

Characterizing correlated noise with single-qubit operations

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Spatially correlated noise poses a significant challenge to fault-tolerant quantum computation by breaking the assumption of independent errors. Existing methods such as cycle benchmarking and quantum process tomography can characterize noise correlations but require substantial resources. We propose straightforward and efficient techniques to detect and quantify these correlations by leveraging collective phenomena arising from environmental correlations in a qubit register. In these techniques, single-qubit state preparations, single-qubit gates, and single-qubit measurements, combined with classical postprocessing, suffice to uncover correlated relaxation and dephasing. Specifically, we use that correlated relaxation is connected to the superradiance effect, which we show to be accessible by single-qubit measurements. Analogously, the established parity oscillation protocol can be refined to quantify correlated dephasing without requiring the preparation of complex and entangled states.

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Introduction. Scalable quantum computation faces a considerable challenge in decoherence, which corrupts the integrity of computational processes by introducing errors. To conquer these errors, quantum error-correcting codes are developed that distribute quantum information across many physical qubits in a way that enables error reversal [1]. The threshold theorems for fault-tolerant quantum computation establish that quantum error-correcting codes can indeed suppress the effects of decoherence, as long as the error rate per physical qubit stays below some critical value, called the noise threshold [2–5]. These results have underpinned the feasibility of scalable quantum computation and have guided efforts to reduce error rates below noise thresholds in experimental platforms [6–9].

Noise due to decoherence in quantum systems is often correlated both spatially and temporally, rather than occurring locally [10–16]. Spatially correlated noise causes errors that affect multiple qubits simultaneously, while temporal correlations stem from memory effects (non-Markovian noise), where past errors can impact future ones. Although standard codes are typically designed under the assumption of independent errors, fault tolerance can still be achieved in the presence of sufficiently short-range correlations, albeit with adjustments of the threshold values [17–19]. In practice, the presence of correlations necessitates a more nuanced approach to detection and correction, an area that remains far from understood. Moreover, the accurate characterization of the spatial structure of noise correlations is essential, as deviations from locality can significantly impact the performance

of fault-tolerant quantum computation [20,21], and in extreme cases, render error correction ineffective [22].

Many recent works have provided valuable insights into spatially correlated noise [23–27]; however, these efforts are often restricted to a single type of noise or few-qubit systems [28–32]. Moreover, other approaches rely on methods, such as cycle benchmarking [33] and process tomography [34], that require significant overhead and access to a complete quantum hardware and software stack, resulting in potentially lengthy design feedback loops.

In this Letter, we present simple and efficient methods that bound and quantify correlated noise. This is particularly important for applications that can only tolerate few correlated errors, and our methods provide an experimentally efficient tool for verifying whether these conditions are met. In contrast to existing methods, our approach requires minimal technical resources, comparable to those needed for measuring standard single-qubit relaxation (T_1) and decoherence (T_2) times, and is inherently platform-agnostic. In fact, they allow the extension of the conventional single-qubit noise spectroscopy techniques to also characterize spatial correlations with a modest increase in the number of data points. This is possible by leveraging collective phenomena associated with spatial correlations [35]; effects akin to superradiance—where radiation from a group of emitters is enhanced by the correlations—signal correlated decay. Similarly, the related concept of superdecoherence, where collective behavior amplifies the degradation of quantum coherences, and the potentially emerging decoherence-free subspaces [36], where collective behavior reduces this degradation, can be used to detect correlated dephasing.

Formalism. We assume the Hamiltonian $H = H_S(t) + H_B + H_{SB}$, with $H_S(t)$ describing the qubit register, H_B the collection of environments, and

