

Multitarget Rydberg gates via spatial blockade engineering

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Multitarget gates offer the potential to reduce gate depth in syndrome extraction for quantum error correction. Although neutral-atom quantum computers have demonstrated native multiqubit gates, existing approaches that avoid additional control or multiple atomic species have been limited to single-target gates. We propose single-control multitarget $C(Z^{\otimes N})$ gates on a single-species neutral-atom platform that requires no extra control and has gate durations comparable to standard CZ gates. Our approach leverages tailored interatomic distances to create an asymmetric blockade between the control and target atoms. Using a graphics processing unit-accelerated pulse synthesis protocol, we design smooth control pulses for CZZ and CZZZ gates, achieving fidelities of up to 99.55% and 99.24%, respectively, even in the presence of simulated atom-placement errors and Rydberg-state decay. Our approach is most effective for $N = 2$ (CZZ) and $N = 3$ targets (CZZZ); for larger N , increasing spatial crowding of the targets introduces significant challenges for maintaining the required blockade asymmetry. This work presents a practical path to implementing low-overhead multitarget gates in single-species neutral-atom systems, significantly reducing the resource overhead for syndrome extraction. To motivate the impact of these gates, we apply a greedy scheduling algorithm and we demonstrate that our proposed gates can reduce the number of atom reconfiguration costs by up to 50% for color code syndrome extraction of code distances greater than 5.

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I. INTRODUCTION

Quantum computing research has advanced to the point where many error correction experiments have been demonstrated in several platforms [1–13]. However, achieving large-scale fault-tolerant operation remains challenging, with a key obstacle in neutral-atom platforms being the long clock time determined by the measurement of multiqubit observables of the form $\hat{Z}^{\otimes N}$, where $N = 4$ for the surface code and $N = 6$ for the triangular color code. Typically, these measurements are performed by applying a roughly depth- N circuit that maps this observable to the measurement of $\hat{Z}$ on an ancilla. An alternative approach would be the application of an N -qubit single-control multitarget gate of the form $(C(Z^{\otimes N}))$ [14], which could not only reduce the clock time of the logical processor but also reduce errors such as atom heating induced by moving atoms in-between gates. In neutral atoms, the Rydberg blockade [15] enables multiqubit gates, but

most naturally, a permutationally symmetric $(C^{\otimes N})Z$ gate is realized [12,16]. Proposals for multitarget gates, which single out the control atom from the targets, so far involve additional experimental controls such as local microwave control or additional lasers compared to the conventional single-species, single-Rydberg-state blockade mechanism used for standard CZ gates [17–21].

In this work, we show that an asymmetric blockade for realizing CZZ and CZZZ gates can be achieved through purely geometric positioning on single-species neutral-atom hardware, hence removing the requirements of additional laser or microwave fields. We propose an arrangement where one control atom strongly blockades multiple target atoms, while target-target blockade is kept negligible, by placing the control at the center and targets on a perimeter (cf. Fig. 1), and then numerically engineering a pulse sequence. This approach aims to retain the operational simplicity of using a single global drive pulse, similar to standard CZ operations [12].

Specifically, we numerically determine the pulse sequence required to implement CZZ and CZZZ gates with high fidelity on timescales comparable to today's two-qubit gates. We examine the robustness of this geometric approach, accounting for realistic noise channels including Rydberg-state decay and atom-position fluctuations. We show that CZZ and CZZZ multiqubit gates achieve the optimal depth for readout of the

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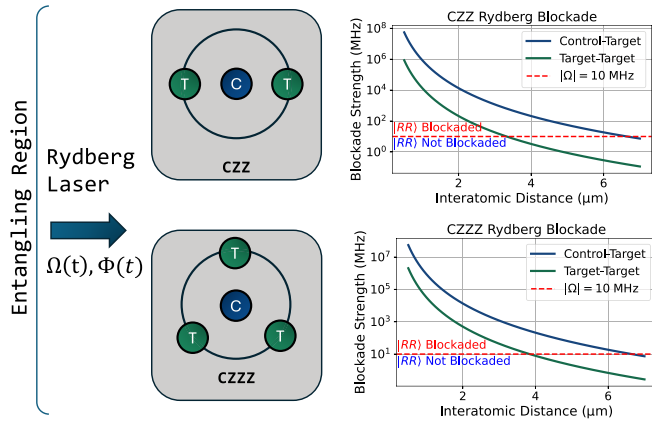


FIG. 1. Geometric arrangements and asymmetric blockade engineering for multiqubit gates. (Top row) A symmetric CZZ gate relies on placing all atoms close together for mutual Rydberg blockade [12]. For the asymmetric CZZ ($N = 2$ targets) and CZZZ ($N = 3$ targets) gates proposed here, the single control atom (C) is placed at the center, while the N target atoms (T) are distributed evenly on the perimeter of a 2D circle around the control. (Bottom row plots) Calculated blockade strength vs interatomic distance for control-target (C-T) and target-target (T-T) pairs in the CZZ and CZZZ configurations. This geometry ensures shorter C-T distances (strong blockade) compared to T-T distances (weak blockade). Blockade strengths below the drive Rabi frequency $|\Omega|$ (dashed line, e.g., 10 MHz) are considered negligible. Asymmetric blockade, required for $C(Z^{\otimes N})$ gates, emerges when the T-T is below $|\Omega|$, while the C-T strength remains well above it.

surface code and color code stabilizers, bounded by the weight of a code's data qubits. Hence, we expect our work to enable a depth-optimal readout of stabilizers in topological codes in neutral atoms.

II. BACKGROUND

A. Neutral-atom quantum computing

Neutral-atom systems employ laser cooling and optical trapping techniques to confine individual atoms in highly ordered arrays. The qubit is typically encoded in long-lived hyperfine or electronic ground state, with single-qubit gates implemented via direct laser pulses on individual atoms. Multi-qubit gates are realized by coupling qubits to a highly excited Rydberg state using a global laser drive on the $|1\rangle \leftrightarrow |r\rangle$ transition [12]. By temporarily exciting one or more qubits into a Rydberg state, strong atom-atom interactions emerge, creating a “blockade” region that is energetically blocked from multiple simultaneous Rydberg excitations within a certain volume. This effect underpins multiqubit entangling operations between atoms that lie within the blockade radius [22–24].

One of the appealing features of neutral-atom quantum computing is the ability to rearrange and scale qubit arrays with relative ease by shifting optical traps. Large qubit registers with flexible geometries have been demonstrated [25]. Single-qubit gate fidelities are extremely high, with reported error rates on the order of $10^{-5}\%$ to $10^{-3}\%$, whereas current two-qubit gate implementations show error rates around 0.5%.

Measurement operations have also been demonstrated with typical error rates on the order of 0.5% [12].

Neutral-atom platforms have demonstrated remarkable promise, with clear pathways to scaling out to larger systems [8], road maps toward fault tolerance [26], and experiments even demonstrating logical computation [5] and magic state distillation [7].

B. Rydberg blockade mechanism

The Rydberg blockade is central to neutral-atom quantum computing, enabling multiqubit entanglement in a single step via strong dipole-dipole interactions. When an atom is excited to a high-lying Rydberg state $|r\rangle$, nearby atoms experience a large energy shift if they also attempt to occupy $|r\rangle$. Provided the interaction $|V_{ij}|$ far exceeds the Rabi frequency Ω , only one atom in the blockade region can be excited at a time [23,27]. The blockade strength typically decays as d^{-6} with interatomic separation d , making atom placement and laser parameters crucial for maintaining high-fidelity operations.

Considering an array of atoms, driven by an on-resonant laser to excite the atoms from the $|1\rangle$ state to the $|r\rangle$ state, the atom Hamiltonian in the rotating-wave frame can be written as [16]

$$H(t) = \sum_{j=1}^n \frac{\Omega_j(t)}{2} |r\rangle_j \langle 1|_j + \text{H.c.} + \sum_{j < k} V_{jk} |r\rangle_j \langle r|_k \langle r|_j \langle r|_k, \quad (1)$$

where V_{jk} is the dipole-dipole interaction energy between atoms j and k when both are in $|r\rangle$ [16]. The atoms $|1\rangle_j$ and $|r\rangle_j$ are coupled by the laser with time-dependent Rabi frequency $\Omega_j(t) = |\Omega_j(t)|e^{i\phi(t)}$. $\Omega(t)$ is taken to be complex, with $|\Omega_j(t)|$ encoding amplitude and $e^{i\phi(t)}$ encoding phase. We absorbed the detuning of the Rydberg laser into the time dependence of the phase. $V_{i,j}$ represents the interatomic blockade strength and is highly dependent on interatomic distances. If $|V_{ij}| \gg \Omega$ for all pairs within the blockade radius, population of doubly excited Rydberg states is energetically suppressed, ensuring that at most one $|r\rangle$ excitation occurs (“blockaded”). Atoms located outside the blockade radius experience negligible mutual interaction and can be excited simultaneously if desired [16].

