

Factor of 1000 suppression of the depolarization rate in ultracold thulium collisions

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Lanthanides are nowadays extensively used to investigate the properties of strongly correlated matter. Nevertheless, exploiting the Zeeman manifold of a lanthanide atom ground state is challenging due to the unavoidable presence of depolarization collisions. Here, we demonstrate that in the case of the thulium atom, it is possible to suppress this depolarization by a factor of 1000 with a carefully tuned magnetic field, thus opening the way for the efficient use of the Zeeman manifold in quantum simulations.

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I. INTRODUCTION

Since the achievement of the Bose-Einstein condensation of rubidium [1], sodium [2], and lithium [3] atoms, ultracold gases of bosons have attracted significant attention as a test-bed system for exploring the properties of strongly correlated matter. The character of interactions between particles affects the properties of a quantum gas. Lanthanides, due to an incomplete f shell, have a dense spectrum of Feshbach resonances [4–14] causing the strong dependence of interatomic scattering on the magnetic field. Large magnetic moment of lanthanides introduces the dipole-dipole interaction (DDI) in the system [15–19]. In particular, thulium atom with one hole in f shell has $4\mu_B$ magnetic moment in the ground state as well as rich spectrum of Feshbach resonances [20], which is less dense compared to such lanthanides as erbium [21] and dysprosium [22]. These aspects along with a relatively simple level structure make thulium a convenient ultracold research tool.

The anisotropy and the long range of DDIs modify the scattering properties and many-body behavior making it possible to observe, in particular, quantum droplets [23–26] and supersolids [27–29]. The large spin manifold of highly magnetic atoms makes it possible to exploit spin richness to explore phenomena like quantum magnetism [30] and exotic topological phases [31,32], to realize long-range-interacting spin-lattice models [19,33–35], and to study nonclassical spin states within a highly controlled system [36,37]. Exploiting

internal degrees of freedom allows to create artificial field [38] and study dark lattices in gauge theories [39], as well as fermionization of bosons, which can happen in specially arranged lattices or special interaction regimes [40], and potentially formation of magnetic phases previously mostly considered with fermions [41,42].

When two magnetic atoms collide, the projections m_{F1} and m_{F2} of their respective total angular momenta $\vec{F}_1$ and $\vec{F}_2$ may change after the collision. Collisions in which $m_F = m_{F1} + m_{F2}$ is decreased (relaxation collision) usually lead to the loss of the atomic pair from the trap, as the Zeeman energy transfers to the kinetic energy of atoms. Collisions in which m_F is conserved (depolarization collision) leave atoms in the trap but lead to the depolarization of the initial state. We note that this mechanism is not the only possible mechanism of depolarization. In case of atoms with zero orbital momentum in the ground state, it has been demonstrated that spin-exchange interaction may play a significant role [43], as well as dynamics of evaporation cooling [44] in case of fermions. Nevertheless, the significant magnetic moment in the ground state makes the dipolar mechanisms to dominate in the depolarization.

Preparing the atoms in the stretched state $m_F = 2F$ eliminates the depolarization collisions. The dipolar relaxation from the stretched state can be reduced by confining atoms in thin layers [45], loading into optical lattices or applying rf fields [46]. External magnetic field B also modifies the relaxation rate in the stretched state. The Born approximation of the DDI potential gives $\propto \sqrt{B}$ dependency for the relaxation cross sections in the case of two identical bosons and $\propto 1/\sqrt{B}$ in the case of identical fermions; both were experimentally demonstrated for Dy isotopes [47]. Noticeably, the relaxation cross section of bosonic Cr exhibits a dip in the region of small magnetic field, which was described by including interatomic potential and applying Fermi's golden rule with the numerically calculated wave functions [46].

