

# Quantum printing and induced vorticity in superconductors I: Linearly polarized light

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We propose an approach to use linearly polarized light to imprint superconducting (SC) vortices. Within the framework of the generalized time-dependent Ginzburg-Landau equations, we demonstrate the induction of the coherent vortex pairs that move in phase with electromagnetic wave oscillations. The overall vorticity of the superconductor remains zero throughout the cycle. Our results uncover rich multiscale dynamics of SC vorticity and suggest optical applications for various types of structured light. In a departure from classical laser printing, the laser printing proposed here can be viewed as *quantum printing* where we induce quantum excitations in the SC liquid.

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## I. INTRODUCTION

Since the invention of laser printing several decades ago, the challenge of printing in exotic materials has continuously evolved. Innovations include laser printing on bubbles [1], plasmonic nanoparticles [2], dielectric metasurfaces [3], nanostructure coloration [4], etc. With the development of structured light endowed with quantum numbers (QNs), the approach has now advanced to printing QNs onto coherent quantum matter. Recent progress in advancing optical control indeed demonstrates that the idea of light transfer of QN is feasible. Examples include light spin angular momentum transfer to phonons [5–7], orbital angular momentum transfer to magnets [8] and spiral Higgs waves [9], and induction of skyrmions [10] and polariton vortices [11,12]. In superconductors, transferring various QNs from the light phase to the coherent quantum electron liquid holds the potential to open pathways to the manipulation and control of superconducting states. In standard laser printing, one produces classical printed objects—letters and images, by means of the transfer of energy to paper. We consider the setup where the magnetic field of the light can steer and print quantized vortices in superconducting electron fluid. We view this process as a quantum version of printing—*quantum printing*. The setup we use focuses on the light with frequency on the order of 0.1–10 THz (here, we do include microwave in the possible source of light to be applied as well; hence, the light term is used in a general sense). The typical wavelength of the light beam would be on the order of a micron or larger. Hence, we

can look at the effects we describe as questions of stirring electronic hydrodynamics with light where we can use the effective long-wavelength description of the quantum fluid, i.e., the hydrodynamic description applies.

Specifically, in this work, we simulate the behavior of a superconducting film subject to incident light using the time-dependent Ginzburg-Landau (TDGL) model with dynamic vector potential created by Gaussian and Laguerre-Gaussian beams [13] to test this hypothesis.

This work is divided into two parts. To set the stage, the first part, presented here, focuses on a linearly polarized Gaussian beam. In the second part, which will be published in the article “Quantum printing and induced vorticity in superconductors II: Laguerre-Gaussian beam” [14], we will elaborate on various kinds of QNs printed into SC hydrodynamics, including spin angular momentum and orbital angular momentum.

In this study, we report the predictions for the real-time motion of coherent SC states with zero *total* QN transfer. The emphasis is given since we will demonstrate that even though the net QN transfer is zero, there are regions in the beam spot where vortices will be generated due to time-dependent out-of-plane magnetic flux. In our simulations of supercurrent vorticity, we track the dynamical interaction between light, superflow, and vortices, as illustrated in the schematic diagram of Fig. 1. In order to analyze the complicated dynamics of the superconducting state, we also examine the interplay between the length scales and timescales of the beam and the superconducting fluid. We describe the effect of the change of coherence length and of the light spot size on the induced vorticity. We also demonstrate various dynamical regimes that emerge as the result of the fast drive where damping is not efficient and see evidence for light-induced oscillations (nonequilibrium regime) to equilibrium relaxation where damping is efficient and the relaxation dominates.

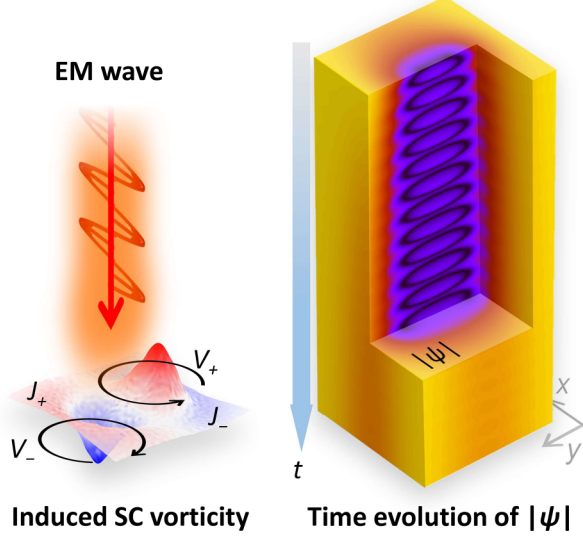
Generation of vortices in a superconducting state via transfer of the angular momentum of an electromagnetic beam has been proposed in Refs. [15,16]. The generation method focused on the superconducting condensation process during

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## High frequency light → SC dynamics



## Low frequency light → SC dynamics

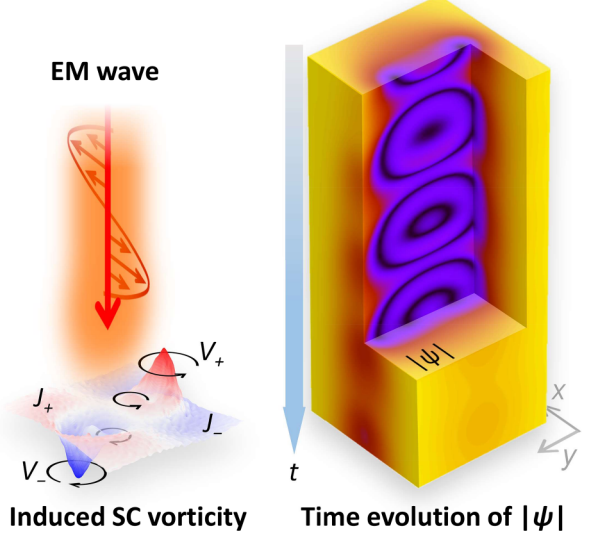


FIG. 1. Linearly polarized light-induced SC vorticity and time evolution of order parameter  $|\psi|$ . The snapshots of vorticity show the light-induced vortex-antivortex pair (labeled  $V_+$  and  $V_-$ ) and supercurrent (labeled  $J_+$  with positive vorticity and  $J_-$  with negative vorticity).

the quench of hot spots in these earlier works. We find that, indeed, structured light can induce the vortices even without relying on the thermal quench. We also find that vortices can be generated without angular momentum transfer, given the structured intensity of light provides locally nonzero magnetic flux through the plane normal to the propagation of the beam.

The paper is organized as follows. We describe the simulation method in Sec. II A, and the light source and the simulation conditions in Sec. II B. Next, we uncover the typical imprinted order parameter pattern by linearly polarized light in Sec. III A and show the evolution of linearly polarized light-induced supercurrent and vortices in Sec. III B. In this section, we demonstrate long-lived cyclic vortex-antivortex pair (VP) generation-recombination processes and nonrepeatable VP generation-recombination processes dependent on the optical frequencies and amplitude of optically induced flux. Thereafter, we discuss the effect of changing the beam's width and edge effects in Sec. III C. Finally, a discussion of the simulation results with the validity of the TDGL model for conventional SC materials and the proposal of the experimental configuration with advanced light sources is presented in Sec. IV.

## II. METHOD: TIME-DEPENDENT GINZBURG-LANDAU THEORY

### A. Generalized time-dependent Ginzburg-Landau theory

Our simulations are based on the generalized time-dependent Ginzburg-Landau theory (gTDGL). We adopt the dimensionless unit form of gTDGL, expressed as [17–20]

$$\frac{u}{\sqrt{1 + \gamma^2 |\psi|^2}} \left( \frac{\partial}{\partial t} + i\mu + \frac{\gamma^2}{2} \frac{\partial |\psi|^2}{\partial t} \right) \psi = (1 - |\psi|^2) \psi + (\nabla - i\mathbf{A})^2 \psi, \quad (1)$$

where  $\psi$  is the complex order parameter, consisting of the amplitude  $|\psi|$  and phase  $\theta_s$ . The terms  $(\nabla - i\mathbf{A})^2$  and  $(\frac{\partial}{\partial t} + i\mu)$

represent the covariant Laplacian with vector potential  $\mathbf{A}$  and the covariant time derivative with scalar potential  $\mu$ , respectively. The parameter  $\gamma = 2\tau_E \Delta_0$  represents the influence of inelastic scattering with the scattering time  $\tau_E$ , where  $\Delta_0$  is the superconducting gap in the absence of external perturbations at temperature  $T$ . The constant  $u$  is the ratio of the relaxation times of the order parameter and the current. For an SC with a spherically symmetric Fermi surface in normal state,  $u = \pi^4/14\zeta(3) \approx 5.79$  [18]. The electric potential  $\mu$  abides the Poisson equation

$$\begin{aligned} \nabla^2 \mu &= \nabla \cdot \left( \mathbf{J}_s - \frac{\partial \mathbf{A}}{\partial t} \right) \\ &= \nabla \cdot \text{Im}[\psi^*(\nabla - i\mathbf{A})\psi] - \frac{\partial(\nabla \cdot \mathbf{A})}{\partial t}, \end{aligned} \quad (2)$$

and the total current density reads as

$$\mathbf{J} = \mathbf{J}_s + \mathbf{J}_n = \text{Im}[\psi^*(\nabla - i\mathbf{A})\psi] - \nabla \mu - \frac{\partial \mathbf{A}}{\partial t}, \quad (3)$$

where  $\mathbf{J}_s = \text{Im}[\psi^*(\nabla - i\mathbf{A})\psi]$ . The vorticity of supercurrent  $\omega_{v,s}$  is  $\nabla \times \mathbf{J}_s$ . In Eqs. (1) and (2) and throughout the paper, the temperature is implicitly included in the units of timescale and length scale, which is encoded by the parameter  $\epsilon(\mathbf{r}) = 1 - T(\mathbf{r})/T_c(\mathbf{r})$  with the local transition temperature  $T_c(\mathbf{r})$ . We measure distance in units of the superconducting coherence length  $\xi(T) = \xi_0/\sqrt{\epsilon}$ , time in units of  $\tau_{GL}(T) = \mu_0 \sigma \lambda_L(T)^2 = \mu_0 \sigma \lambda_{L,0}^2/\epsilon = \tau_{GL,0}/\epsilon$ , where  $\sigma$  is the conductivity in the normal state and  $\lambda_L(T) = \lambda_{L,0}/\sqrt{\epsilon}$  is the London penetration depth, magnetic field in units of  $B_{c2}(T) = \epsilon B_{c2,0} = \epsilon \Phi_0/2\pi \xi_0^2$ , vector potential in units of  $A_0(T) = \xi(T) B_{c2}(T)$ , current density in units of  $J_0(T) = \xi(T) B_{c2}(T)/\mu_0 \lambda_L(T)^2$ , and scalar potential  $\mu$  in units of  $V_0(T) = \xi(T) J_0(T)/\sigma(T)$ . Units  $\xi$ ,  $\tau_{GL}$ ,  $\lambda_L$ , etc. are considered at temperature  $T$  throughout the paper. The restrictions on  $T$  are discussed in Sec. IV A.