In the simplest two-qubit scenario, this blockade-induced nonlinearity directly yields a *controlled-Z* (CZ) gate. With atoms i and j in the strong-blockade regime ($|V_{ij}| \gg \Omega$), the doubly excited state $|r\rangle_i |r\rangle_j$ is energetically inaccessible. Second-order perturbation theory, treating $|r\rangle_i |r\rangle_j$ as off-resonant, shows that in the subspace $\{|10\rangle, |01\rangle, |11\rangle\}$, an effective interaction $\frac{\Omega^2}{4V_{ij}} |11\rangle \langle 11|$ appears. Driving the $|1\rangle \leftrightarrow |r\rangle$ transition for a time t satisfying $\frac{\Omega^2}{4V_{ij}} t = \pi$ applies a π phase only to the $|11\rangle$ component, implementing a CZ gate (up to single-qubit phases) [12,16]. In many systems, the pulse is applied globally to all atoms within an entangling region, offering a path toward scalable, parallel entangling operations [12]. Achieving high gate fidelity requires fine-tuned pulse shaping and precise atom placement to maintain $|V_{ij}| \gg \Omega$ where needed, while also minimizing spurious interactions between distant qubits.

While asymmetric blockade has been explored in previous proposals [17–21], those approaches typically rely on additional hardware overhead or operate on longer timescales. For example, Young *et al.* [21] utilize additional strong microwave fields to dress Rydberg states, predicting the generation of a 25-atom GHZ state with an error of 7.8% including Rydberg decay errors, but no atom-motion or misplacement errors. Su [18] employs multiple Rydberg levels to engineer asymmetric interactions, reporting fidelities above 99% for two-qubit gates that decrease to approximately 94% for four-qubit configurations, although notably the operation time remains invariant with qubit number. The electromagnetically induced transparency (EIT)-based scheme of Müller *et al.* [20] enables parallelized entanglement of mesoscopic ensembles on microsecond timescales but requires electromagnetically induced transparency configurations. Dual-species arrays [28,29] offer another pathway to asymmetric interactions through interspecies Förster resonances. In contrast, our approach relies purely on geometric spatial engineering ($1/R^6$ scaling) using single-species atoms and a single global laser pulse, achieving gate times of 250–600 ns comparable to standard two-qubit operations. This means our approach can be directly implemented in current hardware such as that in Ref. [12] realizing standard CZ gates while avoiding the need for microwave dressing, multiple atomic species, or additional Rydberg levels.

C. Atom rearrangement

Neutral-atom quantum processors offer a unique capability: They can physically rearrange qubits (i.e., individual atoms) during a computation to achieve flexible connectivity. However, this reconfigurability comes at a significant cost. Rearrangement significantly increases both the overall computation time and the likelihood of atom loss. Moving a single atom via optical tweezers typically requires between hundreds to tens of thousands of microseconds [30,31], while a two-qubit entangling CZ gate can be executed on the order of a few hundred nanoseconds [12]. This disparity spans several orders of magnitude in time.

Frequent shuttling of atoms not only adds significant latency to the overall circuit run-time but also introduces additional error channels. Atom loss and decoherence can worsen significantly, given the finite lifetime of optical traps. Although neutral atoms have exceedingly long lifespans when compared to gates, even these long lifetimes can be compromised by the durations required for reconfiguration [31].

Because rearrangement is so time-consuming and error-prone [31], high-depth routines such as repeated syndrome extraction become prohibitively expensive. Thus, reducing rearrangement overhead while preserving high-fidelity gates remains crucial.

D. Impact of $C(Z^{\otimes N})$ gates on error correction

A primary motivation for developing multitarget gates is their potential to streamline quantum error correction (QEC) protocols. A key step in QEC is syndrome extraction, which involves measuring multiqubit stabilizer operators to detect errors [14,32]. Many important QEC codes, such as surface codes [33] or color codes [34], utilize stabilizers involving

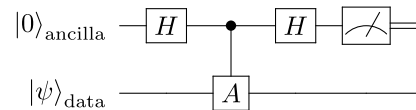
interactions between multiple data qubits (e.g., weight 4 or higher). Color codes are especially prominent in the neutral-atom field, with breakthroughs in neutral atoms utilizing the color code [5,7].

Standard approaches typically decompose the measurement of a weight- k stabilizer into a sequence of k individual two-qubit entangling gates (like CZ) involving an ancilla qubit [32]. This inherently requires at least k entangling layers in the circuit. Executing these layers sequentially can contribute significantly to the circuit depth and may necessitate substantial atom rearrangement in architectures like neutral atoms, increasing execution time and error susceptibility [9,30].

Multitarget gates, such as the $C(Z^{\otimes N})$ gates developed here, offer an alternative paradigm. For a stabilizer measurement involving k data qubits, a $C(Z^{\otimes k-1})$ or similar multitarget gate could potentially implement the required collective phase logic in fewer entangling layers. This reduction in entangling layers could significantly decrease circuit depth and potentially alleviate demands on atom rearrangement.

E. Stabilizer measurements

To determine whether a quantum state $|\psi\rangle$ is in the $+1$ eigenspace of a stabilizer operator A , one can use the measurement circuit shown below.



An ancilla is prepared into the state $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ using a Hadamard gate. The subsequent controlled- A operation entangles the ancilla and data qubits. A final Hadamard on the ancilla maps the resulting state to

$$\frac{1}{2}[|0\rangle \otimes (|\psi\rangle + A|\psi\rangle) + |1\rangle \otimes (|\psi\rangle - A|\psi\rangle)]. \quad (2)$$

Measuring the ancilla projects the data qubits into one of the two eigenspaces of A . An outcome of $m = 0$ corresponds to the $+1$ eigenspace, signaling that the stabilizer check has passed. Conversely, an outcome of $m = 1$ corresponds to the -1 eigenspace, indicating an error.

The motivation for developing higher-weight $CZ^{\otimes N}$ operators is that the practical complexity of this circuit is dominated by the controlled- A gate. For the multiqubit stabilizers, the fundamental drivers of error correction, this single operation must be decomposed into the underlying supported gate set, often a two-qubit entangling gate, resulting in significant atom reconfiguration demands.

III. METHODS: PULSE OPTIMIZATION OF ASYMMETRIC RYDBERG GATES

We target a single global pulse on single-species hardware similar to that in Ref. [25], implementing $C(Z^{\otimes N})$ gates using similar settings to those used for global CZ gates.

A. $C(Z^{\otimes N})$ gates

Many previous neutral-atom gate designs focus on $C^{\otimes N}Z$ (multicontrol Z) operations [12], whereby a -1 phase is applied to homogeneous $|1 \dots 1\rangle$ states. While such gates are

often useful, many multiqubit operations instead require a single control qubit that applies phase shifts to multiple targets. An illustrative example is the two-target case, $C(Z^{\otimes 2})$, which assigns distinct phases to basis states like $|101\rangle$ compared to $|011\rangle$. In contrast, a CCZ gate (i.e. two-control, one-target) acts nontrivially only on $|111\rangle$ and thus cannot implement these different phase evolutions. However, realizing $C(Z^{\otimes N})$ gates on neutral atoms is nontrivial due to the nature of the Rydberg blockade. If all atoms lie within a single blockade radius and one single laser drive is used, any multi- $|1\rangle$ configuration couples *symmetrically* to the Rydberg state, preventing distinct phases from being applied to different patterns of excitation. Consequently, a simple all-in-one-blockade arrangement cannot yield $C(Z^{\otimes N})$. We propose to break this symmetric coupling by spatially arranging qubits such that the control-target and target-target interactions differ. In particular, we place the control qubit at the center of a circle, with each target positioned sufficiently far apart on the perimeter so that

(1) *Control-target blockade*. The distance d_{0j} between the control and each target is small enough to ensure a large van der Waals interaction $V_{0j} \gg \Omega$, establishing a strong blockade for each control-target pair.

(2) *Weak target-target coupling*. Any two targets j, k are separated sufficiently so that V_{jk} is negligible, allowing them to be simultaneously excited without undesired cross-blockade.

