

Strong coupling in a three-mode superconducting-spin hybrid quantum system

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We demonstrate strong coupling in a solid-state hybrid quantum system comprising a fixed-frequency coplanar-waveguide resonator, a superconducting transmon qubit, and an ensemble of nitrogen-vacancy centers in diamond. Frequency-domain spectroscopy reveals a characteristic three-mode avoided crossing, consistent with excitations being distributed across all three subsystems through hybridized eigenmodes. At higher probe powers, we observe nonlinear features and signatures of transmon-¹⁴N nuclear-spin interactions, highlighting the accessibility of higher-excitation manifolds in this architecture. These results establish an experimentally accessible regime of hybrid cavity QED that integrates superconducting and spin degrees of freedom, providing a platform for exploring complex multicomponent dynamics and developing hybrid quantum interfaces.

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I. INTRODUCTION

Hybrid quantum systems combine complementary physical platforms to realize functionality unattainable in any single constituent [1–3]. Among these platforms, superconducting circuits equipped with long-lived spin ensembles unite distinct and complementary strengths [4–6]: Superconducting qubits offer strong nonlinearities and fast control [7], while solid-state spin ensembles provide long coherence times and large collective coupling strengths [8,9]. Integrating these elements in a single device enables coherent conversion, storage, and manipulation of quantum information across disparate degrees of freedom and forms a promising route toward scalable hybrid quantum networks [2,10–13].

A key step toward such architectures is the realization of coherent interactions involving multiple quantum components. Experimental progress has been established in strong coupling between microwave resonators and spin ensembles, such as color centers [14–19], ferromagnets [20–22], and doped ions [14,23], and between spin ensembles and superconducting qubits [24–27], based on which the

feasibility of quantum state transfer, storage, and retrieval is tested. These efforts have primarily relied on direct two-body interactions [24,26] or mediator-based coupling protocols [25], which suffer from scalability challenges due to increasing hardware control overhead and accumulated errors. To overcome these limitations, an efficient route is to operate in a multiple-resonance regime, which enables energy and quantum information to be simultaneously shared among all relevant modes. However, resolving the full hybridized eigenmode structure of a superconducting qubit, microwave resonator, and solid-state spin ensemble in a single, simultaneously strongly coupled device has remained experimentally challenging. Such a regime requires compatibility between local flux tuning of the superconducting circuit, global magnetic-field tuning of the spin ensemble, and probe powers low enough to preserve the transmon response while still resolving the ensemble spectrum.

In this work, we realize strong coupling among a fixed-frequency bus resonator, a superconducting transmon qubit, and an ensemble of negatively charged nitrogen-vacancy (NV[−]) centers in diamond, as shown in Fig. 1. By tuning the transmon into resonance with both the resonator and the spin ensemble, we access a regime in which all coupling rates exceed relevant decoherence rates. Frequency-domain spectroscopy reveals a three-mode avoided crossing with well-resolved polaritonic branches, confirming that excitations are coherently and collectively shared across all three subsystems through their hybridized eigenmodes. At higher probe powers, we observe nonlinear responses,

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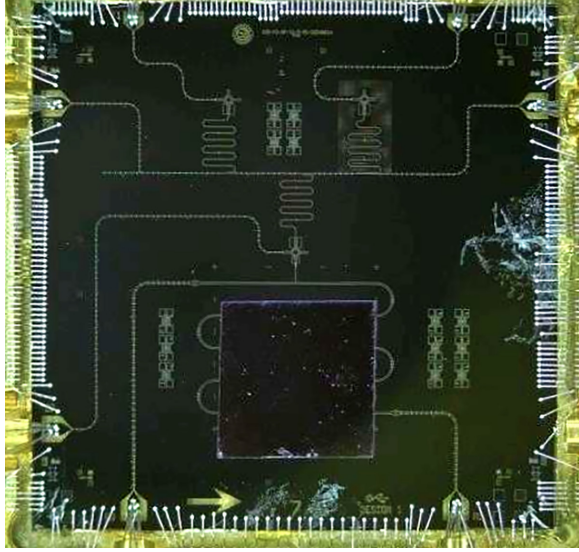


FIG. 1. Optical micrograph of the chip consisting of a CPW bus resonator, a transmon, and a diamond placed on it. The superconducting circuit is fabricated on a sapphire substrate with a 200 nm tantalum base layer; the CPW resonator, readout resonator, and control lines are patterned by laser lithography, and the aluminum Josephson junctions are defined by electron-beam lithography. The diamond crystal, cut along the (111) plane, is the small black square positioned at the center of the CPW resonator.

including supersplitting, multiphoton-transition signatures, and hyperfine-assisted features, consistent with access to excitation manifolds beyond the simple one-excitation picture. Together, these results establish a unified solid-state platform in which superconducting, photonic, and spin degrees of freedom are spectroscopically resolved within a single simultaneously strongly coupled device.

II. EXPERIMENT

Our hybrid system, illustrated in Figs. 2(a) and 2(b), comprises a superconducting transmon qubit, a fixed-frequency

coplanar-waveguide (CPW) bus resonator, and an ensemble of NV^- centers. The transmon is capacitively coupled to the bus resonator, with its transition frequency tunable via a dc flux bias (Φ). The diamond containing the NV^- ensemble is positioned at the antinode of the magnetic field of the resonator to maximize coupling. By applying an external magnetic field (B), the NV^- transition frequencies can be tuned through the Zeeman effect, allowing controlled resonance with the bus resonator. The system's spectral response is probed using frequency-domain microwave transmission measurements under weak excitation conditions.

The individual couplings between the bus resonator and each subsystem are shown in Figs. 2(c) and 2(d). Detailed device characterization, pairwise coupling measurements, and extraction of the relevant linewidths are provided in Appendix A. With the diamond loaded, the bus frequency is 3.007 GHz and has linewidth (FWHM) $\kappa/2\pi = 0.171$ MHz. The bus-transmon spectroscopy is measured in the absence of an external magnetic field, which ensures that the frequency of the ensemble is far detuned from the bus and effectively decoupled. A clear avoided crossing is observed in Fig. 2(c), corresponding to a bus-transmon coupling strength of $g_t/2\pi = 17.490 \pm 0.208$ MHz. The probe power is then increased by 55 dB to measure the bus-ensemble spectroscopy, so that the bus-transmon coupling is suppressed while the bus-ensemble system is still in the linear-response regime. By optimizing the angle and amplitude of the external magnetic field, the NV^- ensemble is tuned to resonance with the bus mode. From the resonance spectra [Fig. 2(d)], the normal-mode splitting suggests the collective bus-ensemble coupling of $\Omega_e/2\pi = 6.597 \pm 0.092$ MHz.

III. RESULTS

Reducing the probe power to single-excitation level places the system in the regime where the bus resonator coupled to the qubit-ensemble collective subsystem can be observed. The transmon frequency is swept and the transmitted signal is recorded, yielding the spectrum shown in Fig. 3. It reveals the system smoothly transforming from nearly a

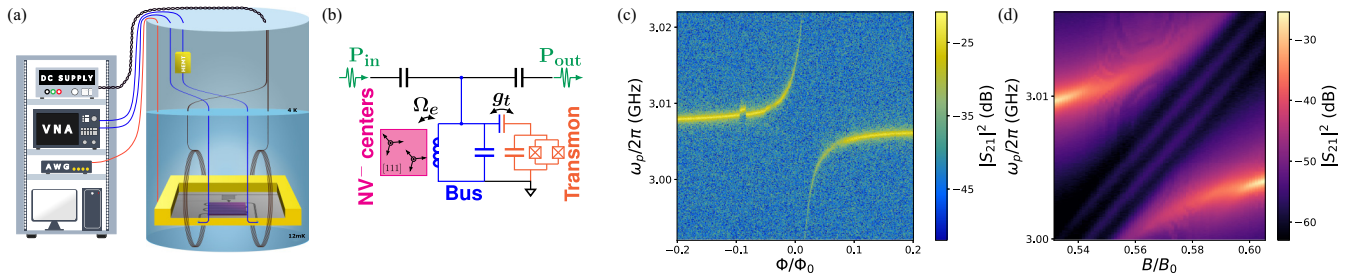


FIG. 2. (a) Schematic illustration of the setup, including control and measurement systems, and the hybrid quantum system. The chip is mounted in the mixing chamber of a dilution refrigerator operating at about 12 mK. Input lines are equipped with cryogenic attenuators and filters to suppress thermal noise, while the output signal is routed through a circulator to cryogenic and then room temperature amplifier before detection by a vector network analyzer. A magnetic field is applied using a three-dimensional Helmholtz coil, and the transmon frequency is tuned by a dc bias line. (b) Circuit diagram of the hybrid quantum system. (c) Bus-transmon transmission spectra as dc bias (Φ) is changed in the diamond-loaded case. The small discontinuity is caused by a flux-jump event. (d) Bus-ensemble transmission spectra as the NV^- ensemble is tuned into resonance with the bus mode via adjusting magnetic-field tuning (B), when the transmon is decoupled. Additional weak peaks between the polaritons arise from the hyperfine structure of NV^- centers associated with the intrinsic ^{14}N nuclear spins.

