

Unified wake topology map for He II counterflow past a cylinder

Yingxuan Hu¹, Wenlin Huang¹, Shihao Yang¹, Limin Qiu^{1,2,*} and Shiran Bao^{1,2,†}

¹*Institute of Refrigeration and Cryogenics, Zhejiang University, Hangzhou 310027, China*

²*Zhejiang Key Laboratory of Refrigeration and Cryogenic Technology, Hangzhou 310027, China*



(Received 5 February 2026; revised 5 April 2026; accepted 14 July 2026; published 10 August 2026)

Thermal counterflow of superfluid ^4He past a cylinder produces quasisteady eddies not only downstream but also anomalously upstream. However, the mechanism and organizing principles behind the observed multistable wake topologies (zero-, two-, four-, and six-vortex states) have remained unclear. We show that the full spectrum of reported normal-fluid wake states is captured numerically with a two-fluid model coupled to Vinen's vortex-line-density equation. Our simulations further reveal that the superfluid component can also develop anomalous upstream eddies, a feature not previously reported. We trace these behaviors to a self-organized zone of enhanced mutual-friction dissipation near the cylinder shoulders that reshapes the effective obstacle, drives upstream eddies in both components, and suppresses intrinsic wake oscillations in the normal fluid. Guided by this mechanism, we perform systematic parameter scans and construct a unified phase diagram in terms of the normal-fluid Reynolds number Re_n and a dimensionless interaction number N , separating inertia- and mutual-friction-controlled transitions and delineating the parameter windows for the discrete wake topologies. These results turn a striking phenomenology into a predictive map and establish mutual-friction feedback as a robust route to unusual wake structures in quantum fluids.

DOI: [10.1103/klbl-7xnz](https://doi.org/10.1103/klbl-7xnz)

I. INTRODUCTION

Flow around a circular cylinder is among the most paradigmatic problems in fluid mechanics. In classical viscous fluids, the canonical wake sequence—from steady recirculation to the formation of the Kármán vortex street—has been mapped out in detail and organized by well-defined control parameters such as the Reynolds number (Re) [1,2]. Replacing the working fluid with superfluid ^4He (He II) below the lambda point at 2.17 K fundamentally transforms the flow physics. He II is a quantum fluid described by Landau-Tisza two-fluid model [3]: An inviscid superfluid component coexists with a viscous normal component composed of thermal quasiparticles. The superfluid supports quantized vortex lines with circulation fixed to $\kappa \simeq 9.97 \times 10^{-4} \text{ cm}^2/\text{s}$ [4]. As vortices move relative to the normal fluid, mutual friction between the two fluids arises from scattering of thermal quasiparticles off the vortex cores [5–7]. This coupling allows the evolving vortex tangle to feed back on the two-fluid motion and reshape wake formation, opening the door to flow states with no classical analog.

A particularly striking manifestation occurs in thermally driven counterflow. When a heat flux is applied, the normal fluid carries entropy away from the heater, while the superfluid flows oppositely to maintain zero net mass flux [8]. Zhang and Van Sciver discovered that, in such counterflow

past a cylinder, large-scale quasisteady vortex pairs can appear not only downstream but also anomalously upstream of the obstacle [9]. This behavior sharply contrasts with the classical cylinder wake, where steady eddies reside only downstream. Subsequent experiments by different groups extended these observations across a range of conditions, revealing that the wake is not described by a single configuration but instead exhibits pronounced multistability: Depending on parameters, the flow can settle into states with no vortices, or with two, four, or six large-scale vortices [10–13]. These findings suggest that mutual-friction coupling can reorganize the wake topology and introduce robust wake states.

Considerable effort has been devoted to explaining the anomalous upstream eddies. Sergeev and Barenghi introduced an ideal point-vortex model showing that stationary upstream vortex configurations can exist for specific circulation values [14]. While insightful, this inviscid framework omits the mutual-friction feedback that is central to He II counterflow. Later, Soullaine *et al.* achieved the first numerical reproduction of the anomalous four-vortex wake using two-fluid simulations and showed that its onset is governed by a dimensionless mutual-friction intensity [15]. Some recent work examined how heat flux, boundary conditions, and modeling choices influence the evolution and stability of this four-vortex state [16,17]. Despite these advances, a central gap remains: Experiments demonstrate a family of distinct, long-lived wake topologies and transitions among them, whereas most numerical studies have focused on a single representative state. A unified framework is still lacking that (1) captures all the observed wake topologies and (2) organizes the transitions into a predictive phase diagram. Meanwhile, the superfluid wake has remained largely unexplored: Existing reports focus on normal-fluid streamline patterns, leaving open whether the superfluid can also develop analogous wake structures.

*Contact author: limin.qiu@zju.edu.cn

†Contact author: srbao@zju.edu.cn

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International](https://creativecommons.org/licenses/by/4.0/) license. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI.