This arrangement suppresses target-target blockade while ensuring the control-target blockade remains strong, thus enabling distinct phase accumulations required by $C(Z^{\otimes N})$.

B. Atom placement for $C(Z^{\otimes N})$ gates

A key step in implementing multitarget gates such as $C(Z^{\otimes N})$ is choosing a geometric layout of the atoms that ensures strong blockade between the control qubit and each target, but sufficiently negligible blockade among the targets themselves. One effective approach is to place the control atom at the center of a circle and distribute the N target atoms uniformly around the perimeter. Thus,

(1) the control atom is at the origin, providing shortest distances (and therefore strong dipole-dipole interactions) to all targets;

(2) the targets are as far apart from one another as possible, reducing intertarget interactions to an insignificant level.

The control atom is located at the origin, and we place each target atom j ($j = 1, \dots, N$) at

$$\mathbf{x}_j = (R \cos \theta_j, R \sin \theta_j, 0), \quad \theta_j = \frac{2\pi}{N} j, \quad (3)$$

for some chosen radius R . The *control-target* blockade effect is then

$$V_{0j} = \frac{C_6}{\|\mathbf{x}_0 - \mathbf{x}_j\|^6} = \frac{C_6}{R^6}, \quad (4)$$

where $\mathbf{x}_0 = (0, 0, 0)$ denotes the control atom's location. For an appropriately chosen R , we have $V_{0j} \gg \Omega$, ensuring strong blockade between the control qubit and any target, and $V_{jk} \approx 0$, ensuring the targets do not blockade each other. We still consider the three-dimensional (3D) arrange-

ment, because noise randomly perturbs along the z axis as well. Formally, any two targets $j \neq k$ on the circle are separated by

$$d_{jk} = \|\mathbf{x}_j - \mathbf{x}_k\| = 2R \sin\left(\frac{\pi}{N}\right). \quad (5)$$

To keep the interaction $V_{jk} = C_6/d_{jk}^6$ negligible compared to the Rabi frequency Ω , one must choose R large enough so that

$$\frac{C_6}{(2R \sin(\frac{\pi}{N}))^6} \ll \Omega. \quad (6)$$

Hence, pushing the targets radially outward can break the target-target symmetry while preserving sufficiently strong control-target blockade and keeping target-target interactions low. However, as N increases, maintaining these conditions grows more challenging, as more target atoms are placed on the perimeter. For instance, a $C(Z^{\otimes 4})$ gate exhibits the limit of a reduced target-target blockade of $\frac{C_6}{8R^6}$ against the target-control blockade $\frac{C_6}{R^6}$, making the system increasingly sensitive to atom motion and demanding more complex pulse shaping. At small N , such as a $C(Z^{\otimes 2})$ (two targets), choosing R is much less sensitive, as the relative target-target blockade is $\frac{C_6}{64R^6}$. As N grows and R must be increased, $|V_{0j}| = C_6/R^6$ diminishes, and the system breaks down once target-target distances become comparable to or shorter than control-target distances.

In simulation, we find that beyond $C(Z^{\otimes 2})$ or $C(Z^{\otimes 3})$, the added complexity and diminishing control-target blockade strength for larger N in this specific circular geometry make achieving high fidelity under realistic noise conditions increasingly difficult with a single global pulse. This geometric constraint inherently limits the direct applicability of this simple circular arrangement to $N = 2$ and $N = 3$ target gates in practice for current experimental parameters and desired fidelities. This “spatial crowding” effect severely constrains the parameter space, making it difficult for our optimization to find a high-fidelity global pulse that satisfies the $V_{CT} \gg \Omega \gg V_{TT}$ hierarchy. With lower variance in atom positioning, $C(Z^{\otimes 4})$ and $C(Z^{\otimes 5})$ may be possible, as they maintain target-target distances less than control-target distances. Higher-weight operators are not possible in our proposed setup due to target-target distances being less than or equal to the target-control distance.

C. Modeling noise channels and atomic perturbations in neutral-atom hardware

High-fidelity multiqubit gates on neutral-atom hardware must account for a variety of imperfections, including spontaneous decay from the Rydberg state, realistic control dynamics, and small perturbations of atoms from their ideal positions in 3D space. In our approach, we incorporate all of these noise sources into a numerical pulse-optimization routine that aims to generate pulses robust against hardware-induced deviations and is well suited to near-term control hardware.

We model two dominant categories of imperfections: (1) *Rydberg decay* via local amplitude-damping-type Kraus maps and (2) small random displacements in the atomic

positions (in all three spatial dimensions) that modify the interatomic interaction strength.

Decay/amplitude damping. With entangling gates being on the order of hundreds of nanoseconds, and logical states $|0\rangle, |1\rangle$ having second-long lifespans, thermal decay of these states is near-zero. However, the Rydberg state has a finite lifespan and is a dominant source of error in entangling gates [12].

To model this noise channel, at each short time step Δt , after evolving under the system Hamiltonian, we apply a local Kraus channel that transfers population from the excited Rydberg state $|r\rangle$ to the ground states $|0\rangle$ and $|1\rangle$, and leakage out of the computational subspace to $|e\rangle$. We do not track amplitudes of $|e\rangle$ and model this as amplitude damping. Concretely, for each qubit i , we define

$$p = \gamma \Delta t, \quad (7)$$

where γ is the decay rate and p is partitioned into three decay probabilities p_0, p_1 , and p_2 for the channels $|r\rangle \rightarrow |0\rangle$, $|r\rangle \rightarrow |1\rangle$, and $|r\rangle \rightarrow |e\rangle$, respectively (with $p_0 + p_1 + p_2 = p$). The associated Kraus operators acting on the three-level system $\{|0\rangle, |1\rangle, |r\rangle\}$ are given by

$$\begin{aligned} E_0 &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \sqrt{1-p} \end{pmatrix}, \\ E_1 &= \begin{pmatrix} 0 & 0 & \sqrt{p_0} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \\ E_2 &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \sqrt{p_1} \\ 0 & 0 & 0 \end{pmatrix}. \end{aligned} \quad (8)$$

Modeling the $|r\rangle \rightarrow |e\rangle$ transition comprises omitting the p_2 Kraus channel and represents the amplitude lost from $|r\rangle$ to $|e\rangle$. This amplitude is permanently lost. In this way, population decays from the Rydberg state $|r\rangle$ into both the computational basis states and a leakage state, thus approximating a continuous amplitude-damping process.

Atomic position perturbations. The Rydberg blockade is highly sensitive to interatomic spacing, and small positional errors along all three spatial directions can substantially degrade gate fidelity. Atoms exhibit systematic trap errors, resulting in deviations in atom positioning (i.e., atom-placement errors) that arise from imperfections in the trapping potential. In addition to these systematic errors, inherent atomic motion arises both from photon recoil during Rydberg excitation and decay and from the intrinsic thermal and quantum zero-point fluctuations present even in an ideal harmonic trap [25,35].

To account for all these contributions in our optimization procedure, at each optimizer iteration we draw random displacements

$$\delta_i = (\delta x_i, \delta y_i, \delta z_i) \quad (9)$$

for each atom i . These displacements update the nominal coordinates as

$$\mathbf{r}_i \rightarrow \mathbf{r}_i + \delta_i, \quad (10)$$

thereby modifying the pairwise blockade interaction

$$V_{ij} = \frac{C_6}{\|\mathbf{x}_i + \delta_i - (\mathbf{x}_j + \delta_j)\|^6}. \quad (11)$$

The displacements δx_i , δy_i , and δz_i are sampled from a Gaussian distribution with widths σ_x , σ_y , and σ_z , allowing the optimizer to learn a more robust gate solution that tolerates typical three-dimensional positional variations. We optimize to maximize the average fidelity across these samples:

$$\bar{\mathcal{L}} = \frac{1}{N_{\text{shots}}} \sum_{m=1}^{N_{\text{shots}}} [1 - \mathcal{F}_m], \quad (12)$$

where $\mathcal{F}_m$ is the fidelity for the m th displacement sample. This batched approach yields control pulses that maintain high fidelity over typical atom-motion distributions.

D. Gate pulse synthesis

1. Base pulse synthesis

We target the generation of time-dependent Rabi pulses $\Omega(t)$ with a phase profile $\Phi(t)$ that maximizes a gate fidelity metric

$$\mathcal{F}(U_{\text{target}}, \mathcal{E}_{\text{Real}}(\Omega, \Phi)), \quad (13)$$

where U_{target} is our target gate and $\mathcal{E}_{\text{Real}}$ is the learned evolution over input basis states $\{|0\rangle, |1\rangle, |+\rangle, |i\rangle\}^{\otimes N}$.