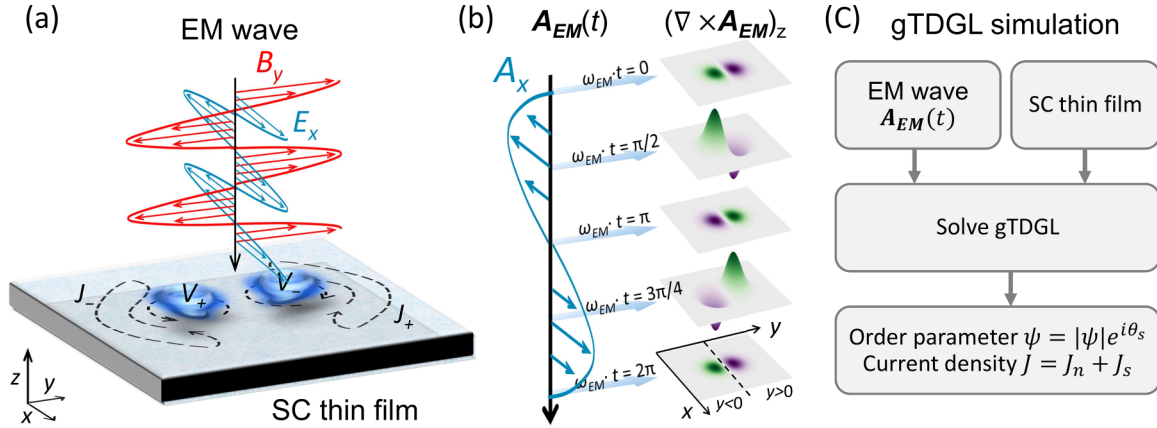


FIG. 2. (a) Schematic diagram of applied EM wave and SC thin film. (b)  $A_{EM}$  and profile of the corresponding  $(\nabla \times A_{EM})_z$  at different time frames. (c) Flowchart of the gTDGL simulation.

We shine light normally on a square-shaped quasi-two-dimensional superconducting thin film with thickness  $d$ , assumed to be much less than  $\xi$ , and length  $L$ . In a quasi-2D case, the effective penetration depth is played by the Pearl length  $\Lambda = \lambda_L^2/d$  [21].

Throughout this work, we use  $\gamma = 10$ , which lies within the commonly observed range for various metals [18] and is frequently adopted in theoretical studies of the gTDGL model [19,20,22]. In the simulations, we neglect screening effects, choosing the thickness to be sufficiently small to result in a large effective penetration depth  $\Lambda$ . The boundary conditions and other details are discussed in Refs. [17–19] and Appendix A. Figure 2(c) demonstrates the schematic diagram of gTDGL simulations. The dynamic  $A$  and the aforementioned parameters of the SC thin film are set as input for the solver in the Python-based open source pyTDGL package [19], which carries out the numerical solution of the gTDGL equations. Finally, the simulation yields the order parameter and current density as an output.

### B. Time-dependent vector potential

In this work, we demonstrate the widely used light source carrying 0-QN as the initial case. The results of higher-order QN modes are elaborated in the second paper of this series [14]. Specifically, the light source considered here is linearly polarized in the  $x$ -direction Gaussian structured beam incident along the  $z$  direction onto an SC thin film positioned at  $z = 0$ . Thus, the vector potential is given by

$$A_{EM}(\mathbf{r}, t) = \mathbf{E}(\mathbf{r}, t)/i\omega_{EM}, \quad (4)$$

with

$$\mathbf{E}(\mathbf{r}, t) = E_0(\mathbf{r})e^{i\omega_{EM}t - ikz}\hat{\mathbf{e}}_x, \quad (5)$$

where  $\omega_{EM}$  and  $k$  are angular frequency (hereinafter referred to as frequency) and wave number of the light, respectively. The parameter  $\mathbf{r}$  is the radius vector in the plane of the film with the origin in the beam's center, and  $E_0(\mathbf{r}) = E_{amp}e^{-|\mathbf{r}|^2/w_0^2}$  is a Gaussian distributed amplitude, where  $E_{amp}$  is the amplitude in the center of the beam and  $w_0$  is the radius of the spot of the beam. The period of the electromagnetic (EM) wave  $\tau_{EM}$  is equivalent to  $2\pi/\omega_{EM}$ .

A simple but key observation is that gradient of the vector potential gives rise to a locally nonzero oscillating  $z$  component of the magnetic field

$$\begin{aligned} B_z(\mathbf{r}, t) &= (\nabla \times A_{EM})_z = -\frac{\partial A_{EM,x}}{\partial y} \\ &= 2\frac{E_{amp}}{i\omega_{EM}}\frac{y}{w_0^2}e^{-|\mathbf{r}|^2/w_0^2}e^{i\omega_{EM}t}, \end{aligned} \quad (6)$$

with maxima of its absolute value located at  $y_0 = \pm w_0/\sqrt{2}$  on the  $y$  axis. Due to the antisymmetry of  $B_z$  in  $y$ , the total flux through the film is zero. However, locally flux is not zero [see Fig. 2 (b)], and the values of the flux through the half planes  $y \leq 0$  are

$$\Phi_{y \leq 0} = \pm \Phi_B = \pm \frac{\sqrt{2}E_{amp}}{i\omega_{EM}}w_0e^{i\omega_{EM}t}. \quad (7)$$

When the amplitude of  $\Phi_B \gtrsim \Phi_0$ , we might expect to see the generation and annihilation of vortices due to the time-dependent magnetic flux.

As we use a number of symbols, for the convenience of the reader we provide a comprehensive list here. The SC order parameter is  $\psi$ , with amplitude  $|\psi|$  and phase  $\theta_s$ .  $A_{EM}$  denotes the vector potential of the applied electromagnetic wave. Characteristic times are  $\tau_{GL}$  (GL relaxation),  $\tau_{EM}$  (period of the EM wave), and  $\tau_{healing}$  (healing of  $\psi$ ), with corresponding frequencies  $\omega_{GL}$  and  $\omega_{EM}$ . Length scales include the coherence length  $\xi$ , London penetration depths  $\lambda_L$ , Pearl length  $\Lambda$ , and the sample size  $L$ ;  $\mathbf{r}$  is the position relative to the beam center,  $w_0$  the Gaussian waist, and  $\lambda_{EM}$  the wavelength. Currents are expressed in units of  $J_0$ , with total current  $\mathbf{J}$  and supercurrent labels  $J_{\pm}$  for positive/negative vorticity. Label of Vortices  $V_{\pm}$  carry winding numbers  $\pm 1$ ,  $VP_j$  denotes the  $j$ -th vortex/antivortex pair, and  $\omega_{v,s}$  is the SC current vorticity. Field quantities are  $E_{amp}$  (field amplitude at the beam center),  $\mathbf{E}$  (field distribution),  $B_z$  (out-of-plane field),  $B_{c2}$  (critical field),  $\Phi_B$  (flux), and  $\Phi_0$  (flux quantum). Normal-state parameters are  $\sigma$  (conductivity),  $r_n$  (resistivity), and  $v_F$  (Fermi velocity).

We emphasize that the simulations are carried out in thin films, i.e.,  $d \ll \xi$ , which allows to neglect variations of the order parameter, current, and vector potential in  $z$  direction. Additionally, as we neglect the screening effects, the Pearl

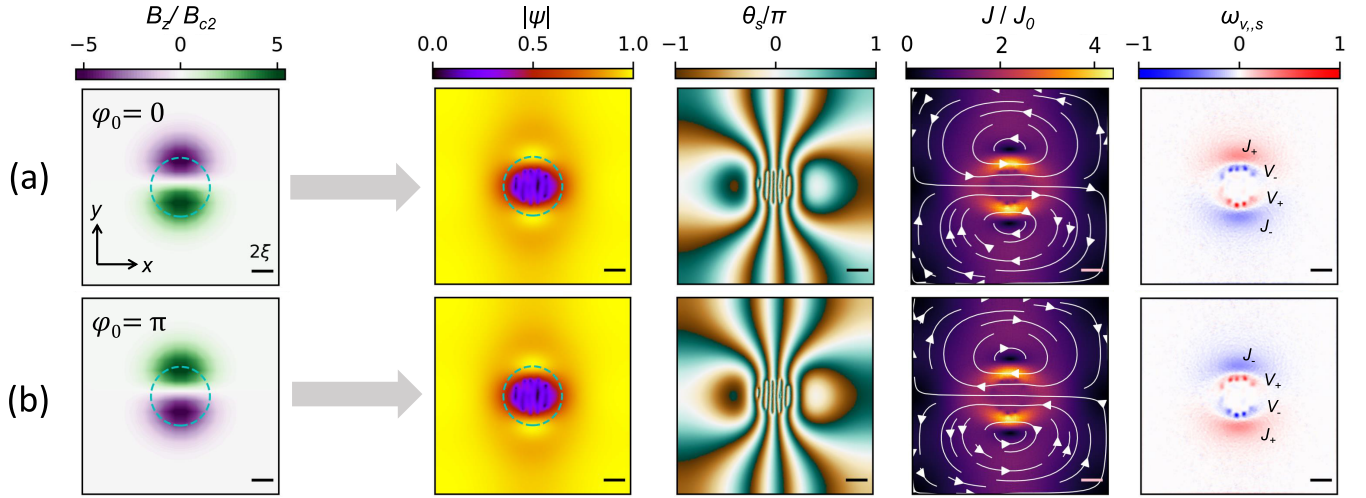


FIG. 3. The SC state excited by the different sign of  $B_z$ . The sign of  $B_z$  is tuned by adding the phase  $\phi_0$  into Eq. (5). Panels (a) ( $\phi_0 = 0$ , first row) and (b) ( $\phi_0 = \pi$ , second row) represent the magnetic flux from the same light source with 0 and  $\pi$  phase shift, respectively. The profiles of  $B_z/B_{c2}$  are shown on left side of gray arrows. The corresponding imprint distribution of the SC state is shown at right side of gray arrows, including  $|\psi|$ ,  $\theta_s$ ,  $J/J_0$ , and vorticity. The green dashed circles and inset axis at the left figures represent the beam spot and direction of coordinate. The notations  $J_+$ ,  $J_-$ ,  $V_+$ , and  $V_-$  represent supercurrent density and vortex with “+” and “−” subscripts, respectively. Both panels (a) and (b) show five VPs in the snapshots.

length is supposed to be much larger than the sample size or the effective size of the light spot. Unless specified differently, in simulations, we consider SC thin films with  $L = 20\xi$ ,  $d = 0.02\xi$ , and  $T = 0.99T_c$ , which for  $\xi = 100$  nm corresponds to  $L = 2 \mu\text{m}$  and  $d = 2$  nm. Further details about  $w_0$  and screening length of the materials are discussed in Sec. IV C.

### III. RESULTS

Here, we begin by presenting the typical imprinted vorticity distribution and SC motion with continuous light source, and giving the picture how coherent state response is dependent on the amplitude of light. Furthermore, we focus on investigating the imprinted profile of SC with different conditions of light source.

#### A. Optical imprint and vorticity distribution

We first demonstrate the imprinting of vortices by Gaussian linearly polarized light in Fig. 3. It represents a snapshot of the simulations at time  $2\tau_{GL}$  for a beam of light at frequency  $\omega_{GL}/8$  and  $w_0 = 3\xi$ , which produces  $\Phi_B$  of amplitude  $16\Phi_0$ . Figures 3(a) and 3(b) are produced for the initial phases of the EM wave  $\phi_0 = 0$  and  $\phi_0 = \pi$ , respectively. We show in Fig. 3 that the phase of the magnetic field directly controls the formation of vortices of certain orientation: In Fig. 3(a),  $B_z$  produces the vortices distribution which is opposite to that in Fig. 3(b).

As expected, the density plots for the magnitude of the order parameters are the same, while the current densities and the winding numbers of vortices change sign. In the profile of vorticity, the distributions of  $V_+$ ,  $V_-$ ,  $J_+$ , and  $J_-$  are opposite between subplots (a) and (b). The appearance of vortices is induced by the excessive  $\Phi_B(t)$ . In these simulations, the maximum induced current density exceeds the critical current density  $J_c \approx 0.48J_0$  (see Appendix B), leading to significant

suppression of the amplitude of the order parameter in the region of the beam spot. This imprint pattern is presented by the vorticity distribution.