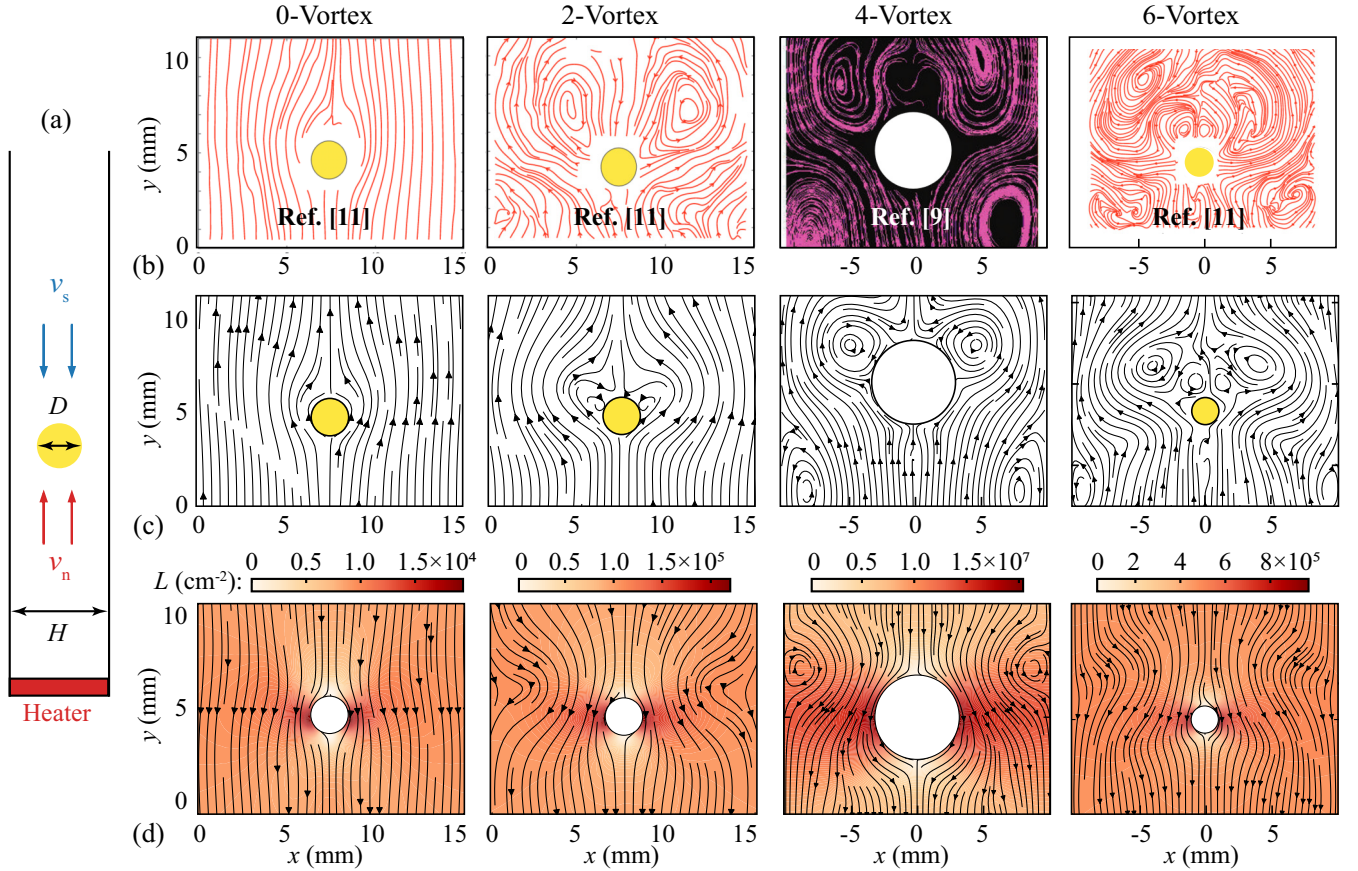


FIG. 1. (a) Schematic of the computational domain. (b) Experimentally reconstructed streamline patterns from particle tracks [9,11]. (c) Simulated normal-fluid streamlines at matched T , q , and system sizes. (d) Corresponding superfluid streamlines overlaid on a colormap of the vortex-line density $L(x, y)$. Parameters are zero-vortex case at $T = 1.94$ K, $q = 50$ mW/cm², blockage ratio $B \equiv D/H = 10\%$; two-vortex case at $T = 1.94$ K, $q = 170$ mW/cm², $B = 10\%$; four-vortex case at $T = 2.03$ K, $q = 1120$ mW/cm², $B = 31.75\%$; and six-vortex case at $T = 2.10$ K, $q = 167$ mW/cm², $B = 10\%$.

In this article, we fill this gap using a unified continuum framework [18–20]. Specifically, we solve the He II two-fluid equations coupled to Vinen’s vortex-line-density equation, so that the vortex tangle and the mutual friction it mediates evolve self-consistently with the flow. Within this description, we reproduce the experimentally observed multistable normal-fluid wake spectrum and further show that the superfluid component can also develop anomalous upstream eddies, a feature not previously reported. We trace this coupled behavior to a self-organized, strongly dissipative region near the cylinder shoulders that, through nonlinear mutual-friction feedback, reshapes the effective obstacle and drives the upstream recirculation that sustains the anomalous eddies. Building on this mechanism, we construct a unified phase diagram in terms of selected dimensionless control parameters, which organizes the discrete wake topologies and separates inertia- and mutual-friction-controlled transitions, providing a mechanism-based framework for wake-topology bifurcations in He II counterflow.

