

Intermittency-driven subcritical fiber breakage in turbulence

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Turbulence fragments slender fibers in oceans, process industries, and the atmosphere, shaping size spectra, transport, and impact. Mean-field estimates compare typical turbulent bending with a material failure threshold and predict a critical span below which fibers should be effectively unbreakable. We show that this conclusion changes when turbulent intermittency is included. In the quasistatic inertial-range regime, the fiber relaxes elastically faster than the turbulent load changes, so its curvature is set by the velocity increment across the fiber; refined similarity relates the amplitude of that increment to the coarse-grained dissipation at the fiber scale. Rare fluctuations of the velocity increment across the fiber therefore generate rare curvature bursts, opening a subcritical breakage channel for fibers whose mean-field curvature lies far below threshold. In a log-normal cascade model, the resulting hazard decays algebraically with span, rather than with the much stronger Gaussian suppression obtained from a nonintermittent model. The same rare-event statistics can be mapped onto an effective mean dissipation: the mean dissipation that an equivalent nonintermittent model would need to reproduce the intermittent exceedance probability. Under realistic conditions, this effective dissipation can exceed the bare mean by orders of magnitude. Intermittency can therefore control fiber fragmentation in nominally subcritical regimes.

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I. INTRODUCTION

Turbulent flows continuously stretch, fold, and break material structures [1]. This fragmentation process is especially important for slender solid filaments. Such objects appear as marine microplastic and textile fibers in the upper ocean [2–4], as pulp fibers in papermaking and biorefining flows [5,6], as glass wool and synthetic fibers in industrial suspensions [7,8], and as fiberlike atmospheric particles, including fibrous aerosols and airborne microplastics [9]. In all these settings, breakage changes the length distribution of the suspended phase. That length distribution in turn affects transport, residence times, mechanical feedback on the carrier flow, and environmental or health impacts [10,11].

Marine microplastic fibers provide a useful example of why a fiber-scale fragmentation theory is needed. Their abundance at the ocean surface is controlled not only by sources and transport, but also by length-dependent residence at the air-sea interface and by subsequent fragmentation [2,3]. A theory that predicts only whether typical turbulent stresses exceed a material threshold can miss an important part of this problem. Observed fibers are often much shorter than the nominal mean-field span at which turbulent bending becomes comparable to the material strength. The question addressed

here is whether those apparently subcritical fibers are truly inactive, or whether turbulent intermittency can make their breakage observable on finite times.

A recent step in this direction is the turbulence-informed kinetic theory of inertial-range fiber fragmentation developed in Ref. [12]. That work bridges slender-beam mechanics and inertial-range turbulent forcing, introduces a load-to-curvature map, identifies a turbulence-defined critical span, and embeds the resulting supercritical and subcritical hazards in a population-balance description of the evolving fiber-length distribution. It also shows how inertial-range fragmentation can provide a mechanistic route to the power-law length spectra reported for environmental microfibers. What remains to be isolated is the extreme-tail problem at fixed, nominally subcritical span: For a prescribed material threshold and a finite observation time, how likely is it that intermittent velocity increments across the fiber drive the curvature above threshold, and can their effect be mapped onto an effective mean dissipation in an equivalent nonintermittent description? These questions are addressed here by setting aside the population-balance dynamics and deriving the subcritical hazard directly as a rare-event first-passage probability.

The physical picture is summarized in Fig. 1. Let κ denote the instantaneous midspan curvature induced by the flow, and let κ_b denote the material breaking curvature. Long fibers are easier to bend than short fibers. In a nonintermittent, Gaussian description, a short fiber has a narrow curvature distribution and the probability of entering the breaking region is extremely small. Real turbulence, however, is intermittent [13]: Dissipation and velocity increments fluctuate over many scales, and their distributions develop broad tails. In the quasistatic regime, the fiber relaxes elastically faster than the

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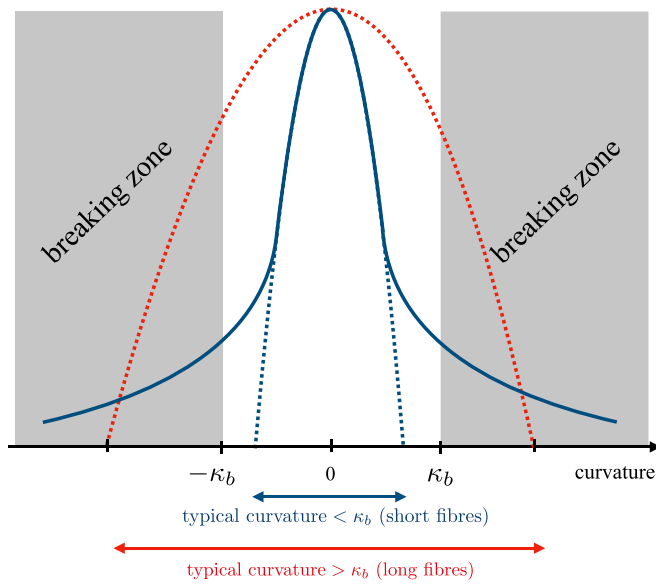


FIG. 1. Schematic probability densities of instantaneous midspan curvature for “short” (blue) and “long” (red) fibers, both centered at zero curvature. The gray shaded regions indicate the breaking zones, where the curvature magnitude exceeds the material breaking curvature κ_b . For short fibers, the typical curvature is below threshold, so the nonintermittent Gaussian baseline (blue dotted curve) lies almost entirely in the unbreakable region. Turbulent intermittency fattens the tails (solid blue curve), creating a finite probability for curvature excursions to enter the breaking zones and thereby opening an intermittency-driven subcritical breaking channel. For long fibers, the typical curvature can already exceed the threshold, as illustrated by the wider Gaussian (red dotted curve). In that regime, typical fluctuations around the mean-field scale already reach the breaking zones, so intermittent tail corrections are not shown explicitly. The distinction between short and long fibers is made quantitative below through the mean-field critical span.

turbulent load changes, so the curvature is set by the velocity increment across the fiber. Through refined similarity, the amplitude of that increment is controlled by the coarse-grained dissipation at the fiber scale. The curvature distribution therefore develops enhanced tails, and rare velocity-increment events across the fiber can push the curvature past the material breaking threshold even when the typical curvature remains well below it.

