

Experimental demonstration of entanglement forging for solving Max-Cut problem

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In noisy intermediate-scale quantum (NISQ) era, distributed quantum computing (DQC) has emerged as a promising direction to address limitations in qubit scalability and noise resilience. Entanglement forging is proposed as an important DQC protocol that enables the integration of the quantum computing resources and exhibits significant efficiency in realizing quantum states and dynamics with shallow entanglement. Here, we experimentally demonstrate the entanglement forging process for solving an eight-vertex Max-Cut problem. We execute parametrized quantum circuits on a four-qubit nuclear magnetic resonance quantum processor and successfully achieve the entanglement forging of the ground state for the eight-qubit Max-Cut Hamiltonian by using variational quantum eigensolver. The ground state reconstructed in experiment achieves a fidelity of 95.88% compared with theoretical prediction, which provides three correct solutions to the Max-Cut problem in parallel. Our work demonstrates the feasibility and efficiency of entanglement forging protocol in solving combinatorial optimization problems on NISQ devices, while also advancing the development of distributed quantum computing.

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I. INTRODUCTION

Distributed quantum computing (DQC) is an advanced quantum technology that connects spatially independent quantum processors (or quantum nodes) into a network architecture [1–5]. This technology not only facilitates the communication between quantum processors [6–8], but also enables the reuse of qubits and the integration of multiple quantum computing resources [9–11], laying a crucial foundation for the future development of quantum networks [12]. According to different interaction mechanisms between the quantum processors, DQC can be classified into protocols based on quantum teleportation [13–15] or circuit cutting [16–19]. Teleportation-based DQC physically establishes entanglement or correlation between different quantum processors, which is generally applied in quantum repeaters [20–22]. Circuit-cutting-based DQC executes quantum circuits separately on the respective quantum processors and combines their measurement results into an outcome of the entire quantum network through classical postprocessing. This protocol can be used to integrate the computational power of noisy intermediate-scale quantum (NISQ) devices

while only needing to overcome their respective decoherence noise [23–25].

Entanglement forging has been proposed in recent years as a circuit-cutting-based DQC protocol [26–29]. In this framework, the Schmidt decomposition is utilized to split a $2N$ -qubit quantum state into a weighted sum of the product states of the two N -qubit subsystems [30,31]. Therefore, all quantum operations are only required to be performed on the N -qubit subsystems, while the entanglement between them is synthesized classically by postprocessing [26]. Compared to the general quantum algorithms performed on the $2N$ -qubit system, the depth and width of the quantum circuits in the entanglement forging process are effectively reduced [32–34], and the robustness against decoherence and quantum control errors is also improved [35–37]. Nevertheless, entanglement forging sometimes requires a significant number of circuit executions, corresponding to the number of terms in the Schmidt decomposition, which depends on the entanglement between the subsystems [38,39]. This somewhat limits the application scenarios of entanglement forging. Fortunately, in the field of quantum computing and quantum simulation, there are numerous tasks where the target states exhibit shallow entanglement [40–42], in which the entanglement forging is expected to achieve very good efficiency and performance.

In this work, we experimentally demonstrate that the entanglement forging method is applicable for solving the Max-Cut problem. Generally, Max-Cut problem has a limited number of solutions, with the corresponding ground state exhibiting low degeneracy. Therefore, the Schmidt decomposition for the entanglement forging process can be truncated to a few terms, and the number of circuit executions is also limited. By

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utilizing a nuclear magnetic resonance (NMR) quantum processor, we experimentally implement parametrized quantum circuits (PQCs) on the four-qubit quantum subsystems and realize the entanglement forging of the ground state for an eight-qubit Max-Cut Hamiltonian via variational quantum eigensolver (VQE) [43,44]. Our work highlights an application scenario for entanglement forging and demonstrates the advantages of distributed quantum computing in optimizing quantum states with shallow entanglement.