II. NUMERICAL METHOD

As illustrated in Fig. 1(a), the computational domain is a two-dimensional channel of length L_c and width H containing

a circular cylinder of diameter D . We model the He II flow using the two-fluid hydrodynamics [3,21] in which the normal and superfluid components with velocities $\mathbf{v}_n$ and $\mathbf{v}_s$ (densities ρ_n and ρ_s) are coupled through a volumetric mutual-friction force $\mathbf{F}_{ns}$ as [6,22,23]

$$\rho_n \frac{D_n \mathbf{v}_n}{Dt} = -\frac{\rho_n}{\rho} \nabla p - \rho_s \nabla \mu + \nabla \cdot (\eta_n \nabla \mathbf{v}_n) - \mathbf{F}_{ns}, \quad (1)$$

$$\rho_s \frac{D_s \mathbf{v}_s}{Dt} = -\frac{\rho_s}{\rho} \nabla p + \rho_s \nabla \mu + \mathbf{F}_{ns}, \quad (2)$$

$$\frac{\partial}{\partial t} (\rho s) + \nabla \cdot (\rho s \mathbf{v}_n) = \frac{1}{T} \mathbf{F}_{ns} \cdot \mathbf{v}_{ns}, \quad (3)$$

where $D_n/Dt \equiv \partial/\partial t + \mathbf{v}_n \cdot \nabla$ and $D_s/Dt \equiv \partial/\partial t + \mathbf{v}_s \cdot \nabla$, $\rho = \rho_n + \rho_s$ is the total density, p is the pressure, η_n is the normal-fluid viscosity, s is the specific entropy, and $\mathbf{v}_{ns} = \mathbf{v}_n - \mathbf{v}_s$ is the relative velocity. The chemical potential gradient is given by $\nabla \mu = -s \nabla T - \frac{\rho_n}{2\rho} \nabla |\mathbf{v}_{ns}|^2$ [3].

At the coarse-grained level, the mutual-friction force is modeled as $\mathbf{F}_{ns} = \frac{\rho_n \rho_s}{3\rho} B_L(T) \kappa L \mathbf{v}_{ns}$, where $B_L(T)$ is the tabulated mutual-friction coefficient [24] and $L(\mathbf{r}, t)$ is the vortex-line density, whose evolution is described by Vinen’s

equation [5,6]

$$\frac{\partial L}{\partial t} + \nabla \cdot (\mathbf{v}_L L) = \alpha_v |\mathbf{v}_{ns}| L^{3/2} - \beta_v L^2 + \gamma_v |\mathbf{v}_{ns}|^{5/2}, \quad (4)$$

where α_v , β_v , and γ_v are temperature-dependent empirical coefficients, and the vortex drift velocity is taken as $\mathbf{v}_L \simeq \mathbf{v}_s$ [5,6,25,26]. Note that instead of solving Eq. (4), Soullaine *et al.* assumed instantaneous local equilibrium value $L = (\alpha_v/\beta_v)^2 |\mathbf{v}_{ns}|^2$ in their work [15]. This reduction is valid only when the vortex relaxation dynamics is fast compared with flow advection [27].

We impose counterflow by prescribing the inlet normal-fluid velocity using the applied heat flux q as $|\mathbf{v}_{n,0}| = q/(\rho_s T)$, together with $\mathbf{v}_{s,0} = -(\rho_n/\rho_s)\mathbf{v}_{n,0}$ to enforce zero net mass flux. This again differs from Ref. [15], which modeled the heat input as a volumetric source term. On the cylinder surface and channel walls, we apply no slip for the normal fluid, $\mathbf{v}_n = \mathbf{0}$, and impermeability for the superfluid, $\mathbf{v}_s \cdot \hat{\mathbf{n}} = 0$. Although this continuum model does not resolve individual quantized vortices [28–30], it has been extensively validated as a reliable description of the coupled two-fluid dynamics on length scales larger than the mean intervortex spacing [22,23,31–33]. Numerical details (discretization, boundary conditions, and convergence tests) are provided in the Supplemental Material [34].

III. VALIDATION AND WAKE OBSERVATIONS

To validate the model and demonstrate its predictive capability, we performed numerical simulations at the same bath temperature T , heat flux q , and channel blockage ratio $B \equiv D/H$ as in the experiments of Refs. [9,11]. Figures 1(b) and 1(c) directly compare the experimentally rendered streamline patterns with the simulated normal-fluid streamline topologies under these matched conditions. Since the experimental streamlines were reconstructed from tracer-particle trajectories in He II, where the particles may also interact with quantized vortices, they should be regarded as a qualitative indicator of the large-scale wake structure rather than as a point-by-point measurement of the coarse-grained normal-fluid velocity field. With this caveat, the simulations reproduce the full experimentally observed spectrum of quasisteady wake states, namely, the zero-, two-, four-, and six-vortex configurations. At low q , the wake is vortex-free and nearly potential. As q increases, a steady downstream recirculation forms (the two-vortex state). In this case, the experimental recirculation zone appears larger than that in the simulation, likely due to deviations of the particle-based streamline reconstruction from the true normal-fluid flow field. With further increase in q , the wake does not develop periodic vortex shedding as in classical cylinder flows; instead, an additional *upstream* vortex pair emerges. For small B , the wake can further organize into a symmetric six-vortex configuration at sufficiently large q .