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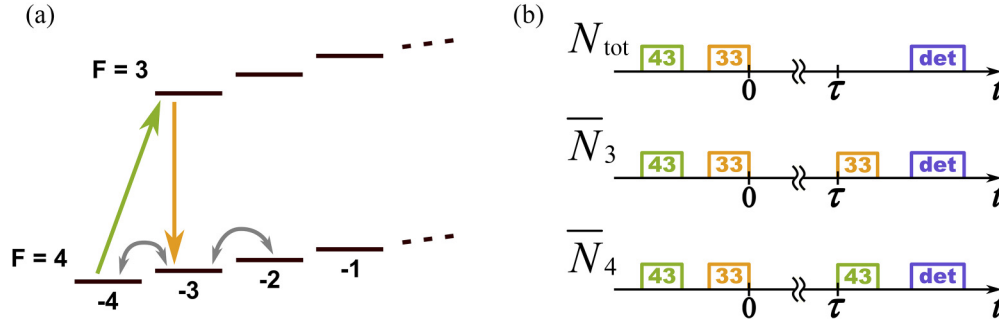


FIG. 1. (a) The scheme of experimental pulses. The colored arrows present π pulses, and the gray curved arrows represent spin-exchange and relaxation channels. (b) The scheme of the microwave π pulses driving the selected transitions to obtain N_{tot} , $\bar{N}_3$, and $\bar{N}_4$.

When atoms are not initially prepared in one of the stretched states $m_F = \pm 2F$, depolarization collisions emerge. Although they conserve both Zeeman energy and kinetic energy of atoms, they complicate the description of the dipolar collisions. The depolarization can be mitigated in three-dimensional optical lattices [48] or by applying different ac Stark shifts to the Zeeman sublevels [49]. In the latter case, the Dy bosons in the state $|J_1 = 8, m_J = -7\rangle$ exhibit the dip in the relaxation cross section similar to the Cr mentioned above.

Here, we demonstrate the existence of a magnetic-field-dependent resonance, suppressing the depolarization rate in thulium by more than 1000 times. This type of suppression to our knowledge has never been predicted nor observed before. The well-established radio-frequency protocol for spin state initialization [50] was used to manipulate the spin mixture and probe the population of different sublevels. The evolution of populations made it possible to determine the collision rates that strongly depend on the magnetic field.

II. COLLISION RATE MEASUREMENT

A. Atomic cloud preparation

The measurements of collision rates were typically performed with 2.0×10^6 atoms at temperature of $2.5 \mu\text{K}$ prepared in a single-beam optical dipole trap (ODT) with $(\omega_x, \omega_y, \omega_z) = 2\pi(326, 326, 2)$ Hz frequencies. The production of such atomic cloud is described in previous works [18,20,51–54]. The Zeeman slower and two-dimensional optical molasses operating at the strong transition $4f^{13}(^2F^0)6s^2 \rightarrow 4f^{12}(^3H_5)5d_{3/2}6s^2$ with a wavelength of 410.6 nm and a natural width of $\Gamma = 2\pi\gamma = 2\pi \times 10.5 \text{ MHz}$ provided the precooling of atoms. Then, the atoms were loaded into the magneto-optical trap operating at the weaker transition $4f^{13}(^2F^0)6s^2 \rightarrow 4f^{12}(^3H_6)5d_{5/2}6s^2$ with a wavelength of 530.7 nm and a natural width of $\Gamma = 2\pi\gamma = 2\pi \times 345.5 \text{ kHz}$. The reduction of magneto-optical trap light intensity provided the polarization of atoms at the lowest magnetic sublevel $|F = 4; m_F = -4\rangle$ of the ground state. After cooling down to $22.5 \pm 2.5 \mu\text{K}$, the atoms were loaded and evaporated in the ODT formed by a linearly polarized laser beam waist of $40 \mu\text{m}$ and a wavelength of 1064 nm .