#### B. Dynamics of light-induced supercurrent and vortex

In Fig. 4, we illustrate the results of simulations for a Gaussian beam of spot size  $2w_0 = 6\xi$ , with  $\omega_{EM} = \omega_{GL}/40$  and amplitude  $E_{amp} = 2A_0\omega_{GL}$ , which correspond to the amplitude of  $\Phi_B = 3.2\Phi_0$ . The choice of  $\omega_{EM}$  here is dictated by the effective relaxation time of the healing of  $|\psi|$  after the recombination of vortices, which is about  $31\tau_{GL}$  (see Appendix C). Under this condition, the spot size may reach the sub-diffraction limit due to the spatiotemporal mismatch between light and material response (see Sec. IV C). However, based on experimental intensity profiles [23–25] and the approximation in Appendix D, we adopt a Gaussian distribution in the simulation. The center of the beam hits the center of the film, where we locate the origin of the coordinates. After an initial delay, we observe the generation and recombination of VPs. In Fig. 4(b), we illustrate this process in the snapshots of the order parameter amplitude, phase, magnitude of the current density, and SC current vorticity  $\omega_{v,s}$  at  $t = \tau_1, \dots, \tau_5$  in the second cycle of the EM wave [see Fig. 4(a)]. As can be seen, the VP motion occurs along the  $y$  direction. This allows us to illustrate the VP evolution in Fig. 4(a), where in the top row we plot the time dependence of the electric field and  $\Phi_B$ , in the second and third rows we plot the magnitude and phase of the order parameter along the line of the VP motion ( $x = 0$  in this case) as a function of time, and in the fourth and the bottom row we plot the magnitude of the full current and the vorticity of the SC current, respectively, along the line  $x = 0$  as a function of time.

In the first half of the cycle,  $|\psi|$  starts to decrease first at  $y = 0$ . Since  $B_z$  is zero at  $y = 0$ , we explain this by that locally current density exceeds the critical current density, i.e.,

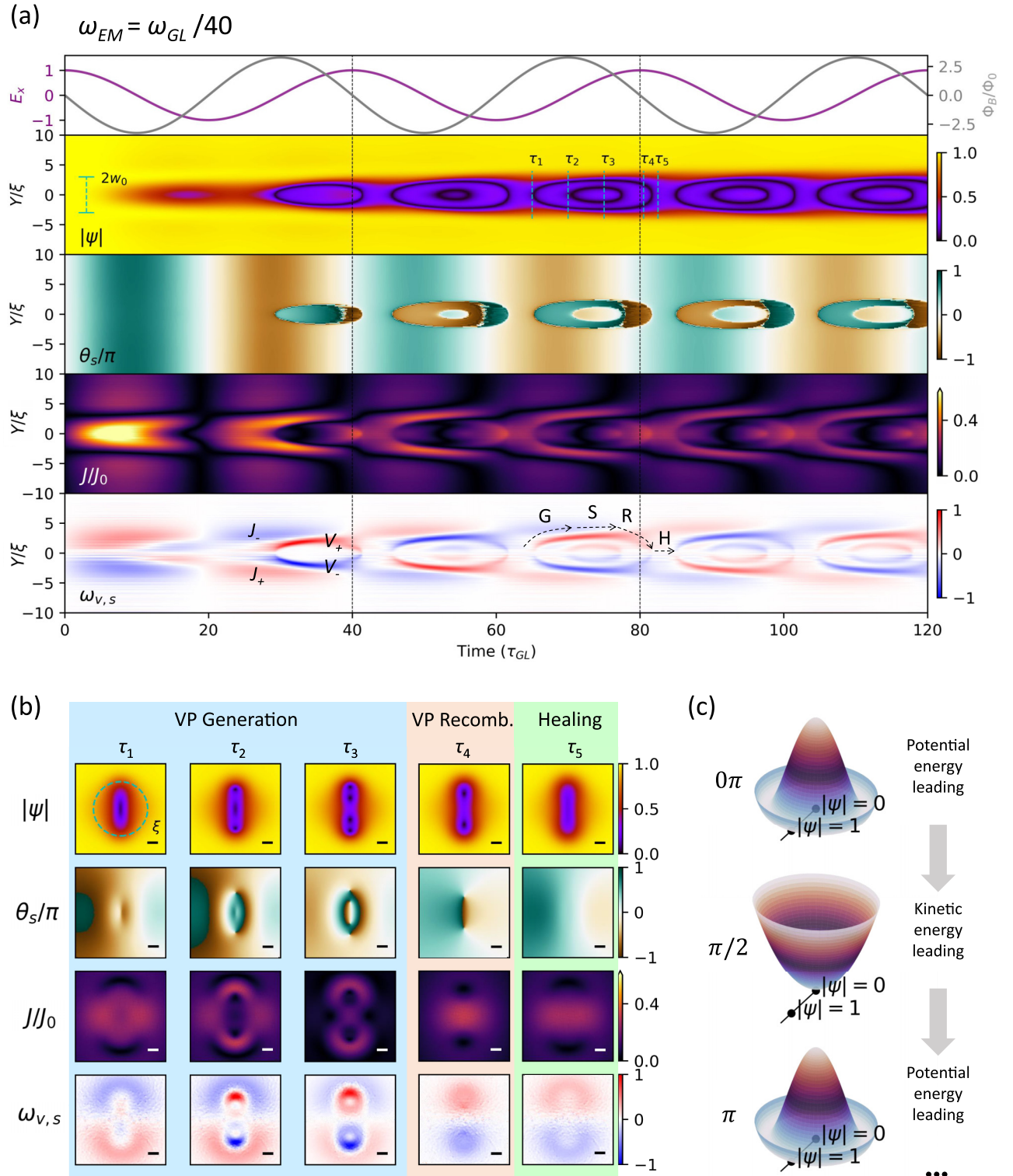


FIG. 4. (a) The SC time evolution driven by a light source with  $\omega_{EM} = \omega_{GL}/40$ . In the top row, the purple curve represents the normalized  $E_0$  referenced on the left axis and the gray curve is the out-of-plane flux  $\Phi_B/\Phi_0$  referenced on the right axis. The second to fourth subplots show the vertical slice at the middle of the sample for  $|\psi|$ ,  $\theta_s/\pi$ , and  $\omega_{v,s}$ . In the subplot of  $|\psi|$ , the left green dashed line marked  $2w_0$  is the size of the spot and the right dashed lines marked  $\tau_1$  to  $\tau_5$  represent selected moments corresponding to different stages of VP evolution, detailed in panel (b). The dashed curves labeled G, S, R, and H denote the duration of the generation, quasi-steady state (dwell time of VP<sub>1</sub>), recombination, and healing processes, respectively. The black dashed lines crossing all subplots at 40 and 80  $\tau_{GL}$  mark the interval of EM wave period. The complete dynamical results of  $|\psi|$ ,  $\theta_s$ ,  $J$ , and  $\omega_{v,s}$  are also demonstrated in the Supplemental Material [28]. (b) Snapshot of  $|\psi|$ ,  $\theta_s$ ,  $J$ , and  $\omega_{v,s}$  at different stages of the VP life time. (c) Schematic diagram of the cycling of free energy density.

$J/J_c > 1$ . Subsequently, one VP ( $VP_1$ ) is generated during the second half cycle of the EM wave. In the third half cycle, some time after  $VP_1$  is generated,  $|\psi|$  in the center of the spot decreases significantly, which can be interpreted as a precursor for the second VP ( $VP_2$ ).  $VP_2$  fully appears in the fourth EM wave half cycle, after which this process becomes (almost) periodic. The windings of the phase around vortices, clearly visible in Fig. 4 (b), are reflected as jumps in the phase in Fig. 4 (a).

Let us discuss the evolution of VPs in more detail on the instance of their motion that occurs during the fourth half cycle of the EM wave. The outer pair,  $VP_1$ , appears at the moment of time  $\tau_1$ , when  $\Phi_B \approx 1.9\Phi_0$ , and the vortices in the pair move relatively fast toward positions at  $y_{\pm} \approx \pm 0.7w_0$  (whereas the maxima of  $B_z$  are located at  $y_0 = \pm w_0/\sqrt{2} \approx 0.71w_0$ ) until time  $\tau_2$  ( $\tau_2 - \tau_1 = 5\tau_{GL}$ ). After that, the vortices in this pair remain around positions at  $y_{\pm}$  until time  $\tau_3$ . Shortly after time  $\tau_2$ , when  $\Phi_B$  reaches  $3.2\Phi_0$  after passing its maximum, the second pair,  $VP_2$ , is generated and follows a similar path as  $VP_1$  until time  $\tau_3$ . After time  $\tau_3$ , the vortices and antivortices in both pairs start moving toward each other, marking the beginning of the recombination process. The outer  $VP_1$  annihilates somewhat after the flux reaches zero value. We also note that the recombination process is associated with the change of sign of the supercurrent vorticity away from the vortices [ $J_{\pm}$  in Fig. 4(a)]. Overall, there is a significant reduction of the order parameter around the center of the beam. While this happens due to complicated dynamics of the current distribution, this can partially be explained by noting that the  $A^2\psi$  term in Eq. (1) can be interpreted as the modification to the mass term in the Lagrangian from which the gTDGL equation is derived, effectively decreasing local  $T_c$  [see Fig. 4(c)].

In Fig. 5(a), we show the results of the simulations in which we change the EM wave frequency to  $\omega_{EM} = \omega_{GL}/10$ . To keep the same amplitude of the flux, we correspondingly increased the amplitude of the EM wave by four times, i.e.,  $E_{amp} = 8A_0\omega_{GL}$ . This time we observe generation of only one VP, which first happens during the seventh half cycle of the EM wave. Subsequently, it undergoes recombination, after which the process of generation and recombination reoccurs almost periodically. A VP is generated almost at times when  $\Phi_B$  reaches its maximum/minimum value of  $\pm 3.2\Phi_0$ . After that, the vortices in the pair move relatively fast for about  $1.9\tau_{GL}$  at a distance of about  $0.45w_0$  each and recombine significantly after the flux changes the sign when  $\Phi_B = \mp 1.9\Phi_0$ . We note that during the generation-recombination cycle of the VP, the electric field grows from almost zero, reaches its maximum/minimum value, and goes to intermediate values without changing the sign at the recombination time. This is unlike the previous case, where at the generation time of  $VP_1$   $E_x$  has intermediate values, then changes sign, and reaches values close to its maximum/minimum at the recombination time. While the data support our conjecture that the generation of a VP is induced by a sufficiently strong  $z$  component of the magnetic field, this observation highlights the complex interplay of timescales set by the EM wave and gTDGL equation.

When we further increase  $\omega_{EM}$  to  $\omega_{GL}/2$ , there is no generation of VPs at all for the intensity of the beam that pro-

duces the same amplitude of  $\Phi_B = 3.2\Phi_0$ . When the intensity increases such that the amplitude of  $\Phi_B$  equals  $9.6\Phi_0$ , we can see a cycle of the VP generation-recombination process [see Fig. 5 (b)], after which the light simply destroys SC locally leaving a “scar” in the SC order parameter.

We further investigate the dependence of the VP generation-recombination processes on  $\omega_{EM}$  and the amplitude of  $\Phi_B$ . In Fig. 6, we present a phase diagram in terms of the generation of long-lived VP cycles (LLVP), i.e., periodic occurrence of VPs, short-lived VP cycles (SLVP), i.e., occurrence of only a few of VP cycles, and zero VP cycles (0VP), i.e., no occurrence of VPs. In the range we demonstrate, there is no generation of VP cycles at any frequency if the amplitude of  $\Phi_B < 2\Phi_0$ , and the generation of periodically occurring LLVP cycles is impossible for  $\tau_{EM} < 2\tau_{GL}$ . While we cut our numerical investigation at  $\Phi_B = 10\Phi_0$ , we expect that for a sufficiently strong intensity of the beam, i.e., sufficiently large amplitudes of  $\Phi_B$ , the SC state inside the region potentially available for the generation of VPs will be completely destroyed. Additionally, for larger amplitudes of  $\Phi_B$ , VPs could be generated on the sides of the  $x = 0$  line, as illustrated in the inset snapshot at  $4\tau_{GL}$  in Fig. 5(b).

While the results might visually resemble the formation and movement of kinematic vortices induced by large currents in thin films of intermediate width [20,26,27], our data also support the conjecture about Abrikosov’s mechanism of vortex formation.