The fact that these discrete, multistable states emerge in the normal fluid within one continuum framework highlights vortex-mediated mutual friction as the key nonlinear mechanism controlling wake-topology selection. The essential ingredient is the strong spatial inhomogeneity of the counterflow around the cylinder. As the incoming flow is

diverted, the counterflow is funneled through the two shoulder corridors, producing a locally large relative velocity $\mathbf{v}_{ns}$; because the vortex-line density rises rapidly with $\mathbf{v}_{ns}$ toward its local-equilibrium scaling $L_{eq} \propto |\mathbf{v}_{ns}|^2$ [6,35], the vortex tangle concentrates near the shoulders. This concentration is directly visible in Fig. 1(d), which plots $L(x, y)$ together with the superfluid streamlines for the same (T, q, B) as the corresponding normal-fluid cases in Fig. 1(c). Since the mutual-friction force scales as $\mathbf{F}_{ns} \propto L\mathbf{v}_{ns}$, the shoulder-localized tangle creates a compact zone of enhanced dissipation that acts as a permeability barrier, effectively “thickening” the obstacle and redirecting the incoming normal flow. Together with the lateral confinement imposed by the channel walls, this diverted flow can close into an upstream recirculating cell and thereby sustains the anomalous upstream vortex pair. This phenomenon is a close analog of elastic blockage in viscoelastic wakes, where a localized buildup of nonlinear stresses upstream of a confined obstacle similarly rejects the inflow and induces an upstream recirculation zone [35–38].

The mutual-friction dissipation also explains the striking downstream wake stability. In a classical viscous cylinder wake, periodic Kármán shedding sets in already at low Reynolds number, $\text{Re} \simeq 47$, when inertia overcomes viscous diffusion [2]. In sharp contrast, He II counterflow experiments report a stable two-vortex configuration [Fig. 1(c)] persisting up to $\text{Re}_n \equiv \rho_n v_{n,0} D / \eta_n \approx 1554$ [11]. This dramatic stabilization arises because, as q increases, the mutual friction $\mathbf{F}_{ns}$ rapidly overwhelms the normal-fluid viscous term $\nabla \cdot (\eta_n \nabla \mathbf{v}_n)$ across the computational domain (see Supplemental Material [34]). The separated shear layers therefore undergo strong mutual-friction damping, suppressing the shear-layer instability that drives Kármán vortex shedding [39].

In Fig. 1(d), we also show the superfluid response under the same conditions. Although the superfluid is inviscid, it can sustain eddylike, coarse-grained circulation on length scales larger than the mean intervortex spacing, where partial vortex polarization and bundling generate a nonzero averaged vorticity [40]. As q increases, the resulting mutual-friction barrier increasingly deflects the superfluid streamlines and promotes an upstream recirculation tendency. However, because the superfluid does not satisfy a no-slip condition at solid boundaries, the near-wall superfluid speed remains large and readily erodes closed recirculating cells; robust upstream superfluid eddies therefore appear only under strong confinement (large B), where the wall geometry cooperates with the shoulder barrier to trap a recirculating cell (see the case with $B = 31.75\%$). In contrast, the superfluid shows no detached downstream wake, consistent with the absence of a viscous boundary layer on the cylinder and hence the lack of boundary-layer separation as a downstream wake-forming mechanism [1].

IV. WAKE TOPOLOGY MAP

To locate transitions among the discrete wake states and place them on a common footing, we construct a wake-topology phase diagram by scanning the control-parameter space (T, q, B) . We surveyed the temperature range $T = 1.9\text{--}2.1$ K and performed systematic sweeps in q and