The aim of this paper is to make this subcritical-intermittency mechanism quantitative. We focus on fibers whose span lies in the inertial range of turbulence and whose bending dynamics are quasistatic: The fiber effectively equilibrates to each turbulent load before that load changes appreciably. This is the regime relevant to long, slender, overdamped microfibers in near-surface oceanic turbulence and to several industrial suspensions. In this regime, the fiber does not behave as a pointlike object sampling a smooth velocity-gradient tensor. Instead, it samples velocity differences across its own span. Refined similarity then relates the amplitude of those velocity differences to the dissipation coarse grained at the fiber scale.

We show that the apparent separation between supercritical fibers and subcritical fibers breaks down once intermittency

is included. Supercritical fibers are those whose mean-field curvature already reaches the material threshold. Subcritical fibers are those whose mean-field curvature lies below it. In a Gaussian mean-field model, the hazard of the subcritical branch is suppressed so strongly that breakage is essentially absent. Within the K62/log-normal cascade closure used as the main solvable benchmark, the hazard remains small, but it is controlled by rare velocity-increment events and decays as a power law with the span. The dependence of this result on the intermittency closure is made explicit in Appendixes C and D. The same rare-event enhancement can be summarized by an effective mean dissipation. This effective quantity is not an additional material parameter and does not change the breaking threshold. It is the mean dissipation that an equivalent nonintermittent model would need in order to reproduce the same rare-event probability generated by the intermittent cascade.

The paper is organized as follows. Section II defines the turbulent and elastic setting and derives the quasistatic curvature scaling. Section III formulates breakage as a rare-event first-passage process. Section IV gives the Gaussian mean-field baseline. Section V replaces that baseline by a log-normal cascade for the coarse-grained dissipation. Section VI derives the intermittent hazard and its effective-dissipation representation. Section VII discusses assumptions and limitations. Section VIII summarizes the results. Appendix A gives the parameter estimates used for the Antarctic Circumnavigation Expedition (ACE) markers in Fig. 2, and Appendix B gives the algebraic subcritical exponents. Appendix C rewrites the rare-event estimate in direct multifractal-spectrum language, and Appendix D gives the corresponding estimate for a generic stretched-exponential far tail of the coarse-grained dissipation.

II. PHYSICAL SETTING AND QUASISTATIC FIBER BENDING

We consider a turbulent carrier flow of an incompressible Newtonian fluid with density ρ , kinematic viscosity ν , and velocity field $\mathbf{u}(\mathbf{x}, t)$. The flow is maintained in a statistically steady state by forcing at an injection scale L_{inj} , and its mean energy dissipation rate is $\bar{\varepsilon}$. The Kolmogorov length is $\eta = \nu^{3/4} \bar{\varepsilon}^{-1/4}$ [13].

For separations in the inertial range, the relevant fluctuating quantity is the dissipation coarse grained over a region of size r , denoted by ε_r . Refined similarity gives the longitudinal velocity increment at scale r as [13–15]

$$\delta u_r \sim (\varepsilon_r r)^{1/3}, \quad (1)$$

with ensemble mean $\langle \varepsilon_r \rangle = \bar{\varepsilon}$. As a reference clock for the turbulent forcing at scale r , we use the mean-dissipation eddy-turnover time $\tau_r \sim \bar{\varepsilon}^{-1/3} r^{2/3}$.

The fibers considered here have length L , radius a , and bending rigidity γ . We assume that the span lies in the inertial range and identify the turbulent separation with the fiber length, $r = L$. We therefore write ε_L , δu_L , and τ_L for the coarse-grained dissipation, velocity increment, and eddy-turnover time at the fiber scale. This regime differs from sub-Kolmogorov fiber models, where the flow is smooth at the fiber scale and the forcing can be approximated by

a locally uniform velocity-gradient tensor [16]. Here, the fiber samples an inertial-range velocity difference across its own span.

The hydrodynamic and elastic assumptions are $a \ll L$ and $\text{Re}_a \ll 1$, where Re_a is the Reynolds number based on the fiber radius. The first condition makes the body slender, while the second allows a resistive-force description of the local drag. We also neglect fiber inertia, as appropriate for strongly overdamped microfibers in water. The inertial-range condition is $\eta \ll L \ll L_{\text{inj}}$. The quasistatic condition, namely, the requirement that elastic bending relaxation be faster than the evolution of the eddy-scale turbulent load, provides the additional mechanical upper bound derived below.

At leading order, the balance between viscous drag, elastic bending, and hydrodynamic forcing on the fiber is described, after projection onto the plane normal to the fiber centerline, by the Euler-Bernoulli equation [17,18]

$$\xi_{\perp} \partial_t \mathbf{r} = -\gamma \partial_s^4 \mathbf{r} + \partial_s(T \partial_s \mathbf{r}) + \mathbf{f}_h. \quad (2)$$

Here, $s \in [0, L]$ is arclength along the centerline, $\mathbf{r}(s, t)$ is the centerline position, and $T(s, t)$ is a Lagrange multiplier enforcing inextensibility, $|\partial_s \mathbf{r}| = 1$. The coefficient $\xi_{\perp}$ is an effective transverse drag coefficient obtained from slender-body or resistive-force theory in Stokes flow [19,20]. Its weak logarithmic dependence on aspect ratio is absorbed into prefactors throughout this scaling theory. The hydrodynamic forcing density is $\mathbf{f}_h(s, t)$. Boundary conditions and details of the load profile change numerical factors of order unity in the estimates below, but not the span and dissipation exponents.

A characteristic bending relaxation time follows by balancing viscous drag and elastic bending for a transverse perturbation of span L . This gives

$$\tau_b(L) \sim \frac{\xi_{\perp} L^4}{\gamma}. \quad (3)$$

The quasistatic regime is therefore the limit in which the turbulent load is slow compared with elastic relaxation: Before the fiber-scale velocity increment has changed appreciably, the fiber has already adjusted to the corresponding elastic shape. In formulas, it is defined by $\tau_b(L) \ll \tau_L$. Using the eddy-turnover scaling above and Eq. (3), this condition is equivalent, at the level of scaling, to

$$L < L_{\text{QS}}, \quad L_{\text{QS}} \sim \left(\frac{\gamma \bar{\varepsilon}^{-1/3}}{\xi_{\perp}} \right)^{3/10}. \quad (4)$$

For spans satisfying the quasistatic condition, the time derivative in Eq. (2) is subleading and the dominant transverse balance is $\gamma \partial_s^4 \mathbf{r} \sim \mathbf{f}_h$.