Due to the complex, nonconvex nature of the control landscape, the quality of converged solutions can vary significantly depending on the initial guess for the pulse parameters. A single optimization run may get trapped in a local minimum, yielding a low-fidelity gate. To overcome this, we developed a graphics processing unit (GPU)-accelerated pulse-optimization pipeline. This approach allows us to instantiate and optimize hundreds of candidate pulses in parallel, each beginning from a different random initialization. By broadly sampling the solution space, we can select the highest-performing candidates for further refinement. This parallel search strategy enables the discovery of the high-fidelity solutions presented.

We discretize the total evolution time T into n small segments, each of duration $\Delta t = T/n$. In segment k , the Hamiltonian is parametrized by Ω_k and Φ_k . We then perform a Trotterized evolution for each segment:

$$U_k = \exp(-i \Delta t H_k(\Omega_k, \Phi_k)), \quad (14)$$

where

$$H_k = \frac{\Omega_k}{2} (e^{i\phi_k} D_+ + e^{-i\phi_k} D_-) + \sum_{i < j} V_{ij} n_i n_j \quad (15)$$

and $D_{\pm}$ are the collective raising/lowering operators $|1\rangle \leftrightarrow |r\rangle$, while n_i projects onto the Rydberg state of atom i . After each segment, we apply the Kraus operators $\{E_0, E_1, E_2\}$ to model amplitude damping. For a given pulse schedule $\{\Omega_k, \phi_k\}$, the final operation is

$$U_{\text{real}} = \left(\prod_{k=1}^n K(\Delta t) U_k \right), \quad (16)$$

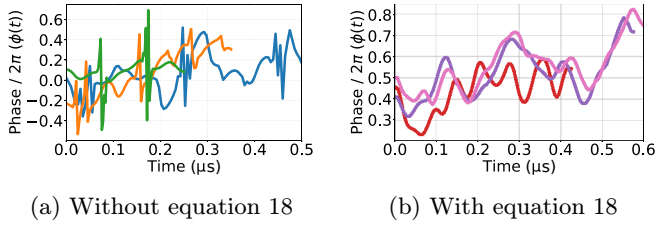


FIG. 2. Two sets of three samples of pulses from training. Each line is an individual high-fidelity pulse for a $C(Z^{\otimes 2})$ gate and has no relation to the other. Panel (a) has no smoothing penalty and panel (b) has the smoothing penalty included. Panel (a) shows examples without the smoothing term. In contrast, panel (b) shows that the loss function enforces smooth pulses by penalizing large phase jumps, thereby discouraging unrealistic trajectories.

with $K(\Delta t)$ denoting the combined effect of Kraus operations in each substep.

Due to its success in related quantum-control tasks [36,37], we use the Adam optimizer [38]. The base loss function is set to maximize fidelity, i.e., $1 - \mathcal{F}(U_{\text{target}}, \mathcal{E}_{\text{Real}})$.

2. Ensuring realistic pulse shapes

High-fidelity solutions found via unconstrained optimization can exhibit rapid, large-amplitude variations in their pulse profiles, making them difficult or impossible to implement on real hardware. To address this, we include a smoothness penalty that discourages abrupt changes in amplitude or phase between consecutive time segments. Specifically, we define

$$\gamma_i = |\Phi_{i+1} - \Phi_i|, \quad i = 1, \dots, n-1, \quad (17)$$

where Φ_k represents either the phase or amplitude at time slice k . We then introduce the cost function

$$C_{\text{smooth}} = \sum_{i=1}^{n-1} f(\gamma_i), \quad \text{where} \quad f(d_i) = \begin{cases} \lambda_s \gamma_i^2, & \text{if } \gamma_i \leq \tau, \\ \lambda_s \tau^2 + \lambda_b (e^{B(\gamma_i - \tau)} - 1), & \text{if } \gamma_i > \tau, \end{cases} \quad (18)$$

which penalizes small parameter jumps quadratically and large jumps exponentially. During the early stages of training, we keep λ_s high initially and reduce to 0 over the first half of training. We still penalize large discontinuities via λ_b .

Figure 2 illustrates two sets of converged pulses with and without this smoothing term. The penalty effectively encourages the optimizer to yield solutions that are both high fidelity and smooth, thereby being implementable on real hardware. By balancing hardware feasibility with pulse-design freedom, our smoothness cost ensures that the final schedules achieve strong performance without relying on nonphysical control trajectories.

The hyperparameters $\lambda_s, \lambda_b, B, \tau$ are chosen heuristically to penalize unrealistic changes in phase trajectories. The exponential term acts as a soft regularizer against large interstep phase changes, which would correspond to abrupt updates that are typically more challenging to realize on arbitrary waveform generators (AWGs). The parameters λ_s and τ play the role of gradient regularizers, where τ sets the soft cap

of per-time-step phase increment that is acceptable (with larger increments increasingly penalized), while λ_s controls the strength of this smoothness preference. To prevent unrealistic trajectories, λ_s and τ act as gradient regularizers. For our work, we set $\tau = 0.25$ as a heuristic threshold on the allowable per-time-step phase increment, which yields phase smoothness on a comparable scale to experimentally implemented smooth-amplitude gate of Evered *et al.* [12]. More restrictive control hardware motivates stronger regularization, whereas more capable waveform generation allows for less. In real experiments, the hyperparameters can be further fine-tuned to match the measured hardware transfer function and operational stability limits.

IV. RESULTS: $C(Z^{\otimes N})$ GATES

A. Pulse generation for $C(Z^{\otimes N})$: speed limit under ideal conditions

We begin by benchmarking and validating that our proposed gate sets are numerically viable and the potential atom coordinates that enable them. We characterize the $C(Z^{\otimes N})$ gate set in an ideal scenario, with no dynamic atom motion, control constraints, or thermal decay. We establish a baseline for these gates being possible, and how quickly these gates can be performed while still achieving high fidelity.

We begin by setting a fixed Rabi frequency, $\Omega_{\text{max}} = 10 \times 2\pi$ MHz, motivated by the multi-MHz regime of previous experiments [12,39]. Simultaneously, we explore various control-target radii, varying the Rydberg blockade strength both between control-target and target-target. This is captured in the radius parameter. Ideally, control-target is $\gg \Omega$ and target-target is $\ll \Omega$, although this is not physically realistic and trade-offs exist in the radii of the system.

We provide a given total gate duration T and discretize the evolution into $n = 100$ segments, each characterized by a constant drive parameter $\{|\Omega_{\text{Max}}|\}$ and the learnable parameter $\{\Phi\}$.

$$\mathcal{F}(\mathcal{E}_{\text{Real}}, C(Z^{\otimes N})). \quad (19)$$

Each optimization run evaluates the Trotterized evolution and backpropagates the fidelity gradient over the control parameters. By sweeping across different fixed values of T , over 50 random initializations to avoid converging in a local minimum, and various radii, we can identify atom placements that are capable of high-fidelity $C(Z^{\otimes N})$ gate and their minimum durations.

In Fig. 3, we illustrate the final fidelity of a $C(Z^{\otimes 2})$ gate for various interatomic distances R . Each dataset was obtained by optimizing based on the parameters discussed under Eq. (1). The rapid fidelity increase seen for certain intermediate radii between 3.25 and 3.75 μm indicates a performant atom configuration where the control-target blockade is sufficiently large, yet target-target blockade remains sufficiently suppressed. In contrast, excessively small or large radii force the optimizer to either extend the gate duration or accept lower fidelity outcomes. This analysis confirms that even in a noiseless scenario, choosing an appropriate distance R impacts our gates' performance significantly, and more importantly, the $C(Z^{\otimes 2})$ gate is possible at high fidelity.

