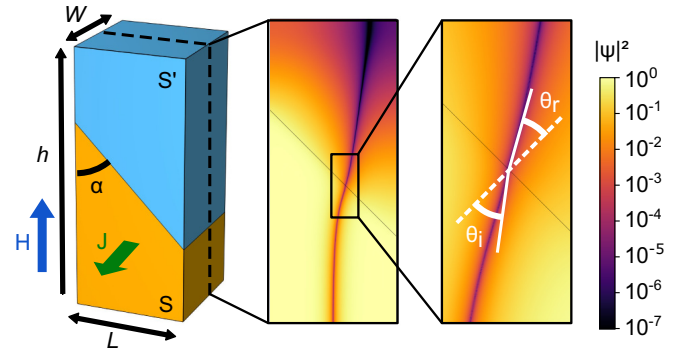


FIG. 1. An oblique view of the system (left): A superconducting-normal metal bilayer with a tilted interface described by the angle α subject to an applied dc current density $\mathbf{J}$ parallel to the boundary and a vertical magnetic field $\mathbf{H}$. A cross-sectional view of the system reveals a vortex refracted at the interface (center), which permits to define incident and refraction angles θ_i and θ_r , respectively (right). The two panels on the right show the color-coded order parameter $|\psi|^2$ (see legend) for which bright (black) encodes the superconducting state (normal state).

tilted by an angle α relative to the vertical. The superconductor S is taken to be a type-II superconductor in the dirty limit with a Ginzburg-Landau parameter $\kappa = 10$. In the following, we treat the effective mass $m_{S/N}$, the normal conductivity $\sigma_{S/N}$, and the diffusion constant $D_{S/N}$ as characteristic parameters. The effective mass parameter is taken as a free phenomenological parameter and set to $m_N/m_S = \sigma_S/\sigma_N = 0.65$. Likewise, the diffusion constant is fixed as $D_{S/N} = 7.1$, i.e., assuming $D_N > D_S$. The coefficient a_N was set to 0.003, indicating that the temperature is slightly above the critical temperature of N: $T > T_{cN}$. In the following, a magnetic field of magnitude $H = 1$ in the given units is applied vertically.

Figure 1 shows a vortex bending at the S/N interface, characterized by an incident (i) angle θ_i and a refracted (r) angle θ_r . To understand the behavior observed in the simulations, we develop an analytical model describing the refractionlike bending of a vortex line crossing an interface tilted by an angle α with respect to the vertical (Fig. 1). Near the vortex core, the order parameter modulus increases linearly with distance to the core r [40], allowing the approximation $|\psi| = \gamma_{S/N} r$, where γ_S and γ_N are constants. From Eq. (3), at the vicinity of the vortex core where ψ tends linearly to 0, we obtain $\mathbf{n} \cdot \nabla \psi_S / m_S = \mathbf{n} \cdot \nabla \psi_N / m_N$, and therefore,

$$\frac{1}{m_S} |\nabla \psi_S| \sin(\theta_i) = \frac{1}{m_N} |\nabla \psi_N| \sin(\theta_r), \quad (4)$$

where the angles θ_i and θ_r are defined in Fig. 1. Continuity of the order parameter across the interface gives $\gamma_S \cos(\theta_i) = \gamma_N \cos(\theta_r)$, from which we deduce

$$|\nabla \psi_S| \cos(\theta_i) = |\nabla \psi_N| \cos(\theta_r). \quad (5)$$

Dividing Eq. (4) by Eq. (5), we get the refraction law

$$\frac{1}{m_S} \tan(\theta_i) = \frac{1}{m_N} \tan(\theta_r). \quad (6)$$

Figure 2 shows cross-sectional views of the vortex line at the S/N interface for various angles α and effective masses

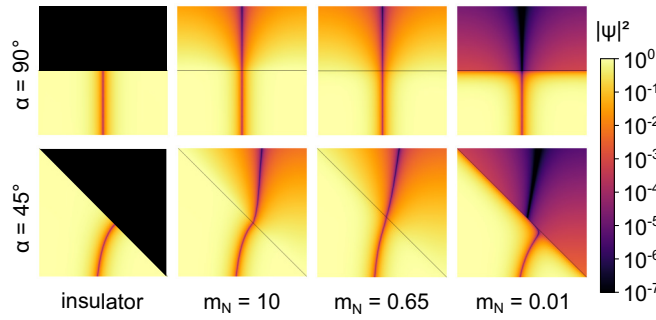


FIG. 2. Cross-sectional view of the crossing of an S/N interface by a vortex for various effective Cooper pair masses in the normal metal, m_N . The normal metal is on top of the superconductor.