In the same quasistatic limit, the transverse hydrodynamic load per unit length is proportional to the relative velocity between the fluid and the fiber. Since the fiber relaxes rapidly, the scale separation implies $|\partial_t \mathbf{r}| \ll |\mathbf{u}|$. The transverse load at the scale of the entire fiber is then estimated as $\mathbf{f}_h \sim \xi_{\perp} \delta \mathbf{u}_L$, while classical beam theory gives the induced midspan curvature as $\kappa(L) \sim f_h L^2 / \gamma$. Combining these two estimates gives

$$\kappa(L) \sim \frac{L^2 \xi_{\perp}}{\gamma} \delta u_L. \quad (5)$$

Finally, using Eq. (1) at $r = L$, we obtain the fluctuating curvature scaling

$$\kappa(L) \sim \frac{\xi_{\perp}}{\gamma} \varepsilon_L^{1/3} L^{7/3}. \quad (6)$$

It is useful to define the corresponding mean-field, or reference, curvature by replacing ε_L with $\bar{\varepsilon}$:

$$\bar{\kappa}(L) \sim \frac{\xi_{\perp}}{\gamma} \bar{\varepsilon}^{1/3} L^{7/3}. \quad (7)$$

Thus, the fluctuating curvature can be written as

$$\kappa(L) \sim \bar{\kappa}(L) \left(\frac{\varepsilon_L}{\bar{\varepsilon}} \right)^{1/3}. \quad (8)$$

III. BREAKAGE AS A RARE-EVENT FIRST-PASSAGE PROCESS

We idealize bending failure as the first passage of the midspan curvature through a fixed, material-dependent threshold $\kappa_b > 0$. The breaking zone is $|\kappa| > \kappa_b$. For a fiber of span L , the breaking time is

$$T(L) = \inf \{t > 0 : |\kappa(t; L)| > \kappa_b\}. \quad (9)$$

In the quasistatic regime, the curvature at scale L loses memory over a time of order τ_L . We therefore model successive samples separated by τ_L as approximately independent. This renewal approximation gives a Poisson rare-event process [21], in close analogy with threshold-based breakup rates used for aggregates in turbulence [22–24]. The hazard rate at fixed span is

$$h(L) \propto \frac{1}{\tau_L} p(L, \kappa_b), \quad (10)$$

where the instantaneous curvature-exceedance probability is

$$p(L, \kappa_b) \equiv \mathbb{P}(|\kappa| > \kappa_b). \quad (11)$$

The proportionality constant depends on details of the sampling convention and is not important for the scaling arguments. Equation (10) states that the hazard is the product of an eddy sampling rate and a failure probability per sample.

The mean-field critical span L_c is defined by comparing the reference curvature with the material threshold, as in the inertial-range critical-span construction of Ref. [12]:

$$\bar{\kappa}(L_c) \sim \kappa_b. \quad (12)$$

Using Eq. (7), this gives

$$L_c \sim \left(\frac{\gamma \kappa_b}{\xi_{\perp} \bar{\varepsilon}^{1/3}} \right)^{3/7}. \quad (13)$$

For a generic span L , the threshold ratio is therefore

$$\frac{\kappa_b}{\bar{\kappa}(L)} \sim \left(\frac{L_c}{L} \right)^{7/3}. \quad (14)$$

The supercritical and subcritical branches are then $L > L_c$ and $L < L_c$, respectively. On the supercritical branch, typical curvatures already probe the breaking zone. On the subcritical branch, typical curvatures lie below threshold, and failure depends on the far tail of the curvature distribution. This critical-span construction is the same organizing scale used in

kinetic descriptions of inertial-range fiber fragmentation [12]; here, it is used as the reference scale relative to which the extreme-tail exceedance probability is measured. Combining the inertial-range, quasistatic, and subcritical restrictions gives the working domain

$$\eta \ll L < \min(L_c, L_{QS}). \quad (15)$$

For near-surface oceanic parameters representative of the ACE measurements, Appendix A gives critical spans on the order of 10 cm. The corresponding Kolmogorov length is much smaller, so the inertial-range description is the relevant one for those estimates rather than a sub-Kolmogorov velocity-gradient description.

For a Poisson process with rate $h(L)$, Eq. (10) gives the mean breaking time

$$\langle T(L) \rangle = \frac{1}{h(L)} \simeq \frac{\tau_L}{p(L, \kappa_b)}. \quad (16)$$

The sensitivity to intermittency is therefore entirely encoded in the probability with which the curvature samples the breaking zone.

IV. GAUSSIAN MEAN-FIELD BASELINE

We first consider a nonintermittent baseline. Suppose that the longitudinal velocity increment across the fiber is a centered Gaussian process with variance $\sigma_u^2(L) \equiv \langle \delta u_L^2 \rangle \propto (\bar{\varepsilon}L)^{2/3}$. Since Eq. (5) is linear in δu_L , the curvature is also Gaussian. Its variance is

$$\begin{aligned} \sigma_\kappa^2(L) &\equiv \langle \kappa^2(L) \rangle \sim \left(\frac{L^2 \xi_\perp}{\gamma} \right)^2 \sigma_u^2(L) \\ &\sim \left(\frac{\xi_\perp}{\gamma} \right)^2 \bar{\varepsilon}^{2/3} L^{14/3} \sim \bar{\kappa}^2(L). \end{aligned} \quad (17)$$

Numerical factors set by the beam shape, drag law, and boundary conditions are absorbed into the scaling constant.

For a subcritical span such that $\kappa_b \gg \sigma_\kappa(L)$, the exceedance probability is controlled by the Gaussian tail. Up to algebraic prefactors, the large-deviation estimate is

$$p_G(L, \kappa_b) = \mathbb{P}(|\kappa| > \kappa_b) = 2 \mathbb{P}(\kappa > \kappa_b) \sim \exp \left[-\frac{\kappa_b^2}{2\sigma_\kappa^2(L)} \right]. \quad (18)$$

The corresponding Gaussian hazard is

$$h_G(L) \sim \tau_L^{-1} \exp \left[-\frac{\kappa_b^2}{2\sigma_\kappa^2(L)} \right]. \quad (19)$$

The Gaussian mean breaking time is

$$\langle T(L) \rangle_G \sim \tau_L \exp \left[\frac{\kappa_b^2}{2\sigma_\kappa^2(L)} \right]. \quad (20)$$

Deep below the critical span, the ratio $\kappa_b/\sigma_\kappa(L)$ is large. The Gaussian model then predicts an extremely rapid suppression of the hazard and astronomically large breaking times. This is the baseline against which the intermittent calculation should be compared. It is not meant as a complete model of turbulent fluctuations; it is a mean-field reference in which the cascade variability of ε_L is switched off.