$$H_{SB} = \sum_{\alpha,j} A_{\alpha,j} \otimes B_{\alpha,j} \quad (1)$$

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the system-bath interactions. A quantum register is typically described by $H_S(t) = H_0 + H_{\text{ct}}(t)$, where $H_0 = \sum_{\alpha} \omega_{\alpha} Z_{\alpha}/2$ is the Hamiltonian of N qubits, with ω_{α} being the frequency of qubit α and Z_{α} the Pauli-Z matrix acting on qubit α . The time-dependent Hamiltonian $H_{\text{ct}}(t)$ describes the single-qubit control as well as the qubit-qubit interactions for multiqubit control. In what follows, we set $H_{\text{ct}}(t) = 0$; see the Supplemental Material [37] for the case $H_{\text{ct}}(t) \neq 0$. In Eq. (1), $A_{\alpha,j}$ is a Hermitian one-qubit system operator [38] to which the environments connect through the Hermitian bath operators $B_{\alpha,j}$. Greek indices denote the qubit locations, while Latin letters are used for the different noise effects that are caused by the different environments, unless stated otherwise. Within the weak coupling theory [32,39] that we will utilize in the coming discussion, the only relevant information from the environments is the bath correlation functions

$$\langle B_{\alpha,j}(t) B_{\beta,k}(0) \rangle = \delta_{jk} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} S_{\alpha\beta,j}(\omega) e^{-i\omega t}. \quad (2)$$

We emphasize that in the weak coupling limit, this form describes general correlated noise without any additional assumptions and further specifications on the nature of the baths, e.g., their microscopic origin. The appearance of δ_{jk} , which signifies the mutually uncorrelated nature of the different environments, arises as a consequence of the weak coupling assumption [40]. Spatial correlations emerge from the self-interactions of a particular bath j , which is described by the noise power spectrum $S_{\alpha\beta,j}(\omega)$.

We transform to the interaction frame with respect to H_0 , which leads to the state of the register to be expressed as $\tilde{\rho}(t) = e^{iH_0 t} \rho(t) e^{-iH_0 t}$, with $\rho(t)$ being the reduced state in the laboratory frame. The dynamics of the register's state in the interaction frame is derived by tracing out the environments, using the time-convolutionless framework of open quantum systems [37], which yields the quantum master equation

$$\frac{d}{dt} \tilde{\rho}(t) = \sum_j \tilde{D}_j[t, \tilde{\rho}(t)]. \quad (3)$$

The right-hand side describes the detrimental effects of the environments through the explicitly time-dependent dissipators $\tilde{D}_j[t, \tilde{\rho}(t)]$ that go beyond the Lindblad form, incorporating both non-Markovian and spatially correlated noise processes. The different noise contributions originating from bath j are additive due to Eq. (2), which provides the opportunity to study the competition between the potential local and nonlocal effects. The explicit form of the dissipators depends on the one-qubit system operator $A_{\alpha,j}$, appearing in Eq. (1), and the noise power spectrum $S_{\alpha\beta,j}(\omega)$.

Correlated relaxation. Applying our formalism, we first turn to transverse coupling to the environment. Without loss of generality, we assume the system operator $A_{\alpha,\text{relax}} = X_{\alpha}$, the Pauli-X matrix acting on qubit α , in H_{SB} . This coupling allows energy exchange between the register and its environment, leading to qubit relaxation. The dissipator for correlated relaxation is

$$\begin{aligned} \tilde{D}_{\text{relax}}[t, \tilde{\rho}(t)] = & -i[H_{XY}(t), \tilde{\rho}(t)] \\ & + \sum_{\substack{\alpha,\beta \\ i,j}} \gamma_{\alpha\beta}^{(ij)}(t) \left(\Pi_{\beta}^{(i)} \tilde{\rho}(t) \Pi_{\alpha}^{(j)} - \frac{1}{2} \{ \Pi_{\alpha}^{(j)} \Pi_{\beta}^{(i)}, \tilde{\rho}(t) \} \right). \end{aligned} \quad (4)$$