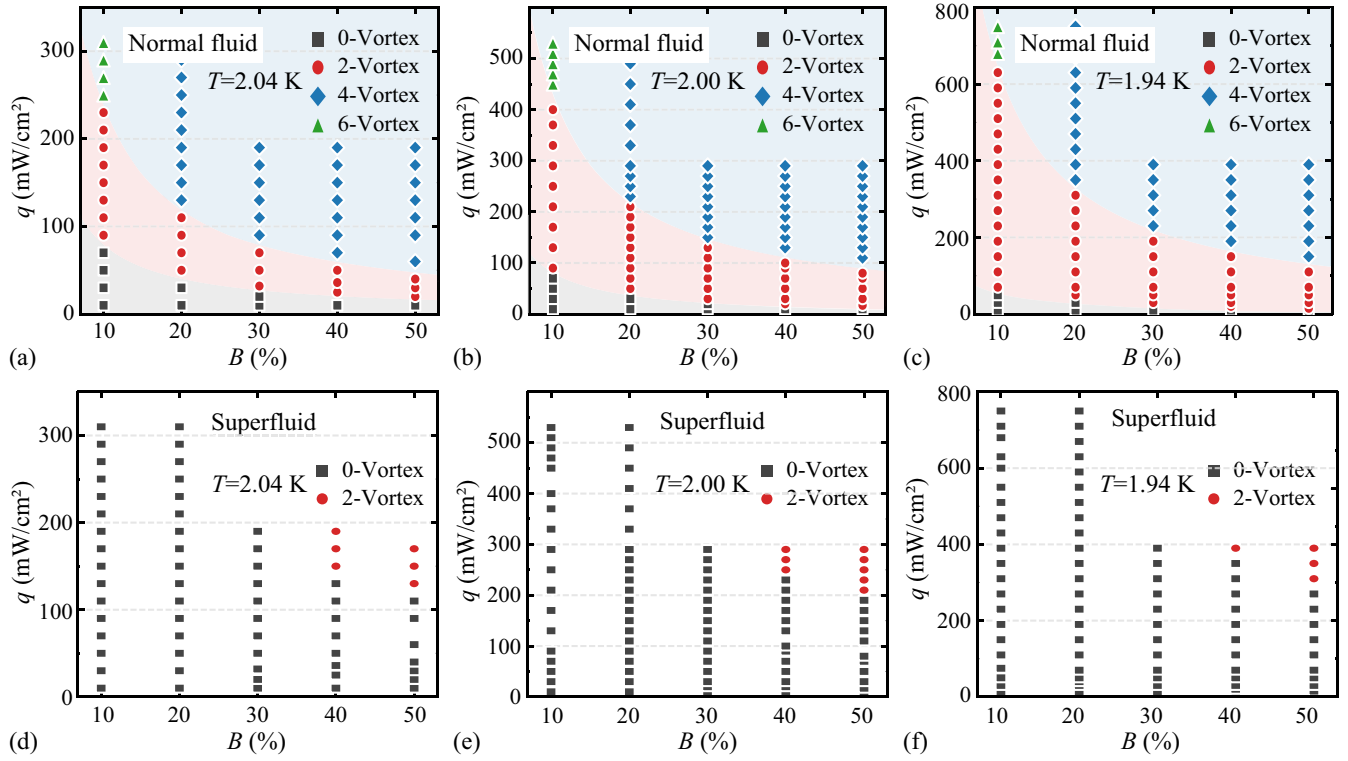


FIG. 2. Wake-topology phase diagrams in the (q, B) plane at $T = 2.04, 2.00$, and 1.94 K for the normal fluid [panels (a)–(c)] and the superfluid [panels (d)–(f)]. The shaded backgrounds indicate the different regimes, i.e., viscosity-dominated (gray), inertia-dominated (red), and mutual-friction-dominated (blue) regions. Symbols denote the observed wake topologies.

B to resolve the wake-state transitions. Figure 2 presents the resulting normal-fluid and superfluid wake topology maps at three representative temperatures, $T = 1.94, 2.00$, and 2.04 K. For each displayed (T, B) , we increased q in steps of 20 mW/cm^2 and refined the increment to 5 mW/cm^2 near transition thresholds.

For the normal fluid, at fixed blockage ratio B , increasing q drives a systematic progression from a viscous-dominated, nearly potential-flow state (zero-vortex) to an inertia-driven separated wake characterized by a downstream vortex pair (two-vortex), and then to a mutual-friction-controlled regime in which an additional upstream vortex pair nucleates, producing the anomalous four-vortex topology. The six-vortex state, with two upstream vortices accompanied by four quastable vortices in the wake, appears only in a restricted corner of the parameter space at the smallest B and sufficiently large q . Meanwhile, increasing B shifts the onset of both the two-vortex and four-vortex states to lower heat flux in every panel, demonstrating that stronger confinement makes wake separation and upstream-pair nucleation accessible at smaller imposed q . In contrast, the superfluid exhibits only a single transition, from the zero-vortex state to the anomalous upstream two-vortex state, and only at sufficiently large B and high q .

The two sequential transitions of the normal fluid in Fig. 2 imply two distinct competitions. The first transition, from the zero-vortex state to the two-vortex wake, is set by the balance between normal-fluid viscous diffusion and inertial advection, and is therefore naturally characterized by the normal-fluid Reynolds number Re_n . The second transi-

tion, from the two-vortex wake to the anomalous four-vortex topology, occurs once mutual friction becomes significant. To quantify this competition, we compare the characteristic force-density scales: $F_{\text{in}} \sim \rho_n v_{n,0}^2 / D$ for inertial advection and $F_{\text{ns}} \sim \frac{\rho_n \rho_s}{3\rho} \kappa B_L L_0 v_{\text{ns},0}$ for mutual friction. Using the Vinen local-equilibrium scaling $L_0 \simeq (\alpha_v / \beta_v)^2 v_{\text{ns},0}^2$ together with the inlet counterflow velocity $v_{\text{ns},0} = (\rho / \rho_s) v_{n,0}$, the ratio $F_{\text{ns}} / F_{\text{in}}$ defines a dimensionless interaction number N given by

$$N = \left[\frac{\rho^2}{3\rho_s^2} B_L \kappa (\alpha_v / \beta_v)^2 \right] v_{n,0} D. \quad (5)$$

This form mirrors that of Soullaine *et al.* [15]: All temperature dependence is absorbed into a single prefactor, leaving the scaling controlled by the kinematic product $v_{n,0} D$.