m_N in the static case with $\mathbf{J} = 0$. The top row corresponds to $\alpha = 90^\circ$, where the applied magnetic field is orthogonal to the interface. In that case, the vortex crosses the boundary without tilting for all masses. The bottom row shows the distribution of the order parameter for $\alpha = 45^\circ$. Here, the vortex is deviated when crossing the interface, with a deviation angle $\theta_i - \theta_r$ depending on the mass parameter m_N of the metal. The leftmost column shows the results obtained for an insulator in place of the metal. This corresponds to the limit $m_N \rightarrow \infty$ and $a_S \rightarrow \infty$, with $\sqrt{a_S}/m_S \rightarrow 0$ [22]. Boundary condition $\mathbf{n} \cdot \nabla \psi = 0$ with the insulator imposes a vortex core perpendicular to the boundary, as observed in Fig. 2 (bottom left panel).

Figure 3(a) compares refraction angles θ_r extracted from simulations (symbols) and analytical predictions based on Eq. (6) (lines) for various m_N . In contrast to refraction of electromagnetic waves in the optical regime, there is no critical angle beyond which the refraction of vortices ceases. The deviation angle $\theta_i - \theta_r$ vanishes in both normal and grazing incidence. However, not all possible incident angles could be reached in the simulation due to the vortex leaving the

system when α became too small. Reaching more incident angles should be possible in a larger system or under stronger magnetic fields. Still, the excellent quantitative agreement between numerical and analytical models shown in Fig. 3(a) motivates broader FEM exploration beyond analytical reach, particularly in dynamic regimes.

Decreasing the angle $\alpha < 90^\circ$ increases both the incident and refraction angles, as the vortex tends to align with the applied magnetic field. However, repulsion from the S/N boundary in the superconductor favors perpendicular vortex alignment, reducing θ_i . This effect is stronger for a larger jump in effective Cooper pair mass, where the vortex requires longer distances to reorient along the magnetic field. The refraction angle is less affected due to weaker boundary interaction arising from reduced Cooper pair density. Figure 3(b) shows the relationship between the angle α and the deviation angle $\theta_i - \theta_r$ for different mass parameters. Considering Eq. (6), it is clear that the deviation angle is negative when m_N is larger than m_S , and positive in the opposite case. Having $\theta_i - \theta_r = 0$ corresponds to a straight vortex.

In the insulating limit $m_N \rightarrow \infty$, Eq. (6) yields normal incidence ($\theta_i = 0$), consistent with the insulator boundary conditions. In the opposite limit $m_N \ll m_S$, corresponding to highly conductive metals, Eq. (6) predicts $\theta_i = 90^\circ$ for all $\theta_r \neq 0$. Correspondingly, the simulated Cooper pair density $|\psi|^2$ in Fig. 2 for $m_N = 0.01$ and $\alpha = 45^\circ$ shows that the vortex core tends to remain at the interface before entering the superconducting region, resulting in an apparent displacement between the vortex core in the metal and in the superconductor. The feature resembles the Goos-Hänchen effect in light transmission [41]. In the present system, the large difference in Cooper pair density between the normal metal and the superconducting regions creates an energy barrier similar to the Bean-Livingston barrier [42], which pins a segment of the vortex along the interface [11]. The vortex eventually enters the superconductor when its line energy, proportional to its length [40], outweighs the barrier. As $\alpha < 90^\circ$ decreases, the reduced line energy cost favors longer interfacial segments, thereby enhancing this displacement. This effect is similar to vortex trapping by twin planes [12].