The first term is the contribution due to the Hamiltonian H_{XY} induced by the environmental effects, which describes bath-mediated symmetric and antisymmetric (Dzyaloshinskii-Moriya) exchange interactions. (For details on H_{XY} and on the derivation of the master equation, see the Supplemental Material [37].) The second term describes the correlated decay processes, with $i, j \in \{1, 2\}$ and $\Pi_{\alpha}^{(1)} = \Pi_{\alpha}^{\dagger} = (X_{\alpha} + iY_{\alpha})/2$ and $\Pi_{\alpha}^{(2)} = \Pi_{\alpha} = (X_{\alpha} - iY_{\alpha})/2$ the local raising and lowering operators, respectively. The time-dependent rates are calculated via $\gamma_{\alpha\beta}^{(ij)}(t) = \int_{-\infty}^{\infty} d\omega/2\pi S_{\alpha\beta}(\omega) F_{\alpha\beta}^{(ij)}(\omega, t)$, where $F_{\alpha\beta}^{(ij)}$ denotes the filter function of the particular rate [37]. We recognize $\gamma_{\alpha\beta}^{(12)}(t) \equiv \gamma_{\alpha\beta}^{\uparrow}(t)$ as the correlated absorption rates and $\gamma_{\alpha\beta}^{(21)}(t) \equiv \gamma_{\alpha\beta}^{\downarrow}(t)$ as the correlated emission rates. The remaining contributions are the nonsecular terms with the rates obeying the constraint $\gamma_{\alpha\beta}^{(11)}(t) = [\gamma_{\beta\alpha}^{(22)}(t)]^*$, which ensures that the master equation preserves the Hermiticity of the density matrix. These terms are typically neglected under the secular approximation because the rates $\gamma_{\alpha\beta}^{(11)}(t)$ oscillate much more rapidly than the absorption and emission rates. However, as the number of qubits increases, the contributions of the nonsecular terms can accumulate, rendering the secular approximation invalid; therefore, we avoid using it here.

The correlated relaxation dissipator in Eq. (4) is reminiscent of the one appearing in the master equation used to describe the collective spontaneous emission of an ensemble of excited atoms, that is, superradiance [41–43]. This motivates us to study the evolution of the qubit energies $W_{\alpha}(t) = \omega_{\alpha} \langle Z_{\alpha} \rangle(t)/2$, as well as the intensities of the radiation emitted by the qubits,

$$I_{\alpha}(t) = -\frac{d}{dt} W_{\alpha}(t) = -\frac{\omega_{\alpha}}{2} \sum_j \text{Tr}\{Z_{\alpha} \tilde{D}_j[t, \tilde{\rho}(t)]\}. \quad (5)$$

Here, we used that $\langle Z_{\alpha} \rangle(t) = \text{Tr}[Z_{\alpha} \rho(t)] = \text{Tr}[Z_{\alpha} \tilde{\rho}(t)]$, so that the derivative and the intensity can be obtained using Eq. (3). The contributions to the intensity are additive with respect to the different decoherence effects, whether they are local or nonlocal. Using the relaxation dissipator in Eq. (4), it is straightforward to calculate the intensities,

$$I_{\alpha}(t) = \omega_{\alpha} [\gamma_{\alpha}^{\downarrow}(t) \langle \Pi_{\alpha}^{\dagger} \Pi_{\alpha} \rangle - \gamma_{\alpha}^{\uparrow}(t) \langle \Pi_{\alpha} \Pi_{\alpha}^{\dagger} \rangle] + I_{\text{corr}}^{(\alpha)}(t). \quad (6)$$

The first two terms represent the independent emission and absorption by the qubits, which we denote by $I_{\text{ind}}^{(\alpha)}(t)$. The negative sign indicates energy entering the system. In these terms, the local emission rate $\gamma_{\alpha}^{\downarrow}$ and absorption rate $\gamma_{\alpha}^{\uparrow}$ are understood to include all local contributions, even those arising independently of the correlated environments (e.g., from additional local baths). The last term in Eq. (6) appears due to the buildup of pair correlations between the qubits,

$$\begin{aligned} I_{\text{corr}}^{(\alpha)}(t) = & \omega_{\alpha} \sum_{\beta \neq \alpha} (Q_{\alpha\beta}^{\downarrow}(t) - Q_{\beta\alpha}^{\uparrow}(t)) \\ & \times (\langle \tilde{\Pi}_{\alpha}^{\dagger}(t) \tilde{\Pi}_{\beta}(t) \rangle - \langle \tilde{\Pi}_{\alpha}(t) \tilde{\Pi}_{\beta}^{\dagger}(t) \rangle) + \text{c.c.}, \end{aligned} \quad (7)$$