B. Populating the Zeeman sublevels

Thulium has only one stable isotope ^{169}Tm with LS bond in the ground state $4f^{13}(^2F^0)6s^2$ ($L = 3, S = 1/2, J = 7/2$), which has two hyperfine components $F = 4$ (lower) and $F = 3$ (upper) due to the nonzero nuclear spin $I = 1/2$. In the presence of an external magnetic field, both hyperfine components split into the Zeeman manifolds [see Fig. 1(a)] separated by frequencies $g_F B \mu_B / h$, where g_F is the Lande g factor of F level, B is the absolute value of the magnetic field, h is the Planck constant, and μ_B is the Bohr magneton. Since g_F takes different values for the hyperfine components ($g_{F=4} = 0.999$ and $g_{F=3} = 1.284$), a nonzero magnetic field splits their Zeeman manifolds differently, thus allowing frequency-selective addressing of every specific $|F = 4\rangle \rightarrow |F = 3\rangle$ transition with a microwave field. To generate this field, the exact copy of the microwave setup presented in Ref. [50] was exploited. Figure 1(a) presents a sequence of the two π pulses populating the $|4, -3\rangle$ spin state. Similarly, a sequence of π -pulse pairs with adjusted frequencies and duration can populate any specific Zeeman sublevel of the ground state in the ladderlike manner.

C. Measurement of collision rates

In the lowest $|4, -4\rangle$ spin state, the depolarization and relaxation processes are suppressed. Contrarily, in the $|4, -3\rangle$ spin state, both processes are allowed. The corresponding spin dynamics was investigated exploiting the $|4, -3\rangle \rightarrow |3, -3\rangle$ and $|3, -3\rangle \rightarrow |4, -3\rangle$ π pulses (35 and $50 \mu\text{s}$ duration, respectively). Figure 1(b) presents the three pulse sequences that provide the number of atoms N_{tot} , $\bar{N}_3$, and $\bar{N}_4$ after the absorption imaging. The two subsequent $|4, -3\rangle \rightarrow |3, -3\rangle$ and $|3, -3\rangle \rightarrow |4, -3\rangle$ π pulses transferred all the atoms to the $|4, -3\rangle$ spin state. The probe beam was tuned to the wide 410.6 nm transition, so the absorption imaging would contain all the atoms in the $|F = 4\rangle$ Zeeman manifold. Therefore, N_{tot} is the total population of all sublevels of $|F = 4\rangle$, $\bar{N}_3$ is the number of atoms in all the sublevels except $|4, -3\rangle$, and $\bar{N}_4$ is the number of atoms in all the sublevels except $|4, -4\rangle$ after the τ time period.

Figure 2 presents the experimentally measured $N_{\text{tot}}(\tau)$, $\bar{N}_3(\tau)$, and $\bar{N}_4(\tau)$ dependencies corresponding to $B = 2.98 \text{ G}$ [panel (a)] and $B = 0.565 \text{ G}$ [panel (b)]. For the smallest τ , $\bar{N}_3$ is predictably negligible, because all the atoms were in the