V. INTERMITTENT CURVATURE STATISTICS

We now incorporate intermittency through a multiplicative cascade for the coarse-grained dissipation [25–27]. This choice is used as a K62/log-normal benchmark: It is analytically transparent, but it is not assumed to be a universal representation of turbulent intermittency. The dissipation at scale L is generated by the product of independent positive multipliers along the cascade from L_{inj} down to L . Writing $L = L_{\text{inj}}2^{-n}$, we take

$$\varepsilon_L = \bar{\varepsilon} \prod_{k=1}^n \beta_k, \quad (21)$$

with $\beta_k > 0$ and $\mathbb{E}[\beta_k] = 1$. Thus,

$$\chi_L \equiv \ln \left(\frac{\varepsilon_L}{\bar{\varepsilon}} \right) = \sum_{k=1}^n \ln \beta_k. \quad (22)$$

For large-scale separations, the central limit theorem motivates an approximately Gaussian distribution for χ_L . In the Kolmogorov-Obukhov 1962 log-normal model [14,15], we write

$$\chi_L \sim \mathcal{N}(\mu_\chi(L), \sigma_\chi^2(L)), \quad \sigma_\chi^2(L) = \mu \ln \left(\frac{L_{\text{inj}}}{L} \right). \quad (23)$$

The dimensionless parameter μ measures the intermittency strength of the coarse-grained dissipation; we use the conservative value $\mu \simeq 0.2$ from standard estimates [28]. The constraint $\langle \varepsilon_L \rangle = \bar{\varepsilon}$ fixes the mean of χ_L as $\mu_\chi(L) = -\sigma_\chi^2(L)/2$. Equivalently, $\varepsilon_L = \bar{\varepsilon} \exp(\chi_L)$.

Combining Eq. (8) with the log-normal model gives a log-normal model for the curvature amplitude. Setting $Z_L = \chi_L/3$, one has

$$\begin{aligned} Z_L &\sim \mathcal{N}(\mu_Z(L), \sigma_Z^2(L)), \\ \mu_Z(L) &= \frac{1}{3}\mu_\chi(L), \quad \sigma_Z^2(L) = \frac{1}{9}\sigma_\chi^2(L). \end{aligned} \quad (24)$$

The curvature amplitude is then

$$\kappa(L) \simeq \bar{\kappa}(L) \exp Z_L. \quad (25)$$

Equation (25) should be understood as a closure for the fiber-scale curvature amplitude associated with the instantaneous quasistatic bending response to the turbulent load. A raw geometric curvature extracted from a time-evolved fiber configuration may also contain dimensionless response factors associated with the instantaneous load profile, orientation, and projection onto bending modes. In the present scaling theory, these factors are assumed to be absorbed into the calibration of $\bar{\kappa}(L)$, or, in direct comparisons with resolved data, to be accounted for by using an appropriately response-normalized quasistatic curvature amplitude.

The sign of the curvature is immaterial for bending failure, so we apply the log-normal model to $|\kappa|$. For positive argument $\kappa > 0$, the probability density is

$$\mathcal{P}_L(\kappa) = \frac{1}{\kappa \sigma_Z(L) \sqrt{2\pi}} \exp \left[-\frac{(\ln[\kappa/\bar{\kappa}(L)] - \mu_Z(L))^2}{2\sigma_Z^2(L)} \right]. \quad (26)$$

The exceedance probability is

$$p_\beta(L, \kappa_b) = \mathbb{P}(|\kappa| > \kappa_b) = \frac{1}{2} \operatorname{erfc} \left[\frac{\ln[\kappa_b/\bar{\kappa}(L)] - \mu_Z(L)}{\sqrt{2}\sigma_Z(L)} \right]. \quad (27)$$

For strongly subcritical thresholds, $\kappa_b \gg \bar{\kappa}(L)$, the complementary-error-function argument in Eq. (27) is large. Keeping the leading quadratic contribution gives

$$p_\beta(L, \kappa_b) \sim \exp \left[-\frac{\ln^2[\kappa_b/\bar{\kappa}(L)]}{2\sigma_Z^2(L)} \right]. \quad (28)$$

The nonzero mean $\mu_Z(L)$ and the algebraic prefactor in the asymptotic expansion of the complementary error function give subleading, model-dependent corrections to prefactors and to the local algebraic exponent at finite scale separation. The leading result in Eq. (28) is the weak-intermittency large-deviation form used below.

Equation (28) is the central statistical difference between the intermittent and Gaussian pictures. The Gaussian baseline in Eq. (18) has a tail controlled by κ_b^2 . The intermittent model has a tail controlled by the square of $\ln[\kappa_b/\bar{\kappa}(L)]$. For moderately subcritical thresholds, the two descriptions can be comparable. For strongly subcritical thresholds and sufficiently deep inertial ranges, however, the log-normal tail is much larger than the Gaussian tail. At fixed L , turbulent intermittency therefore enhances the probability of a breaking curvature burst by many orders of magnitude.

The log-normal construction above uses a random multiplicative cascade to obtain a closed expression. The rare-event mapping itself is broader: Appendix C formulates the same threshold condition in terms of a generic multifractal spectrum, while Appendix D gives the hazard obtained when the far tail of $\varepsilon_L/\bar{\varepsilon}$ is modeled by a stretched exponential rather than by a log-normal law.

VI. SUBCRITICAL HAZARD AND EFFECTIVE DISSIPATION

The previous section gives the probability that one quasistatic eddy-time sample produces a curvature above threshold. We now translate that single-sample probability into a rate and then into an equivalent mean-field representation. These are distinct steps. First, the intermittent exceedance probability is inserted into the Poisson sampling relation to obtain a hazard and a mean first-passage time. Second, a reference rarity level is used to define the mean dissipation that a nonintermittent model would need in order to reproduce the same probability. This construction keeps the material threshold fixed and assigns the statistical enhancement only to an effective mean dissipation in the equivalent model.