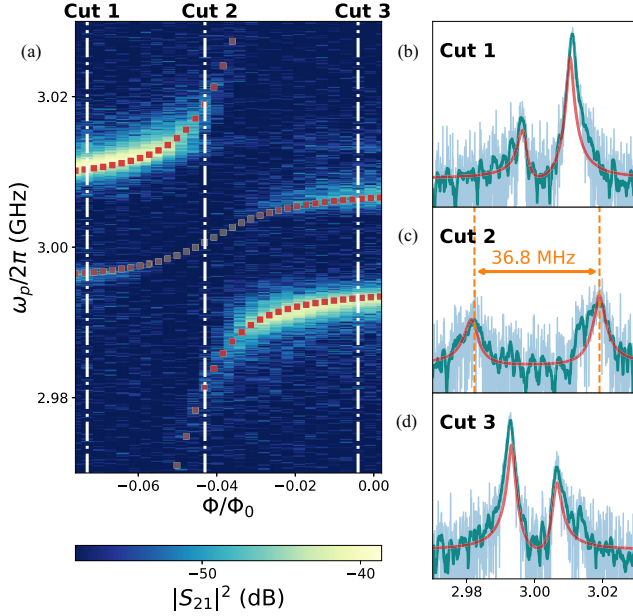


FIG. 3. (a) Transmission spectroscopy of the hybrid system at a probe power of -70 dBm as the transmon frequency is swept across resonance with the bus and NV^- ensemble. Red dotted lines: transition frequencies of the hybrid system under one-excitation approximation, with color shade indicating the proportion of photons in the resonator. (b)–(d) Transmission coefficients of three cuts in panel (a), highlighting the hybridized polaritonic modes in three resonance cases. Pale blue lines: measured data. Green lines: Savitzky-Golay smoothed data, shown for visual clarity. Red lines: theoretical simulations using Eq. (4).

two-mode bus-ensemble hybridization (Cut 1) to a three-mode hybridization at triple resonance (Cut 2), and back again (Cut 3) as the transmon detunes. Specifically, when the transmon frequency approaches the resonant frequency of the bus and NV^- ensemble, the bus-ensemble polaritons begin to dress the transmon states. As the hybridized energy levels reorganize, one bright polariton shifts outward with its frequency moving away, the other polariton loses resonator character and becomes dark in transmission, and simultaneously a lower-frequency bright polariton emerges. Due to the increased effective coupling between the bus and collective transmon-spin subsystem, on resonance [28], the two bright-mode polaritons exhibit avoided crossing with a minimal separation approximately 36.8 MHz, exceeding the splitting when bus couples to transmon alone (approximately 34.2 MHz) or to NV^- ensemble alone (approximately 13.2 MHz). These pronounced spectral features provide spectroscopic evidence for hybridization among all three subsystems, and the measured mean bright-polariton linewidth on resonance, i.e., Γ_h , is about 6.0 MHz (see details in Appendix C), which further confirms that the hybrid system is in the strong-coupling regime.

To elucidate the experimental observations further, a simplified Hamiltonian capturing the essential three-mode interactions is considered. The resonator field is described as a single harmonic mode, while the transmon is approximated as a qubit [7]. The spin ensemble is strongly coupled to

the bus resonator, and in the low-excitation limit, it can be approximated as a harmonic oscillator with the collective mode [29,30]. The system Hamiltonian is described by pairwise couplings between the bus resonator and qubit, and between the bus resonator and ensemble,

$$H = \hbar\omega_r a^\dagger a + \hbar\omega_t \frac{\sigma_t^z}{2} + \hbar\omega_s S^\dagger S + \hbar g_r (a^\dagger \sigma_t^- + a \sigma_t^+) + \hbar\Omega_e (a^\dagger S + a S^\dagger), \quad (1)$$

where a , S ($a^\dagger$, $S^\dagger$) are the annihilation (creation) operators for the bus mode and the collective ensemble mode, respectively, $\sigma_t^{z,\pm}$ are Pauli operators for the transmon qubit, and ω_r , ω_t , ω_s are the transition frequencies of the resonator, transmon, and spin ensemble, respectively.

Diagonalizing Eq. (1) in the bright-dark basis yields three hybridized eigenmodes; the derivation and the relation between the bright/dark basis and the measured transmission spectrum are provided in Appendix B. Using parameters independently extracted from the pairwise bus-transmon and bus-ensemble spectra, we diagonalize the one-excitation Hamiltonian to calculate the transition frequencies from the ground state to the hybrid eigenmodes. These calculated frequencies, shown as red dotted lines in Fig. 3(a), are not fitted point by point, but provide a constrained comparison between the measured polariton branches and the eigenfrequencies expected from the three-mode model. Specifically, at triple resonance $\omega_r = \omega_t = \omega_s$, the bright and dark eigenmodes are

$$|1_\pm\rangle = \frac{|1\rangle|g\rangle|G\rangle \pm |0\rangle|B\rangle}{\sqrt{2}}, \quad |1_D\rangle = |0\rangle|D\rangle, \quad (2)$$

with eigenenergies $E_\pm = \hbar(\omega_r \pm \Omega_h)$ and $E_D = \hbar\omega_r$, respectively. Here, the symmetric and asymmetric states

$$|B\rangle = \frac{g_t|e\rangle|G\rangle + \Omega_e|g\rangle|E\rangle}{\Omega_h}, \quad |D\rangle = \frac{\Omega_e|e\rangle|G\rangle - g_t|g\rangle|E\rangle}{\Omega_h}, \quad (3)$$

where $|0, 1\rangle$ represent the zero- and one-photon states in the bus, $|g, e\rangle$ are the ground and first levels of transmon, $|G, E\rangle$ are the ground and first collective states of NV^- ensemble, and $\Omega_h = \sqrt{g_t^2 + \Omega_e^2}$ is the hybrid coupling strength between bus and collective transmon-ensemble (transmon qubit and NV^- qubits). The experimentally extracted parameters give $\Omega_h/2\pi = 18.693 \pm 0.197$ MHz, which is remarkably close to half of the observed polariton splitting.

The emergence of a mode with strongly suppressed resonator weight provides a characteristic signature of the three-mode hybridized manifold. In the corresponding pairwise resonant configurations, the weak-probe transmission is described by two bright polaritonic branches, whereas simultaneous three-mode resonance produces a bright-dark-bright structure through the hybridized eigenstates. However, as demonstrated above, the transmon and NV^- ensemble constitute an effective three-level quantum system with a common ground state and states $|B\rangle$ and $|D\rangle$. It is the coupling of this three-level system to the bus resonator that enables the dark mode observation. Importantly, the NV^- triplet cannot account for the three-level feature because the applied magnetic field detunes one state far from the resonant levels.

To further characterize the dissipation rate of the system, the transmission coefficient is evaluated using the same model. By solving the Heisenberg-Langevin equations for the steady state, we obtain the transmission S_{21} scaling with

$$S_{21}(\omega_p) \propto \frac{1}{\Delta_r + \frac{g_r^2}{\Delta_t} + \Omega_e^2 \int d\omega \frac{\varrho(\omega)}{\Delta_\omega}}, \quad (4)$$

where $\Delta_r = \omega_r - \omega_p - i\kappa/2$, $\Delta_t = \omega_t - \omega_p - i\gamma_t/2$, and $\Delta_\omega = \omega - \omega_p - i\gamma_s/2$ are the complex detunings of the probe frequency ω_p from the resonator, transmon, and spin frequencies, respectively, and $\varrho(\omega)$ is the spectral density profile of the ensemble [31]. Using the parameters of the individual subsystem measurements, $|S_{21}|^2$ is simulated with results shown in Figs. 3(b)–3(d), where the line shape agrees with experimental data. Using the measured on-resonance bright-polariton linewidth to infer an effective collective transmon-ensemble linewidth through $\Gamma_h = (\kappa + \Gamma_{te})/2$, we estimate an effective hybrid cooperativity $C = 4\Omega_h^2/(\kappa\Gamma_{te}) \approx 692$. This value should be understood as a phenomenological figure of merit for the resolved hybrid manifold and further supports operation in the strong-coupling regime. Further details of the linewidth extraction and cooperativity estimate are given in Appendix C.

Small discrepancies between the experimental data and theoretical lines remain, which can be attributed to power broadening of the transmon under our spectroscopy conditions and additional dissipation channels introduced by the applied magnetic field (most notably increased loss in the aluminum Josephson junction). Both effects broaden the spectra and degrade the signal-to-noise ratio even after extensive averaging, reducing the visibility of fine spectral features. Nonetheless, the pronounced three-mode avoided crossings remain clearly resolved, demonstrating coherent energy exchange and strong coupling among the bus resonator, transmon, and NV^- ensemble.

Our hybrid quantum system has demonstrated sensitivity to both probe and control fields under low-power conditions, prompting an investigation into the transition from linear to nonlinear behavior with increasing probe power. Consequently, the probe power is varied from -70 to -30 dBm in 10 dB increments. The corresponding resonance traces are shown in Fig. 4, while the complete power-dependent spectra and simulations at higher probe powers are provided in Appendix D. As probe power increases, the original polariton peaks broaden progressively and eventually merge, while additional spectral features emerge between them. These changes are consistent with the nonlinear response of the system, including supersplitting, multiphoton transitions [7,32], and saturation-induced evolution away from the weak-probe three-mode regime. Above -30 dBm, the transmon becomes thermalized and gradually decouples from the resonator. Consequently, the system response evolves from a three-mode hybridized regime toward one dominated by the coupling between the bus resonator and spin ensemble [33]. This evolution results in a power-dependent response that departs from the linear three-mode model, consistent with access to higher-excitation manifolds and the breakdown of the low-excitation approximation.