### C. Imprint profile and edge effect

Edge effects play a significant role in vortex nucleation and dynamics, particularly in finite-sized superconducting thin films. In this work, we consider an SC thin film surrounded by a vacuum environment and apply Neumann boundary conditions at the SC-vacuum interface (Appendix A). In the previous section, we considered cases where the spot size  $2w_0$  was significantly smaller than the sample size and larger than but comparable to  $\xi$ , in which no vortices were observed to nucleate from the sample edges. In this section, we change  $w_0$  and report some basic observations.

In Fig. 7(a), we consider the case when  $\xi < 2w_0 < L$  for three values of  $w_0$ :  $w_0 = 2\xi$  with peak magnitude of  $\Phi_B = 6.4\Phi_0$  (S1),  $w_0 = 4\xi$  with peak magnitude of  $\Phi_B = 12.8\Phi_0$  (S2),  $w_0 = 8\xi$  with peak magnitude of  $\Phi_B = 50.9\Phi_0$  (S3). All snapshots for  $|\psi|$  and  $\omega_{v,s}$  in Fig. 7 are at  $t = 2\tau_{GL}$ , and the frequency for all cases is  $\omega_{EM} = \omega_{GL}/8$ .

In case S1, two VPs are generated on the sides of the  $x = 0$  line. In case S2, the flux increases twice and there are three VPs generated at  $t = 2\tau_{GL}$ . In case S3, the distance from the edge of the spot to the edges of the film is  $2\xi$ .  $\Phi_B$  is 8 times larger than in case S1, and the maximum and minimum of  $B_z$  are located at  $y = \pm 8\xi/\sqrt{2} \approx 5.66\xi$ . This results in a large magnetic flux in the areas close to the edges at  $y = \pm 10\xi$ . Consequently, in combination with the reduction of the barrier for a vortex to enter from an edge, vortices and antivortices are generated close to these edges, as well as a number of VPs in the center of the beam.

Accordingly, if the beam spot is close to the edge, we observe the vortex generation at the edges even if the region is outside the beam spot. We interpret this as a combined effect

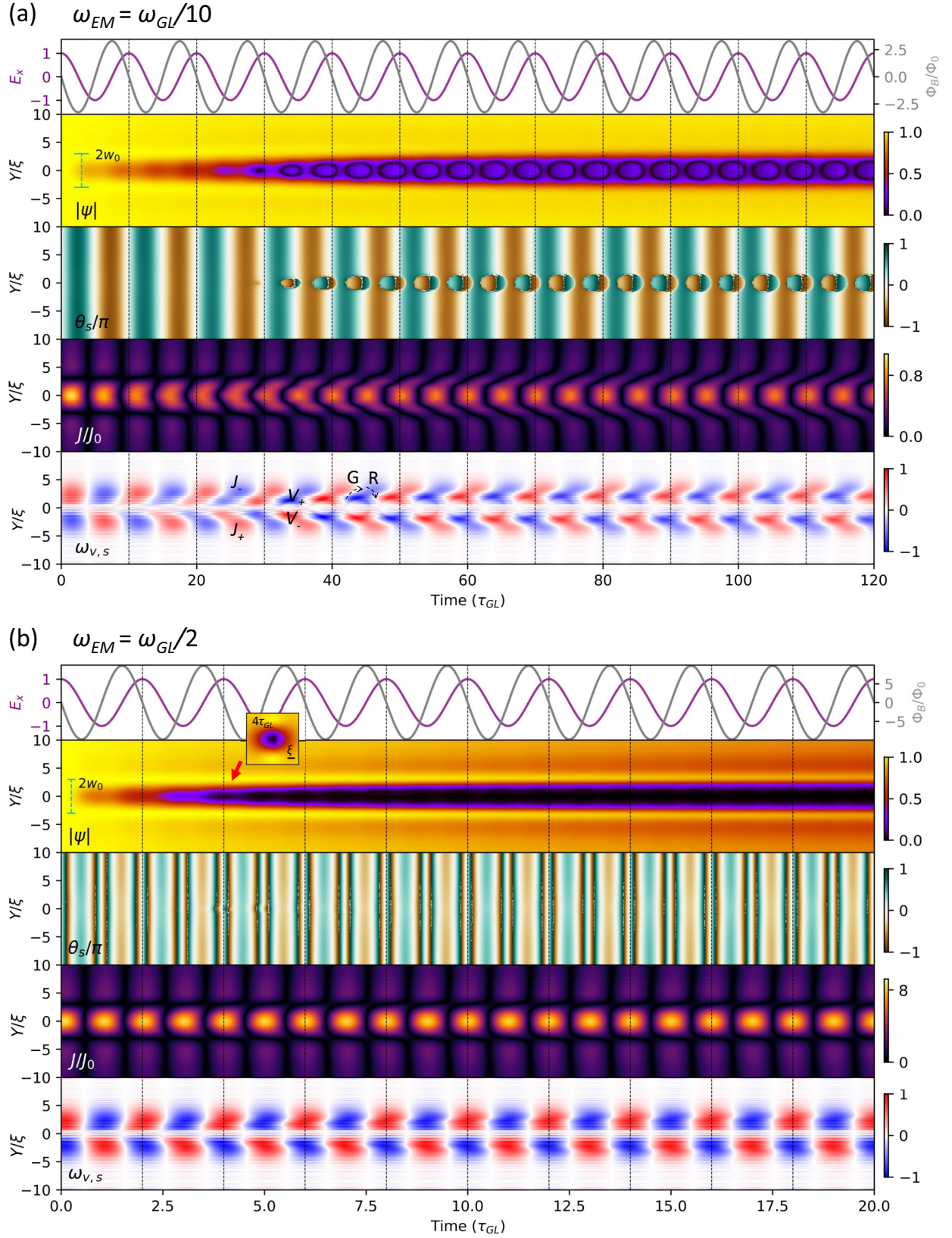


FIG. 5. Similar simulations as Fig. 4 but with higher  $\omega_{EM}$ : (a)  $\omega_{EM} = \omega_{GL}/10$  and (b)  $\omega_{EM} = \omega_{GL}/2$ . The red arrow insets in panel (b) indicate a VP life cycle, and the inset of  $|\psi|$  shows the profile of VPs at  $4\tau_{GL}$ . The simulation condition of  $E_{amp}$  in panel (a) is 4 times larger than the case of Fig. 4, and in panel (b) is 60 times larger. The complete dynamical results for  $|\psi|$ ,  $\theta_s$ ,  $J$ , and  $\omega_{v,s}$  are also demonstrated in the Supplemental Material [28].

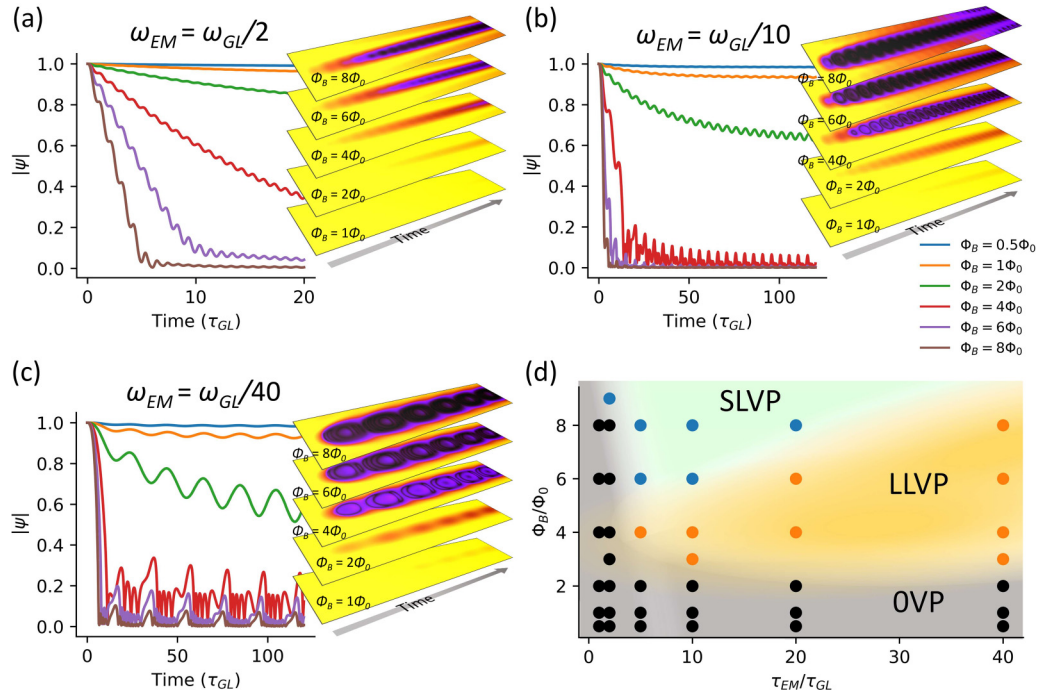


FIG. 6. The time traces of  $|\psi|$  at  $x, y = (0, 0)$  for (a)  $\omega_{EM} = \omega_{GL}/2$ , (b)  $\omega_{EM} = \omega_{GL}/10$ , and (c)  $\omega_{EM} = \omega_{GL}/40$ . The yellow inserts show the time-dependent profile along  $x = 0$  with different  $\Phi_B$ . (d) The results of  $\tau_{EM}$  and  $\Phi_B$  dependent on state of VP cycle. The black points, blue points, and orange points represent OVP, SLVP, and LLVP, respectively.

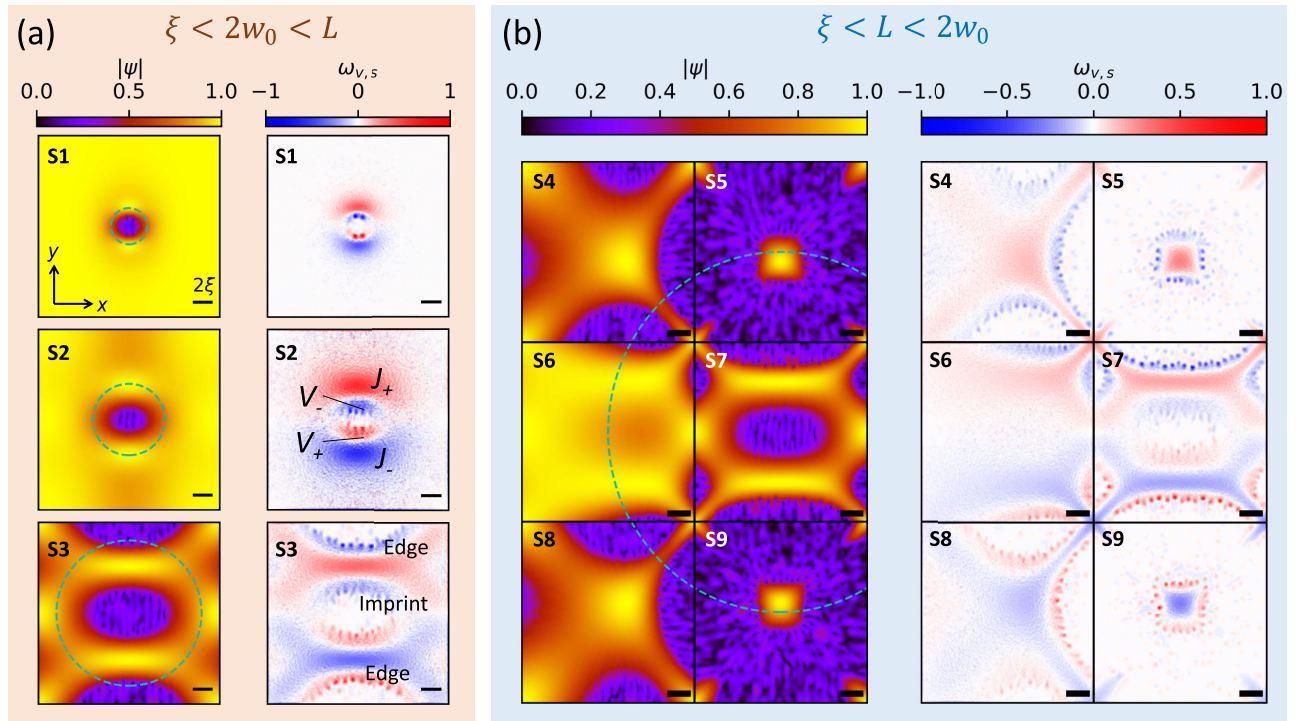


FIG. 7. The distribution of supercurrent and vortex depends on different light spot sizes  $w_0$ , marked by green dashed circles in each sample.  $|\psi|$  shows the profile of SC states, and  $\omega_{v,s}$  exhibits supercurrent and vortices with faint colored area and colored dots, respectively. Panel (a) demonstrates cases where  $w_0$  is smaller than size of sample  $L (=20\xi)$ .  $w_0$  of samples S1–S3 are, respectively,  $2\xi$ ,  $4\xi$ , and  $8\xi$ . The inset axis in S1 indicates direction of coordinate for all samples. The notations  $J_+$ ,  $J_-$ ,  $V_+$ , and  $V_-$  mark the supercurrent and vortices, as mentioned in the main text. The vortex clusters in S3 labeled “Edge” are vortices generated at the edge and the one labeled “Imprint” indicates the optical imprint led generation. Panel (b) presents the cases where  $2w_0 > L$ .  $w_0$  for all samples is  $20\xi$ . Six SC samples, S4–S9, are positioned at different locations under the same light spot. Those six samples are simulated independently. All results in this figure are snapshots at  $\tau_{GL}$  with light source  $\omega_{EM} = \omega_{GL}/8$  and fixed  $E_{amp}$ .