II. THEORETICAL MODEL

The entanglement forging method is based on Schmidt decomposition [30,31], where a $2N$ -qubit bipartite quantum state $|\psi\rangle$ can be decomposed into the superposition of product states formed by the two N -qubit quantum subsystems A and B, i.e.,

$$|\psi\rangle = (\hat{U}_A \otimes \hat{U}_B) \sum_{n=1}^{2^N} \lambda_n |b_n\rangle \otimes |b_n\rangle. \quad (1)$$

Here, $\{|b_n\rangle\}$ represents a set of orthogonal basis for the N -qubit subsystems, typically chosen as the computational basis vectors. λ_n denotes the Schmidt coefficients, which are non-negative real numbers satisfying the normalized condition $\sum_{n=1}^{2^N} \lambda_n^2 = 1$. The unitary operators $\hat{U}_A$ and $\hat{U}_B$ act on the two subsystems to transform the fixed basis $\{|b_n\rangle\}$ into any desired basis set. Therefore, Eq. (1) offers a representation of ansatz state for the bipartite quantum system. The unitary operators $\hat{U}_A$ and $\hat{U}_B$ along with the Schmidt coefficients λ_n can be utilized as the variational parameters for generating any $2N$ -qubit state.

In the entanglement forging process, quantum operations can be independently executed on the two subsystems to generate each product state in Eq. (1), using which the bipartite state $|\psi\rangle$ can be classically reconstructed. The complexity of this process is closely related to the entanglement between the two subsystems, which is characterized by the von Neumann entropy of the subsystem $\hat{\rho}_A$ or $\hat{\rho}_B$ and fully determined by the Schmidt coefficients as

$$S(\hat{\rho}_A) = S(\hat{\rho}_B) = - \sum_{n=1}^{2^N} \lambda_n^2 \ln \lambda_n^2. \quad (2)$$

When the Schmidt coefficients are uniformly distributed, i.e., $\lambda_n = 1/\sqrt{2^N}$, the two subsystems exhibit maximum entanglement between them. In this situation, the entanglement forging requires the implementation of 2^N product states, which is not a scalable process. Therefore, the entanglement forging method is not universally efficient. Nevertheless, there exist certain tasks of quantum computing and quantum simulation in which the target states exhibit weak entanglement between two subsystems. In such scenarios, we can efficiently realize distributed quantum computing and quantum simulation by entanglement forging, significantly reducing the resources required for quantum operations.

Here, we demonstrate that entanglement forging can be effectively applied to solve the Max-Cut problem [45]. Consider an undirected graph $G = (V, E)$, where V represents the set of vertices labeled by

$j = 1, 2, \dots, |V|$, and $|V|$ denotes the total number of the vertices. E represents the set of edges, with each edge $(j, k) \in E$ having an associated weight $w_{jk} = w_{kj}$. The Max-Cut problem aims to find solutions to partition the graph's vertices into two complementary subsets, S and T , which maximize the total weight of the edges between the two sets. We can assign values $x_j = 0$ or $x_j = 1$ to all vertices. For vertices $j \in S$, we set $x_j = 0$; otherwise, $x_j = 1$. Then, the cost function for the Max-Cut problem is defined as

$$C(\mathbf{x}) = \sum_{j,k=1}^{|V|} w_{jk} x_j (1 - x_k). \quad (3)$$

Here, $\mathbf{x} = (x_1, x_2, \dots, x_{|V|})$. The cost function can be mapped to an Ising Hamiltonian by making $x_j \rightarrow (1 - \hat{\sigma}_z^{(j)})/2$:

$$\begin{aligned} \hat{H}_C &= \sum_{j,k=1}^{|V|} \frac{w_{jk}}{4} (1 - \hat{\sigma}_z^{(j)})(1 + \hat{\sigma}_z^{(k)}) \\ &= -\frac{1}{2} \sum_{j < k} w_{jk} \hat{\sigma}_z^{(j)} \hat{\sigma}_z^{(k)} + \frac{1}{2} \sum_{j < k} w_{jk}, \end{aligned} \quad (4)$$

where $\hat{\sigma}_z^{(j)}$ is the Pauli-Z operator of the j th spin. Therefore, the Max-Cut problem is equivalent to minimizing the energy and finding the ground state of the Ising Hamiltonian:

$$\hat{H} = \sum_{j < k} w_{jk} \hat{\sigma}_z^{(j)} \hat{\sigma}_z^{(k)}. \quad (5)$$