where c.c. is the complex conjugate. The time-dependent factors $Q_{\alpha\beta}^{\downarrow(\uparrow)}(t)$ are obtained from the noise power spectrum with filter-function integrals, and they are related to the correlated decay rates by $\gamma_{\alpha\beta}^{\downarrow(\uparrow)}(t) = Q_{\alpha\beta}^{\downarrow(\uparrow)}(t) + [Q_{\beta\alpha}^{\downarrow(\uparrow)}(t)]^*$ [37]. The correlated intensity is an initial-state-dependent quantity that leads to superradiance or subradiance in the context of

collective spontaneous emission of atoms [42]. In terms of the qubits, it reveals the presence of correlated incoherent noise.

Measuring the intensity directly in quantum computing architectures would be impractical, as one typically does not have the means to place detectors into the register's environment. Fortunately, this is not necessary. Due to the conservation of energy, by initializing the system in the fully inverted state, thereby fixing its initial total energy, and subsequently measuring the local expectation values $P_\alpha(t) = \langle \Pi_\alpha^\dagger \Pi_\alpha \rangle$ for each qubit over time, one can determine the energy leaving the system. These measurements are routinely performed, as the same data are commonly used in relaxation time ($T_{1\alpha}$) acquisitions. Remarkably, a more careful analysis of these data also reveals information about the spatial correlations of the noise. Specifically, using $I_\alpha = -\omega_\alpha \dot{P}_\alpha/2$ with $\langle \Pi_\alpha \Pi_\alpha^\dagger \rangle = 1 - P_\alpha$, Eq. (6) shows that the correlated intensity $I_{\text{corr}}^{(\alpha)}(t)$ can be extracted by deconvolving $P_\alpha(t)$ [37]. After reconstructing $I_{\text{corr}}^{(\alpha)}(t)$, a characterization of correlated decay is possible: Equation (7) yields a lower bound on the correlation strength,

$$|I_{\text{corr}}^{(\alpha)}(t)| \leq 2\omega_\alpha \sum_{\beta \neq \alpha} |Q_{\alpha\beta}^\downarrow(t) - Q_{\beta\alpha}^\uparrow(t)|. \quad (8)$$

If the rates saturate to their constant value on a timescale much shorter than the total relaxation timescale, then $|Q_{\alpha\beta}^\downarrow(t) - Q_{\beta\alpha}^\uparrow(t)| \approx |S_{\alpha\beta}(\omega_\beta)|/2$, and since Eq. (8) is valid for all times, we can lower bound the correlations by $\sup_t |I_{\text{corr}}^{(\alpha)}(t)|$. Furthermore, $I_{\text{corr}}^{(\alpha)}$ identifies the qubits that are involved in correlated decay, providing a proxy for the spatial extent of the correlations.

A numerical illustration of these ideas is presented in Fig. 1. We consider $N = 5$ qubits with no direct couplings between them and with normally distributed frequencies around the average frequency $\bar{\omega}$, initialized in the fully inverted state $|11111\rangle$. We note that while uniform qubit frequencies amplify correlated effects, tuning the frequencies may not always be possible. For this reason, we consider an example with unequal qubit frequencies to demonstrate that the correlated effects remain detectable even in this case. For concreteness, here we assume a straightforward model for the spatial correlations: $S_{\alpha\beta}(\omega) = e^{i\theta_{\alpha\beta}} \xi_{\alpha\beta} S(\omega)$, stemming from the general properties of the correlated noise spectrum [44]. The phase $\theta_{\alpha\beta}$ and the real dimensionless factor $\xi_{\alpha\beta} \leq 1$ determining the correlation length are generally functions of frequency, depending on the microscopic details of the environment. However, for simplicity, we assume that they are independent of frequency [32]. We consider Ohmic relaxation at zero temperature, with an exponential cutoff, so that $S_{\alpha\beta}(\omega \geq 0) = \lambda\omega e^{-\omega/\omega_c} \xi_{\alpha\beta}$ and $S_{\alpha\beta}(\omega < 0) = 0$.