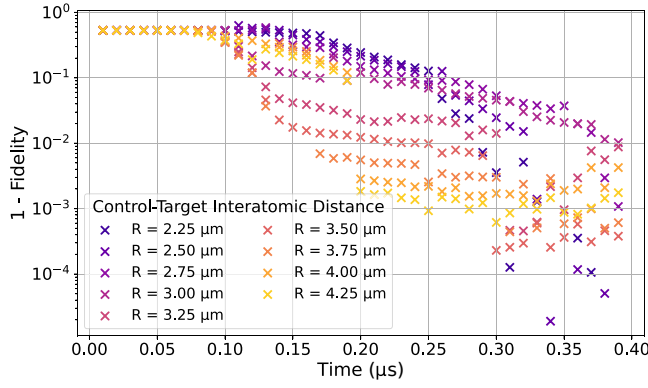


FIG. 3. Achieved fidelities for a $C(Z^{\otimes 2})$ gate under ideal conditions (no atomic motion, no decay), plotted against the total evolution time T . Each color/marker denotes a distinct atom-to-control radius R , ranging from 2.25 to 4.25 μm . For intermediate radii (3.25, 3.50, and 3.75 μm), the gate rapidly attains high fidelity within 0.15 μs . Smaller or larger radii require longer durations to surpass 80% fidelity.

Meanwhile, in Fig. 4 we show our simulations for a $C(Z^{\otimes 3})$ gate under various evolution times and radii. Here, no clear “optimal” radius emerges, and longer gate times are required to attain high-fidelity solutions. This underscores the added geometric and timing constraints of a three-target blockade arrangement, wherein crosstalk between control atoms grows more significant and saturates the achievable fidelity for short pulses. However, we find that high-fidelity solutions at fidelities $>99.9\%$ do exist for $C(Z^{\otimes 3})$ gates, with radii between 3 and 4 μm demonstrating the best fidelities found, aligning well with current neutral-atom hardware atom positioning parameters [12,25].

B. Understanding noise channels

To understand where the dominant sources of noise persist in our ideal pulses, we make use of a Pauli transfer map

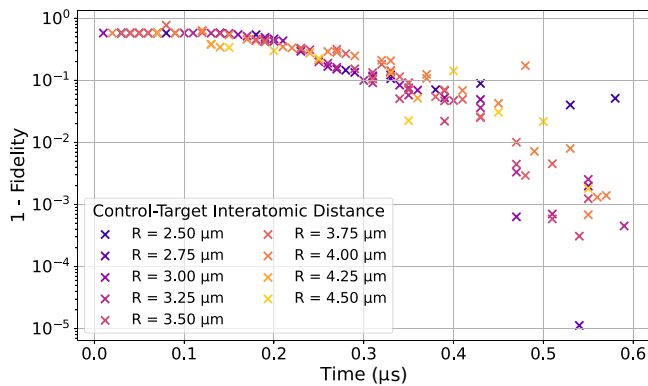


FIG. 4. Achieved fidelities for a $C(Z^{\otimes 3})$ gate under ideal conditions (no atomic motion, no decay), plotted against the total evolution time T . Each color/marker denotes a distinct atom-to-control radius R , ranging from 2.5 to 4.5 μm . No clear good interatomic distance emerges, and slower gate times when compared to Fig. 3 are required to implement the $C(Z^{\otimes 3})$ gate.

(PTM) [41,42]. PTMs map Pauli operators to Pauli operators, allowing us to identify unwanted channels within a gate. We compare our learned unitaries to the ideal comparable $C(Z^{\otimes N})$ gate.

We analyze our pulses with $>99\%$ fidelity from Figs. 3 and 4. We observe that the key error channels are

(1) $X \leftrightarrow Y$ coupling predominantly on the control qubit, with lower-weight coupling on the target qubits, resulting in coherent phase errors on the control.

(2) $Z \leftrightarrow I$ coupling, from residual Rydberg population.

At lower radii (2.0 μm), the dominant noise channels are a coherent $X \leftrightarrow Y$ error on the control qubit, indicating a phase error. Another dominant error source is represented by an error channel between $Z \leftrightarrow I$. These couplings represent amplitude dampening in the form of residual Rydberg population. When we evaluate on Pauli transfer maps, we only consider the states $|0\rangle$ and $|1\rangle$; hence, residual population in the $|r\rangle$ state is equivalent to amplitude decay. At higher radii ($>4.0 \mu\text{m}$), we observe a similar pattern, however, with $I \leftrightarrow Z$ errors being the dominant source of error. We observe a consistent error structure in the Pauli transfer map, with residual $|r\rangle$ population and coherent errors being the dominant persistent errors.

We visualize the average Pauli transfer map for the ideal and learned $C(Z^{\otimes 2})$ gate in the Appendix.

The PTM highlights the two primary physical processes limiting fidelity in this scheme: coherent phase accumulation errors likely arising from imperfect pulse shaping (manifesting as $X \leftrightarrow Y$) and incoherent population loss from residual Rydberg- state population (manifesting as $Z \leftrightarrow I$). Understanding the relative contribution of these distinct physical error mechanisms is imperative for tailoring pulse sequences to mitigate the dominant error source for specific parameter regimes.

C. Realistic pulse generation with noise and atom motion

While the idealized pulse designs of Section IV A provide a lower bound on gate durations and potential fidelities, real hardware faces additional noise mechanisms and control constraints that can significantly degrade performance. In particular, neutral-atom devices exhibit atom-position fluctuations, spontaneous decay out of Rydberg states, and pulse shaping constraints. We now extend our pulse-optimization approach to account for these challenges, using a robust training procedure that simultaneously samples thousands of perturbed Hamiltonians and penalizes large, rapid changes in pulse parameters. This procedure ensures that our multitarget Rydberg gates remain realistic and high fidelity under realistic conditions.

We randomly sample 3D displacements $\delta x, \delta y, \delta z$ from Gaussian distributions for each qubit with parameters described in Table I [25,35]. Decay parameters and all other relevant parameters for our training procedure and model are described in Table I.

Combining the smoothness cost with an average infidelity across all sampled perturbations yields the final training loss:

$$\mathcal{L}_{\text{total}} = \underbrace{[1 - \langle \mathcal{F} \rangle_{\text{batch}}]}_{\text{average infidelity over displacements/decay}} + C_{\text{smooth}}, \quad (20)$$

TABLE I. Final parameters used in pulse training and experiments.

Parameter	Value
Rydberg lifetime	88 μs [12]
Atomic species	Rb
Small jump penalty	$\lambda_s = 0.01 \rightarrow 0$
Large jump penalty	$\lambda_b = 1.0$
Exponent factor	$B = 2.0$
Gate duration range	$T = 250\text{--}600$ ns
Global Rabi drive	$\Omega_{\text{max}} = 2\pi \times 10$ MHz
C_6 Rydberg blockade	$862\,690 \times 2\pi$ MHz μm^6 [25]
Ramp-up time	10 ns
Displacement samples	10 000
Kraus P values	$p_0 = 0.1354$, $p_1 = 0.2504$, $p_2 = 0.6142$ [12]
Atom motion (x, y)	$\sigma_x = \sigma_y = 0.014$ μm [35,40]
Atom motion (z)	$\sigma_z = 0.16$ μm [35,40]

where $\langle \mathcal{F} \rangle_{\text{batch}}$ is the mean fidelity on a random set of perturbed Hamiltonians. We set our finite Rydberg lifetimes to 88 μs [12]. We set a Gaussian ramp-up time to Ω_{Max} of 10 ns, describing the time to ramp up the Rabi drive to Ω_{Max} at the beginning of the pulse, and ramp down at the end of the gate [12]. This is visualized in Fig. 7, and all $|\Omega|$ drives follow this pattern of ramp up-drive-ramp down.

D. Pulse results

Based on the parameters outlined above, we discover realistic, high-fidelity pulses that can implement the desired gate set.

We define per-CZ error rate as the infidelity of a pulse, divided by the number of targets, specifically error per CZ = $\frac{1 - \text{fidelity}}{N}$.

We demonstrate up to 99.55% fidelity $C(Z^{\otimes 2})$ gates, as seen in Fig. 5. For a coarse-grained comparison with standard two-qubit operations, we can define a “per-CZ” error rate by dividing the gate infidelity by the number of targets, yielding a value of 0.225% per target for this gate. The pulse profiles discovered are smooth and performant under realistic atom motion.

As for $C(Z^{\otimes 3})$ gates, we demonstrate up to a 99.24% fidelity pulse in Fig. 6, resulting in an approximated comparative per-CZ error rate of 0.253%. We observe that the pulses learned for $C(Z^{\otimes 3})$ gates are more complex and challenging to

TABLE II. Fidelity, gate time, and control-target radius of various learned pulses. The reported fidelity is the average over random displacements described in Table I.