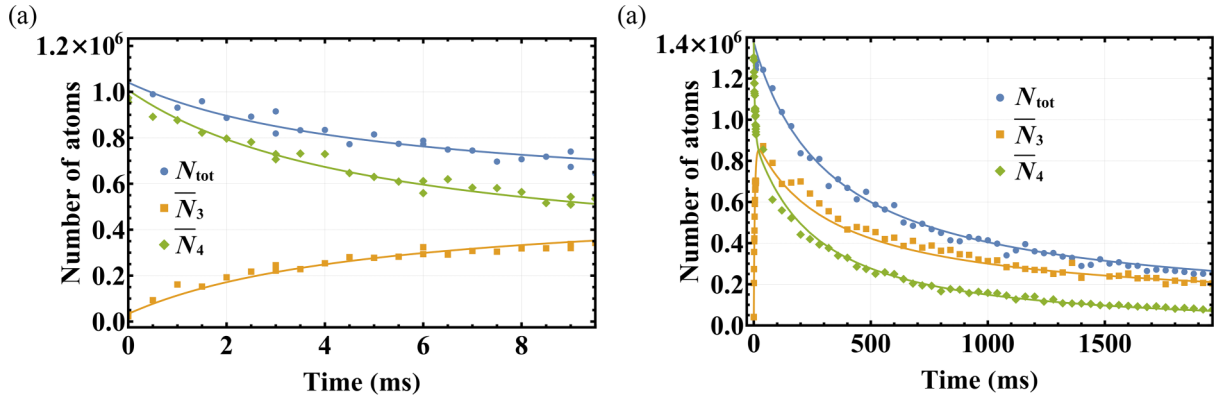


FIG. 2. The number of detected atoms vs time: (a) in the magnetic field $B = 2.98$ G at short times and (b) in $B = 0.565$ G. Every experimental value was obtained from a single absorption image. Solid lines in panel (a) correspond to the fit by system (1) accounting for the relations (2) providing the N_{tot} , $\bar{N}_3$, and $\bar{N}_4$ values. Solid lines in panel (b) correspond to the fit by the system with next-order collisions (see Appendixes).

$|4, -3\rangle$ state and were removed from the $|F = 4\rangle$ manifold. Correspondingly, $\bar{N}_4$ was similar to N_{tot} . However, for larger τ , $\bar{N}_3$ increases revealing a gradual appearance of atoms not in the $|4, -3\rangle$ state. Correspondingly, $\bar{N}_4$ becomes smaller than N_{tot} demonstrating the gradual appearance of atoms particularly in the $|4, -4\rangle$ state. This behavior indicates the presence of spin dynamics processes.

Considering only the two-body collisions, there are depolarization processes like $|4, -3\rangle \otimes |4, -3\rangle \rightarrow \frac{1}{\sqrt{2}}(|4, -4\rangle \otimes |4, -2\rangle + |4, -2\rangle \otimes |4, -4\rangle)$ and relaxation processes leading to losses from the trap. If we consider the dynamics only at short times (while populations of the all states but $|4, -3\rangle$ are negligible), then the number of atoms in the $|4, -4\rangle$ and $|4, -2\rangle$ states is relatively small and only collisions of atoms in $|4, -3\rangle$ state are important. In this case, we can describe dynamics by the system

$$\begin{aligned}\dot{N}_3 &= -\alpha(\beta_{\text{loss}} + \beta_{\text{depol}})N_3^2, \\ \dot{N}_2 &= \dot{N}_4 = \alpha \frac{\beta_{\text{depol}}}{2} N_3^2,\end{aligned}\quad (1)$$

where $\alpha = \frac{\bar{\omega}^3}{(2\pi)^3} \left(\frac{\pi m}{kT}\right)^{\frac{3}{2}}$, $\bar{\omega} = (\omega_x \omega_y \omega_z)^{\frac{1}{3}}$, T is the atomic cloud temperature, and m is the mass of atom. N_4 , N_3 , and N_2 are the populations in the states $|4, -4\rangle$, $|4, -3\rangle$, and $|4, -2\rangle$, respectively, β_{depol} is a depolarization spin-exchange rate, and β_{loss} is a spin-relaxation rate leading to losses from the trap. The experimental data correspond to

$$\begin{aligned}N_{\text{tot}} &= a_4 N_4 + a_3 N_3 + a_2 N_2, \\ \bar{N}_3 &= a_4 N_4 + a_2 N_2, \\ \bar{N}_4 &= a_3 N_3 + a_2 N_2,\end{aligned}\quad (2)$$

where a_4 , a_3 , and a_2 are the detection efficiencies for N_4 , N_3 , and N_2 , respectively (see Appendixes). Figure 2(a) demonstrates the numerical solution to the system (1) fitted the experimental decay data calculated according to Eq. (2) with parameters N_{03} , N_{04} , β_{depol} , and β_{loss} with initial conditions $N_3(0) = N_{03}$, $N_4(0) = N_{04}$, and $N_2(0) = 0$.