of the relatively large magnetic field close to the edge and reduction of the barrier for a vortex to enter from the edges into the sample. We also note that the overall neutrality of the structure is preserved; i.e., the total vorticity of the sample is zero.

In the figure of vorticity, the profile of vortices ( $V_+$ ), antivortices ( $V_-$ ), and supercurrents with positive/negative curl ( $J_+/J_-$ ) could be clearly discerned, as indicated in Fig. 7(a). If  $2w_0 < L$ , in the upper half of the film at  $2\tau_{GL}$ , we predominantly see antivortices,  $V_-$ , in the distribution of vortices and supercurrents with positive curl,  $J_+$ . For the lower half of the film, the situation is vice versa. Similarly, with different coherence lengths, if  $\xi$  is smaller than  $2w_0$ , then the distribution of vortices follows the results when changing the spot size (see Appendix E).

In Fig. 7(a), samples S1 and S2 exhibit a spot size  $2w_0$  sufficiently small such that it is far from the edge, resulting in distribution patterns that are nearly identical, except for variations in vortex numbers. In the case of S3, the distance between the edge of the spot and the edge of the sample is merely  $2\xi$ . The order of  $J_+$ ,  $V_-$ ,  $V_+$ , and  $J_-$  from top to bottom is also broken. There are two more noticeable vortex clusters nucleating at the upper and lower sides, which are induced by the edges of SC. The cluster size is roughly  $6\xi$ .

As a result, vortex nucleation in our system occurs near the edge of the illuminated region, where the magnetic field gradient is strong and local suppression of  $\psi$  facilitates vortex entry. The vortices then migrate toward the maximum field region, reaching equilibrium at a location distinct from that predicted by the constant-field scenario. This behavior is clearly seen in sample S3, where vortices originating from the SC edge redistribute around the field peak, rather than remaining pinned at the boundary.

In Fig. 7(b), we consider the case where  $2w_0 > L$ . We shine a Gaussian beam with  $w_0 = 20\xi$  on six thin film samples S4–S9 positioned with respect to the beam as depicted in the figure. We note that the samples are not connected and treated independently in the simulations. The frequency  $\omega_{EM} = \omega_{GL}/8$  is the same as for subfigure (a), and the amplitude of  $\Phi_B = 480\Phi_0$ .

For S4–S9, the overall vorticity still roughly follows the order  $V_-$ ,  $J_+$  at the upper part of the beam, and  $V_+$ ,  $J_-$  at the lower part. In sample S7, we observe VPs with some of the vortices and antivortices entered from the edges, while in the rest of the samples, we see only vortices/antivortices entered from the edges.

## IV. DISCUSSION

### A. Applicability of the gTDGL model

The gTDGL equations are microscopically justified only within a very small range of temperatures around  $T_c$ . Nevertheless, it often gives qualitatively correct results at much lower temperatures. Very close to  $T_c$ , microscopic calculations put certain bounds for the validity of the gTDGL theory. In the time domain, changes in the superconducting condensate should occur on a timescale much slower than  $\tau_E$ , the inelastic electron-phonon scattering time [18,29]; i.e., the theory is microscopically valid as long as  $\tau_{GL} \gg \tau_E$ . In clean samples,

$\tau_E^{-1} \sim k_B T^3 / \hbar \Omega_D^2$ , where  $\Omega_D$  is the Debye temperature. For low-temperature superconductors such as Nb, it is typically a few GHz. Superconductors like Nb<sub>3</sub>Al and MgB<sub>2</sub> can have  $\tau_E^{-1} \sim 10$  GHz, and for cuprates  $\tau_E^{-1}$  might be  $\sim 100$  GHz. However, for dirty samples,  $\tau_E^{-1}$  might be larger [30]. Experiments on dirty NbN thin films report  $\tau_E^{-1} \sim 100$  GHz [31].

Application of TDGL theory to driven processes can be justified for frequencies satisfying  $\omega_{EM}\tau_{GL}(T) < 1/\epsilon$  [16], which implies

$$\omega_{EM}\tau_{GL,0} < 1. \quad (8)$$

In the following, we want to estimate whether there is a parameter space within validity of the gTDGL model that corresponds to the ratios of  $w_0/\xi$  used in the simulations. Let us represent the spot size in terms of  $\lambda_{EM}$  as  $w_0 = \alpha_\lambda \lambda_{EM}$ , where  $\alpha_\lambda$  is a real number greater than 1/2 due to the diffraction limit in free space, and in terms of  $\xi(T)$  as  $w_0 = \alpha_\xi \xi$ , where  $\alpha_\xi > 0$ . In the simulations presented above,  $\alpha_\xi$  ranged from 3 to 20. We then can write for the zero temperature Ginzburg-Landau coherence length ( $\xi_0 \approx \xi \sqrt{\epsilon}$ )

$$\xi_0 \approx \frac{\alpha_\lambda}{\alpha_\xi} \sqrt{\epsilon} \frac{2\pi c}{\omega_{EM}}. \quad (9)$$

Using Table I, for example, for Nb, determining maximum  $\epsilon_{\max}$  below which gTDGL is still microscopically valid from  $\tau_{GL}/\epsilon_{\max} = \tau_E$ , we find  $\epsilon_{\max} \approx 10^{-4}$  (see Table I for other SC materials). Then, for  $\omega_{EM} = 10$  THz, from Eq. (9) we find at  $\epsilon = 10^{-4}$ ,  $10^{-6}$   $\alpha_\lambda/\alpha_\xi = 0.02, 0.2$ , respectively. And, correspondingly, for  $\alpha_\lambda = 1/2$ ,  $\alpha_\xi \approx 24.8, 2.48$ . Analogically for Pb, for  $\omega_{EM} = 4$  THz and  $\epsilon = 10^{-4}, 10^{-6}$ , we find  $\alpha_\xi \approx 28, 2.8$ , respectively. However, using as the criterion for inducing LLVP cycles  $\omega_{EM}\tau_{GL}(T) < 2\pi/5$  (see Fig. 6; LLVP happens for  $\tau_{EM}$  greater than  $5\tau_{GL}$ ), we conclude that at these frequencies only SLVP cycles are possible. Indeed, using Eq. (9), the criterion  $\omega_{EM}\tau_{GL} < 2\pi/5$  can be rewritten as

$$\frac{\xi_0}{\tau_{GL}} > v_0 = \frac{\alpha_\lambda}{\alpha_\xi} \frac{5c}{\sqrt{\epsilon}} = \frac{\alpha_\lambda}{\alpha_\xi} \frac{1500 \mu\text{m}}{\sqrt{\epsilon} \text{ ps}}. \quad (10)$$

For Pb, at  $\epsilon = 4 \times 10^{-3}$  and  $\alpha_\lambda = 1/2$ , to satisfy this criterion,  $\alpha_\xi$  must be larger than  $10^4$ , which is far from the values of  $\alpha_\xi$  for which we presented simulations.

While it is hard to satisfy criterion in Eq. (10) in the range of microscopic validity of gTDGL, we emphasize that factor 1500 on the right-hand side of Eq. (10) is derived from the simulations for  $\alpha_\xi = 3$ ,  $\gamma = 10$ , and  $u = 5.79$ . The numerical factor can be different for different sets of  $\alpha_\xi$ ,  $\gamma$ , and  $u$ . Our observation is that for  $\gamma = 0$ , as in the standard TDGL, while the winding of the order parameter in the phase is present, the amplitude of the order parameter is significantly suppressed for the range of frequencies presented in Sec. III B, making it impossible to claim that there are any VPs, as a result of Fig. 9(c). In this regard, we note that  $\gamma \sim \sqrt{\epsilon}$ , and it is possible that larger values of  $\gamma$  weaken the criterion in Eq. (10).

From the above discussion, it follows that in the region of microscopic validity of the gTDGL model, the LLVP phase in Fig. 6 is hardly achievable for the normal optical techniques such as the conventional optical microscopy technique in free space for the ratios of  $w_0/\xi$  used in the present simulations. However, we see that as temperature lowers down

TABLE I. Properties of conventional SC materials. The parameters  $\xi_0$ ,  $\lambda_{L,0}$  (or  $\Lambda_0$  for thin film),  $r_n$ ,  $v_F$ ,  $\Delta_0$ ,  $\epsilon_{\max}$  and  $\gamma_{\max}$  are referenced from several references [18,32–45], and  $\tau_{GL,0}$ ,  $\omega_{GL,0}/2\pi$ , and  $\xi_0/\tau_{GL,0}$  are calculated in current work. The parameter  $r_n$  is referenced from the resistivity near  $T_c$ . Note that the timescale  $\tau_{GL,0}$  is related to relaxation time of supercurrent [29], which is much shorter than the relaxation time of the order parameter, especially for the gTDGL model (see Appendix C).

| SC type | Material  | $\xi_0$<br>(nm) | $\lambda_{L,0}$ or $\Lambda_0$<br>(nm) | $r_n = 1/\sigma$<br>( $\mu\Omega$ cm) | $\tau_{GL,0}$<br>(fs) | $\omega_{GL,0}/2\pi$<br>(THz) | $\tau_E^{-1}$<br>(GHz) | $\xi_0/\tau_{GL,0}$<br>( $\mu\text{m}/\text{ps}$ ) | $v_F$<br>( $\mu\text{m}/\text{ps}$ ) | $\Delta_0$<br>(meV/THz) | $\epsilon_{\max}$  | $\gamma_{\max}$ |
|---------|-----------|-----------------|----------------------------------------|---------------------------------------|-----------------------|-------------------------------|------------------------|----------------------------------------------------|--------------------------------------|-------------------------|--------------------|-----------------|
| Type I  | Al        | 1600            | 16                                     | $\sim 0.5^a$                          | 15                    | 64                            | 0.02                   | 25                                                 | 2.0                                  | 0.175/0.04              | $4 \times 10^{-9}$ | 340             |
|         | Pb        | 83              | 37                                     | $\sim 2.3^b$                          | 75                    | 13                            | 50                     | 1.1                                                | 1.8                                  | 2.8/0.7                 | $4 \times 10^{-3}$ | 17              |
| Type II | Nb        | 38              | 39                                     | $2^c$                                 | 96                    | 10                            | 1.4                    | 0.4                                                | 1.4                                  | $\sim 2.5/0.60$         | $1 \times 10^{-4}$ | –               |
|         | YBCO      | 1.4             | 210                                    | $\sim 150^d$                          | 37                    | 27                            | 690                    | 0.04                                               | $> 0.7$                              | 20/4.8                  | 0.025              | –               |
|         | Nb (2 nm) | 10–100          | $10^3\text{--}5 \times 10^5$           | 153                                   | 1.6–820               | 1–600                         | –                      | 0.1–60                                             | –                                    | 0.18                    | –                  | –               |

<sup>a</sup> $r_n$  of 300-nm Al.