In the following, we discuss the influence of a constant transport current of magnitude $J = 0.27$, applied as shown in Fig. 1, on the position of the vortex near the interface. The transport current drives vortex motion to the right by exerting a Lorentz force on it. Figure 4(a) shows cross sections of moving vortices for various angles α and mass m_N . Remarkably, even for $\alpha = 90^\circ$, where no static tilt is observed in Fig. 2, a dynamic tilt emerges in the presence of the normal metal. This tilt arises from two key mechanisms in the metal: reduced vortex viscosity and preferential vortex nucleation. Vortex viscosity stems from energy dissipation associated with vortex motion. Here, two dissipation phenomena are taken into account [31]: normal current and induced electric field dissipation, and the relaxation of the order parameter. The dissipated power density associated with the latter in the given units is $W_T = (1/D_{S/N})|(\partial_t + i\kappa\phi)\psi|^2$. Here, $D_N > D_S$, resulting in lower dissipation in the metal and hence reduced viscosity. The power density dissipated via normal currents is given by $W_{BS} = \sigma_{S/N}(\partial_t \mathbf{A} + \nabla \phi)^2$, with $\sigma_N > \sigma_S$, here, which increases dissipation and thus vortex viscosity in the metal.

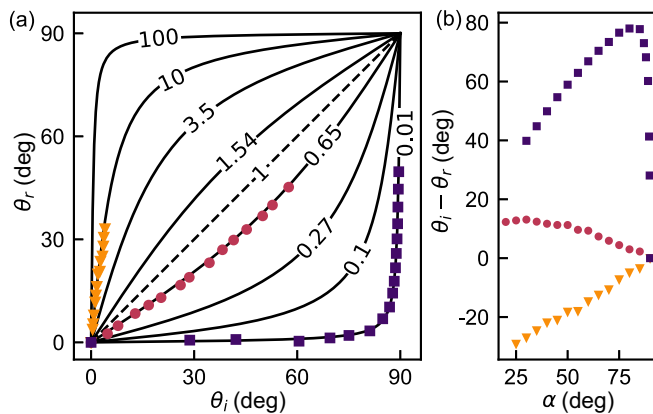


FIG. 3. (a) Refraction angle θ_r as a function of incident angle θ_i for various effective Cooper pair masses m_N in the metal at selected values for the effective mass. Solid lines correspond to analytical refraction law solutions [Eq. (6)], with m_N indicated on top, and symbols are results extracted from simulations. (b) Deviation angle $\theta_i - \theta_r$ as a function of interface inclination α . The color coding indicates different values of m_N consistent with panel (a).