Gate	Fidelity	Time (ns)	Radius (μm)
$C(Z^{\otimes 2})$	0.9955	575	3.5
$C(Z^{\otimes 2})$	0.9913	400	3.5
$C(Z^{\otimes 2})$	0.9880	375	3.5
$C(Z^{\otimes 2})$	0.9821	325	3.5
$C(Z^{\otimes 3})$	0.9924	575	4.0
$C(Z^{\otimes 3})$	0.9821	525	4.0
$C(Z^{\otimes 3})$	0.9818	575	3.5

implement in real hardware, with the resultant pulses having sharper changes in phase when compared to the $C(Z^{\otimes 2})$ gate.

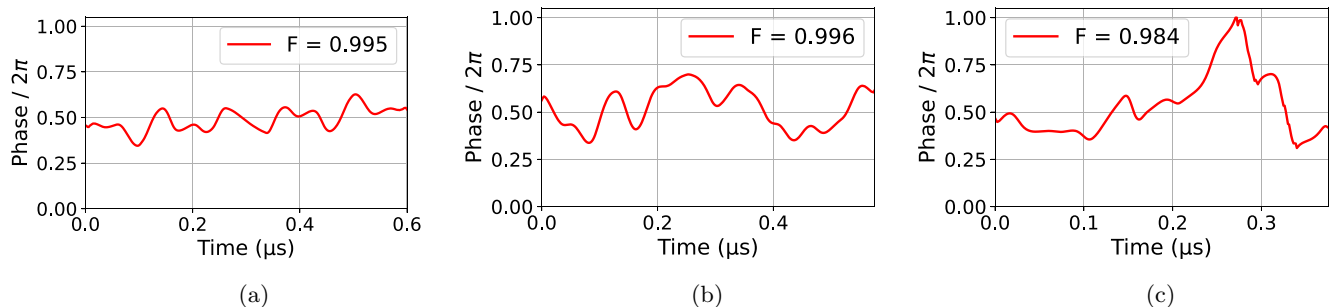
Implementing these optimized pulse shapes, particularly the phase modulations shown in Figs. 5 and 6, requires precise control over the driving laser field. While the smoothness constraint ensures the absence of instantaneous jumps, faithfully implementing over the gate duration ($\sim 250\text{--}600$ ns) necessitates AWGs with sufficient bandwidth and resolution. Evered *et al.* [12] make use of a Spectrum M4i.6631-x8 AWG, which demonstrates that hardware specifications sufficient for implementing our simulated pulses are utilized in state-of-the-art experiments.

We report a set of $C(Z^{\otimes 2})$ and $C(Z^{\otimes 3})$ gate fidelities, times, and interatomic radii in Table II based on parameters in Table I. Corresponding pulses for various pulses are visualized in Figs. 5 and 6, with the Rabi drive amplitude from Fig. 7(b).

These pulses are evaluated with Rydberg decay, atom motion, and target real control constraints. The resultant pulses discovered have timescales of hundreds of nanoseconds, comparable with state-of-the-art gate durations [12]. These pulses not only allow for single effective CZ gate fidelities of as low as 0.20%, but also can enable reductions in circuit depth by expanding the underlying gate set. These pulse fidelities compete with current state-of-the-art fidelities per-CZ of $\approx 0.5\%$ [12]. These pulses align well with syndrome extraction and can potentially be used to reduce syndrome extraction depth over various error correcting codes.

E. Experimental considerations

While our simulations demonstrate the theoretical potential for high-fidelity $C(Z^{\otimes N})$ gates ($N = 2, 3$) using geometric

FIG. 5. Pulses for $C(Z^{\otimes 2})$ with fidelity reported. Each pulse is trained and evaluated with thermal noise and atom motion.

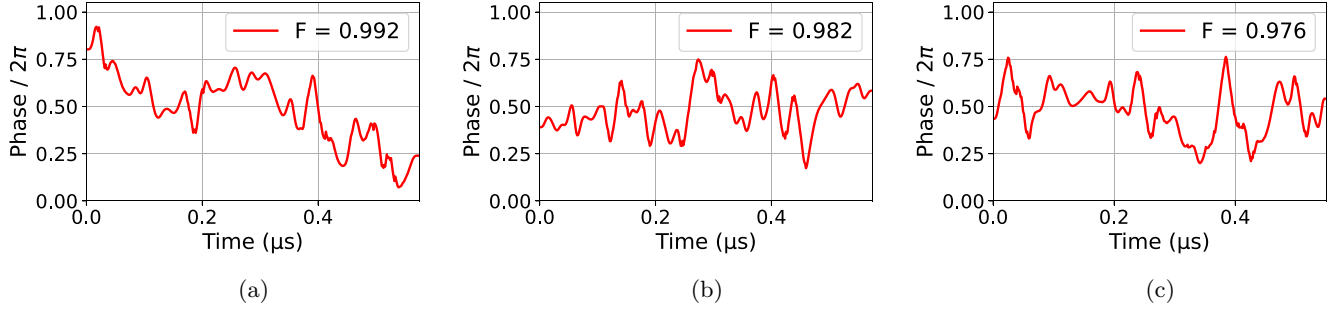


FIG. 6. Pulses for $C(Z^{\otimes 3})$ with fidelity reported. Each pulse is trained and evaluated with noise and atom motion.

blockade engineering under modeled noise, realizing these gates experimentally requires careful consideration of several practical challenges.

Our scheme relies on precise relative positioning between the central control atom and the perimeter target atoms. The simulations incorporated static position errors sampled from Gaussian distributions with standard deviations $\sigma_x \approx \sigma_y \approx 0.014 \mu\text{m}$ and $\sigma_z \approx 0.16 \mu\text{m}$ (Table I), based on Refs. [35,40]. Achieving and maintaining sufficiently consistent precision across multiple atoms over the course of long computations is demanding, requiring highly stable trapping potentials and robust calibration routines to compensate for drifts, thermal effects, or differential light shifts [31,43,44]. Beyond spatial errors, variations in laser intensity and Doppler shifts can induce quasistatic variations in the local Rabi frequency [45,46], reducing fidelity. Jandura *et al.* [45] developed pulses robust to such imperfections while maintaining high logical-qubit performance, and Fromenteil *et al.* [46] introduced resonant pulse sequences achieving error suppression at order ε^4 . Recent benchmarking of high-fidelity Rydberg gates [39] identifies Rydberg decay—which depends on the blockade strength and hence interatomic positioning—and laser frequency noise, rather than intensity inhomogeneity, as dominant error sources.

Figure 8 provides a visual depiction of the scheduling process for a distance-3 color code. The initial six-way symmetry of the stabilizers is broken down into three conflict-free layers when CZZ gates are available, illustrating the packing efficiency achieved by the algorithm. This reduction is a direct consequence of the asymmetric blockade engineering enabling these multitarget operations.

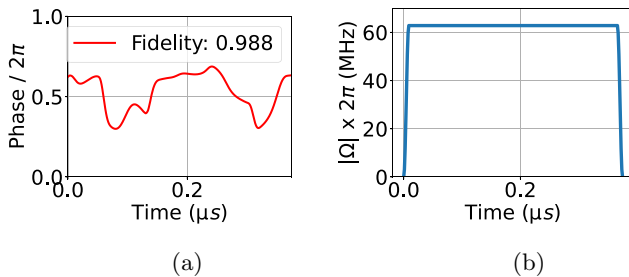


FIG. 7. (a) Phase pulse for $C(Z^{\otimes 2})$ gate at 98.8% fidelity. (b) Omega drive with 10 ns ramp-up, drive at Ω_{Max} , and ramp-down for 10 ns.

Having demonstrated the physical feasibility and high fidelity of our multitarget gates, we now return to our core motivation: quantifying their direct impact on the efficiency of syndrome extraction schedules for topological codes.

V. APPLICATION TO SYNDROME EXTRACTION SCHEDULES

Having demonstrated the physical feasibility of high-fidelity $C(Z^{\otimes N})$ gates, we now analyze their direct impact on reducing the circuit depth of syndrome extraction, a critical bottleneck in QEC. While multitarget gates can implement high-weight stabilizers in fewer operations than a sequence of two-qubit gates, they introduce a nontrivial scheduling problem. Stabilizers must be executed in parallel layers, where no two gates in a single layer share a physical qubit.

The minimum number of layers required is fundamentally limited by the structure of the QEC code itself. Specifically, the lower bound on the schedule depth is the maximum degree of any data qubit in the code's stabilizer graph—that is, the maximum number of stabilizers any single data qubit participates in. A data qubit must be addressed once for each stabilizer it belongs to, and it can only be part of one gate per layer. This bound holds regardless of the power of the available multitarget gate set.

ALGORITHM 1. Syndrome extraction with multitarget gates (generic stabilizer code).