Substituting Eq. (28) into the Poisson estimate gives the intermittent subcritical hazard

$$h_\beta(L) \sim \tau_L^{-1} \exp \left[-\frac{\ln^2[\kappa_b/\bar{\kappa}(L)]}{2\sigma_Z^2(L)} \right]. \quad (29)$$

The corresponding mean breaking time is

$$\langle T(L) \rangle_\beta \sim \tau_L \exp \left[\frac{\ln^2[\kappa_b/\bar{\kappa}(L)]}{2\sigma_Z^2(L)} \right]. \quad (30)$$

Comparing Eqs. (20) and (30) shows how intermittency changes the subcritical first-passage problem. In the Gaussian model, the exponent scales with the square of the threshold measured in units of the rms curvature. In the intermittent model, the exponent scales with the square of the logarithmic threshold ratio. Deep below L_c , this difference reduces the predicted mean breaking time by many orders of magnitude.

For fixed material threshold, Eqs. (14), (23), and (28) imply an algebraic subcritical hazard. Up to scale-independent prefactors and subleading logarithmic corrections, one obtains

$$p_\beta(L, \kappa_b) \propto L^\alpha, \quad h_\beta(L) \propto L^\zeta, \quad \langle T(L) \rangle_\beta \propto L^{-\zeta},$$

$$\alpha = \frac{49}{2\mu}, \quad \zeta = \alpha - \frac{2}{3}. \quad (31)$$

Appendix B gives the derivation and also shows how the local exponent approaches Eq. (31). Although these exponents are large for realistic μ , the key point is qualitative: The intermittent hazard is algebraic in span, whereas the nonintermittent Gaussian hazard is suppressed much more violently. This algebraic span dependence is the K62/log-normal result; it is not assumed to be universal. Appendix D shows that stretched-exponential dissipation tails lead instead to stretched-exponential hazards.

It is useful to rephrase this result in a renormalization language [29]. We integrate out the intermittent fluctuations of ε_L and ask which effective mean dissipation would make a nonintermittent model reproduce the same rare-event level. Choose a small reference exceedance probability per eddy-time sample, $p^* \ll 1$. The reference span L^* on the subcritical branch is defined by

$$p_\beta(L^*, \kappa_b) = p^*. \quad (32)$$

Using Eq. (16), this is equivalently

$$p^* = \frac{\tau_{L^*}}{\langle T(L^*) \rangle}. \quad (33)$$

We define the effective dissipation by requiring the mean-field critical span computed with ε_{eff} to coincide with L^* :

$$L_c(\varepsilon_{\text{eff}}) = L^*. \quad (34)$$

Since Eq. (13) gives $L_c(\varepsilon) \propto \varepsilon^{-1/7}$, we obtain

$$\frac{\varepsilon_{\text{eff}}}{\bar{\varepsilon}} \sim \left(\frac{L_c}{L^*} \right)^7. \quad (35)$$

Thus, the intermittent cascade is mapped onto a single effective-dissipation parameter in the equivalent nonintermittent model.

Within the log-normal model, this ratio can be expressed in terms of the intermittency parameter, the inertial-range depth $\Lambda = L_{\text{inj}}/L_c$, and the chosen rarity level. We also write $\alpha^* = \operatorname{erfc}^{-1}(2p^*)$. Solving Eq. (32) asymptotically for weak intermittency and a large but finite inertial range gives

$$\ln \left(\frac{\varepsilon_{\text{eff}}}{\bar{\varepsilon}} \right) \simeq \alpha^* \sqrt{2\mu} \ln \Lambda - \frac{\mu}{2} \ln \Lambda. \quad (36)$$

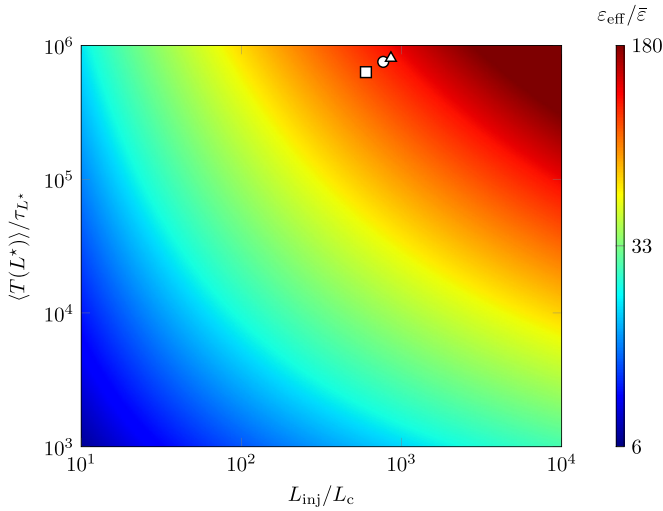


FIG. 2. Effective dissipation ratio $\varepsilon_{\text{eff}}/\bar{\varepsilon}$ as a function of the outer-to-critical scale ratio L_{inj}/L_c and of the dimensionless mean waiting time at the reference span, $\langle T(L^*) \rangle/\tau_{L^*} = 1/p^*$. Both axes are logarithmic. Colors use a logarithmic scale in $\varepsilon_{\text{eff}}/\bar{\varepsilon}$; the colorbar ticks show the minimum and maximum values, together with an intermediate level. Symbols indicate ACE near-surface fiber classes: circle, regenerated cellulose/viscose-rayon; square, cotton; and triangle, wool. They are evaluated at the neuston depth $z \simeq 0.2$ m using the ACE median diameter $d = 16.7$ μm . For each class, the probability level p^* is chosen so that $\langle T(L^*) \rangle = 30$ days. The estimates used to place the symbols are given in Appendix A. The color map is the K62/log-normal effective-dissipation estimate.

In terms of dimensional input parameters, the same expression is

$$\frac{\varepsilon_{\text{eff}}}{\bar{\varepsilon}} \simeq \exp \left\{ \text{erfc}^{-1} \left[2 \frac{\tau_{L^*}}{\langle T(L^*) \rangle} \right] \sqrt{2\mu \ln \left(\frac{L_{\text{inj}}}{L_c} \right)} - \frac{\mu}{2} \ln \left(\frac{L_{\text{inj}}}{L_c} \right) \right\}. \quad (37)$$

The effective-dissipation ratio is therefore controlled by three dimensionless ingredients: the intermittency strength μ , the inertial-range depth L_{inj}/L_c , and the dimensionless waiting time at the reference span.

Figure 2 shows that representative combinations of inertial-range depth and waiting time can produce effective dissipations up to 2 orders of magnitude larger than the bare mean. The ACE symbols provide illustrative parameter locations derived in Appendix A. Under the Poisson rare-event construction, this enhancement translates into a large reduction of the mean breaking time for subcritical fibers. Configurations that would be effectively unbreakable in a Gaussian mean-field theory can therefore be promoted to observable breakage on experimental or environmental timescales.