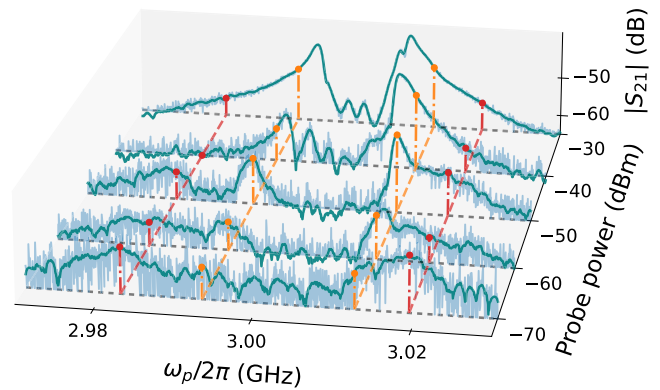


FIG. 4. Transmission spectra of the hybrid system at various probe powers when the transmon, bus resonator, and NV^- ensemble are tuned into triple resonance. Pale blue lines: raw data. Green lines: Savitzky-Golay smoothed data, shown for visual clarity. Red dots/lines: positions of peaks at probe power -70 dBm. Orange dots/lines: positions of peaks at probe power -50 dBm. Red and orange lines are to illustrate broadening and shifting of polaritons at higher powers.

Furthermore, preliminary spectral features consistent with ^{14}N coupling to the transmon are observed at probe power of -50 dBm, as shown in the inset of Fig. 5. In addition to power broadening and polariton peak splitting under high probe power, the hyperfine splitting of NV^- centers [34] is clearly resolved as two faint horizontal lines near 3 GHz between the polariton branches. When the transmon is tuned into resonance, the primary avoided crossing transforms into multiple avoided crossings as fine structures. As analyzed in Appendix E, these fine lines are consistent with hyperfine-resolved NV^- transitions participating in the hybridized resonator-transmon-ensemble response. Although the present spectral resolution does not yet allow a full coherent-control demonstration, these features suggest a possible route toward interfacing superconducting circuits with hyperfine-resolved spin degrees of freedom.

IV. CONCLUSION AND OUTLOOK

To conclude, our frequency-domain measurements reveal well-resolved polaritonic branches whose structure provides direct spectroscopic evidence of three-mode hybridization

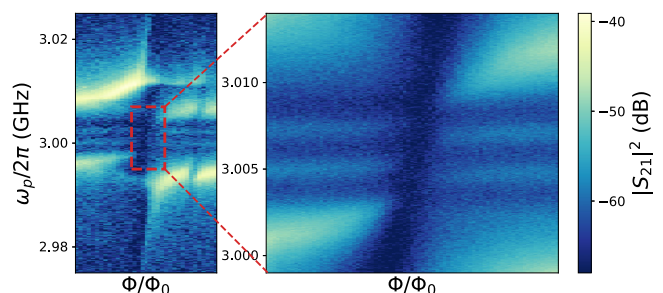


FIG. 5. Full transmission spectrum of the hybrid system showing fine spectral features arising from the hyperfine structure of the NV^- ensemble. Inset: Magnified view of the highlighted region.

of the transmon, bus resonator, and NV^- ensemble. These results are enabled by an improved hybrid platform that combines a tunable superconducting qubit with a low-broadening diamond spin ensemble, allowing access to a simultaneously strong-coupled superconducting-spin regime. Under stronger probing, additional resonances consistent with multiphoton excitations, nonlinear hybrid dynamics, and hyperfine-resolved NV^- features emerge, illustrating the system's accessibility to excitation manifolds beyond the weak-probe limit. Together, these measurements establish a superconducting-spin cavity-QED platform in which photonic, superconducting, and spin degrees of freedom can be spectroscopically resolved within a single hybrid device.

These capabilities may offer a platform for coherent multicomponent control in hybrid quantum devices. Through the bus resonator, collective excitations shared by the transmon and NV^- ensemble could be prepared and probed, providing a basis for future studies of state transfer, storage, and nonlinear hybrid dynamics [35]. The transmon tunability also provides a route to higher-excitation manifolds and nonlinear circuit-spin dynamics, opening future opportunities to investigate driven many-body effects [36], coherence, fidelity, and scaling in multicomponent quantum devices [37,38]. Realizing such applications will require improved control of the resonator-ensemble coupling, including fast tunability, to take full advantage of spin coherence.

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DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

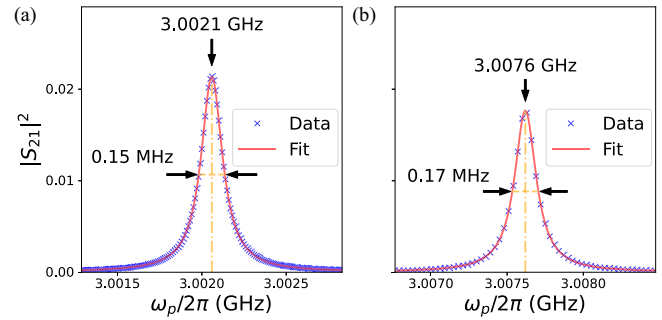


FIG. 6. Transmission spectra of the bus resonator with diamond [panel (a)] and with diamond plus magnetic field [panel (b)]. The central frequencies (dash-dotted lines) and FWHM values (dashed lines) are indicated for each case.

APPENDIX A: DEVICE AND MEASUREMENT DETAILS

1. Bus resonator

The bus resonator (see the bottom half of the chip in Fig. 1) is characterized through transmission measurements under three distinct conditions: the bare resonator, the resonator with a diamond sample placed on it, and the resonator with the diamond under an external magnetic field applied. The bare resonator exhibited a resonant frequency of around 3.0961 GHz, consistent with the designed value. When the diamond is introduced, the frequency is shifted to around 3.0021 GHz, primarily due to the dielectric loading of the diamond sample, and the measured FWHM is 0.154 MHz [see Fig. 6(a)]. After the superconducting chip has been subjected to external magnetic field, the frequency further adjusted to around 3.0076 GHz, and the FWHM slightly increases to 0.171 MHz [see Fig. 6(b)]. These parameters are extracted via least-squares fitting of the spectroscopic data, yielding a standard deviation of $\sim 10^{-5}$ MHz. Owing to this exceptionally small statistical uncertainty, a simulation precision of 0.1 MHz is adopted, which comfortably exceeds the fitting error.

2. Transmon

The transmon qubit (see the center cross-shaped pattern of the chip in Fig. 1) is initially characterized by transmission line measurements, revealing a sweet spot frequency of 3.230 GHz and anharmonicity of 203.1 MHz. Through time-domain experiments, we determined the coherence times with $T_1 \approx 10.3 \mu\text{s}$ (energy relaxation time) and $T_2 \approx 6.58 \mu\text{s}$ (dephasing time). The coupling between the transmon and the bus resonator is evaluated in the absence of the diamond sample, yielding a coupling strength of around 17.9 MHz (see Fig. 7). Upon introducing the diamond, the coupling strength remained nearly unchanged [see Fig. 2(c)].

To quantify the transmon's contribution to the hybrid system, we employ the Jaynes-Cummings (JC) model, which describes the coherent interaction between a two-level system (the transmon) and a quantized cavity mode (the bus resonator). Under weak probing conditions, the transmission spectrum reveals two distinct polariton states, with their fre-

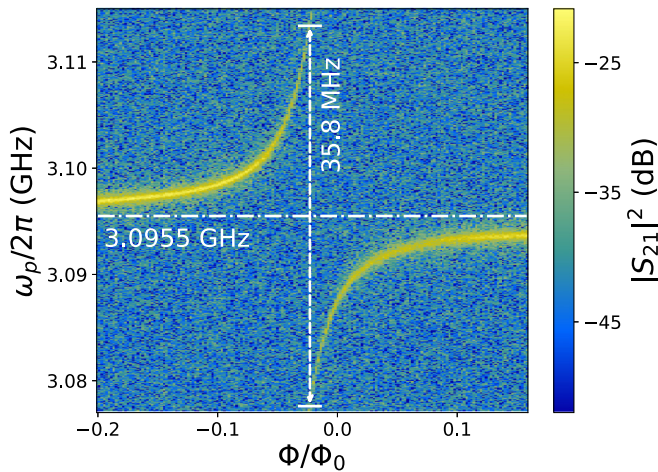


FIG. 7. Transmission spectra of the transmon coupled to the bus resonator without the diamond. The resonance frequency (dash-dotted line) and coupling strength (dashed line) are indicated.

quencies given by

$$\omega_{\pm} = \frac{\omega_r + \omega_t}{2} \pm \frac{1}{2} \sqrt{(\omega_r - \omega_t)^2 + 4g_t^2}, \quad (\text{A1})$$

where ω_r and ω_t are the resonant frequencies of the bus resonator and transmon, respectively, and g_t is the coupling strength between them. These spectral peaks exhibit Lorentzian line shapes characterized by center frequencies $\omega_{\pm}$ and FWHM defined by

$$\Gamma_+ = \kappa \cos^2 \theta_1 + \gamma_t \sin^2 \theta_1, \quad (\text{A2})$$

$$\Gamma_- = \kappa \sin^2 \theta_1 + \gamma_t \cos^2 \theta_1, \quad (\text{A3})$$

where θ_1 is the mixing angle satisfying $\tan(2\theta_1) = 2g_t/(\omega_r - \omega_t)$, κ is the decay rate of the bus resonator, and γ_t is the decay rate of the transmon.