First, we consider the case of a common bath with a correlation length much longer than the register's size, i.e., $\xi_{\alpha\beta} = 1$. In Fig. 1(a), the total frequency-weighted intensity $\sum_\alpha I_\alpha/\omega_\alpha$ is shown, as well as its deviation from the local contributions, that is, the total frequency-weighted correlated intensity. This quantity provides a lower bound on the ℓ_1 norm of the off-diagonals of the correlated relaxation rate matrices $\gamma_{\alpha\beta}^{\downarrow(\uparrow)}$, provided $\theta_{\alpha\beta}$ is frequency independent [37]. Figure 1(b) shows that $I_{\text{corr}}^{(\alpha)}$ are all nonzero, indicating that each qubit participates in correlated relaxation due to the long correlation

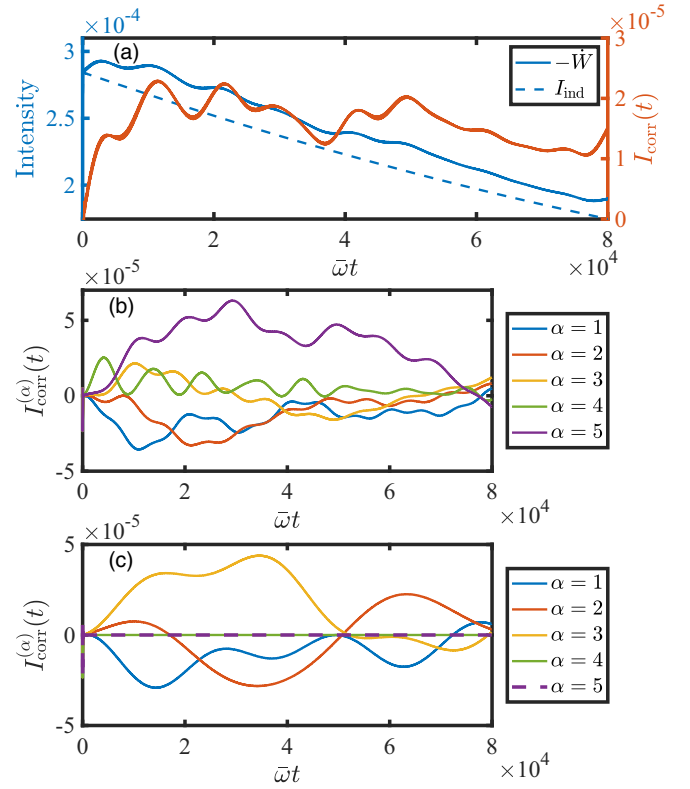


FIG. 1. The intensities of radiation leaving the $N = 5$ qubit system in the presence of correlated relaxation. The qubit frequencies are drawn from the Gaussian distribution with mean $\bar{\omega}$ and standard deviation $\Delta = 10^{-3}\bar{\omega}$. This choice reflects variations on a given physical qubit register, arising, e.g., from variability in chip fabrication for solid-state qubits, or other type of qubit-to-qubit variability. The strength of the Ohmic noise is chosen as $\lambda/\bar{\omega} = 10^{-5}$ and the cutoff frequency as $\omega_c/\bar{\omega} = 10$. (a) The derivative of the total energy (blue solid line), the total local contribution of the intensity (blue dashed line), and the total correlated contribution (red solid line). Notice the solid line's broadening due to the nonsecular terms. (b) The correlated intensities, with a bath correlation length encompassing all qubits. (c) As in panel (b), with three out of five qubits experiencing correlated relaxation.

length. In contrast, Fig. 1(c) depicts a second scenario, in which only a subset of qubits lie within the correlation length of the bath. In this case, only those qubits exhibit nonzero correlated intensities, while the others dissipate their energy independently.