VII. DISCUSSION

The mechanism identified here rests on three ingredients. First, the fiber must be long enough to sample inertial-range velocity differences, rather than only a smooth velocity gradient. Second, the bending dynamics must be quasistatic, so

that the curvature is set by the instantaneous velocity increment across the fiber through the turbulent load. Third, the turbulent dissipation at the fiber scale must be intermittent, because refined similarity relates the amplitude of those velocity increments to the coarse-grained dissipation. Under these conditions, the breakage problem becomes a first-passage problem for the far tail of a curvature distribution whose amplitude is controlled by the cascade.

The log-normal cascade used above is a convenient analytical realization of intermittency, not a claim that the K62 form is exact. In particular, the algebraic subcritical law is a consequence of this closure, whereas the first-passage formulation can be evaluated with other tail models. Other multifractal descriptions would replace the quadratic large-deviation function in Eq. (28) by a different rate function. Direct numerical simulations also show that extreme one-point dissipation and enstrophy tails can deviate from log-normality and exhibit stretched-exponential behavior [30]. Such changes can modify numerical exponents, prefactors, the functional form of the far-tail hazard, and finite-range corrections. They do not remove the central point: A broad intermittent tail converts a nominally inaccessible threshold into a rare but finite event channel.

The Poisson clock in Eq. (10) uses the turnover time based on the mean dissipation. This choice separates the statistics of burst amplitudes from the sampling rate and gives a transparent first estimate. A more detailed model could use a local, event-conditioned turnover time depending on ε_L . That refinement would introduce additional model dependence in prefactors and possibly in subleading exponents, but it would not eliminate the rare velocity-increment events that drive subcritical failure.

The effective dissipation in Eq. (37) should also be interpreted carefully. The material threshold κ_b is not changed. Rather, ε_{eff} is the value of the mean dissipation that a nonintermittent mean-field model would need in order to reproduce a specified intermittent exceedance probability. In that sense, intermittency lowers the apparent mean-field critical span and increases the effective dissipation assigned to the equivalent mean-field description, but it does not alter the material property itself.

Finally, the estimates for ACE fibers in Appendix A are parameter placements, not a complete environmental fragmentation model. They use representative material strengths, a near-surface dissipation estimate, and a prescribed one-month waiting time to show where the fiber classes fall in the dimensionless plane of Fig. 2. A predictive oceanic model would need source terms, residence times, transport, surface-retention effects, aging, biofouling, and length-dependent removal. The present calculation isolates one component of that larger problem: the effect of turbulent intermittency on the breakage hazard at fixed span.

VIII. CONCLUSIONS

We have shown that turbulent intermittency opens a subcritical channel for fiber breakage. In the quasistatic inertial-range regime, the curvature of a slender fiber is set by the velocity increment across the fiber. Refined similarity then relates the amplitude of that increment to the

coarse-grained dissipation at the fiber scale. Rare fluctuations of the velocity increment across the fiber are therefore converted into rare curvature bursts. Their probability inherits the broad tails of the intermittent coarse-grained dissipation, allowing fibers well below the mean-field critical span to cross a fixed material breaking threshold.

A nonintermittent Gaussian model predicts a subcritical hazard that is suppressed by an exponential of the squared threshold measured in rms-curvature units. The intermittent log-normal cascade replaces this by a large-deviation form involving the square of a logarithmic threshold ratio. On the subcritical branch, this produces an algebraic hazard in span, with exponents controlled by the intermittency parameter. The algebraic law is therefore a closure-dependent K62/log-normal prediction; the Appendixes show how the same first-passage construction is rewritten for a direct multifractal spectrum and for stretched-exponential dissipation tails. The same effect can be mapped onto an effective dissipation in an equivalent nonintermittent model; this effective value may exceed the bare mean by orders of magnitude under realistic parameter choices.

The main implication is that subcritical fibers should not automatically be treated as inactive. Their typical curvature is below threshold, but rare velocity-increment events can dominate breakage statistics over long observation times. Accounting for intermittency is therefore essential for quantitative predictions of turbulent fiber fragmentation in natural and industrial flows.

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DATA AVAILABILITY

No new experimental or numerical data were generated in this work. The parameter estimates used for Fig. 2 are obtained from the published ACE measurements cited in the text and from the assumptions listed explicitly in Appendix A.

APPENDIX A: ACE NEAR-SURFACE MICROFIBER PARAMETERS

This Appendix lists the parameter choices used to place the ACE markers in Fig. 2. The ACE surface samples are dominated by millimetric cellulose-based fibers, including regenerated cellulose/viscose-rayon and cotton, and by natural animal fibers such as wool. We use the representative ACE median diameter $d = 16.7 \mu\text{m}$ reported in Ref. [2].

1. Near-surface dissipation at the neuston depth

During ACE, the expedition-wide median neutral wind speed was $U_{10} = 7.25 \text{ m s}^{-1}$ [4]. We use the standard air-sea stress closure $\tau_a = \rho_a C_D U_{10}^2$, where τ_a is the air-side stress, ρ_a is the air density, and C_D is the drag coefficient. We take $C_D = 1.3 \times 10^{-3}$. The air-side friction velocity is $u_{*a} = \sqrt{C_D} U_{10}$. Continuity of interfacial momentum flux gives the

water-side value $u_{*w} = u_{*a} \sqrt{\rho_a / \rho_w}$, where ρ_w is the water density. Using $\rho_a / \rho_w \simeq 1.2/1025$, one obtains $u_{*w} \simeq 9 \times 10^{-3} \text{ m s}^{-1}$.

For the wave-affected near-surface layer, we use the parametrization

$$\bar{\varepsilon}(z) \equiv \varepsilon_t(z) = A \frac{u_{*w}^3}{\kappa_K z}. \quad (\text{A1})$$

Here, $\kappa_K = 0.4$ is the von Kármán constant, the representative neuston depth is $z = 0.2 \text{ m}$, and $A \in [20, 100]$. This gives $\bar{\varepsilon}(0.2 \text{ m}) \in [1.8, 9] \times 10^{-4} \text{ m}^2 \text{ s}^{-3}$. Throughout the placement estimate, we use the central value $\bar{\varepsilon}(0.2 \text{ m}) = 4 \times 10^{-4} \text{ m}^2 \text{ s}^{-3}$.