By fitting the data, we obtain the bus frequency $\omega_r/2\pi = 3.0070$ GHz, the coupling strength $g_t/2\pi = 17.5 \pm 0.2$ MHz, and transmon decay rate $\gamma_t/2\pi = 4.0 \pm 0.1$ MHz, yielding a value that reflects the system's damping dynamics and coherence limitations. Using the above parameters and the theoretical transmission function

$$S_{21}(\omega_p) \propto \frac{1}{\omega_r - \omega_p - i\kappa/2 - \frac{g_t^2}{\omega_t - \omega_p - i\gamma_t/2}}, \quad (\text{A4})$$

where ω_p is the probe frequency, the transmission spectrum can be reconstructed, as shown in Fig. 8. We observe that the extracted transmon decay rate γ_t corresponds to a coherence time of ~ 0.25 μs , which is shorter than the directly measured T_1 and T_2 times (ranging from 6 to 10 μs). This discrepancy arises from distinct experimental conditions. The T_1/T_2 measurements characterize the intrinsic coherence of the transmon, performed at its flux sweet spot and in the absence of the diamond sample. In contrast, the spectroscopy used to extract γ_t was conducted under different circumstances: (1) The bus resonator was loaded with the diamond, introducing additional loss channels; (2) the transmon was biased away from its sweet spot, increasing its sensitivity to flux noise; and (3) the spectroscopic technique itself is subject to

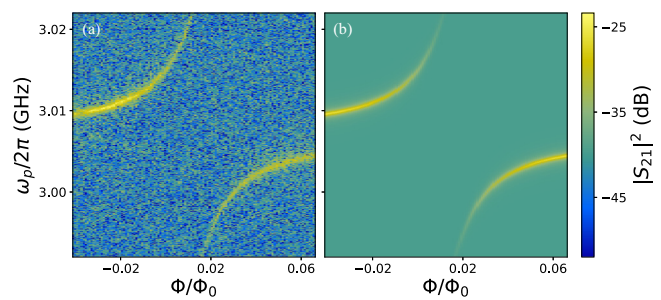


FIG. 8. Transmission spectra of the transmon coupled to the bus resonator with the diamond: experimental data [panel (a)] and theoretical fit [panel (b)].

power broadening and averaging effects that increase the fitted linewidth. Therefore, γ_t represents an effective decay rate for the transmon within the coupled system under these specific, loss-enhanced conditions, rather than its intrinsic coherence limit.

We verify that the transmon remains in the linear-response regime at -70 dBm probe power. The power-dependent spectroscopy result is shown in Fig. 9. The measurement was taken under resonant conditions between the transmon and the resonator, with the diamond absent. At low probe power (around -70 dBm), the spectrum shows the characteristic vacuum Rabi splitting, manifesting as two distinct peaks. Increasing the probe power first induces a supersplitting of each primary peak, followed by the appearance of additional resonances [32]. With a further increase in power, the transmon undergoes thermalization and eventually decouples from the resonator, causing the spectrum to collapse back to a single peak corresponding to the bare resonator frequency.

3. NV⁻ spin ensemble

The diamond is shown as the black square on top of the bus resonator in Fig. 1, and its orientation is illustrated in Fig. 10. With the crystalline [111] axis aligned with the laboratory z axis and the in-plane magnetic field applied along the

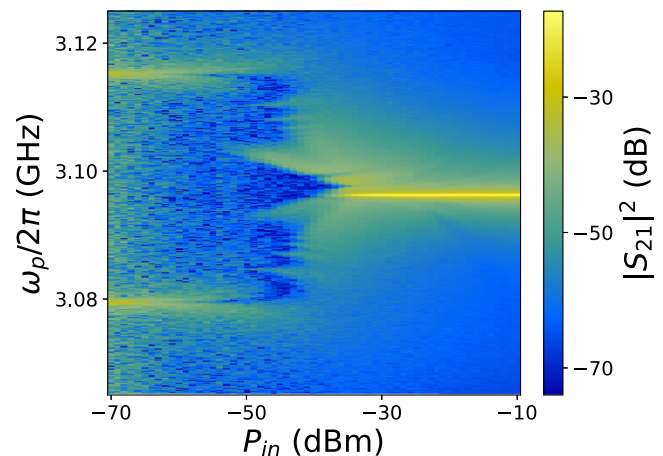


FIG. 9. Power-dependent spectroscopy of the bus-transmon system without the diamond.

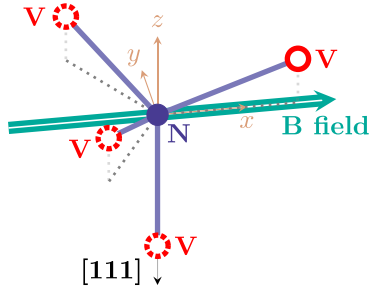


FIG. 10. Schematic for orientation of the NV^- center with respect to the laboratory axes and applied magnetic field.

laboratory x axis, the four possible NV orientations respond differently as the field increases from zero. One subensemble is aligned most closely with the magnetic field and therefore couples strongly at low field. A second pair of subensembles becomes resonant only at higher field values, while the fourth orientation remains perpendicular to the field and does not couple at all. As a result, in our experiment we effectively address only a single NV^- subensemble.

The resonance between the NV^- ensemble and the bus resonator is characterized through spectroscopy, as illustrated in Fig. 2(d). When the ensemble is in resonance with the resonator, two distinct polariton peaks emerge, with a separation of 13.5 MHz directly yielding the collective coupling strength (see Fig. 11). The FWHM of these peaks is measured to be 1.7 MHz for both. Employing the relation $\text{FWHM} \approx (\kappa + \Gamma_{\text{ens}})/2$ for the polariton linewidth [30], where Γ_{ens} is the spin ensemble decay rate, the contribution of the NV^- ensemble to the dissipation is evaluated as $\Gamma_{\text{ens}}/2\pi \approx 3.2$ MHz.

To fit the data, we consider the transmission spectrum (in the linear-response regime) derived in Ref. [30],

$$|S_{21}(\omega_p)|^2 \propto \left| \frac{1}{\omega_r - \omega_p - i\kappa/2 - \Omega_e^2 \int d\omega \frac{\varrho(\omega)}{\omega - \omega_p - i\gamma_s/2}} \right|^2, \quad (\text{A5})$$

where ω_p is the probe frequency, κ is the cavity decay rate, γ_s is the spin decoherence rate, and $\varrho(\omega)$ is the spectral den-

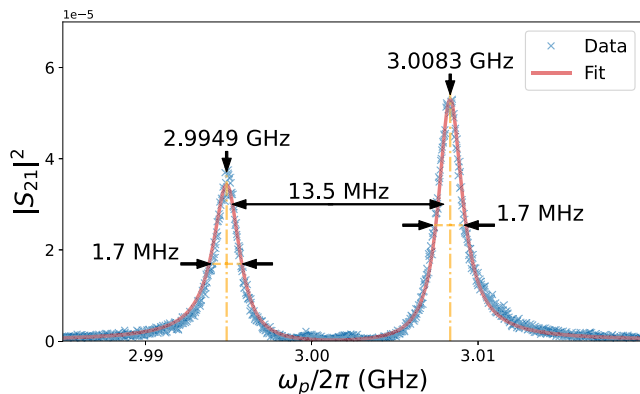


FIG. 11. Transmission spectra of the NV^- ensemble coupled to the bus resonator.

sity profile associated with the NV^- ensemble following a q -Gaussian distribution [31]:

$$\varrho(\omega) = \rho_0 \left[1 - (1 - q) \frac{(\omega - \omega_s)^2}{\Delta^2} \right]^{\frac{1}{1-q}}, \quad (\text{A6})$$

which reduces to Gaussian for $q = 1$ and Lorentzian for $q = 2$. By fitting the experimental data (picking $\gamma_s/2\pi = 1$ kHz as a reasonable small value), we obtain the bus frequency $\omega_r/2\pi = 3.0020$ GHz, the ensemble central frequency $\omega_s/2\pi = 3.0012$ GHz, the collective coupling strength $\Omega_e/2\pi = 6.6 \pm 0.1$ MHz, the width of $\varrho(\omega)$ as $\Gamma_e/2\pi = 3.4 \pm 0.2$ MHz, and $q = 1.9591 \pm 0.0660$. The fit result is shown in Fig. 11.

Based on typical single-spin coupling strengths of a few hertz for CPW resonators and the collective coupling Ω_e , we estimate that the effective number of participating spins is on the order of 10^{12} . This estimate is approximate because the coupling is weighted by the spatial microwave-field profile and by the inhomogeneous distribution of NV^- centers within the diamond.

APPENDIX B: THREE-MODE HYBRIDIZATION THEORY

1. The approximated Hamiltonian

The bus resonator is modeled as a harmonic oscillator with frequency ω_r and annihilation operator a . The transmon is generally treated as an anharmonic oscillator with fundamental frequency ω_t , anharmonicity parameter δ_t , annihilation operator b , and nonlinear term $b^\dagger b^\dagger b b$, which goes beyond the two-level approximation and captures the transmon's intrinsic multilevel nature. The NV^- ensemble consists of a collection of N spins, which can be viewed as N spin 1/2 described by Pauli operators. The bus-transmon and bus-ensemble couplings are capacitive and magnetic-dipole couplings, respectively. The Hamiltonian of this hybrid quantum system can be written as

$$H = \hbar\omega_a a^\dagger a + \hbar\omega_t b^\dagger b + \hbar \frac{\delta_t}{2} b^\dagger b^\dagger b b + \hbar \sum_{j=1}^N \omega_j \frac{\sigma_j^z}{2} + \hbar g_t (a^\dagger b + a b^\dagger) + \hbar \sum_{j=1}^N g_j (a^\dagger \sigma_j^- + a \sigma_j^+), \quad (\text{B1})$$

where we have used the rotating-wave approximation.

In the experiment, the mean number of excitations in the NV^- spin ensemble is much lower than the number of spins, which allows us to make the Holstein-Primakoff approximation to describe the spin ensemble as a harmonic oscillator. This approximation is also valid in our case where the spin ensemble has narrow inhomogeneous broadening feature [29,30]. The transmon is simply truncated as a two-level system, as the drive field is weak. Consequently, Eq. (B1) is approximated as Eq. (1).