If there are d solutions to the Max-Cut problem, then the ground state of Hamiltonian $\hat{H}$ will exhibit a degeneracy of d . Typically, the number of solutions to the Max-Cut problem is limited, which means that the ground states within the degenerate subspace are superpositions of a small number of product states. Thus, it is possible to implement the quantum operations on two N -qubit subsystems independently and achieve entanglement forging of the ground states for a $2N$ -qubit Max-Cut Hamiltonian. The general process is depicted in Fig. 1. First, we divide the $2N$ spins, which represent the vertices of the graph, into two subsystems A and B, as shown in Fig. 1(a). Then, the Max-Cut Hamiltonian can be decomposed into a sum of Kronecker products of the two subsystems' Hamiltonians, i.e., $\hat{H} = \sum_i^L \hat{H}_i^A \otimes \hat{H}_i^B$, and the optimization problem is now reformulated as finding the minimum value of $\sum_i^L \langle \hat{H}_i^A \otimes \hat{H}_i^B \rangle$. Using the entanglement forging representation in Eq. (1), we decompose the expectation value of the Max-Cut Hamiltonian into the quantities of two subsystems:

$$\begin{aligned} \sum_i^L \langle \hat{H}_i^A \otimes \hat{H}_i^B \rangle &= \sum_i^L \sum_{m,n=1}^K \lambda_m \lambda_n \langle b_m | \hat{U}_A^\dagger \hat{H}_i^A \hat{U}_A | b_n \rangle \\ &\quad \times \langle b_m | \hat{U}_B^\dagger \hat{H}_i^B \hat{U}_B | b_n \rangle. \end{aligned} \quad (6)$$

Given the assumption that the ground-state degeneracy is low, the upper bound of the summation here is chosen as an effective truncation number K . Consequently, a VQE can be performed based on Eq. (6) to search for the minimum energy.

As illustrated in Fig. 1(b), we initially prepare the subsystems A and B into the states like $\hat{\rho}_i^{A/B} = (\mathbb{1} + \epsilon \hat{H}_i^{A/B})/2^N$, where ϵ denotes the polarization rate. Then, the unitary

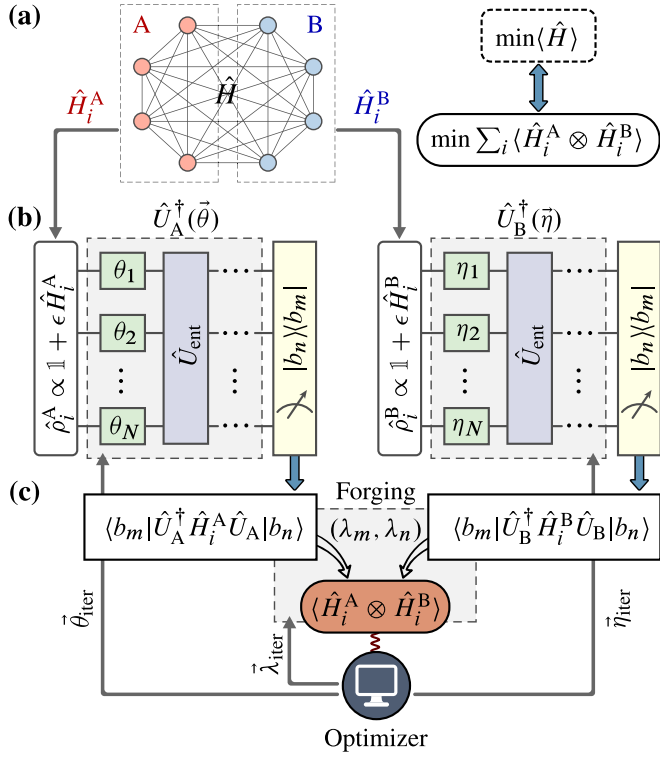


FIG. 1. (a) The vertices of Max-Cut problem can be divided into two subsystems, which transforms the search for the minimum energy of total Hamiltonian into the task of minimizing the sum of Kronecker products of two sub-Hamiltonians. (b) Quantum measurement of the two subsystems, including the preparation of sub-Hamiltonians, a unitary transformation realized by a PQC, and the measurement of $|b_n\rangle\langle b_m|$ for each subsystem. (c) In optimization process, expectation value of the total Hamiltonian is computed from quantum measurement results of the subsystems. The PQC parameters $\vec{\theta}$, $\vec{\eta}$ and Schmidt coefficients $\vec{\lambda}$ are optimized using classical algorithm.