-
- 1: **Input:** A stabilizer code defined by $\{S_1, S_2, \dots\}$, each S_i has an ancilla plus data qubits; limit `max_gate_size`.
 - 2: **Split large stabilizers:** For any S_i with weight $> \text{max_gate_size}$, partition data qubits into subchunks of size $\leq \text{max_gate_size} - 1$.
 - 3: **Collect subchunks:** Each subchunk plus its ancilla forms a multitarget gate (e.g., $C(Z^{\otimes k})$). Gather them into $\mathcal{C}$.
 - 4: **Greedy scheduling with random tie-breaks:**
 1. Sort $\mathcal{C}$ by subchunk size (descending).
 2. **While** $\mathcal{C}$ not empty:
 - new cycle $\text{cycle} = \{\}$.
 - For** $c \in \mathcal{C}$ in sorted order:
 - (a) **If** c has no qubit conflict with cycle , place c in cycle ; else defer.
 - Remove scheduled subchunks from $\mathcal{C}$.
 3. Randomly permute ties; keep best schedule.
 - 5: **Output:** Conflict-free cycles covering every stabilizer subchunk.
-

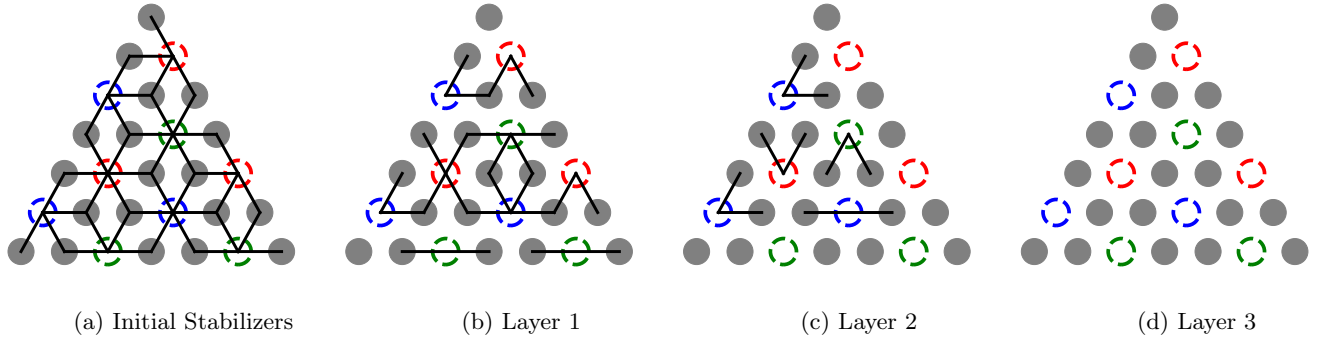


FIG. 8. Colorcode stabilizer scheduling across multiple time cycles. In panel (a), all stabilizers are pending. In panels (b)–(d), each cycle applies $C(Z^{\otimes k})$ gates on subchunks, where $k \leq 2$ for the two-target gates shown. With a max gate size of 3 (allowing CZZ), the total cycles drop from 6 to 3. Ancillas (dashed rings) connect only to data qubits that have not been used in the current cycle.

To solve the scheduling challenge, we employ a heuristic greedy algorithm (Algorithm 1) that packs nonconflicting gates into the minimum number of layers.

To highlight the importance of this data-degree bound, we contrast this result with the surface code. In the standard surface code, each data qubit is involved in exactly four stabilizers (two X type and two Z type), giving it a maximum data qubit degree of 4. As predicted by the theoretical bound, our scheduling algorithm confirms that the syndrome extraction circuit cannot be compressed below four layers, as shown in Fig. 9. This demonstrates that while our multitarget gates are powerful, the ultimate performance gain is codetermined by the inherent structure of the chosen QEC code. Our method is therefore best suited for codes with low data qubit degree.

We applied this scheduling algorithm to the color code, a leading candidate for QEC in neutral-atom systems. A standard implementation using only two-qubit CZ gates requires six distinct layers. As shown in Table III, incorporating CZZ gates (max gate size of 3) immediately reduces the layer count by 50% for many code distances. This highlights a significant reduction in both gate depth and the associated atom rearrangement overhead.

VI. CONCLUSION

We have shown that multitarget, single-control gates $C(Z^{\otimes 2})$ and $C(Z^{\otimes 3})$ can be achieved on a single-species neutral-atom platform in one pulse by deliberately breaking the usual spatial symmetry of the Rydberg blockade. Positioning a control atom at the center and spreading target atoms on a sufficiently large perimeter balances strong control-target coupling against minimal target-target interactions. Our optimized global pulses are robust against Rydberg decay and random atom displacements, achieving gate fidelities up to $\sim 99.55\%$ for $C(Z^{\otimes 2})$ and $\sim 99.24\%$ for $C(Z^{\otimes 3})$. While the specific circular geometry explored here faces inherent challenges in maintaining the required blockade asymmetry for $N > 3$ targets using a single pulse, the demonstrated $C(Z^{\otimes 2})$ and $C(Z^{\otimes 3})$ gates already represent valuable primitives for quantum computing, particularly for QEC.

In the future, it would be interesting whether analytical solutions to our pulse sequences could be found and whether our scheme could be generalized to larger N . In addition, other platforms with Rydberg interactions could make use of our framework, for example, donors in silicon [47,48], trapped ions [49], or even fermionic neutral-atom quantum computing platforms [50,51]. Such pulse optimization could also

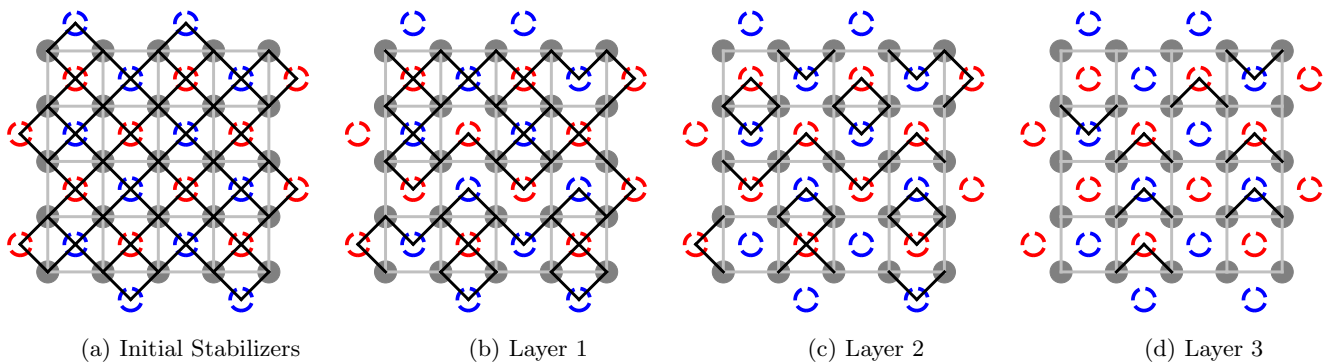


FIG. 9. Surfacecode stabilizer scheduling. Four total cycles remain necessary even if multitarget gates are used, since data qubit participates in up to four stabilizers.