<sup>b</sup> $r_n$  of 100-nm Pb (dimensional  $r_n$  shown in the preprint version [46]).

<sup>c</sup> $r_n$  of 160-nm Nb.

<sup>d</sup> $r_n$  in the *ab* plane.

(i.e.,  $\epsilon$  increases), the criterion for reaching the LLVP phase weakens. Investigation within a more exact model for lower temperatures is thus needed to answer the question about the possibility of reaching the LLVP phase without the need of luxury experimental equipment mentioned in the next section. We note that while the ratios of  $w_0/\xi$  used in the simulations here are rather small for the spot size of the beams respecting the diffraction limit in free space, the mechanism of VP generation demonstrated above might survive for larger ratios of  $w_0/\xi$  and lower temperatures.

### B. Heating effect

Another aspect that needs discussion is the heating of the SC sample due to irradiation from the light source. Since we worked within the gTDGL model, in the following we provide rough estimations in the very vicinity of the transition temperature  $T_c$ . Simultaneously, this is the worst scenario for the heating power as the resistive part of the impedance is the largest there and is approximately equal to the one in the normal state.

Very close to  $T_c$ , the standard theory of propagation of EM waves in bulk metals can be used, and the EM wave penetration length  $\delta$  can also be taken as equal to the one in the normal state,  $\delta = \sqrt{2/\mu_0\sigma_n\omega_{EM}}$ , and the wave number inside the sample is  $k_{in} = 1/\delta = \sqrt{\mu_0\sigma_n\omega_{EM}/2}$ . Then, the power incoming to the SC film is

$$P_{in} = \frac{\pi(E_{amp}w_0)^2k_{in}}{4\mu_0\omega_{EM}} = \frac{\pi(\Phi_B\omega_{EM})^2k_{in}}{8\mu_0\omega_{EM}}. \quad (11)$$

We note that in the case of very thin films, the correct description, at least, would involve reflection from both interfaces. At lower temperatures, the values of  $\delta$  and  $k_{in}$  will be different and can be estimated from a simple extension of the theory of propagation of EM waves in metals to a two-fluid model [47].

Only a fraction of the power in Eq. (11) will be converted to heat, which can be estimated from the time-averaged Joules power density

$$q = \frac{\sigma E_0^2(\mathbf{r})}{2}, \quad (12)$$

where  $\sigma$  is the real part of conductivity. Very close to  $T_c$ , we can take  $\sigma = \sigma_n$ , where  $\sigma_n$  is the normal conductivity at  $T_c$ .

We can then roughly estimate the Joule's power in the area of the beam's spot size as

$$P_q = \frac{\sigma E_{amp}^2}{2} \pi w_0^2 (1 - e^{-2d/\delta}) \delta / 2, \quad (13)$$

where  $d$  is the thickness of the film.

Using values of  $\sigma_n$  from Table I [48],  $\sigma_n = 1/2 \mu\Omega^{-1} \text{cm}^{-1}$  and  $\sigma_n = 1/153 \mu\Omega^{-1} \text{cm}^{-1}$  for 160- and 2-nm-thick Nb films, respectively, we find for  $\omega_{EM} = 1 \text{ THz}$   $\delta = 1.26 \times 10^{-7} \text{ m}$  and  $1.10 \times 10^{-6} \text{ m}$ , respectively. Thus,  $d/\delta = 1.27$  and  $1.8 \times 10^{-3}$ , respectively, for 160- and 2-nm-thick Nb films, which justifies the neglect of the decay of the electric field inside the sample for the 2-nm-thick film. We see that in this case,  $d \ll \delta$ ,

$$P_q/P_{in} = 4d/\delta. \quad (14)$$

To have  $\Phi_B$  equal to 2 flux quanta, which was shown to be close to the threshold after which the generation of VPs occurs, the incoming power is

$$P_{in} = \frac{\pi(\Phi_0\omega_{EM})^2k_{in}}{2\mu_0\omega_{EM}}, \quad (15)$$

which at  $\omega_{EM} = 1 \text{ THz}$  is equal to  $P_{in} \approx 3 \times 10^{-5}$  and  $3.4 \times 10^{-6} \text{ W}$  for 2- and 160-nm-thick Nb films, respectively.

Assuming the film is at temperature below  $T_c$  by  $\Delta T$ , we estimate how much time the affected area remains superconducting from

$$P_q t = C v \Delta T, \quad (16)$$

where  $C$  is the specific heat per mole and  $v$  is the amount of mole contained in the affected volume. From this, we find a number of light cycles before SC condensate starts melting:

$$N = \frac{2C n w_0^2 \Delta T}{\pi N_A \sigma \Phi_B^2 \omega_{EM}}, \quad (17)$$

where we used  $v = n\pi w_0^2 d/N_A$ , where  $n$  is the concentration of electrons and  $N_A$  is Avogadro's number. According to Ref. [49],  $C/T = 33.1 \text{ mJ}/(\text{K mole})$  at  $T = 9 \text{ K}$  ( $T_c/T = 1.01$ ), and we take  $n = 5.56 \times 10^{22} \text{ cm}^{-3}$  for Nb at  $T = 300 \text{ K}$  [35], which is good enough for a rough estimate.

For the 160-nm thin film, for  $\Phi_B = 2\Phi_0$ ,  $\Delta T = 0.09 \text{ K}$ ,  $\omega_{EM} = 1 \text{ THz}$ , and  $w_0 = \lambda_{EM}$  and  $20\xi$  (we take  $\xi = 38 \text{ nm}$ ),

we have  $N \approx 6.6 \times 10^6$  cycles and  $N \approx 1.08$  cycles. In the latter case, by reducing the frequency to  $\omega_{\text{EM}} = 0.1$  THz, the number of cycles increases by 10 times to  $N \approx 10$  cycles.

For the 2-nm thin film, for  $\Phi_B = 2\Phi_0$ ,  $\Delta T = 0.09$  K,  $\omega_{\text{EM}} = 1$  THz, and  $w_0 = \lambda_{\text{EM}}$  and  $20\xi$ , we have  $N \approx 5 \times 10^8$  cycles and  $N \approx 81.4$  cycles. Even if we take  $\Delta T = 0.0009$  K, which corresponds to  $\epsilon = (T_c - T)/T_c \approx 10^{-4}$ , we get  $N \approx 5 \times 10^6$  cycles and  $N \approx 0.8$  cycles. In the latter case, by reducing the frequency to  $\omega_{\text{EM}} = 0.1$  THz, the number of cycles increases by 10 times to  $N \approx 8$  cycles.

As mentioned above, for lower temperatures the provided estimate for heating is rather a gross overestimation, as the resistive part of the impedance decreases significantly from its value right below  $T_c$  [47]. Additionally, as suggested in Ref. [16], one can consider using a substrate to dissipate the heating power from the area affected by the beam spot.

### C. Proposed experimental setup

As mentioned in Sec. IV A, considering the experiment with a continuous wave light source, the narrow range of  $\epsilon$  pushes the experiment in the harsh situation, i.e.,  $T_c - T < 10^{-4}T_c$ . Nonetheless, even if the applicable range of the gTDGL model is extremely narrow, it is still used to simulate the vortex dynamics and provide the phenomenological information at lower temperatures. For instance, the prediction of the kinematic vortex within the gTDGL model [27] found its confirmation in experiments [26]. The cutting edge light sources, such as the few cycle-pulse laser techniques [50,51] and the subwavelength-sized THz light [52–55], have been developed over a decade and beyond several orders of the restriction of the criterion in Eq. (10). The case  $\omega_{\text{EM}}\tau_{\text{GL}} \geq 2\pi/5$  for SLVP [opposite to condition in Eq. (10) for LLVP] can be taken into account with the former technique, and  $\alpha_\lambda < 1/2$  can be achieved with the latter one.

Additionally, we also point out that increasing the energy deposited by the beam on an SC may induce thermal effects, which are undesirable in this context. Thus, one would need to come up with the mitigation strategy to negate the effects of heating. This is not an insurmountable obstacle, but the strategy would depend on specific materials and details of optical technique. The application of aforementioned light sources also efficiently alleviates the thermal effect. The pulse laser technique, which is known for the cold processing of surgeries and materials [56,57], reduces heating due to the ultrashort pulse duration, and subwavelength spot size reduces the heating area directly.

In this section, we propose candidates for experimental configurations that correspond to the scenario of Fig. 5(a) based on these two techniques. Let us focus on a specimen of a 2 nm niobium thin film at  $T = 0.99T_c$  with  $\Lambda(T) = 10 \mu\text{m}$  (corresponding to  $\lambda_{L,0} = 45 \text{ nm}$ ), thin film resistivity  $r_n = 153 \mu\Omega \text{ cm}$ , and  $\tau_{\text{GL}}(T) = 160 \text{ fs}$  ( $\omega_{\text{GL}}/2\pi \approx 6 \text{ THz}$ ). In the context of niobium thin films, the Pearl length, which serves as the effective penetration depth, can range from 1 to  $500 \mu\text{m}$  when the thickness is 2 nm [44,45]. So, we expect that this long penetration depth maintains the vortex dynamics similar to our simulation, which is without considering the screening effect. Since the lower the frequency  $\omega_{\text{EM}}$ , the more light-induced VPs are generated (see Fig. 6), we consider the

case  $\omega_{\text{EM}} = \omega_{\text{GL}}/10 \approx 0.6 \text{ THz}$  for a thin niobium film that corresponds to the scenario of Fig. 5(a). It corresponds to a minimum spot size of  $250 \mu\text{m}$ , which is at the diffraction limit of experimental conditions in free space.

The induced  $B_z$  of linearly polarized light depends on the nonuniform distribution of light intensity. Thus,  $B_z$  increases significantly when the spot size is reduced to less than  $\lambda_{\text{EM}}/2$ . It can be implemented by using near-field THz radiation. Additionally, keeping the photon energy below the SC gap with a small spot size is advantageous as it avoids inducing quasiparticles. Appendix F provides more details of issues about quasiparticles and depairing of SC state induced by intense THz pump beam. For the case of  $\omega_{\text{EM}} = \omega_{\text{GL}}/10$  and maintaining the same spot size as in the simulation ( $0.6 \mu\text{m}$ ), the estimated  $E_{\text{amp}}$  is  $30.4 \text{ kV/cm}$ . These lower frequencies will help avoid photoexcited carriers crossing  $\Delta_0$ . Nowadays, state-of-the-art light sources can achieve transient terawatt energy levels [58] and  $100 \text{ MV/cm}$  of THz electric fields [50,51]. If we consider the intense THz technique combined with the subwavelength spot size, the technique of near-field nanotip THz photoemission that is usually applied by scanning near-field optical microscopy (SNOM), the transient and local THz electric field can achieve orders of amplitude from  $\text{kV/cm}$  to  $\text{MV/cm}$  [52,53]. For the observation of vortices, the size of the tip is determined by the scale of the SC coherence length, which is usually on the order of submicrometer. Instead of utilizing the tip as a probe in SNOM, we consider it as a THz pump for VP generation. Another approach to surpass the diffraction limit involves the techniques of metamaterials [54], near-field THz antenna [55], etc.