Correlated dephasing. We now consider longitudinal coupling to the environment, where the system operator appearing in Eq. (1) is $A_{\alpha,\varphi} = Z_\alpha$. This coupling leads to the fluctuation of the qubit frequencies, and hence to dephasing; the dissipator reads

$$\begin{aligned} \tilde{D}_\varphi[t, \tilde{\rho}(t)] = & -i[H_{ZZ}(t), \tilde{\rho}(t)] \\ & + \sum_{\alpha,\beta} \gamma_{\alpha\beta}^{(\varphi)}(t) \left[Z_\beta \tilde{\rho}(t) Z_\alpha - \frac{1}{2} \{Z_\alpha Z_\beta, \tilde{\rho}(t)\} \right], \quad (9) \end{aligned}$$

where the first term describes the bath-mediated ZZ couplings between the qubits (see the Supplemental Material [37] for details on the derivation of the master equation), while the

second term entails correlated dephasing with time-dependent rates $\gamma_{\alpha\beta}^{(\varphi)}(t) = t \int_{-\infty}^{\infty} d\omega/\pi S_{\alpha\beta,\varphi}(\omega) \text{sinc}(\omega t)$. This dissipator does not contribute to the intensity in Eq. (5); hence, correlated dephasing does not interfere with the correlated relaxation discussed previously.

Considering only the effects from dephasing [Eq. (9)], the master equation [Eq. (3)] turns into decoupled differential equations for the components of the density matrix in the computational basis (the eigenbasis of H_0). Particularly interesting are the equations for the antidiagonal matrix elements $\tilde{\rho}_{\bar{l},l}(t)$, where l is a bitstring of length N representing the state $|l\rangle$ and $\bar{l}$ its complement. These equations read

$$\dot{\tilde{\rho}}_{\bar{l},l}(t) = -2\tilde{\rho}_{\bar{l},l}(t) \sum_{\alpha,\beta} (-1)^{l_\alpha + l_\beta} \gamma_{\alpha\beta}^{(\varphi)}(t), \quad (10)$$

with $l_\alpha \in \{0, 1\}$ the α th bit of l . In the absence of correlations, i.e., $\gamma_{\alpha\beta}^{(\varphi)}(t) = \delta_{\alpha\beta} \gamma_{\alpha}^{(\varphi)}(t)$, the dephasing rate of $\tilde{\rho}_{\bar{l},l}(t)$ is the same for all l and equals the sum of the local dephasing rates.

According to Eq. (10), correlations lead to the separation of the dephasing rates for $\tilde{\rho}_{\bar{l},l}(t)$ depending on k , which we define as the difference between the number of 1s and 0s in l . For instance, the dephasing rate of the far-off-diagonal matrix element $\tilde{\rho}_{0\dots 0,1\dots 1}(t)$, representing the coherence between the ground state $|0\dots 0\rangle$ and the fully inverted state $|1\dots 1\rangle$, is $2 \sum_{\alpha,\beta} \gamma_{\alpha\beta}^{(\varphi)}(t)$. Note that $\gamma_{\alpha\beta}^{(\varphi)}(t) \sim \xi_{\alpha\beta}^{(\varphi)}$, so that $\sum_{\alpha,\beta} \gamma_{\alpha\beta}^{(\varphi)}(t) \sim N^2$ for fully correlated dephasing, as opposed to $\sum_{\alpha,\beta} \gamma_{\alpha\beta}^{(\varphi)}(t) \sim N$ for fully independent dephasing. This enhanced sensitivity of coherence, caused by noise correlations, is known as superdecoherence [45,46]. In contrast, the dephasing rates of the matrix elements with the same number of 1s and 0s in l ($k = 0$) are maximally reduced by the correlations. For a perfectly correlated environment ($\xi_{\alpha\beta}^{(\varphi)} = 1$), these rates vanish, leading to the formation of a decoherence-free subspace [36]. If, in addition to correlated dephasing, there are (correlated) baths that couple transversely to the qubits, leading to dissipators in the form of Eq. (4), superdecoherence persists due to the independent bath assumption [37]. Also, the separation of times over which the antidiagonal matrix elements vanish persists; see Fig. 2(a).