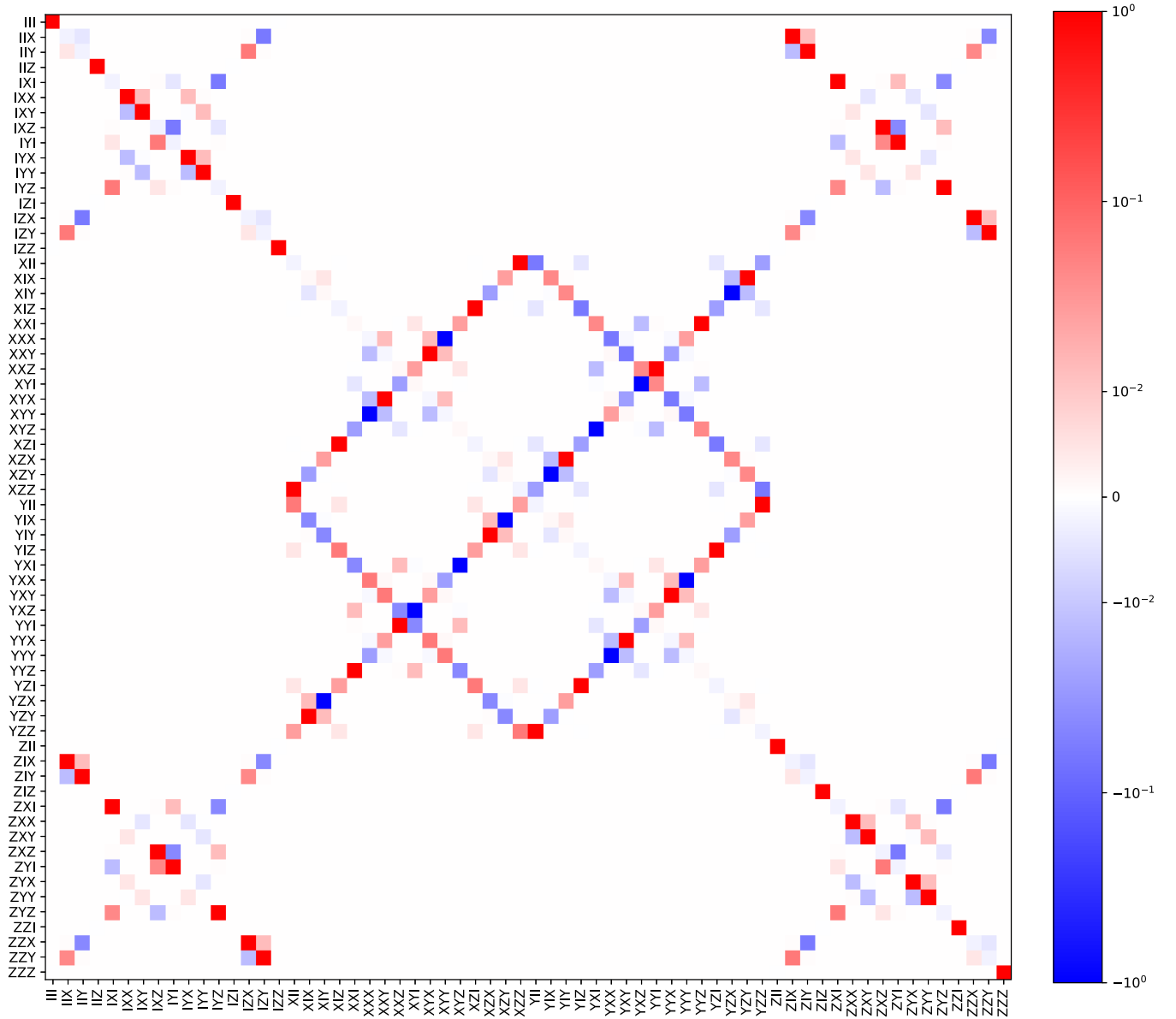


FIG. 10. Logarithmic plot of the average PTM from simulation for the learned $C(Z^{\otimes 2})$ gate. This map visualizes how input Pauli operators (indexed on the Y axis) transform into output Pauli operators (indexed on the X axis) under the learned quantum channel $\mathcal{E}$. The magnitude (color/intensity) of the PTM element $\mathcal{R}_{ij}$ indicates the strength of the $P_i \rightarrow P_j$ mapping. Lightly shaded regions highlight the predominant error channels deviating from the ideal $C(Z^{\otimes 2})$ operation, primarily identified as $X \leftrightarrow Y$ crosstalk and $Z \leftrightarrow I$ residual Rydberg population.

be used for simulating fermionic systems, where multiqubit gates appear in the form of Jordan-Wigner strings and which

have been simulated in trapped-ion platforms [52], which have been shown to be able to implement multiqubit gates [53].

TABLE III. Layer count reduction for color code syndrome extraction. The baseline using only CZ gates requires six layers. Percentages show the reduction from this baseline.

Distance d	Max gate size			
	2 (CZ)	3 (CZZ)	4 (CZZZ)	5
3	6 (0%)	3 (50%)	3 (50%)	3 (50%)
5	6 (0%)	3 (50%)	4 (33%)	3 (50%)
7	6 (0%)	4 (33%)	4 (33%)	3 (50%)
9	6 (0%)	4 (33%)	4 (33%)	3 (50%)

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DATA AVAILABILITY

The data that support the findings of this article are openly available [54]; embargo periods may apply.

APPENDIX: PAULI TRANSFER MAPS

Here, we provide the PTMs discussed in the main text. We compare the PTM learned without motion or decay (Fig. 10) and the ideal one (Fig. 11).

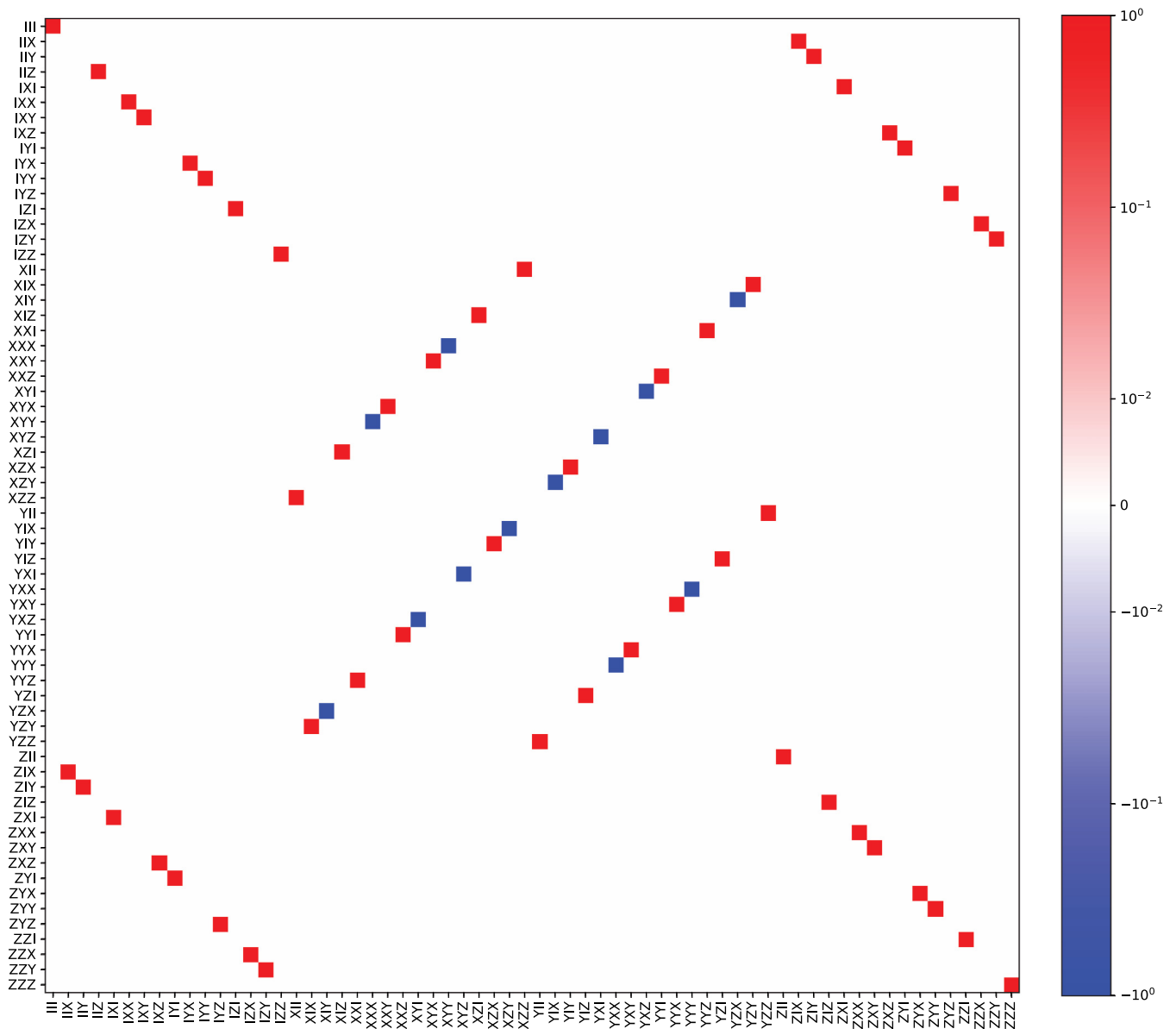


FIG. 11. Theoretical PTM for an ideal, error-free $C(Z^{\otimes 2})$ gate. This map illustrates the perfect transformation where input Pauli operators (Y axis) map deterministically to output Pauli operators (X axis). The PTM is sparse, containing only $+1$ or -1 entries at locations defined by the unitary $C(Z^{\otimes 2})$ evolution, serving as a noiseless reference.

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