In few-cycle THz pulse techniques, the observation of the LLVP phase is impossible as the number of light cycles is limited. However, scenarios similar to the simulations illustrated in Figs. 5(b) and 9 are possible. In such a case, the minimum  $2w_0$  is considered to be  $0.5 \mu\text{m}$  for  $\omega_{\text{EM}} = \omega_{\text{GL}}/2$  in the simulations. To maintain a constant flux similarly as the case of Fig. 5(b), specifically, for a peak value of  $\Phi_B \sim 9.6\Phi_0$ , the required  $E_{\text{amp}}$  is approximately  $448 \text{ kV/cm}$ , which is achievable with intense THz techniques [50,51]. In Appendix C, the simulation of a subcycle optical pulse also demonstrates the generation of a single-cycle VP.

On the probe side, the instruments should be able to resolve space and timescales on the order of  $\xi$  and  $\tau_{\text{GL}}$ , which is achievable by a time-resolved scanning superconducting quantum interference device [59], time-resolved scanning tunneling microscopy, [60], time-resolved nitrogen-vacancy center microscopy [61,62], time-resolved Lorentz microscopy [63,64], time-resolved magneto-optical microscopy [65,66], etc.

As discussed above, our results exhibit vortex motion that is synchronized with an external periodic field, which is distinct from kinematic vortices observed in SC thin films [67]. Kinematic vortices emerge due to phase slips and can persist even in the absence of a periodic driving potential. Similar periodic vortex behavior has also been studied in the context of multiple harmonic generation in superconducting radio-frequency cavities [68–71], where the vortex response is shaped by pinning effects and cavity boundaries. In contrast, the vortex motion in our simulations is directly driven by the incident light field, without relying on pinning or

geometric confinement, indicating a fundamentally different mechanism.

## V. SUMMARY

We propose an approach to quantum steering of a coherent electron state by Gaussian structured light. The nonuniformity of the vector potential of a linearly polarized Gaussian beam leads to a nonzero  $z$  component of the magnetic field antisymmetric with respect to the coordinate in the direction perpendicular to the polarization of the electric field. The maximum and minimum of the magnetic field are located away from the center of the beam and are the spots where the amplitude of time-dependent magnetic field is maximal. While total magnetic flux is zero, the upper and lower half planes in the plane of the film are threaded by oscillating fluxes of different signs. This creates a situation where the barrier to introduce a vortex into the bulk can be overcome by generation of a VP. During half-period of the EM wave, vortices in such VPs move toward the position of maximum/minimum of the magnetic field and back to recombine at the beam's center. In this approach, light induces a dynamical vortex state with zero total vorticity and can transfer quantum numbers (such as spin and orbital angular momentum) to the superconducting condensate, which will be illustrated in the follow-up paper. This process can be viewed as quantum printing where we imprint the quantum numbers of the light field.

Based on the gTDGL theory, we show that depending on the frequency of the incident linearly polarized Gaussian beam, the generation of VPs may occur in a periodic fashion (LLVP phase) or can last for a few cycles (SLVP phase). Although in the region of microscopic validity of the gTDGL model for the ratios of the spot size and coherence length used in the simulations, we showed that the LLVP phase is not achievable for the beams of spot sizes that comply with the diffraction limit, the criterion for the LLVP phase weakens as temperature lowers down, suggesting that the LLVP phase for rather small ratio  $w_0/\xi$  and spot sizes complying with the diffraction limit might be achievable at lower temperatures within more exact theory. Simulations for larger ratios  $w_0/\xi$  within the range of validity of the gTDGL model are required to conclude about the possibility of the LLVP phase at temperatures close to  $T_c$ .

Although vortices in VPs induced by the beam generate and recombine, one can perhaps prohibit their recombination by separating them using the Lorentz force in the presence of a current flowing through the film perpendicular to the motion of vortices in VPs or employing the inverse Faraday effect in superconducting condensate [16,72], potentially leading to controlled generation of vortices with EM radiation.

We also discuss a broader context of the presented work. Generation of vortices under light is expected on general grounds. The light provides transient electric and perpendicular magnetic fields, which in turn generate and move vortices in VPs. The approach is similar to the printing of matter in which the recipient substance receives ink to produce the copy.

In this report, light can be used to not only transfer energy and oscillating magnetic fields along optical axis to SC materials, but also transfer quantum numbers (such as the spin and orbital angular momentum of the beam) to SC and induce patterns of vorticity. The further results of momentum transfer are presented in the follow-up paper [14]. Overall, this process of induction of topological excitations in a quantum fluid can be viewed as the *quantum printing* where we imprint the quantum numbers of the light field on a coherent electron fluid—the SC.

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## DATA AVAILABILITY

Data sets are available from the corresponding author on request and Python package LG-TDGL (GitHub repository link: [74]).

## APPENDIX A: BOUNDARY CONDITION AT SUPERCONDUCTOR-VACUUM INTERFACES

In our simulations, we model the SC thin film as surrounded by an infinite vacuum region. This boundary condition has a minor influence when the beam spot size is smaller than the sample. However, when the spot size approaches or exceeds the sample dimensions, boundary effects become significant and strongly influence vortex nucleation at the sample edges. In this Appendix, we briefly outline the boundary conditions used in our model and provide a discussion of the associated edge effects under light excitation.

We use Neumann boundary conditions for both the order parameter  $\psi$  and scalar potential  $\mu$  [19]:

$$\hat{n} \cdot (\nabla - i\mathbf{A})\psi = 0, \quad (\text{A1})$$

$$\hat{n} \cdot \nabla\mu = 0. \quad (\text{A2})$$

We initialize the system with a uniform order parameter,  $\psi(\mathbf{r}, t = 0) = 1$ .

Under vector potential  $\mathbf{A}$  at the boundary, the effect of polarization becomes evident. For instance of  $x$ -polarized light, as shown in Fig. 7, the Neumann boundary condition along the  $x$  axis ( $\hat{n} = \pm\hat{y}$ ) yields

$$\frac{\partial \psi}{\partial y} = i\psi A_y = 0, \quad (\text{A3})$$

whereas along the  $y$  axis ( $\hat{n} = \pm\hat{x}$ )

$$\frac{\partial \psi}{\partial x} = i\psi A_x = \frac{\psi E_x}{\omega_{\text{EM}}}. \quad (\text{A4})$$

This shows that the order parameter is more strongly influenced along the  $x$  axis due to the nonzero  $A_x$ .

In typical cases of SC barriers involving a constant magnetic field, as modeled by the Bean-Livingston barrier [75] and numerically studied by Hernández and [76], vortices tend to nucleate at the SC-vacuum boundary due to suppression of the order parameter and then migrate inward to reach equilibrium. Similarly to the work of Hernández and [76], vortex nucleation in our system can initiate when the superconducting order parameter  $\psi$  is locally suppressed to zero at the sample boundary, allowing vortices to enter the superconductor. In their study, which considered a constant magnetic field, vortices always nucleated at the SC-vacuum boundary and then propagated inward to reach equilibrium in the bulk of the SC region.

In contrast, our system is driven by a finite-sized time-varying electromagnetic field from light, which induces a spatially nonuniform magnetic field with a well-defined maximum. Specifically, the induced magnetic field reaches its peak at a position  $y = \pm w_0/\sqrt{2}$ . Part of the vortices tend to nucleate near the edges of sample and then propagate toward the region of maximum field, as seen in sample S3 in Fig. 7(a).

This edge effect becomes particularly evident in the cases of samples S5, S7, and S9, where the incident light corresponds to a total flux of approximately  $480 \Phi_0$ . In S5 and S9 (beam periphery), the superconducting state is significantly suppressed, leading to extensive vortex nucleation near the edges. In contrast, in the center region (S7), located at the node between positive and negative  $B_z$ , superconducting states are less suppressed and fewer vortices are generated. This spatial distribution of vortices reveals how boundary-induced suppression and light-carried magnetic field together govern the nucleation of vortices in finite-sized superconductors.

#### APPENDIX B: CRITICAL CURRENT DENSITY $J_c$

In the gTDGL approach, the driven current can be used to unpair Cooper pairs when it is higher than the typical critical current, which is usually a few  $0.1J_0$  [18,20]. We present here a simulation (Fig. 8) that clearly demonstrates the presence of the critical current density of the SC thin film in our simulation.

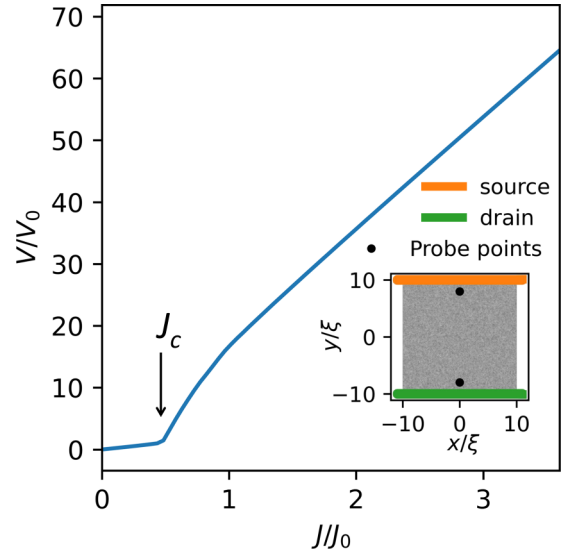


FIG. 8. The  $I$ - $V$  curve of an SC thin film.  $J_c$  is estimated as  $0.48J_0$ .

#### APPENDIX C: RELAXATION TIME OF HEALING PROCESS OF $|\psi|$

In this Appendix, we proceed to investigate the relaxation time of the healing process of  $|\psi|$  affected by the optical pulse. From the results of the typical light-VP interaction in Sec. III B, the intrinsic time duration of generation, recombination, and healing delay the synchronization with the light oscillation. For example, the VP cycle behaves almost “in phase” with amplitude of  $\Phi_B$  when  $\omega_{\text{EM}} = \omega_{\text{GL}}/40$ , but it turns to be “out of phase” when  $\omega_{\text{EM}} = \omega_{\text{GL}}/10$ . The approach of single-cycle optical pulse-induced VP generation enables us to study the equilibration process with minimal intervention and provides insight into the appropriate off-peak interval of the vector potential.

Here, we utilize the second time derivative of the Gaussian distribution for the time-dependent part of  $\Phi_B$ . Relaxation processes are presented in Fig. 9. We consider the relaxation process in two cases:  $\gamma = 10$  and  $\gamma = 0$ , which represent the gTDGL model and the standard TDGL model, respectively. The fitting results of the relaxation time of healing,  $\tau_{\text{healing}}$ , are listed as follows: For the case of  $\gamma = 10$ ,  $\tau_{\text{healing}} = 30.9$  and  $1.6 \tau_{\text{GL}}$  for  $|\psi|$  and  $\theta_s$ , respectively; and for the case of  $\gamma = 0$ ,  $\tau_{\text{healing}} = 5.1$  and  $2.5 \tau_{\text{GL}}$  for  $|\psi|$  and  $\theta_s$ , respectively. Thus, it results in the long healing process of  $|\psi|$  and short of  $\theta_s$ . The relaxation process follows the exponential decay. The decay time of the order parameter is approximately  $u \cdot \tau_{\text{GL}}$  for the TDGL model ( $\gamma = 0$ ) and longer for gTDGL model ( $\gamma \neq 0$ ). The relaxation of  $\theta_s$  shows stronger coherent behavior with light.