2. Injection scale

Since ACE took place in austral summer [4], we use the representative outer scale $L_{\text{inj}} = 100 \text{ m}$ for the surface mixed layer or wave-driven mixing depth relevant to the neuston layer.

3. Material-dependent critical span

In the mean-field picture, L_c separates supercritical spans, for which typical turbulent bending reaches the failure threshold, from subcritical spans, for which failure is rare-event dominated. To place the ACE markers, we estimate L_c for each material by equating the peak viscous bending stress of a slender cylinder to a representative wet or field-aged material strength σ^{mat} . For inertial-range forcing at scale L , the effective strain rate is $\dot{\gamma}_L \sim (\bar{\varepsilon}/L^2)^{1/3}$. The peak stress estimate is

$$\sigma_{\text{max}}(L) \simeq \frac{8}{3\pi} \xi_{\perp}(L) \dot{\gamma}_L \frac{L^3}{d^3}, \quad (\text{A2})$$

where $\xi_{\perp}(L)$ is the span-dependent transverse drag coefficient. We use the seawater dynamic viscosity $\mu_{\text{sw}} = 10^{-3} \text{ Pa s}$, and estimate

$$\xi_{\perp}(L) \simeq \frac{4\pi \mu_{\text{sw}}}{\ln(2L/d) + 0.5}. \quad (\text{A3})$$

The critical span solves $\sigma_{\text{max}}(L_c) = \sigma^{\text{mat}}$. For the three ACE fiber classes, we use the representative material strengths $\sigma_{\text{visc}}^{\text{mat}} = 140 \text{ MPa}$, $\sigma_{\text{cotton}}^{\text{mat}} = 250 \text{ MPa}$, and $\sigma_{\text{wool}}^{\text{mat}} = 110 \text{ MPa}$. These values are used only to place the symbols in Fig. 2. Solving the critical-span condition with the central dissipation value above gives $L_c^{\text{visc}} \simeq 0.129 \text{ m}$, $L_c^{\text{cotton}} \simeq 0.167 \text{ m}$, and $L_c^{\text{wool}} \simeq 0.116 \text{ m}$. The horizontal coordinates in Fig. 2 are therefore $L_{\text{inj}}/L_c \simeq 7.7 \times 10^2$ for regenerated cellulose/viscose-rayon, 6.0×10^2 for cotton, and 8.6×10^2 for wool.

4. Vertical coordinate

The vertical axis in Fig. 2 is the dimensionless mean waiting time at the reference span, $\langle T(L^*) \rangle / \tau_{L^*} = 1/p^*$. To give the ACE markers a direct physical-time interpretation, we choose p^* so that $\langle T(L^*) \rangle \equiv T_{\text{mon}} = 30 \text{ days}$. The Poisson estimate then fixes $1/p^* = T_{\text{mon}} / \tau_{L^*}$. For spans on the order of the critical spans listed above, the eddy-turnover time is on the order of a few seconds. Hence, $p^* \simeq 1.4 \times 10^{-6}$ and $\langle T(L^*) \rangle / \tau_{L^*} = 1/p^* \simeq 7 \times 10^5$. This corresponds to mean breaking times on the order of one month, with variations of about 25–35 days across the three ACE fiber spans.

APPENDIX B: ASYMPTOTIC SCALING EXPONENTS ON THE SUBCRITICAL BRANCH

This Appendix derives the power-law exponents α and ζ used in Sec. VI. The starting point is the large-deviation tail of the curvature-exceedance probability,

$$p_\beta(L, \kappa_b) \sim \exp \left[-\frac{\ln^2[\kappa_b/\bar{\kappa}(L)]}{2\sigma_Z^2(L)} \right]. \quad (\text{B1})$$

The critical span is defined by Eq. (12). Since the reference curvature scales as $L^{7/3}$, the threshold ratio is $\kappa_b/\bar{\kappa}(L) \sim (L_c/L)^{7/3}$. The log-normal variance of Z_L is $\sigma_Z^2(L) = (\mu/9)\ln(L_{\text{inj}}/L)$. Substituting these relations into Eq. (B1) gives

$$\ln p_\beta(L, \kappa_b) \sim -\frac{49}{2\mu} \frac{\ln^2(L_c/L)}{\ln(L_{\text{inj}}/L)}. \quad (\text{B2})$$

It is useful to define $y = \ln(L_c/L)$ and $\lambda = \ln(L_{\text{inj}}/L_c)$. Then, $\ln(L_{\text{inj}}/L) = \lambda + y$, and Eq. (B2) becomes

$$\ln p_\beta(L, \kappa_b) \simeq -\frac{49}{2\mu} \frac{y^2}{\lambda + y}. \quad (\text{B3})$$

The local scaling exponent is defined as $\alpha(L) \equiv d \ln p_\beta(L, \kappa_b)/d \ln L$. Since $d \ln L = -dy$, this is $\alpha(L) = -d \ln p_\beta/dy$. Differentiating Eq. (B3) gives

$$\alpha(L) = \frac{49}{2\mu} \frac{y(2\lambda + y)}{(\lambda + y)^2}. \quad (\text{B4})$$

This expression shows the slow finite-range variation of the local exponent.

Deep in the subcritical inertial branch, with $y \gg \lambda$, the ratio in Eq. (B4) tends to unity. Therefore, $\alpha(L) \rightarrow \alpha = 49/(2\mu)$, and the exceedance probability can be written, up to scale-independent prefactors and subleading logarithmic corrections, as $p_\beta(L, \kappa_b) \propto L^\alpha$.

The hazard follows from $h_\beta(L) \sim \tau_L^{-1} p_\beta(L, \kappa_b)$, with $\tau_L \sim \bar{\varepsilon}^{-1/3} L^{2/3}$. Thus, the local hazard exponent is $\zeta(L) \equiv d \ln h_\beta(L)/d \ln L = \alpha(L) - 2/3$. In the asymptotic regime, this becomes $\zeta = \alpha - 2/3 = 49/(2\mu) - 2/3$. Using Eq. (16), the mean breaking time scales as $\langle T(L) \rangle_\beta \propto L^{-\zeta}$. Thus, the subcritical exponents are controlled by the intermittency parameter μ in the log-normal model. In more general cascades, the same calculation is replaced by the corresponding large-deviation rate function for $\ln \varepsilon_L$.