Thus, the dynamics of the antidiagonal elements carry information on correlated dephasing. To reveal them, we cluster the antidiagonal elements of ρ by k , and define $\rho^{(k)}$ as the sum of all matrix elements $\rho_{\bar{l},l}$ such that the difference between the number of 1s and 0s in l is k . A well-established method for obtaining the absolute value of $\rho^{(N)}(t) = \rho_{0\dots 0,1\dots 1}(t)$ is by parity oscillations [10,47]. Nevertheless, we show that parity oscillations contain low-frequency corrections arising from elements $\rho^{(k)}(t)$ ($k \neq N$). In fact, all $\rho^{(k)}(t)$ can be extracted from these parity oscillations.

Parity oscillations [10,47], here with generalized initial states, are obtained as follows. (1) Prepare an initial state $\rho(0)$ that populates the antidiagonal matrix elements of interest. Possible choices include the equal-superposition state $|+\rangle^{\otimes N}$ and the N -qubit Greenberger-Horne-Zeilinger (GHZ) state $(|0\rangle^{\otimes N} + |1\rangle^{\otimes N})/\sqrt{2}$. (2) Wait for time t . (3) Apply the single-qubit gate $\text{RZ}(\phi)\sqrt{X}^\dagger \text{RZ}^\dagger(\phi)$ to each qubit and measure all qubits in the Z basis.

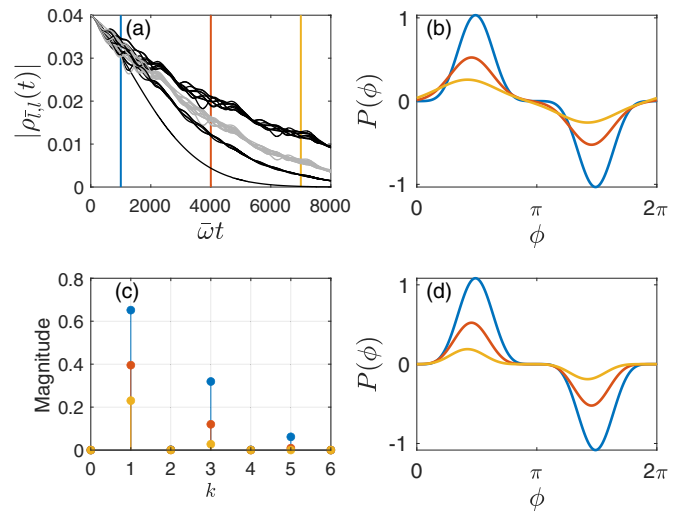


FIG. 2. Five-qubit system with nonuniform frequencies under correlated $1/f$ dephasing and Ohmic correlated relaxation. The parameters are as in Fig. 1 for relaxation and the strength of dephasing is $\lambda_\varphi/\bar{\omega} = 10^{-9}$ with infrared cutoff $\omega_{\text{ir}}/\bar{\omega} = 10^{-6}$. (a) Separation of timescales in the dynamics of the antidiagonal density matrix elements due to the longitudinally coupled environmental correlations. Each line represents one of 16 choices for $\{\bar{l}, l\}$. The additional gray lines correspond to evolution with uncorrelated dephasing, showing the absence of a separation in the timescales. (b) Parity oscillation signal from the initial equal-superposition state after the different idling times that are indicated by the vertical lines in panel (a). (c) The Fourier coefficients of the parity oscillations of panel (b). (d) Parity oscillations with uncorrelated dephasing, i.e., $\xi_{\alpha\beta}^{(\varphi)} \sim \delta_{\alpha\beta}$, showing unchanged signal shape, indicating the absence of spatial correlations in the longitudinally coupled noise.