Guided by this reasoning, we construct the stratified dimensionless phase diagram for the normal fluid in Fig. 3 by replotting all normal-fluid wake-topology points, together with the experimental data, in two complementary projections. In the bottom panel, we show the onset of wake separation into the two-vortex state in the (Re_n, B) plane; the boundary collapses to a nearly constant threshold $\text{Re}_{n,c} \approx 300$, indicating that, within the blockage-ratio range explored here, this kinematic transition is largely insensitive to T and shows only weak dependence on B once viscosity and inertia are properly scaled. In the top panel, we show the onset of upstream-pair nucleation into the anomalous four-vortex state in the (N, B) plane; the boundary similarly collapses near $N_c \approx 33$, indicating that this bifurcation is

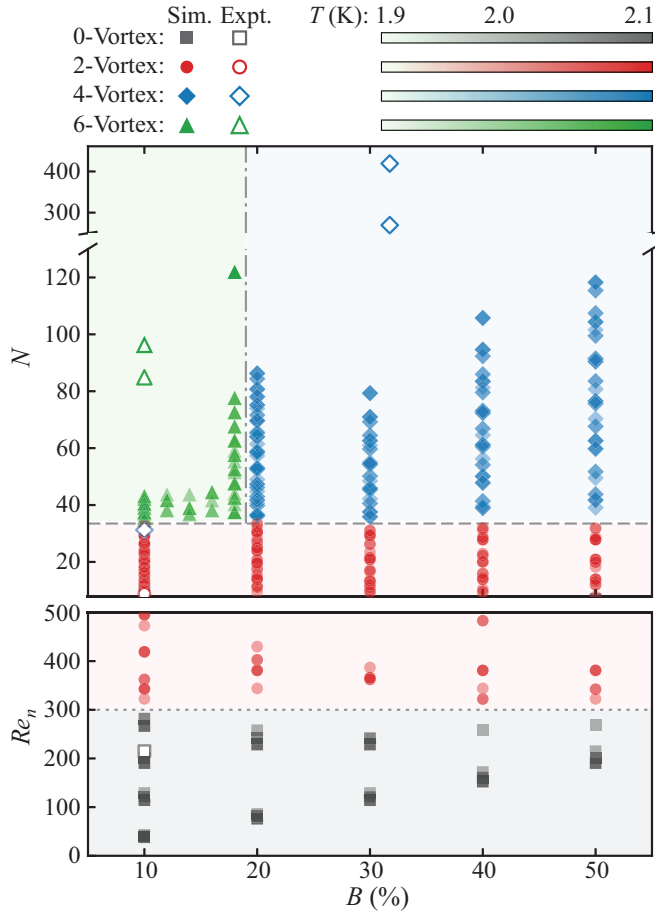


FIG. 3. Stratified dimensionless phase diagram compiling all wake-topology points from Fig. 2 together with the experimental data. Bottom: Separation onset from the zero-vortex to two-vortex state plotted in the (Re_n, B) plane. Top: Wake topology plotted as the interaction number N vs B , showing the two-vortex region below $N_c \simeq 33$, the four-vortex region above N_c at larger B , and the six-vortex region above N_c at small B . Open and filled symbols denote experimental and numerical results, respectively, and symbol opacity encodes the bath temperature T (colorbar).

governed primarily by the balance between inertial advection and vortex-mediated mutual friction and again shows weak dependence on B . The six-vortex topology appears only for $N \gtrsim N_c$ and is further restricted to $B \lesssim 19\%$, indicating that, in addition to requiring sufficiently strong vortex feedback, this higher-order state is geometrically permitted only when confinement is weak enough to provide adequate downstream space for four quasistable wake vortices to coexist with the upstream pair [41,42]. We note that among the seven experimental data points included in Fig. 3, six fall in the phase regions predicted by the simulations. The only outlying point

is an experimental four-vortex point in the upper panel, close to the boundary between the two- and six-vortex states. This apparent mismatch may reflect the limited experimental resolution of the two weaker downstream eddies, which could cause a true six-vortex state to appear experimentally as a four-vortex pattern.

V. CONCLUSION

Overall, these results turn the He II counterflow cylinder wake from a long-standing visual curiosity into a quantitative benchmark for how mutual friction reshapes, stabilizes, and reorganizes macroscopic flow. The key advance is not merely reproducing multiple steady topologies, but demonstrating that their bifurcations can be predicted from a small set of measurable inputs within a continuum framework that retains the essential nonlinearity of vortex-mediated feedback. We note, however, an intrinsic limitation of coarse-grained two-fluid models of this class: Quantized vortices enter only through a smoothed mutual-friction field, so effects associated with individual vortex lines are averaged out. In particular, a moving vortex can generate a macroscopic wake in the normal fluid [7,43–45], and in dense tangles such wakes may overlap and drive large-scale disturbances that could, in principle, promote normal-fluid turbulence even in the presence of strong mutual-friction damping [46,47]. These effects lie beyond the present model's resolution. Nevertheless, within its range of validity, the present study isolates the dominant mutual-friction-controlled mechanism and yields a quantitative wake-topology map, providing a practical baseline for interpreting measurements and suggesting strategies to tailor flow stability in He II transport and thermal management where suppressing or triggering large-scale recirculation is desirable.