operators $\hat{U}_A^\dagger(\vec{\theta})$ and $\hat{U}_B^\dagger(\vec{\eta})$ are realized using two PQCs [46] applied to the subsystems. By measuring the operators $|b_n\rangle\langle b_m|$ on the final states of both PQCs, we obtain the measurement fragments defined as $\langle b_m | \hat{U}_A^\dagger \hat{H}_i^A \hat{U}_A | b_n \rangle$ and $\langle b_m | \hat{U}_B^\dagger \hat{H}_i^B \hat{U}_B | b_n \rangle$. Afterward, the measurement fragments are used to calculate the expectation values $\langle \hat{H}_i^A \otimes \hat{H}_i^B \rangle$ as shown in Fig. 1(c), where the Schmidt coefficients λ_m, λ_n are involved. Finally, a classical optimizer is used to iterate the parameters $\vec{\theta}, \vec{\eta}, \vec{\lambda}$ to search for the minimum value of $\sum_i \langle \hat{H}_i^A \otimes \hat{H}_i^B \rangle$. According to the optimized parameters, we can reconstruct the ground state and obtain the solutions for the Max-Cut problem. Based on Eq. (6), it can be inferred that the number of measurements required for each iteration is $2LK^2$, where L represents the number of terms in the Hamiltonian decomposition and K is the truncation number of the Schmidt decomposition.

III. EXPERIMENTAL PROCEDURES

In order to demonstrate the entanglement forging for solving Max-Cut problem in experiment, we consider a specific Max-Cut problem with eight vertices labeled by

$j = 1, 2, \dots, 8$, and the weights of the edges are given by

$$w = \frac{1}{4} \begin{pmatrix} 0 & 3 & 9 & 8 & 9 & 9 & 9 & 9 \\ 3 & 0 & 10 & 9 & 9 & 9 & 9 & 9 \\ 9 & 10 & 0 & 3 & 9 & 9 & 9 & 9 \\ 8 & 9 & 3 & 0 & 9 & 9 & 9 & 9 \\ 9 & 9 & 9 & 9 & 0 & 3 & 9 & 8 \\ 9 & 9 & 9 & 9 & 3 & 0 & 10 & 9 \\ 9 & 9 & 9 & 9 & 9 & 10 & 0 & 3 \\ 9 & 9 & 9 & 9 & 8 & 9 & 3 & 0 \end{pmatrix}. \quad (7)$$

Here, we divide the eight-qubit system into two four-qubit subsystems and decompose the total Hamiltonian into three terms:

$$\hat{H} = \hat{H}_1^A \otimes \mathbb{1} + \mathbb{1} \otimes \hat{H}_2^B + \hat{H}_3^A \otimes \hat{H}_3^B. \quad (8)$$

The first two terms represent the Hamiltonians of the respective subsystems, i.e.,

$$\hat{H}_1^A = \sum_{1 \leq j < k \leq 4} w_{jk} \hat{\sigma}_z^{(j)} \hat{\sigma}_z^{(k)}, \quad \hat{H}_2^B = \sum_{5 \leq j < k \leq 8} w_{jk} \hat{\sigma}_z^{(j)} \hat{\sigma}_z^{(k)}. \quad (9)$$

The last term denotes the coupling between the two subsystems and can be expressed as the Kronecker product of their respective operators:

$$\hat{H}_3^A = \frac{3}{2} \sum_{j=1}^4 \hat{\sigma}_z^{(j)}, \quad \hat{H}_3^B = \frac{3}{2} \sum_{j=5}^8 \hat{\sigma}_z^{(j)}. \quad (10)$$