#### APPENDIX D: DISCUSSION OF SUB-DIFFRACTION-LIMITED BEAM PROFILE

When investigating optically induced microscale behavior in materials, the mismatch in spatial and temporal scales between light and matter becomes an unavoidable challenge. In recent years, sub-diffraction-limit (sub-DL) imaging techniques have been widely studied. Here, we review several

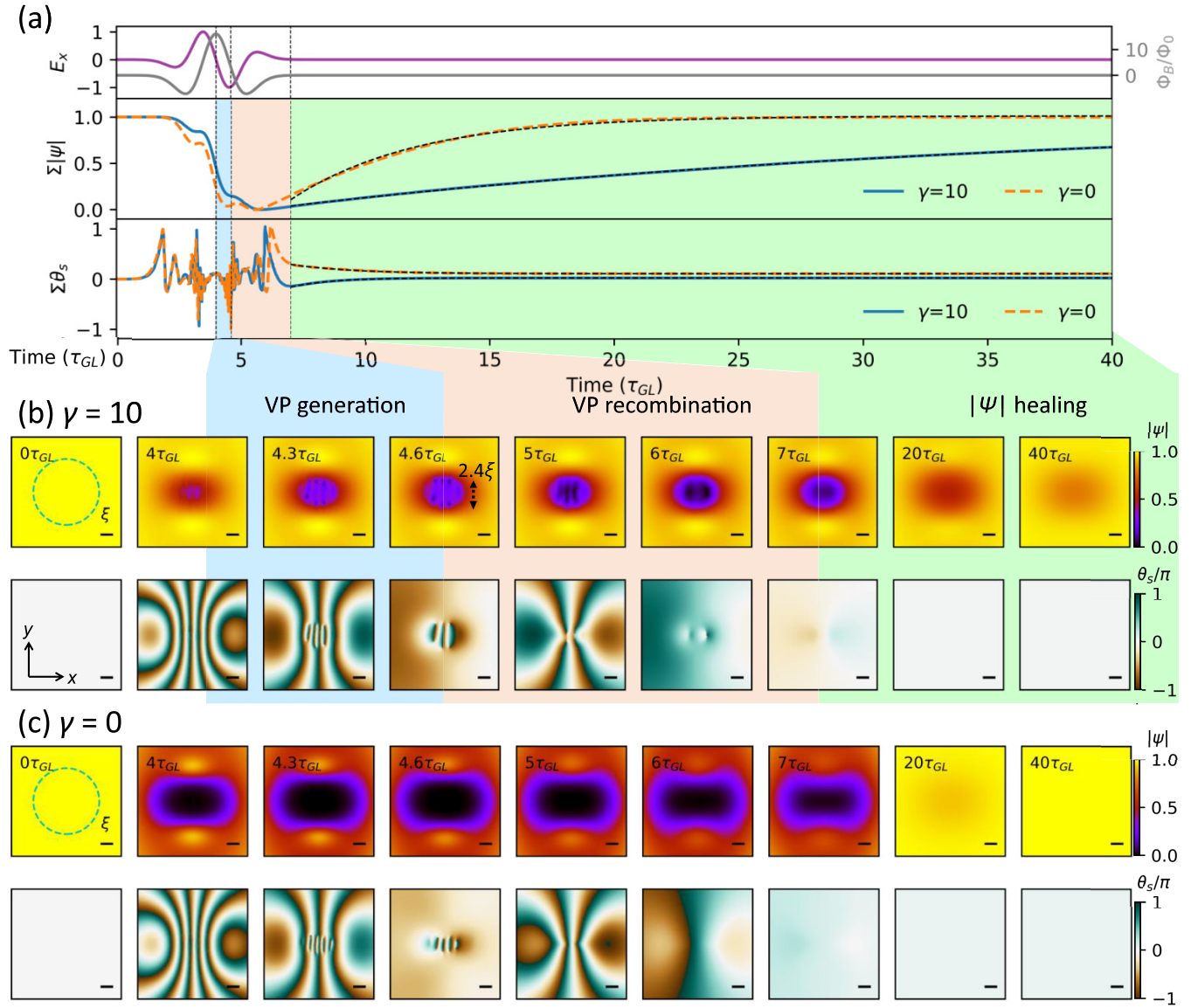


FIG. 9. The time evolution of  $|\psi|$  and  $\theta_s$  with optical pulse. (a) Top subplot shows normalized  $E_x$  and  $\Phi_B$  of the optical pulse. The optical pulse  $E_x$  is generated by the third derivative of the Gaussian distribution, and  $\Phi_B$ , proportional to integral of  $E_x$ , is the second time derivative of the Gaussian distribution. The following two subplots represent total  $|\psi|$  (denoted by  $\Sigma|\psi|$ ) and summation  $\theta_s$  ( $\Sigma\theta_s$ ) across the entire area of sample, respectively. The  $\Sigma|\psi|$  curve is obtained by the min-max normalization, and the  $\Sigma\theta_s$  curve is calculated via the max absolute scaling. The blue solid lines and orange dashed lines represent the SCs with  $\gamma = 10$  and  $\gamma = 0$ , respectively. The time evolution can be categorized into three processes: VP generation from 4.0 to 4.6  $\tau_{GL}$  (blue region), VP recombination from 4.6 to roughly 7  $\tau_{GL}$  until the last VP recombination (pink region), and the healing process of the order parameter after 7  $\tau_{GL}$  (green region). The relaxation time of the healing process is determined by an exponential fit  $c_1 \exp(-t/\tau_{\text{healing}}) + c_2$  after 7  $\tau_{GL}$  with fitting constants  $c_1$  and  $c_2$ , and the fitting curves are shown as the black dashed lines. The selected enlarged snapshots of  $|\psi|$  (upper row) and  $\theta_s$  (lower row) for demonstrating different stages of (b)  $\gamma = 10$  and (c)  $\gamma = 0$ . The green dashed circles and inset axis at the left figures illustrate the beam spot and direction of coordinate.

relevant studies and discuss approximation methods for describing Gaussian beams and sub-DL beam profiles.

There are two conventional methods for measuring beam profiles: the beam profiler and the knife-edge scan. The beam profiler is often limited by the resolution of the charged-coupled device pixels and the bulkiness of the camera system, making it difficult to implement in near-field measurements. In contrast, the knife-edge scan is a more practical approach. It involves moving a sharp edge across the beam

and recording the spatially resolved intensity profile. Similarly, rather than moving the knife, one can move the beam itself. Therefore, scanning a sharp-edged sample using a fixed beam can be considered an equivalent method for reconstructing the beam profile.

We base our consideration of the THz beam profile on SNOM experiments reported by Zhao *et al.* [25], Qui *et al.* [23], and Mastel *et al.* [24], which demonstrate spot sizes of approximately  $\lambda/38$ ,  $\lambda/752$ ,  $< \lambda/1000$ , respectively. These

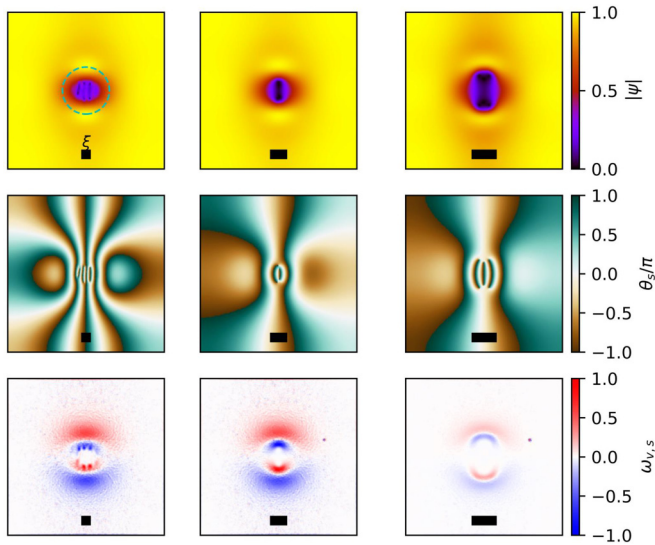


FIG. 10. The simulation for the same spot size with different coherence lengths  $\xi$ . The profiles of  $\psi$ ,  $\theta_s$ , and  $\omega_{v,s}$  are the snapshots at  $2\tau_{GL}$  for the light source  $\omega_{EM} = \omega_{GL}/8$  and  $2w_0 = 0.6\mu\text{m}$ . The values of  $\xi$  are chosen to be 0.1, 0.2, and  $0.3\mu\text{m}$  for the first to third columns, respectively.

experimental profiles show subwavelength beams with intensity distributions that follow either a Gaussian distribution [23] or Lorentzian function [24].

It is well known that the Lorentzian and Gaussian profiles closely overlap in the central region, particularly where the intensity exceeds 40% of the peak value. In our simulations, vortex generation is strongly influenced by this high-intensity central region and is relatively insensitive to the beam profile at the edges. Therefore, the results obtained using sub-DL beams should resemble those using a Gaussian light profile.

#### APPENDIX E: DEPENDENCE OF VORTICES' DISTRIBUTION ON $\xi$

In this Appendix, we demonstrate how the distribution of vorticity depends on the coherence length  $\xi$ . Figure 10 demonstrates the profile of  $|\psi|$ ,  $\theta_s$ , and  $\omega_{v,s}$  for three cases  $\xi = 0.1, 0.2$ , and  $0.3\mu\text{m}$  below the width of spot  $2w_0 = 0.6\mu\text{m}$ . In this simulation, we can observe that the distributions of vorticity are similar.

#### APPENDIX F: QUASIPARTICLE EXCITATION AND DEPAIRING ISSUE OF INTENSE THz SPECTROSCOPY

One of the key issues about optical pump is the generation of photoexcited quasiparticles. This effect can be minimized if the photon energy remains below the superconducting gap  $2\Delta_0$  and outside phonon resonance. For example, the superconducting gaps of Nb, NbN, and YBCO correspond to 0.33, 1.3, and 4.8 THz, respectively. Thus, applying a sub-THz beam could mitigate quasiparticle excitation.

As previously mentioned, THz excitation with photon energies below the SC gap is often employed to minimize

quasiparticle excitation and avoid breaking Cooper pairs. However, intense THz spectroscopy experiments have shown that it is difficult to completely suppress quasiparticle generation. For example, Matsunaga and [77] reported that in NbN, THz pump-probe spectroscopy revealed a high density of excited quasiparticles when the electric field exceeded 50 kV/cm, resulting in gap closure and suppression of superconductivity. Their THz pulse spectrum contained approximately 60%–70% of its intensity below the SC gap, with the remainder above it. Despite the presence of higher-energy components, their optical conductivity data showed that around 50% of the superconducting spectral weight remained at fields below 25 kV/cm field. These results suggest that although quasiparticles are indeed sensitive to intense THz fields, the superconducting state can still persist under well-controlled conditions.

Additionally, quantifying the precise impact of THz-induced depairing remains challenging, as strong electric fields can introduce complex nonlinear effects. Despite the relatively low photon energies of THz beam, intense THz pulses can disrupt superconductivity by breaking Cooper pairs toward the normal state [77–79]. For instance, Glossner *et al.* [79] demonstrated that Cooper pair breakup in high- $T_c$  YBCO occurs under an intense THz field arriving at 30 kV/cm, even when the photon energy ( $< 3\text{ meV}$ ) is much lower than the superconducting gap energy ( $\sim 20\text{ meV}$ ).

Although these studies confirm that intense THz fields can depair superconductivity, they also consistently indicate threshold field strengths on the order of tens of kV/cm. This supports the relevance of our calculated vortex dynamics, which occur within a regime of partial suppression of the SC state, rather than complete destruction, consistent with experimental observations under similar field strengths.

A straightforward approach to mitigate these effects is to lower the driving frequency to the sub-THz range. This not only reduces photon energy but also allows the same amplitude of the vector potential to be maintained with a lower electric field, as described by Eq. (4).

Another strategy is to use a smaller spot size, which is even more effective if it is below the diffraction limit. A smaller spot size increases the gradient of light intensity and the resulting higher temperature gradient, which enhances the local  $B_z$  according to Eq. (6), and also improves the efficiency of thermal dissipation via diffusion. Therefore, this approach not only reduces the required light source intensity but also mitigates thermal issues.

Nowadays, sub-THz-range nanoimaging techniques are already feasible, as demonstrated by the THz-SNOM spectra in Ref. [80] or other techniques mentioned in Sec. IV C, which also include the sub-THz range. Beyond this sub-THz strategy, our estimations suggest that high- $T_c$  YBCO remains a viable candidate, offering a broader range of accessible photon energies.

By comparing the experimental conditions in these prior works with our calculations, we identify a viable parameter space where the predicted vortex dynamics could be observed while minimizing detrimental heating and quasiparticle excitation effects.

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