2. Linear-response regime

The linear-response regime can be approximated as the one-excitation regime to simplify the analysis, where weak driving confines the system dynamics predominantly to

single quantum excitations. Using the single-excitation basis $|1gG\rangle, |0eG\rangle, |0gE\rangle$, the Hamiltonian in Eq. (1) can be written as a 3×3 matrix:

$$H_{\text{excit-1}} = \hbar \begin{pmatrix} \omega_r & g_t & \Omega_e \\ g_t & \omega_t & 0 \\ \Omega_e & 0 & \omega_s \end{pmatrix}. \quad (\text{B2})$$

Analytically solving the eigenenergies and eigenstates of $H_{\text{excit-1}}$ is feasible, as the problem can be transformed to solving a cubic equation. However, the resulting analytical expressions tend to be intricate and offer limited physical intuition. For this reason, we instead investigate the following two cases that are directly related to our experiment.

a. Triple-resonance case

The resonant case $\omega_r = \omega_t = \omega_s$ gives compact, physically transparent eigenvalues and eigenvectors. In this case, Eq. (B2) can be written as

$$H_{\text{excit-1}} = \hbar \begin{pmatrix} \omega_r & g_t & \Omega_e \\ g_t & \omega_r & 0 \\ \Omega_e & 0 & \omega_r \end{pmatrix}, \quad (\text{B3})$$

which yields eigenenergies

$$E_D = \hbar\omega_r, \quad E_{\pm} = \hbar(\omega_r \pm \Omega_h), \quad (\text{B4})$$

where $\Omega_h = \sqrt{g_t^2 + \Omega_e^2}$. The eigenstates can be accordingly solved as

$$|1_D\rangle = |0\rangle|D\rangle, \quad \text{with } |D\rangle = \frac{\Omega_e|eG\rangle - g_t|gE\rangle}{\Omega_h}, \quad (\text{B5})$$

$$|1_{\pm}\rangle = \frac{|1gG\rangle \pm |0\rangle|B\rangle}{\sqrt{2}}, \quad \text{with } |B\rangle = \frac{g_t|eG\rangle + \Omega_e|gE\rangle}{\Omega_h}. \quad (\text{B6})$$

Therefore, on resonance, one mode is dark (no bus photon) and two bright polariton modes are split by $2\Omega_h$. This is the emergent three-mode hybridization studied in the experiment.

b. Detuned case

When the transmon frequency is changed while the bus and ensemble are still in resonance, i.e., $\omega_r = \omega_s \neq \omega_t$, we can numerically solve the eigenproblem of 3×3 Hamiltonian matrix

$$\hbar \begin{pmatrix} \omega_r & g_t & \Omega_e \\ g_t & \omega_t & 0 \\ \Omega_e & 0 & \omega_r \end{pmatrix} \quad (\text{B7})$$

to obtain the eigenenergies $E_D(\omega_t)$, $E_{\pm}(\omega_t)$ and corresponding eigenstates $|1_D(\omega_t)\rangle$, $|1_{\pm}(\omega_t)\rangle$, as a function of ω_t . When probing the system in the linear-response regime, the transmission coefficient $|S_{21}|^2$ attaches maxima near the transition frequencies corresponding to the above eigenenergies.

To understand the transitions between the hybrid modes for this detuned case, we introduce the annihilation operators B and D to $|B\rangle$ and $|D\rangle$, respectively,

$$B = \frac{g_t b + \Omega_e S}{\Omega_h}, \quad D = \frac{\Omega_e b - g_t S}{\Omega_h}, \quad (\text{B8})$$

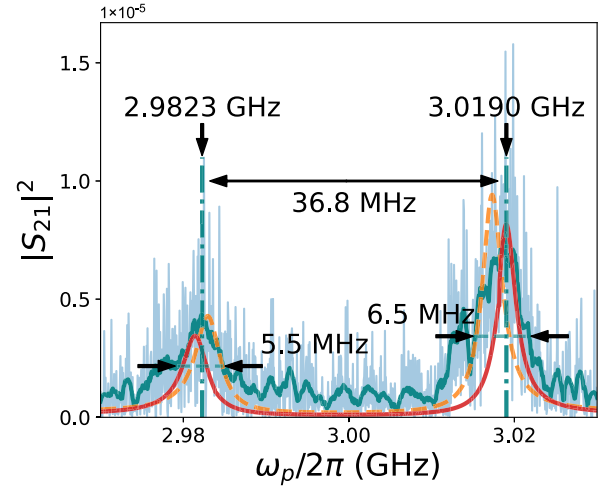


FIG. 12. Transmission spectra of the triple-resonance condition for the hybrid quantum system. Pale blue: experimental data. Green: Savitzky-Golay smoothed data, shown for visual clarity and approximating the peaks. Orange: fitted data. Red: simulation using parameters from individual system characterization.

such that $B^\dagger|0gG\rangle = |B\rangle$ and $D^\dagger|0gG\rangle = |D\rangle$. Since the inner product $\langle B, D \rangle = 0$, we can rewrite the Hamiltonian of Eq. (1) as

$$H = \hbar\omega_r a^\dagger a + \hbar\omega_B B^\dagger B + \hbar\omega_D D^\dagger D + \hbar\Omega_h(a^\dagger B + aB^\dagger) + \hbar\chi(B^\dagger D + BD^\dagger) + \dots, \quad (\text{B9})$$

where $\dots$ denote the transmon anharmonicity term, which is irrelevant in the one-excitation Hamiltonian, and

$$\omega_B = \frac{\omega_t g_t^2 + \omega_s \Omega_e^2}{\Omega_h^2}, \quad \omega_D = \frac{\omega_t \Omega_e^2 + \omega_s g_t^2}{\Omega_h^2}, \quad \chi = \frac{g_t \Omega_e (\omega_t - \omega_s)}{\Omega_h^2}. \quad (\text{B10})$$

Thus, the bus mode a couples to mode B via the JC interaction, while the mode D only couples to B when $\omega_t \neq \omega_s$. This matches the observation in the experiment, where an avoided crossing between two bright modes is shown when detuning ω_t from $\omega_r = \omega_s$, and the dark mode is invisible for the triple-resonance case, then becoming visible when the detuned ω_t leads to a nonzero interaction with mode B .

APPENDIX C: TRIPLE RESONANCE AND PARAMETER ESTIMATION

The spectroscopy experiment result of the hybrid quantum system under a weak probe has been illustrated in Fig. 3(a). The triple-resonance spectra with characterized parameters are shown in Fig. 12. We clearly observe two distinct polariton peaks at 2.9823 and 3.0190 GHz, with a splitting of 36.8 MHz, indicating a collective interaction strength of at least 18.4 MHz between the transmon and the NV^- ensemble coupled to the bus. The FWHM of two peaks is measured to be 5.5 and 6.5 MHz, respectively, which gives average width of polariton peaks as $(5.5 + 6.5)/2 = 6.0$ MHz.

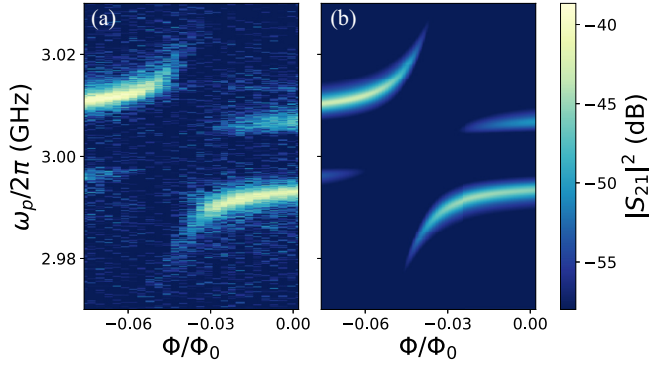


FIG. 13. Transmission spectrum of (a) experimental data and (b) simulation.

The triple-resonance spectra [Fig. 3(c)] are expected to be explained by Eq. (4). However, directly substituting prefitted parameters into the model fails to adequately explain the experimental results for several reasons. First, significant measurement noise obscures the precise signal features. Second, the resonant frequencies ω_r , ω_t , and ω_s exhibit drift during data acquisition. Third, the transmon decay rate γ_t is sensitive to magnetic fields, meaning the external field lifts its value. One may hope to extract all system parameters using a direct global fit of the complete spectrum. This method turns out impractical as the large parameter space leads to prohibitively long fitting times with poor convergence. Therefore, to bridge the experimental observations with the theoretical description of Eq. (4), we employ a stepwise, approximate parameter estimation strategy. We first estimate the key parameters, including ω_r , ω_t , ω_s , and γ_t , using the simplified effective Hamiltonian model of Eq. (1). These values are then fixed with the prefitted parameters of the NV⁻-center ensemble (precisely, q and Γ_q for the q -Gaussian distribution) to simulate the full spectrum using Eq. (4). The results are shown in Figs. 3 and 13.

APPENDIX D: FULL PROBE-POWER-DEPENDENT SPECTRA FOR NONLINEAR RESPONSE

Figure 14 presents frequency-domain spectroscopy of the hybrid system as a function of transmon detuning, measured at different probe powers. Throughout the measurement, the NV⁻ ensemble is kept on resonance with the bus resonator. The line traces shown in Fig. 4 are obtained from vertical cuts through this two-dimensional map.

At present, a unified analytical model that fully captures the nonlinear response of such a hybrid system remains a challenge. Nevertheless, the key phenomena observed, including supersplitting, multiphoton transitions, and the population dynamics of the spin ensemble, can each be addressed within tailored approximations. In the following, we outline the approximations that lead to tractable models for different probe-power regimes, providing a conceptual framework for interpreting the observed spectral evolution.