Therefore, the expectation value of the total Hamiltonian under the entanglement forging state $|\psi\rangle$ is given by

$$\begin{aligned} \langle \hat{H} \rangle = & \sum_{n=1}^K \lambda_n^2 (\langle b_n | \hat{U}_A^\dagger \hat{H}_1^A \hat{U}_A | b_n \rangle + \langle b_n | \hat{U}_B^\dagger \hat{H}_2^B \hat{U}_B | b_n \rangle) \\ & + \sum_{m,n=1}^K \lambda_m \lambda_n \langle b_m | \hat{U}_A^\dagger \hat{H}_3^A \hat{U}_A | b_n \rangle \langle b_m | \hat{U}_B^\dagger \hat{H}_3^B \hat{U}_B | b_n \rangle. \end{aligned} \quad (11)$$

In our experiment, the Schmidt decomposition is truncated to $K = 3$, which allows us to search for multiple Max-Cut solutions in parallel. These basis vectors are chosen as $|b_1\rangle = |0000\rangle$, $|b_2\rangle = |0001\rangle$, and $|b_3\rangle = |0010\rangle$.

The experiment is implemented on an NMR platform. The ^{13}C -labeled iodotriuroethylene dissolved in d-chloroform, which includes a ^{13}C and three ^{19}F nuclear spins, is used here as the four-qubit subsystems. All experimental procedures are conducted on a Bruker Avance III 400 MHz spectrometer at room temperature, under a static magnetic field of 9.4 T. The sample is initially at the thermal equilibrium state $\hat{\rho}_{\text{eq}}$ [47], and its natural Hamiltonian is given by

$$\hat{H}_{\text{NMR}} = \sum_{j=1}^4 \pi \nu_j \hat{\sigma}_z^{(j)} + \sum_{1 \leq j < k \leq 4} \frac{\pi}{2} J_{jk} \hat{\sigma}_z^{(j)} \hat{\sigma}_z^{(k)}, \quad (12)$$

where ν_j represents the chemical shift of the j th spin and J_{jk} is the J -coupling strength between the j th and k th spins [47].

The preparation of the initial state $\hat{\rho}_i^{A/B} = (\mathbb{1} + \epsilon \hat{H}_i^{A/B})/16$ can be achieved by applying a unitary operation $\hat{U}_{\text{ini}}$ and a constant z -direction gradient pulse (GPZ) $\mathcal{G}$ on the