APPENDIX C: DIRECT MULTIFRACTAL-SPECTRUM FORMULATION OF THE SUBCRITICAL HAZARD

This Appendix rewrites the subcritical threshold condition without using a specific random-multiplier construction. The multifractal-spectrum formulation used here is the standard $D(h)$ description of inertial-range turbulence [13,25,27,28,31–33]. To avoid confusion with the hazard rate $h(L)$, we denote the local Hölder exponent of the inertial-range velocity field by $\mathfrak{h}$. At scale L , the multifractal description assigns velocity increments and probabilities of the form

$$\delta u_L(\mathfrak{h}) \sim U \left(\frac{L}{L_{\text{inj}}} \right)^\mathfrak{h}, \quad \mathbb{P}_\mathfrak{h}(L) \propto \left(\frac{L}{L_{\text{inj}}} \right)^{3-D(\mathfrak{h})}. \quad (\text{C1})$$

Here, U is a large-scale velocity, L_{inj} is the injection scale, and $D(h)$ is the multifractal spectrum. We also define the codimension $C(h) = 3 - D(h)$. Refined similarity [14,15,34] gives the corresponding coarse-grained dissipation as

$$\frac{\varepsilon_L}{\bar{\varepsilon}} \sim \left(\frac{L}{L_{\text{inj}}} \right)^{3\mathfrak{h}-1} = \exp[(1-3\mathfrak{h})\lambda_L], \quad \lambda_L \equiv \ln \left(\frac{L_{\text{inj}}}{L} \right). \quad (\text{C2})$$

The breakage condition follows directly from Eqs. (8) and (14). Since $\kappa(L)/\bar{\kappa}(L) \sim (\varepsilon_L/\bar{\varepsilon})^{1/3}$, crossing the material threshold is equivalent, at scaling accuracy, to

$$\frac{\varepsilon_L}{\bar{\varepsilon}} > \left(\frac{L_c}{L} \right)^7. \quad (\text{C3})$$

Combining Eqs. (C2) and (C3) gives the threshold condition

$$1 - 3\mathfrak{h} > a_*(L), \quad a_*(L) \equiv 7 \frac{\ln(L_c/L)}{\ln(L_{\text{inj}}/L)}. \quad (\text{C4})$$

Equivalently, breakage requires $\mathfrak{h} < \mathfrak{h}_*(L)$, with $\mathfrak{h}_*(L) = [1 - a_*(L)]/3$. On the singular branch relevant to strong events, the exceedance probability is controlled by the codimension at this boundary exponent:

$$p_D(L, \kappa_b) \propto \exp[-\lambda_L C(\mathfrak{h}_*(L))]. \quad (\text{C5})$$

The corresponding hazard is therefore

$$h_D(L) \propto \tau_L^{-1} \exp[-\lambda_L C(\mathfrak{h}_*(L))], \quad (\text{C6})$$

where τ_L is the eddy-turnover time used in Eq. (10). The K62/log-normal closure corresponds to the parabolic codimension $C_{K62}(\mathfrak{h}) = (1 - 3\mathfrak{h})^2/(2\mu)$, with μ the intermittency parameter [13–15]. Substituting this codimension in Eq. (C6) recovers the leading log-normal result used in the main text. Other spectra $D(h)$ change the codimension sampled at the threshold, and therefore the quantitative hazard, without changing the first-passage structure of the calculation.

APPENDIX D: STRETCHED-EXPONENTIAL FAR-TAIL ESTIMATE

This Appendix gives a tail-level sensitivity estimate for the subcritical hazard when the far tail of the normalized coarse-grained dissipation is not described by the K62/log-normal closure. The stretched-exponential form below is used as a conditional far-tail model, motivated by numerical evidence that extreme small-scale dissipation and enstrophy statistics need not be log-normal and are often represented by stretched-exponential fits [30,35]. It is not assumed to be a universal law for ε_L . Let $X_L \equiv \varepsilon_L/\bar{\varepsilon}$ be the dissipation coarse grained at the fiber scale and normalized by its mean. For large values of the positive argument X , assume that the survival probability of X_L has the stretched-exponential form

$$S_L(X) \equiv \mathbb{P}(X_L > X) \simeq \mathcal{A}_L(X) \exp[-\mathcal{B}_L X^q]. \quad (\text{D1})$$

Here, $S_L(X)$ is the survival probability at scale L , $\mathcal{A}_L(X)$ is a positive subexponential prefactor, $\mathcal{B}_L > 0$ is the tail-rate coefficient, and $q > 0$ is the stretching exponent. By subexponential, we mean that $\mathcal{A}_L(X)$ does not contribute to the leading X^q rate in the tail. The curvature threshold corresponds to the dissipation threshold $X_L > X_b(L)$, with $X_b(L) = (L_c/L)^7$, as

follows from Eq. (C3). Inserting this threshold into Eq. (D1) gives

$$p_{\text{st}}(L, \kappa_b) \simeq \mathcal{A}_L(X_b(L)) \exp \left[-\mathcal{B}_L \left(\frac{L_c}{L} \right)^{7q} \right]. \quad (\text{D2})$$

The subscript “st” labels this stretched-tail estimate. The associated hazard and mean breaking time are

$$h_{\text{st}}(L) \simeq \tau_L^{-1} \mathcal{A}_L(X_b(L)) \exp \left[-\mathcal{B}_L \left(\frac{L_c}{L} \right)^{7q} \right], \quad (\text{D3})$$

$$\langle T(L) \rangle_{\text{st}} \simeq \tau_L [\mathcal{A}_L(X_b(L))]^{-1} \exp \left[\mathcal{B}_L \left(\frac{L_c}{L} \right)^{7q} \right]. \quad (\text{D4})$$

Thus, a stretched-exponential dissipation tail produces a stretched-exponential dependence of the subcritical hazard on L_c/L , rather than the algebraic dependence obtained from the K62/log-normal closure. If the same effective-dissipation construction is applied at a prescribed rarity level p^* , with L^* determined by $p_{\text{st}}(L^*, \kappa_b) = p^*$, and the subexponential prefactor is neglected, Eqs. (35) and (D2) give

$$\frac{\varepsilon_{\text{eff}}^{\text{st}}}{\bar{\varepsilon}} \simeq \left[\frac{\ln(1/p^*)}{\mathcal{B}_{L^*}} \right]^{1/q}. \quad (\text{D5})$$

Here, $\mathcal{B}_{L^*}$ is the tail-rate coefficient evaluated at the reference span L^* . Equation (D5) makes explicit that the numerical value of the effective dissipation depends on the assumed far-tail model, whereas the rare-event first-passage formulation remains the same.

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