Our model of the system is constructed from the Hamiltonian provided in Eq. (B1), which describes the transmon as an anharmonic oscillator and the spin ensemble via an inhomogeneous Tavis-Cummings model. Within the probe frequency range shown in Fig. 14, the transmon can be accurately modeled as a two-level system (qubit), since higher-energy levels lie outside the spectral window of the probe and therefore remain unpopulated. Consequently, we approximate the Hamiltonian of the system as

$$H = \hbar\omega_a a^\dagger a + \hbar\omega_t \frac{\sigma_t^z}{2} + \hbar \sum_{j=1}^N \omega_j \frac{\sigma_j^z}{2} + \hbar g_t (a^\dagger \sigma_t^- + a \sigma_t^+) + \hbar \sum_{j=1}^N g_j (a^\dagger \sigma_j^- + a \sigma_j^+). \quad (\text{D1})$$

The coherent probe is included through the drive Hamiltonian

$$H_p = \hbar\eta (ae^{i\omega_p t} + a^\dagger e^{-i\omega_p t}), \quad (\text{D2})$$

where η corresponds to the time-independent amplitude implemented experimentally. In both the linear- and nonlinear-response regimes, the transmission spectrum is proportional

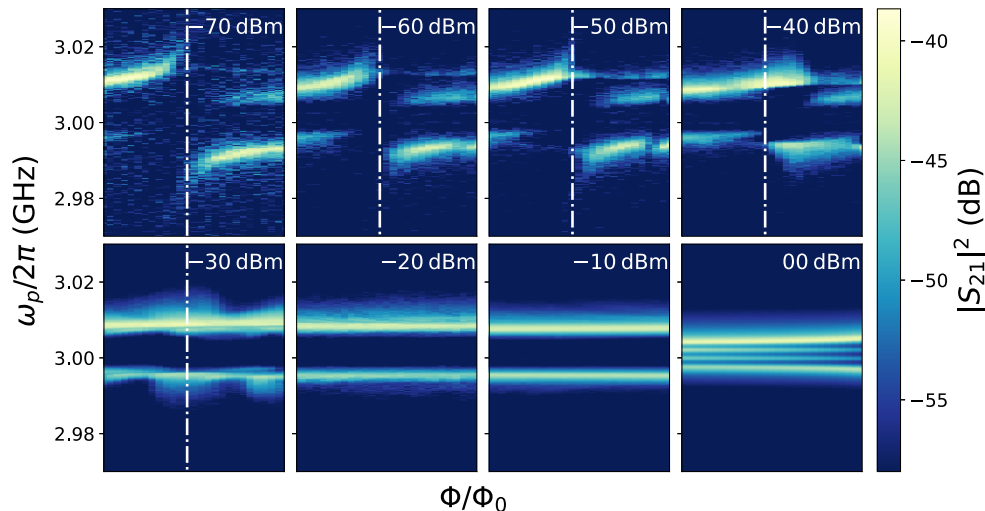


FIG. 14. Full probe-power-dependent spectra for the hybrid system from -70 to 0 dBm. The dash-dotted lines show the cuts in Fig. 4.

to the steady-state expectation value of the resonator field, $|\langle a \rangle_{ss}|$. This quantity can be computed, in principle, by solving the corresponding Lindblad master equation or the equivalent Heisenberg-Langevin equations. Particularly, the Lindblad master equation in the rotating frame at the probe frequency ω_p is

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}[H_\Delta, \rho] + \kappa\mathcal{D}[a]\rho + \gamma\mathcal{D}[\sigma_t^-]\rho + 2\gamma_\phi\mathcal{D}[\sigma_t^z/2]\rho + \sum_j (\gamma_s\mathcal{D}[\sigma_j^-]\rho + \gamma_\parallel\mathcal{D}[\sigma_j^z/2]\rho), \quad (D3)$$

where $\mathcal{D}[L]\rho = L\rho L^\dagger - \{L^\dagger L, \rho\}/2$ is the Lindblad dissipator and H_Δ is the rotated Hamiltonian

$$H = \Delta_r a^\dagger a + \frac{\Delta_t}{2} \sigma_t^z + \sum_j \frac{\Delta_j}{2} \sigma_j^z + g_t(a\sigma_t^+ + a^\dagger\sigma_t^-) + \sum_j g_j(a\sigma_j^+ + a^\dagger\sigma_j^-) + \eta(a + a^\dagger), \quad (D4)$$

with $\Delta_r = \omega_r - \omega_p$, $\Delta_t = \omega_t - \omega_p$, and $\Delta_j = \omega_j - \omega_p$. The dissipation rates are the cavity decay rate κ , the transmon relaxation rate γ and pure-dephasing rate γ_ϕ , and the spin transverse-relaxation rate γ_s and longitudinal-relaxation rate $\gamma_\parallel$.

1. -70 dBm

Under this weak probe condition, the system operates in the linear-response regime where both the transmon and the spin ensemble remain in the low-excitation limit. Within this limit, the operator relations $\langle a\sigma_t^z \rangle = -\langle a \rangle$ and $\langle a\sigma_j^z \rangle = -\langle a \rangle$ for all j hold. Solving the corresponding equations of motions

$$\frac{d}{dt}\langle a \rangle = -i\left(\omega_r - \omega_p - i\frac{\kappa}{2}\right)\langle a \rangle - ig_t\langle \sigma_t^- \rangle - i\sum_j g_j\langle \sigma_j^- \rangle - i\eta, \quad (D5)$$

$$\frac{d}{dt}\langle \sigma_t^- \rangle = -i\left(\omega_t - \omega_p - i\frac{\gamma_t}{2}\right)\langle \sigma_t^- \rangle - ig_t\langle a \rangle, \quad (D6)$$

$$\frac{d}{dt}\langle \sigma_j^- \rangle = -i\left(\omega_j - \omega_p - i\frac{\gamma_s}{2}\right)\langle \sigma_j^- \rangle - ig_j\langle a \rangle \quad (D7)$$

in steady state (by setting the time derivatives to zero), the transmission function given in Eq. (4) can be obtained.

2. -60 and -50 dBm

At increased probe powers, the polariton peaks exhibit a pronounced supersplitting, analogous to the transmon supersplitting reported in Ref. [32]. This spectral feature originates from the strong driving of an effective two-level system formed between the ground state and the hybridized eigenmodes. The underlying process is a textbook example of “dressing of dressed states,” in which an already dressed polariton is further dressed by the coherent probe field, leading to a second-order level repulsion visible as a doublet within each original polariton line.

To capture this effect analytically under resonant conditions, we employ a simplified description that exploits the symmetric coupling of the resonator and the bright mode to

the probe. When the transmon, the resonator, and the spin ensemble are on resonance, the resonator operator a and the bright-mode operator B can be approximated by a common collective lowering operator:

$$a \rightarrow \frac{\Sigma_-}{\sqrt{2}}, \quad B \rightarrow \frac{\Sigma_-}{\sqrt{2}}, \quad (D8)$$

where Σ_- describes the joint low-energy excitation of the coupled resonator-bright-mode subsystem. Substituting this mapping into the full Hamiltonian [Eq. (B1)] and the corresponding Lindblad master equation reduces the problem to an effective few-level model whose dissipative dynamics are governed by

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}[H_{\text{eff}}, \rho] + \Gamma\mathcal{D}[\Sigma_-]\rho, \quad (D9)$$

where Γ is dissipation rate of the hybridized mode. The steady-state expectation value $\langle a \rangle_{ss}$ is then obtained by solving for the steady-state density matrix ρ_{ss} of this reduced model. This met resulting expression reproduces the characteristic supersplit line shapes observed in the spectra, thereby providing a tractable analytical account of the “dressing of dressed states” regime without requiring a full numerical treatment of the many-body hybrid system.

3. -40 and -30 dBm

At these higher probe powers, the system begins to access higher rungs of the JC ladder formed by the resonator mode a and the hybridized bright mode B . As a consequence, additional weak resonances (mini-peaks) appear alongside the main polariton lines, reminiscent of the multiphoton spectral features observed in Ref. [32]. These multiphoton processes require extending the dynamical description beyond the single-excitation subspace. The full set of Heisenberg-Langevin equations derived from the Lindblad master equation must therefore include equations of motion for higher-order operator products that encode the relevant correlations. For instance, beyond the standard variables $\langle a \rangle$, $\langle \sigma_t^- \rangle$, and $\langle \sigma_j^- \rangle$, one now needs to track quantities such as

$$\frac{d}{dt}\langle a\sigma_j^z \rangle, \quad \frac{d}{dt}\langle a^\dagger a\sigma_j^- \rangle. \quad (D10)$$

Importantly, even at these elevated powers the spin ensemble itself remains only weakly excited, so the low-excitation factorization $\langle a\sigma_j^z \rangle = -\langle a \rangle$ continues to hold for all spins j . This observation significantly reduces the number of independent variables, as it eliminates the need to treat $\langle a\sigma_j^z \rangle$ as an independent dynamical quantity. The remaining hierarchy of coupled equations is then truncated at a finite excitation number n_{max} , which yields a closed set of algebraic equations in the steady state. Solving this truncated system provides an approximate but physically transparent description of the multiphoton resonances observed in the measured spectra.

4. -20, -10, and 0 dBm

At these probe powers, the measured spectra become essentially independent of the transmon flux, indicating that the transmon is thermally populated and effectively decoupled from the resonator. Indeed, the populated resonator mode acts

TABLE I. Fitting parameters extracted at different driving powers. Units: ω_r, ω_s in 2π GHz; $\Omega_e, \gamma_\perp$ in 2π MHz; ϵ in GHz^{-2} . Errors represent 1σ confidence intervals.