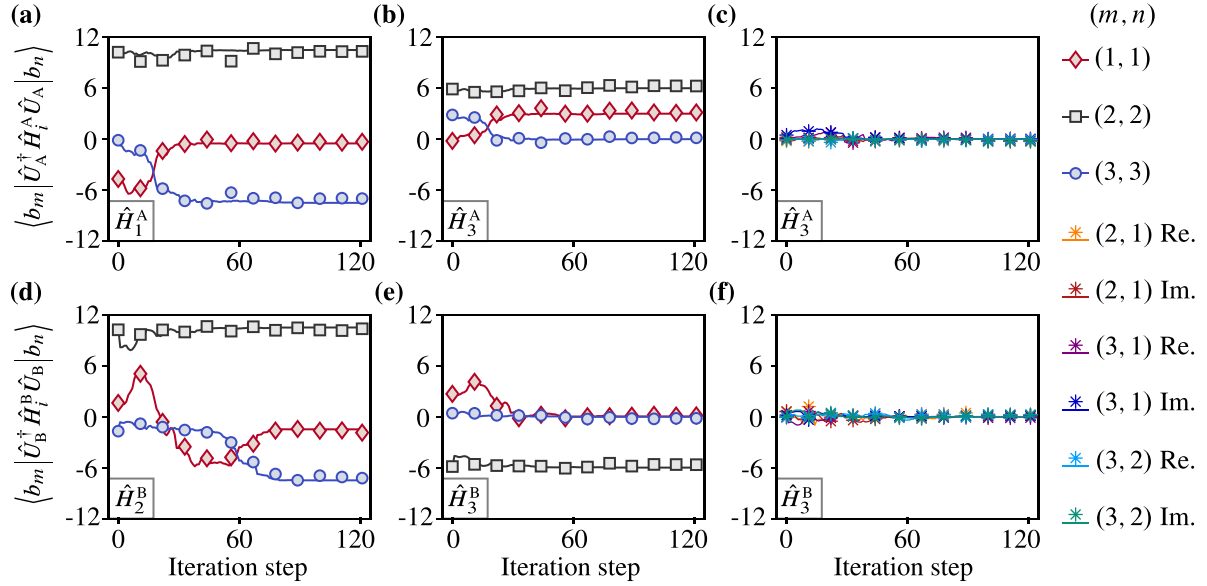


FIG. 2. The measurement fragments depending on iteration steps during the optimization process for subsystem A [panels (a)–(c)] and subsystem B [panels (d)–(f)]. In panels (a), (b), (d), and (e), the measurement results of diagonal elements ($m = n = 1, 2, 3$) for the corresponding sub-Hamiltonians are presented by red diamonds, black squares, and blue circles, respectively. The numerical simulations are shown by solid lines in matching colors. In panels (c) and (f), the measurement results of off-diagonal elements ($m \neq n$) are presented by asterisk markers for experimental values and solid lines of same colors for the corresponding simulated results.

thermal equilibrium state [48], i.e.,

$$\mathcal{G}(\hat{U}_{\text{ini}} \hat{\rho}_{\text{eq}} \hat{U}_{\text{ini}}^\dagger) \rightarrow \hat{\rho}_{\text{ini}} = (\mathbb{1} + \epsilon \hat{\rho}_\Delta)/16. \quad (13)$$

Here, $\hat{U}_{\text{ini}}$ is implemented by a 25 ms gradient ascent pulse engineering (GRAPE) pulse. By optimizing this pulse in advance, we can make the deviation density matrix $\hat{\rho}_\Delta$ as close as possible to $\hat{H}_i^{A/B}$ and maximize the polarization rate ϵ [47]. Next, the parametrized quantum circuits with three layers are used for realizing the unitary transformation $\hat{U}_A^\dagger(\vec{\theta})$ and $\hat{U}_B^\dagger(\vec{\eta})$, which are both implemented using 25 ms shaped pulses. The amplitudes and phases of these shaped pulses can be adjusted at each iteration step according to the circuit parameters $\vec{\theta}$ and $\vec{\eta}$. In the measurement process, all signals are directly acquired by measuring the single quantum coherence of the ^{13}C spin. Therefore, the expectation values of the diagonal terms $|b_n\rangle\langle b_n|$ ($n = 1, 2, 3$) are reconstructed using the measurements after two readout pulses with durations of 1.2 and 20 ms, respectively [47]. The expectation values of the off-diagonal terms $|b_n\rangle\langle b_m|$ ($n \neq m$) are reconstructed using the measurements after three readout pulses with durations of 20, 15, and 20 ms, respectively [47].

After the measurement process, we calculate the energy of the eight-qubit Max-Cut Hamiltonian using the measurement fragments of the four-qubit subsystems according to Eq. (11). Then, an optimization process is employed to iterate the variational parameters $\vec{\theta}$, $\vec{\eta}$, and $\vec{\lambda}$, aiming to minimize the energy value. The optimization process can be executed using a variety of classical optimization algorithms. These include gradient-free algorithms such as Nelder-Mead and simultaneous perturbation stochastic approximation [49], as well as gradient-based algorithms like the active-set method and the interior-point method [50]. In the Supplemental Material [47], we provide the analytical expression of the gradient for this

optimization problem and compare the convergence behavior of different algorithms. In our experiments, the interior-point method is utilized to iterate the variational parameters and minimize the cost function.