ACKNOWLEDGMENTS

The authors thank W. Guo and S. Xiong for valuable discussions. S.B. acknowledges support from the National Natural Science Foundation of China (Grants No. 52441601 and No. 52206028) and the Natural Science Foundation of Zhejiang Province (Grant No. LZ25E060002). L.Q. acknowledges support from the Fundamental Research Funds for the Central Universities (Grant No. K20250227).

DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

- [1] G. K. Batchelor, *An Introduction to Fluid Dynamics* (Cambridge University Press, Cambridge, England, 2000).
- [2] C. H. K. Williamson, Vortex dynamics in the cylinder wake, *Annu. Rev. Fluid Mech.* **28**, 477 (1996).

- [3] L. D. Landau and E. M. Lifshitz, *Fluid Mechanics*, 2nd ed. (Elsevier, Amsterdam, 1987), Vol. 6.
- [4] R. J. Donnelly, *Quantized Vortices in Helium II* (Cambridge University Press, Cambridge, England, 1991).

- [5] W. F. Vinen, Mutual friction in a heat current in liquid helium II. II. Experiments on transient effects, *Proc. R. Soc. A* **240**, 128 (1957).
- [6] W. F. Vinen, Mutual friction in a heat current in liquid helium II. III. Theory of the mutual friction, *Proc. R. Soc. A* **242**, 493 (1957).
- [7] Y. Tang, W. Guo, H. Kobayashi, S. Yui, M. Tsubota, and T. Kanai, Imaging quantized vortex rings in superfluid helium to evaluate quantum dissipation, *Nat. Commun.* **14**, 2941 (2023).
- [8] S. W. Van Sciver, *Helium Cryogenics* (Springer, New York, 2012).
- [9] T. Zhang and S. W. Van Sciver, Large-scale turbulent flow around a cylinder in counterflow superfluid ^4He (He II), *Nat. Phys.* **1**, 36 (2005).
- [10] S. W. Van Sciver, S. Fuzier, and T. Xu, Particle image velocimetry studies of counterflow heat transport in superfluid He II, *J. Low Temp. Phys.* **148**, 225 (2007).
- [11] T. V. Chagovets and S. W. Van Sciver, Visualization of He II counterflow around a cylinder, *Phys. Fluids* **25**, 105104 (2013).
- [12] D. Duda, M. La Mantia, M. Rotter, and L. Skrbek, On the visualization of thermal counterflow of He II past a circular cylinder, *J. Low Temp. Phys.* **175**, 331 (2014).
- [13] S. Fuzier, S. W. Van Sciver, and T. Zhang, PIV measurements of He II counterflow around a cylinder, *AIP Conf. Proc.* **850**, 203 (2006).
- [14] Y. A. Sergeev and C. F. Barenghi, Normal fluid eddies in the thermal counterflow past a cylinder, *J. Low Temp. Phys.* **156**, 268 (2009).
- [15] C. Soullaine, M. Quintard, B. Baudouy, and R. Van Weelden, Numerical investigation of thermal counterflow of He II past cylinders, *Phys. Rev. Lett.* **118**, 074506 (2017).
- [16] Z. Li, Z. Liu, Q. Zhang, M. Dai, Z. Zhang, and Y. He, Evolution of the four-vortex structure of the He II thermal counterflow based on the simplified two-fluid model, *Phys. Fluids* **35**, 095130 (2023).
- [17] H. Yousefi and H. Afshin, Numerical simulation of thermal counterflow in superfluid helium: Investigating the effect of rotation and heat flux on the surface of the cylinder, *Comput. Phys. Commun.* **310**, 109495 (2025).
- [18] Y. Hu, K. Wang, X. Zhi, L. Qiu, and S. Bao, Numerical study on the development length of thermal counterflow in superfluid helium, *Phys. Fluids* **37**, 073612 (2025).
- [19] Y. Hu, W. Huang, S. Yang, K. Wang, X. Zhi, L. Qiu, and S. Bao, Transient heat transfer in superfluid helium (He-II) from circular heaters with finite dimensions, *Int. Commun. Heat Mass Transfer* **169**, 109564 (2025).
- [20] S. Bao and W. Guo, Transient heat transfer of superfluid ^4He in nonhomogeneous geometries: Second sound, rarefaction, and thermal layer, *Phys. Rev. B* **103**, 134510 (2021).
- [21] S. K. Nemirovskii and V. V. Lebedev, Hydrodynamics of superfluid turbulence, *Zh. Eksp. Teor. Fiz.* **84**, 1729 (1983) [*Sov. Phys. JETP* **57**, 1009 (1983)].
- [22] S. K. Nemirovskii, On the closure problem of the coarse-grained hydrodynamics of turbulent superfluids, *J. Low Temp. Phys.* **201**, 254 (2020).
- [23] M. Tsubota, K. Fujimoto, and S. Yui, Numerical studies of quantum turbulence, *J. Low Temp. Phys.* **188**, 119 (2017).