Parameter	−30 dBm	−20 dBm	−10 dBm
ω_r	3.003	3.003	3.002
ω_s	3.001	3.001	3.001
Ω_e	6.82(3)	6.43(3)	6.19(3)
$\gamma_\perp$	5.22(6)	4.71(6)	4.60(5)
ϵ	7.0(21.3)	5.7(18.1)	6.2(18.7)

as the dominant environment for the transmon. The system dynamics therefore reduce to those of the resonator coupled solely to the inhomogeneous spin ensemble. To treat this regime analytically, we adopt a mean-field (semiclassical) approximation in which operator products are factorized, e.g., $\langle a\sigma_j^- \rangle \rightarrow \langle a \rangle \langle \sigma_j^- \rangle$. Substituting these factorizations into the full Heisenberg-Langevin equations, we obtain a closed set of equations

$$\frac{d}{dt} \langle a \rangle = -i\tilde{\Delta}_r \langle a \rangle - i \sum_j g_j \langle \sigma_j^- \rangle - i\eta, \quad (\text{D11})$$

$$\frac{d}{dt} \langle \sigma_j^- \rangle = -i\tilde{\Delta}_j \langle \sigma_j^- \rangle + ig_j \langle a \rangle \langle \sigma_j^z \rangle, \quad (\text{D12})$$

$$\frac{d}{dt} \langle \sigma_j^z \rangle = -\gamma_\parallel (1 + \langle \sigma_j^z \rangle) - 2ig_j (\langle a \rangle \langle \sigma_j^+ \rangle - \text{H.c.}), \quad (\text{D13})$$

where $\tilde{\Delta}_r = \omega_r - \omega_p - ik/2$, $\tilde{\Delta}_j = \omega_j - \omega_p - i\gamma_\perp/2$, with $\gamma_\perp$ the transverse dissipation of the spin, and $\gamma_\parallel$ is the longitudinal relaxation rate of the spin ensemble. Setting the time derivatives to zero yields the steady-state values $\langle a \rangle_{ss}$, following the procedure described in Ref. [33].

We now study this effective model for experimental data acquired at three representative probe powers: −30, −20, and −10 dBm. (Even at −30 dBm, where the transmon still influences the spectrum, the model captures the qualitative trend and highlights the remaining discrepancies.) For a realistic inhomogeneous spin ensemble, one would typically discretize the distribution of spins into many subensembles, each with a well-defined frequency and coupling strength. However, obtaining accurate results in this way requires a large number of subensembles, rendering the numerical procedure computationally expensive. To instead focus on the essential observation of two spectral peaks merging gradually, we consider a more simplified case where all spins are identical, $\omega_j = \omega_s$ and $g_j = \Omega_e/\sqrt{N}$. Under this assumption, the steady-state solution of the above equations yields

$$\tilde{\Delta}_r a - \frac{\Omega_e^2}{\tilde{\Delta}_s (1 + \frac{|a|^2}{\epsilon |\tilde{\Delta}_s|^2})} a + \eta = 0, \quad (\text{D14})$$

where $\epsilon = N\gamma_\parallel/(2\Omega_e^2\gamma_\perp)$ is a parameter characterizing the ensemble. Fitting this simplified model to the experimental spectra, we find good agreement for all three drive powers. The results are shown in Fig. 15, with fitted parameters in Table I.

For the highest probe power (0 dBm), where the hyperfine structure of the NV^- centers is clearly resolved, the spin ensemble is treated as three independent subensembles, each

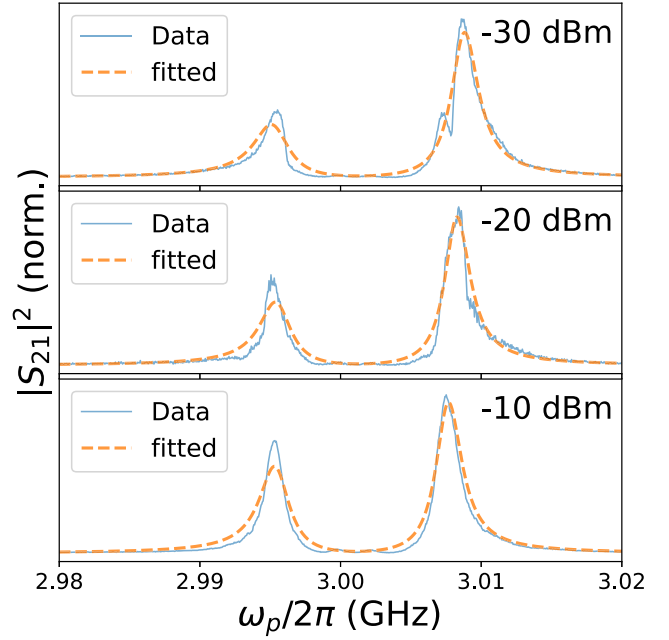


FIG. 15. Experimental spectra (blue) and the steady-state solution of the mean-field model assuming identical spins (orange) at three probe powers. Y axes are normalized transmission.

coupled to the common resonator mode. The same mean-field Heisenberg-Langevin formalism is applied separately to each subensemble to model the resulting composite spectral features.

APPENDIX E: HYPERFINE-RESOLVED SPIN MODEL

The NV^- center in diamond exhibits a hyperfine structure due to the interaction between its electron spin ($S = 1$) and the nuclear spin of the nitrogen atom (^{14}N , $I = 1$). As a result, each electron spin sublevel (e.g., $m_S = 0$ and $m_S = -1$) splits into three hyperfine sublevels, corresponding to the nuclear-spin projections $m_I = -1, 0, +1$. For transitions between electron spin levels, such as from $m_S = 0$ to $m_S = -1$, the selection rules ($\Delta m_I = 0$) allow transitions only between sublevels with the same m_I value [34]. This leads to three distinct transition frequencies. Consequently, the NV^- ensemble is actually a collection of three subensembles, with

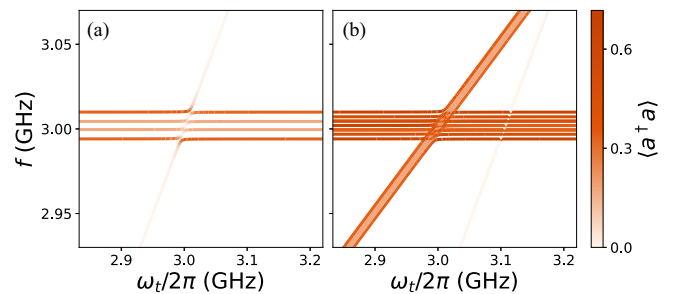


FIG. 16. (a) One-photon and (b) two-photon transitions of the strongly coupled hybrid quantum system.

Hamiltonian approximated as

$$H = \hbar\omega_a a^\dagger a + \hbar\omega_b b^\dagger b + \hbar\frac{\delta_t}{2} b^\dagger b^\dagger b b + \hbar \sum_{l=0,\pm 1} \omega_{s,l} S_l^+ S_l^- + \hbar g_t (a^\dagger b + ab^\dagger) + \hbar \sum_{l=0,\pm 1} \frac{\Omega_e}{\sqrt{3}} (a^\dagger S_l^- + a S_l^+). \quad (\text{E1})$$

Here, the coupling strength Ω_e is equally distributed among three subensembles and $\omega_{s,l} = \omega_s + \alpha l$, with $\omega_s = \omega_r$ and $\alpha = 2.2$ MHz.

To interpret Fig. 5, we numerically solve the eigenvalue problem of this Hamiltonian within the manifold of no

more than two excitations, using determined parameters from experimental data. The eigenstates are dressed states of resonator mode, transmon mode, and three collective modes of subensembles, and can be classified as two sets with one total excitation and two total excitations, respectively. Figure 16(a) shows energies of the one-excitation dressed states changing with detuned transmon frequency, which exhibits two avoided crossing lines and three bright-dark-bright lines indicating that transmon exchanges energy with the composed resonator and the three-subensemble system. These lines constitute the main pattern in Fig. 5. Meanwhile, Fig. 16(b) shows half of the energies in the two-excitation dressed states. These transitions serve as weak two-photon transitions observed in Fig. 5.