IV. EXPERIMENTAL RESULTS

Figure 2 presents the experimental results of the measurement fragments for subsystems A and B during the optimization process. The red diamonds, black squares, and blue circles in Fig. 2(a) represent the experimentally measured diagonal elements, $\langle b_n | \hat{U}_A^\dagger \hat{H}_1^A \hat{U}_A | b_n \rangle$, versus the iteration steps for $n = 1, 2, 3$, respectively. The solid lines with the same colors here denote the corresponding numerically simulated values. The measurement fragments for $\hat{H}_3^A$ include not only diagonal elements, as depicted in Fig. 2(b) with the same labels in Fig. 2(a), but also nondiagonal elements. These nondiagonal elements, presented by asterisks and lines of different colors in Fig. 2(c), all exhibit very small values. Similarly, the measurement fragments of subsystem B from both experiments and numerical simulations are presented in Figs. 2(d)–2(f). Figure 3 presents the reconstructed expectation values of the Max-Cut Hamiltonian versus iteration steps by the red circles with error bars. The blue solid line represents the simulated values, while the black dashed line indicates the theoretical ground state energy, $\langle \hat{H} \rangle = -15$. The measurement values match well with the numerical predictions and converge to stable plateaus after about 100 iterations.

To extract the solutions to the Max-Cut problem, we further reconstruct the ground state of the Max-Cut Hamiltonian using the optimized parameters $\vec{\theta}_{\text{opt}}$, $\vec{\eta}_{\text{opt}}$, $\vec{\lambda}_{\text{opt}}$. This process

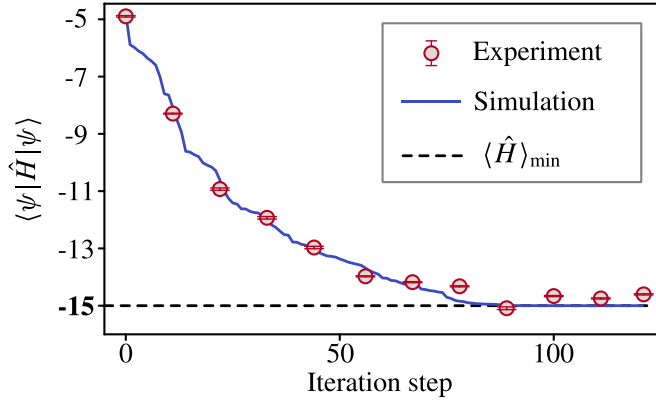


FIG. 3. Convergence of the expectation value for the eight-qubit Max-Cut Hamiltonian vs iteration steps. The black dashed line denotes the theoretical ground-state energy. The red scatters and blue solid line represent the experimental data and simulated values of the Hamiltonian expectation during the optimization process, respectively.

begins with the preparation of the pseudopure state [51]:

$$\hat{\rho}_{\text{pps}} = \frac{1 - \epsilon_{\text{pps}}}{16} \mathbb{1} + \epsilon_{\text{pps}} |b_1\rangle\langle b_1|, \quad (14)$$

which is realized by applying a 25 ms GRAPE pulse and a constant GPZ to the thermal equilibrium state [47]. In order to preserve local phases of the Schmidt decomposition, all phases are referenced with respect to $|b_1\rangle$. Thus, we further prepare the superposition states $(|b_1\rangle + |b_n\rangle)/\sqrt{2}$ ($n \neq 1$) by applying 10 ms GRAPE pulses to $\hat{\rho}_{\text{pps}}$. Afterward, by applying 25 ms GRAPE pulses to realize $\hat{U}_A(\vec{\theta}_{\text{opt}})$ and $\hat{U}_B(\vec{\eta}_{\text{opt}})$, the final states are given by

$$|\phi_n^{A/B}\rangle = \begin{cases} \hat{U}_{A/B}|b_1\rangle, & n = 1, \\ \hat{U}_{A/B}(|b_1\rangle + |b_n\rangle)/\sqrt{2}, & n = 2, 3. \end{cases} \quad (15)$$

These six states, each with an unknown global phase, are obtained through state tomography, as depicted in Fig. 4(a).