- [24] R. J. Donnelly and C. F. Barenghi, The observed properties of liquid helium at the saturated vapor pressure, *J. Phys. Chem. Ref. Data* **27**, 1217 (1998).
- [25] R. T. Wang, C. E. Swanson, and R. J. Donnelly, Anisotropy and drift of a vortex tangle in helium II, *Phys. Rev. B* **36**, 5240 (1987).
- [26] L. Kondaurova, V. L'vov, A. Pomyalov, and I. Procaccia, Structure of a quantum vortex tangle in ^4He counterflow turbulence, *Phys. Rev. B* **89**, 014502 (2014).
- [27] S. K. Nemirovskii, Quantum turbulence: Theoretical and numerical problems, *Phys. Rep.* **524**, 85 (2013).
- [28] C. F. Barenghi, A. V. Gordeev, and L. Skrbek, Depolarization of decaying counterflow turbulence in He II, *Phys. Rev. E* **74**, 026309 (2006).
- [29] A. W. Baggaley, The sensitivity of the vortex filament method to different reconnection models, *J. Low Temp. Phys.* **168**, 18 (2012).
- [30] S. Yui, Y. Tang, W. Guo, H. Kobayashi, and M. Tsubota, Universal anomalous diffusion of quantized vortices in ultraquantum turbulence, *Phys. Rev. Lett.* **129**, 025301 (2022).
- [31] H. Sanavandi, M. Hulse, S. Bao, Y. Tang, and W. Guo, Boiling and cavitation caused by transient heat transfer in superfluid helium-4, *Phys. Rev. B* **106**, 054501 (2022).
- [32] S. Inui, M. Hulse, T. Kanai, and W. Guo, Boiling peak heat flux for steady inhomogeneous heat transfer in superfluid ^4He , *Phys. Rev. B* **108**, 174509 (2023).
- [33] L. Galantucci, M. Sciacca, and C. F. Barenghi, Coupled normal fluid and superfluid profiles of turbulent helium II in channels, *Phys. Rev. B* **92**, 174530 (2015).
- [34] See Supplemental Material at <https://link.aps.org/supplemental/10.1103/kibl-7xnz> for detailed governing equations, boundary conditions, and grid independence tests.
- [35] S. Peng, T. Tang, J. Li, M. Zhang, and P. Yu, Numerical study of viscoelastic upstream instability, *J. Fluid Mech.* **959**, A16 (2023).
- [36] C. C. Hopkins, S. J. Haward, and A. Q. Shen, Upstream wall vortices in viscoelastic flow past a cylinder, *Soft Matter* **18**, 4868 (2022).
- [37] C. C. Hopkins, A. Q. Shen, and S. J. Haward, Effect of blockage ratio on flow of a viscoelastic wormlike micellar solution past a cylinder in a microchannel, *Soft Matter* **18**, 8856 (2022).
- [38] B. Qin, P. F. Salipante, S. D. Hudson, and P. E. Arratia, Upstream vortex and elastic wave in the viscoelastic flow around a confined cylinder, *J. Fluid Mech.* **864**, R2 (2019).
- [39] J. Bertolaccini, E. L  v  que, and P.-E. Roche, Disproportionate entrance length in superfluid flows and the puzzle of counterflow instabilities, *Phys. Rev. Fluids* **2**, 123902 (2017).
- [40] S. Babuin, E. Varga, L. Skrbek, E. L  v  que, and P.-E. Roche, Effective viscosity in quantum turbulence: A steady-state approach, *Europhys. Lett.* **106**, 24006 (2014).
- [41] M. Sahin and R. G. Owens, A numerical investigation of wall effects up to high blockage ratios on two-dimensional flow past a confined circular cylinder, *Phys. Fluids* **16**, 1305 (2004).
- [42] S. Singha and K. P. Sinhamahapatra, Flow past a circular cylinder between parallel walls at low Reynolds numbers, *Ocean Eng.* **37**, 757 (2010).

- [43] D. Kivotides, C. F. Barenghi, and D. C. Samuels, Triple vortex ring structure in superfluid helium II, [Science](#) **290**, 777 (2000).
- [44] L. Galantucci, A. W. Baggaley, C. F. Barenghi, and G. Krstulovic, A new self-consistent approach of quantum turbulence in superfluid helium, [Eur. Phys. J. Plus](#) **135**, 547 (2020).
- [45] L. Galantucci, G. Krstulovic, and C. F. Barenghi, Quantum vortices leave a macroscopic signature in the thermal background, [Phys. Rev. Lett.](#) **136**, 016001 (2026).
- [46] B. Mastracci, S. Bao, W. Guo, and W. F. Vinen, Particle tracking velocimetry applied to thermal counterflow in superfluid ^4He : Motion of the normal fluid at small heat fluxes, [Phys. Rev. Fluids](#) **4**, 083305 (2019).
- [47] S. Yui, H. Kobayashi, M. Tsubota, and W. Guo, Fully coupled two-fluid dynamics in superfluid ^4He : Anomalous anisotropic velocity fluctuations in counterflow, [Phys. Rev. Lett.](#) **124**, 155301 (2020).