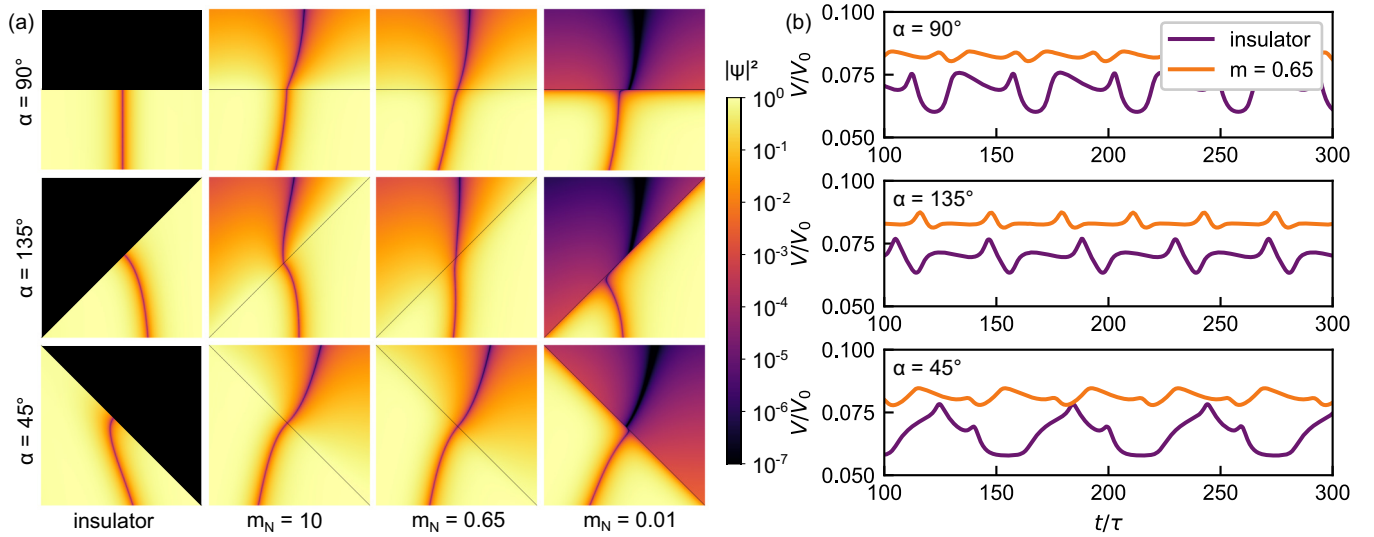


FIG. 4. Close views of the crossing of an S/N interface by a vortex under an applied transport current for various effective Cooper pair masses m_N in S' [panel (a)]. Vortices are moving to the right, inducing voltage oscillations [panel (b)].

For the values of $D_{S/N}$ and $\sigma_{S/N}$ considered here, the reduced dissipation from order parameter relaxation dominates, leading overall to decreased viscosity in the metal and enhanced vortex motion there. Consequently, the vortex line bends toward the direction of motion in the normal metal. See Supplemental Material [34] for more details on those two mechanisms in our system. Additionally, the lower Cooper pair density in the metal reduces the nucleation barrier, favoring vortex entry from the metal, further contributing to the observed tilt.

For low values of the mass parameter [e.g., $m_N = 0.01$, Fig. 4(a)], the previously discussed displacement of the vortex core between normal and superconducting regions exists, even when $\alpha = 90^\circ$. This results from the same mechanism as described in the static regime (Fig. 2). Note, however, that this displacement in the static regime could only be realized for $\alpha \neq 90^\circ$. The combined effect of applied current and varying angle α further modulates this displacement: $\alpha > 90^\circ$ enhances it, while $\alpha < 90^\circ$ reduces it. The angle α also affects the vortex speed, leading to variations in the frequency of voltage oscillations, as shown in Fig. 4(b). Here, voltage is defined as $V = (1/S_A) \int_{S_A} \phi dS - (1/S_B) \int_{S_B} \phi dS$, with S_A and S_B the areas of the two surfaces where current is injected. Two competing mechanisms are at play. First, for $\alpha < 90^\circ$, the vortex length inside the superconductor decreases during its motion, reducing its line energy and creating a geometric force that favors motion, as described in Ref. [18]. Second, the taller edge where the vortex nucleates constitutes a higher energy barrier preventing vortex entry. This asymmetry in entrance barrier leads to a superconducting diode effect [18]. Figure 4(b) shows that by just adding a metal overlayer to, e.g., superconducting wedges [15,16], one increases vortex mobilities. They are relevant for radio-frequency-signal generation [43] and flux generators [20]. Our findings point toward vortex-velocity distributions to be explored in metal-coated trapezoids, helices, and tubes [18–20] and provide an additional degree of freedom in designing static and dynamic

properties in 3D superconductivity. We emphasize that for fully quantitative and microscopically justified results valid across a wider temperature range, or for nonsuperconducting metals with $T_{cN} = 0$, quasiclassical methods based on the Usadel or Eilenberger equations are required [44]. We estimate our findings to stimulate the implementation of these numerically challenging approaches to quantitatively model the time-dependent effects discussed here.

In summary, we have demonstrated that a vortex penetrating a normal metal via proximity effect exhibits refraction, governed by a quantitative law derived analytically and validated through 3D time-dependent Ginzburg-Landau simulations. In the static regime, vortex refraction, in combination with interfacial energy barriers and vortex line tension, results in an apparent displacement of the vortex core across the S/N boundary. Under transport current, modulation of the vortex tilting emerges from the interplay between altered vortex viscosity and preferential nucleation driven by the proximity effect. The vortex refraction law presented in this work shares deep analogies with other domains of physics, such as heat transfer [45], fluid dynamics [29,46], and electro- and magnetostatics [30], offering a unifying perspective across physical systems. Our results provide important insights into the proximity effect of type-II superconductors under applied fields and currents. They pave the way toward an understanding of proximity effects in nonplanar geometries, which we expect to gain importance in future 3D superconducting devices [17,47]. Here, engineered curvatures give rise to advanced functionalities in proximity-coupled S/N systems [48,49].

The simulations have been performed using the facilities of the Scientific IT and Application Support Center of EPFL. We acknowledge support by the SNSF via Project No. 10000845.

Data availability. The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

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