The “parity,” obtained from repetition of the above process, is defined as $P(\phi) = P_{\text{even}}(\phi) - P_{\text{odd}}(\phi)$, with $P_{\text{even/odd}}$ the probability of measuring an even/odd number of -1 s in step (3). A direct computation [37] shows that

$$P(\phi, t) = \sum_{k=-N}^N e^{i(\phi - \pi/2)k} \rho^{(k)}(t), \quad (11)$$

so that $\rho^{(k)} = \frac{\mathcal{F}[P'](k)}{2N+1}$, with $\mathcal{F}[P'](k)$ the Fourier transform of P' , evaluated at k , and $P'(x) = P(\frac{2\pi}{2N+1}x + \frac{\pi}{2})$. Under the assumption that the only nonzero antidiagonal entries of $\rho(t)$ are the far-off-diagonal elements, which is the case for a perfect GHZ state, $|\rho_{0\dots 0,1\dots 1}(t)\rangle$ is half the amplitude of $P(\phi, t)$. In experiments with high-fidelity GHZ states, where additionally dephasing is the dominant noise channel, the above assumption is justified. However, in general, it is not, e.g., for noisy GHZ states, when relaxation cannot be ignored, or for states such as $|+\rangle^{\otimes N}$.

The correlations, if present, lead to distinct decay times for $\rho^{(k)}(t)$ for each k ; the Fourier components of the parity oscillations diminish at different rates. The resulting change in $P(\phi)$ directly reveals the presence of correlations, as illustrated in Fig. 2 using a five-qubit ($N = 5$) example subjected to correlated $1/f$ dephasing ($S_{\alpha\beta,\varphi}(\omega) = \lambda_\varphi/|\omega|$) and Ohmic relaxation as discussed previously. By introducing $\Delta G_{kk'}(t) = \ln \rho^{(k)}(t) - \ln \rho^{(k')}(t)$, we can quantify the cor-

relations through the short-time behavior of $\Delta G_{kk'}$, for any $k \neq k'$. In the absence of correlations, $\Delta G_{kk'}$ is time independent. In the presence of correlated white noise, $\Delta G_{kk'}$ exhibits linear short-time dependence, whereas for correlated colored noise, the leading short-time behavior becomes quadratic [37]. In the white-noise (colored-noise) case, the slope (curvature) of $\Delta G_{kk'}(t)$, denoted by $\mathcal{G}_{s(c)}(k, k')$, directly yields the sum of the off-diagonal elements of $\xi_{\alpha\beta}^{(\varphi)}$:

$$\mathcal{G}_{s(c)}(k, k') = g_{kk'} \lambda^{(\varphi)} \sum_{\alpha \neq \beta} \xi_{\alpha\beta}^{(\varphi)}, \quad (12)$$

where $g_{kk'}$ is a combinatorial factor and $\lambda^{(\varphi)}$ is the strength (total noise power) of white noise (colored noise) [37].

Previous experimental evidence for correlated dephasing has been obtained by creating the GHZ state and observing the degradation of the far-off-diagonal matrix element by parity oscillations [10,48]. The scaling behavior of this degradation with increasing qubit numbers demonstrates correlated dephasing through superdecoherence, as the GHZ coherence is sensitive to the total sum of the dephasing contributions, including both local and correlated noise. Interestingly, the same information, i.e., the aggregate dephasing noise correlations, can be extracted from the generalized parity oscillation protocol we outlined above. This is particularly advantageous because it is challenging to generate high-fidelity many-qubit GHZ states; additionally, precisely due to superdecoherence, the oscillation amplitude quickly becomes very weak, making it difficult to monitor.

We note that “multiple quantum coherences”(MQCs) [37,49,50] offer an alternative to parity oscillations. However, they provide information about ρ different from $\rho^{(k)}$. Furthermore, the validity of the method that relates the MQC signal to the information about ρ has only been demonstrated under restricted noise channels, which are presupposed to have uncorrelated errors, among other limitations [37,51].

Conclusion. We proposed simple methods that reveal the presence of spatially correlated noise in qubit registers. Initializing the system in the fully inverted state and then monitoring the dynamics of the intensity of radiation leaving the system shows the presence of correlated environments that couple transversely to the qubits. Moreover, observing the change in the line shape of parity oscillations from the initial equal-superposition state signals the presence of correlated dephasing. Both procedures require only single-qubit control, making them readily implementable on all currently available platforms, even before the quantum processor in question is fully calibrated or integrated. This leads to shorter design feedback loops, accelerating the development of quantum computing hardware.

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Data availability. The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

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