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- [1] J. Q. You and F. Nori, Atomic physics and quantum optics using superconducting circuits, *Nature (London)* **474**, 589 (2011).
 - [2] G. Kurizki, P. Bertet, Y. Kubo, K. Mølmer, D. Petrosyan, P. Rabl, and J. Schmiedmayer, Quantum technologies with hybrid systems, *Proc. Natl. Acad. Sci. USA* **112**, 3866 (2015).
 - [3] A. A. Clerk, K. W. Lehnert, P. Bertet, J. R. Petta, and Y. Nakamura, Hybrid quantum systems with circuit quantum electrodynamics, *Nat. Phys.* **16**, 257 (2020).
 - [4] P. Rabl, D. DeMille, J. M. Doyle, M. D. Lukin, R. J. Schoelkopf, and P. Zoller, Hybrid quantum processors: Molecular ensembles as quantum memory for solid state circuits, *Phys. Rev. Lett.* **97**, 033003 (2006).
 - [5] K. Tordrup, A. Negretti, and K. Mølmer, Holographic quantum computing, *Phys. Rev. Lett.* **101**, 040501 (2008).
 - [6] A. Imamoglu, Cavity QED based on collective magnetic dipole coupling: Spin ensembles as hybrid two-level systems, *Phys. Rev. Lett.* **102**, 083602 (2009).
 - [7] A. Blais, A. L. Grimsom, S. M. Girvin, and A. Wallraff, Circuit quantum electrodynamics, *Rev. Mod. Phys.* **93**, 025005 (2021).
 - [8] M. V. G. Dutt, L. Childress, L. Jiang, E. Togan, J. Maze, F. Jelezko, A. S. Zibrov, P. R. Hemmer, and M. D. Lukin, Quantum register based on individual electronic and nuclear spin qubits in diamond, *Science* **316**, 1312 (2007).
 - [9] A. Bienfait, J. J. Pla, Y. Kubo, X. Zhou, M. Stern, C. C. Lo, C. D. Weis, T. Schenkel, D. Vion, D. Esteve, J. J. L. Morton, and P. Bertet, Controlling spin relaxation with a cavity, *Nature (London)* **531**, 74 (2016).
 - [10] Z.-L. Xiang, S. Ashhab, J. Q. You, and F. Nori, Hybrid quantum circuits: Superconducting circuits interacting with other quantum systems, *Rev. Mod. Phys.* **85**, 623 (2013).
 - [11] B. Julsgaard, C. Grezes, P. Bertet, and K. Mølmer, Quantum memory for microwave photons in an inhomogeneously broadened spin ensemble, *Phys. Rev. Lett.* **110**, 250503 (2013).
 - [12] X. Zhu, Y. Matsuzaki, R. Amsüss, K. Kakuyanagi, T. Shimo-Oka, N. Mizuochi, K. Nemoto, K. Semba, W. J. Munro, and S. Saito, Observation of dark states in a superconductor diamond quantum hybrid system, *Nat. Commun.* **5**, 3524 (2014).
 - [13] W. Kersten, N. de Zordo, O. Diekmann, T. Reiter, M. Zens, A. N. Kanagin, S. Rotter, J. Schmiedmayer, and A. Angerer, Triggered superradiance and spin inversion storage in a hybrid quantum system, *Phys. Rev. Lett.* **131**, 043601 (2023).
 - [14] D. I. Schuster, A. P. Sears, E. Ginossar, L. DiCarlo, L. Frunzio, J. J. L. Morton, H. Wu, G. A. D. Briggs, B. B. Buckley, D. D. Awschalom, and R. J. Schoelkopf, High-cooperativity coupling of electron-spin ensembles to superconducting cavities, *Phys. Rev. Lett.* **105**, 140501 (2010).
 - [15] Y. Kubo, F. R. Ong, P. Bertet, D. Vion, V. Jacques, D. Zheng, A. Dréau, J.-F. Roch, A. Auffeves, F. Jelezko, J. Wrachtrup, M. F. Barthe, P. Bergonzo, and D. Esteve, Strong coupling of a spin ensemble to a superconducting resonator, *Phys. Rev. Lett.* **105**, 140502 (2010).
 - [16] R. Amsüss, C. Koller, T. Nöbauer, S. Putz, S. Rotter, K. Sandner, S. Schneider, M. Schramböck, G. Steinhäuser, H. Ritsch, J. Schmiedmayer, and J. Majer, Cavity QED with magnetically coupled collective spin states, *Phys. Rev. Lett.* **107**, 060502 (2011).
 - [17] V. Ranjan, G. de Lange, R. Schutjens, T. Debelhoir, J. P. Groen, D. Szombati, D. J. Thoen, T. M. Klapwijk, R. Hanson, and L. DiCarlo, Probing dynamics of an electron-spin ensemble via a superconducting resonator, *Phys. Rev. Lett.* **110**, 067004 (2013).
 - [18] C. Grezes, B. Julsgaard, Y. Kubo, M. Stern, T. Umeda, J. Isoya, H. Sumiya, H. Abe, S. Onoda, T. Ohshima, V. Jacques, J. Esteve, D. Vion, D. Esteve, K. Mølmer, and P. Bertet, Multimode storage and retrieval of microwave fields in a spin ensemble, *Phys. Rev. X* **4**, 021049 (2014).
 - [19] A. Angerer, K. Streltsov, T. Astner, S. Putz, H. Sumiya, S. Onoda, J. Isoya, W. J. Munro, K. Nemoto, J. Schmiedmayer, and J. Majer, Superradiant emission from colour centres in diamond, *Nat. Phys.* **14**, 1168 (2018).
 - [20] X. Zhang, C.-L. Zou, L. Jiang, and H. X. Tang, Strongly coupled magnons and cavity microwave photons, *Phys. Rev. Lett.* **113**, 156401 (2014).
 - [21] M. Goryachev, W. G. Farr, D. L. Creedon, Y. Fan, M. Kostylev, and M. E. Tobar, High-cooperativity cavity QED with magnons at microwave frequencies, *Phys. Rev. Appl.* **2**, 054002 (2014).
 - [22] Y. Tabuchi, S. Ishino, T. Ishikawa, R. Yamazaki, K. Usami, and Y. Nakamura, Hybridizing ferromagnetic magnons and microwave photons in the quantum limit, *Phys. Rev. Lett.* **113**, 083603 (2014).
 - [23] S. Probst, H. Rotzinger, S. Wünsch, P. Jung, M. Jerger, M. Siegel, A. V. Ustinov, and P. A. Bushev, Anisotropic rare-earth spin ensemble strongly coupled to a superconducting resonator, *Phys. Rev. Lett.* **110**, 157001 (2013).
 - [24] X. Zhu, S. Saito, A. Kemp, K. Kakuyanagi, S.-i. Karimoto, H. Nakano, W. J. Munro, Y. Tokura, M. S. Everitt, K.

- Nemoto, M. Kasu, N. Mizuochi, and K. Semba, Coherent coupling of a superconducting flux qubit to an electron spin ensemble in diamond, *Nature (London)* **478**, 221 (2011).
- [25] Y. Kubo, C. Grezes, A. Dewes, T. Umeda, J. Isoya, H. Sumiya, N. Morishita, H. Abe, S. Onoda, T. Ohshima, V. Jacques, A. Dréau, J.-F. Roch, I. Diniz, A. Auffèves, D. Vion, D. Esteve, and P. Bertet, Hybrid quantum circuit with a superconducting qubit coupled to a spin ensemble, *Phys. Rev. Lett.* **107**, 220501 (2011).
- [26] S. Saito, X. Zhu, R. Amsüss, Y. Matsuzaki, K. Kakuyanagi, T. Shimo-Oka, N. Mizuochi, K. Nemoto, W. J. Munro, and K. Semba, Towards realizing a quantum memory for a superconducting qubit: Storage and retrieval of quantum states, *Phys. Rev. Lett.* **111**, 107008 (2013).
- [27] Y. Tabuchi, S. Ishino, A. Noguchi, T. Ishikawa, R. Yamazaki, K. Usami, and Y. Nakamura, Coherent coupling between a ferromagnetic magnon and a superconducting qubit, *Science* **349**, 405 (2015).
- [28] The resonant frequency is slightly shifted to 3.0012 GHz due to multiple adjustments of magnetic field and fridge temperature throughout the experiment.
- [29] Z. Kurucz, J. H. Wesenberg, and K. Mølmer, Spectroscopic properties of inhomogeneously broadened spin ensembles in a cavity, *Phys. Rev. A* **83**, 053852 (2011).
- [30] I. Diniz, S. Portolan, R. Ferreira, J. M. Gérard, P. Bertet, and A. Auffèves, Strongly coupling a cavity to inhomogeneous ensembles of emitters: Potential for long-lived solid-state quantum memories, *Phys. Rev. A* **84**, 063810 (2011).
- [31] K. Sandner, H. Ritsch, R. Amsüss, C. Koller, T. Nöbauer, S. Putz, J. Schmiedmayer, and J. Majer, Strong magnetic coupling of an inhomogeneous nitrogen-vacancy ensemble to a cavity, *Phys. Rev. A* **85**, 053806 (2012).
- [32] L. S. Bishop, J. M. Chow, J. Koch, A. A. Houck, M. H. Devoret, E. Thuneberg, S. M. Girvin, and R. J. Schoelkopf, Nonlinear response of the vacuum Rabi resonance, *Nat. Phys.* **5**, 105 (2009).
- [33] A. Angerer, S. Putz, D. O. Krimer, T. Astner, M. Zens, R. Glattauer, K. Streltsov, W. J. Munro, K. Nemoto, S. Rotter, J. Schmiedmayer, and J. Majer, Ultralong relaxation times in bistable hybrid quantum systems, *Sci. Adv.* **3**, e1701626 (2017).
- [34] Y. Yang, H. H. Vallabhapurapu, V. K. Sewani, M. Isarov, H. R. Firgau, C. Adambukulam, B. C. Johnson, J. J. Pla, and A. Laucht, Observing hyperfine interactions of NV⁻ centers in diamond in an advanced quantum teaching lab, *Am. J. Phys.* **90**, 550 (2022).
- [35] Y. Mao, M. Gong, K. Nemoto, W. J. Munro, and J. Majer, Perspective on witnessing entanglement in hybrid quantum systems, *Appl. Phys. Lett.* **119**, 110501 (2021).
- [36] M. Lei, R. Fukumori, J. Rochman, B. Zhu, M. Endres, J. Choi, and A. Faraon, Many-body cavity quantum electrodynamics with driven inhomogeneous emitters, *Nature (London)* **617**, 271 (2023).
- [37] T. Nadolny and C. Bruder, Macroscopic quantum synchronization effects, *Phys. Rev. Lett.* **131**, 190402 (2023).
- [38] Y. Kurman, K. Hymas, A. Fedorov, W. J. Munro, and J. Quach, Quantum computation with quantum batteries, *Phys. Rev. X* **16**, 011016 (2026).