Finally, the ground state of the eight-qubit Max-Cut Hamiltonian is reconstructed using

$$|\psi\rangle = \sum_{n=1}^3 \lambda_n |\psi_n^A\rangle \otimes |\psi_n^B\rangle, \quad (16)$$

where $|\psi_n^{A/B}\rangle$ are given by

$$\begin{aligned} |\psi_1^{A/B}\rangle &= |\phi_1^{A/B}\rangle, \\ |\psi_2^{A/B}\rangle &= |\phi_2^{A/B}\rangle / \langle \phi_1^{A/B} | \phi_2^{A/B} \rangle - |\phi_1^{A/B}\rangle, \\ |\psi_3^{A/B}\rangle &= |\phi_3^{A/B}\rangle / \langle \phi_1^{A/B} | \phi_3^{A/B} \rangle - |\phi_1^{A/B}\rangle. \end{aligned} \quad (17)$$

Figure 4(b) illustrates the projection probabilities of the reconstructed ground state $|\psi\rangle$ onto the eight-qubit computational basis states $|b_i'\rangle$. The experimental results and numerical simulation results are, respectively, presented using red bars and gray bars. The experimentally reconstructed state exhibits a fidelity of 95.88% compared to the simulated state. The ground state we obtained from the experiment consists of three major components, $|00001111\rangle$, $|00110011\rangle$, and $|00111100\rangle$,

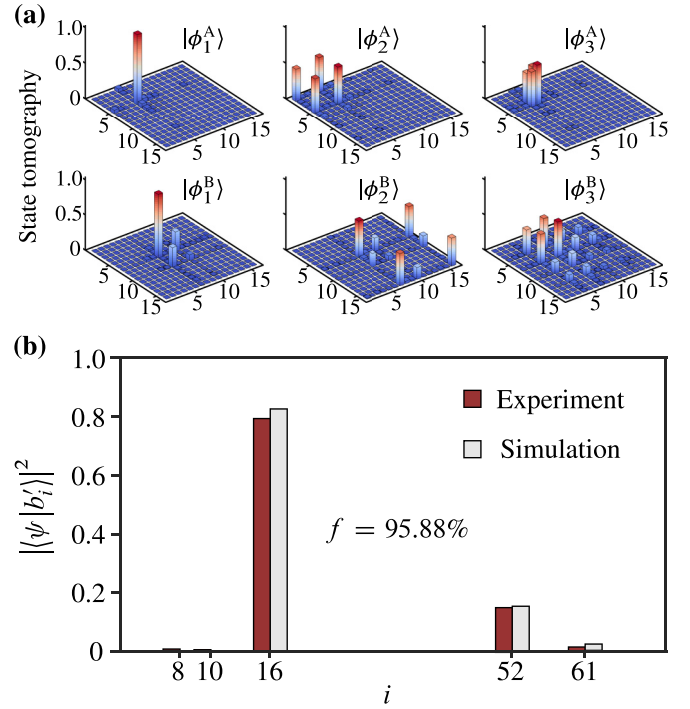


FIG. 4. (a) Absolute values of the density matrices obtained via quantum state tomography for subsystem states $|\phi_n^A\rangle$ and $|\phi_n^B\rangle$ with $n = 1, 2, 3$, respectively. (b) Five dominant projection probabilities of the reconstructed ground state of the Max-Cut Hamiltonian onto eight-qubit computational basis. Red bars denote the experimental data and gray bars are the simulated values. The experimental ground state achieves a fidelity of 95.88% relative to the theoretical prediction.

which correspond to the three solutions of the original Max-Cut problem.

V. CONCLUSION AND DISCUSSION

In summary, our work successfully demonstrates the application of the entanglement forging method to solving an eight-vertex Max-Cut problem on an NMR quantum platform. By combining entanglement forging, parametrized quantum circuit, and variational quantum algorithm, we achieved effective dimensionality reduction in quantum computing for solving the combinatorial optimization problems. Although entanglement forging is not scalable for universal quantum algorithms, it can efficiently integrate distributed NISQ computing resources for optimization tasks targeting states with shallow entanglement. On one hand, the scheme is affected only by the noise within each subprocessor, making it more robust to decoherence and control errors compared to a single large-scale quantum processor. On the other hand, by reducing the dimensionality of the Hilbert space, entanglement forging can also simplify the optimization landscape, leading to improved convergence for the quantum algorithms. It is anticipated that this approach will facilitate the construction of distributed quantum computing networks, enabling quantum computing resources to be applied to solving classically intractable problems in the near future.

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DATA AVAILABILITY

The data that support the findings of this article are openly available [52].

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