At the long times, system (1) can be appended with next-order processes (see Appendixes). While the large number of

additional parameters does not allow to extract all these additional parameters, one could see contribution of additional channels by fitting data at long times [see Fig. 2(b)]. Yet, due to the complication of this model, only Eq. (1) was used for further analysis.

III. MAGNETIC FIELD DEPENDENCE OF COLLISIONS

The rates obtained from fitting the data by system (1) for various values of magnetic field are presented in Fig. 3. Both the depolarization and loss rates demonstrate a resonancelike and nonmonotonic behavior, particularly revealing the 0.90 G value of the magnetic field at which the β_{depol} rate is more than 1000 times less than the 2×10^{-11} baseline (Fig. 3). The depolarization suppression resonance is also reproduced in the loss rate β_{loss} [Fig. 3(b)] by a factor of 50 at the same magnetic field. Overall, the loss rate curve closely resembles the depolarization curve in shape, but appears compressed along the vertical axis.

To validate the model (1), additional experiments were performed with a $|4, -2\rangle \rightarrow |3, -1\rangle$ π pulse to obtain the $\bar{N}_2$ data (see Appendixes). Loss rate values β_{loss} and β_{depol} were calculated with and without the additional data. The obtained values for two magnetic fields are shown in Fig. 3 as red squares and coincide with the initial experimental values within error bars.

In the magnetic field of 0.90 G, the lifetime of the $|4, -4\rangle$ state is around 6.8 s. At the low values of β_{loss} and β_{depol} , system (1) is sufficient to describe spin dynamics at all times, so the lifetime of the $|4, -3\rangle$ state can be defined as $\tau = \frac{e-1}{\alpha(\beta_{\text{loss}} + \beta_{\text{depol}})N_3(0)}$ and is around 2.3 s, which is only 3 times shorter compared to the $|4, -4\rangle$ state.

To check the validity of the two-body model, a series of experiments similar to the one in Fig. 2 with different initial atomic numbers were conducted. Given the constant ODT volume and the same starting temperature, we can explore the dependence of β_{loss} and β_{depol} on the initial number of atoms. Figure 4 shows that β_{loss} and β_{depol} are close to constant values, $\beta_{\text{loss}} = (2.6 \pm 2.3) \times 10^{-12}$ and $\beta_{\text{depol}} = (1.5 \pm 0.4) \times$

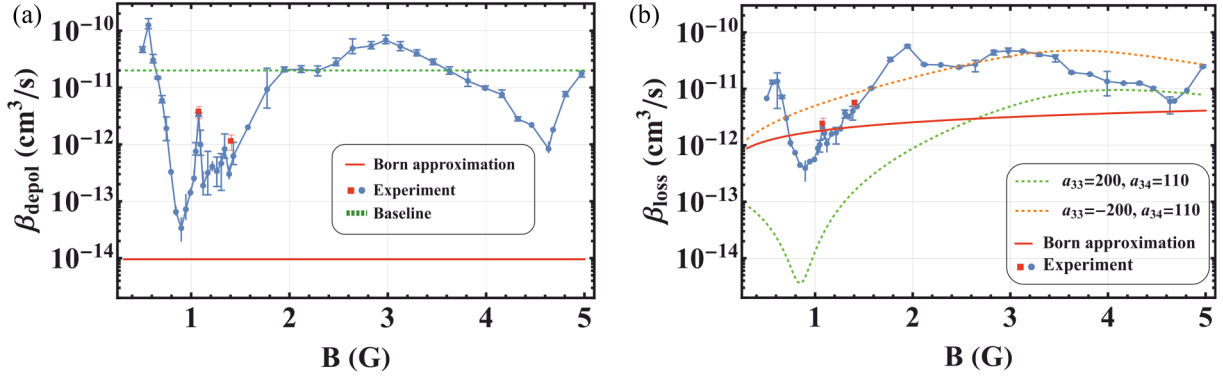


FIG. 3. (a) The dependence of β_{depol} on the magnetic field. The green dashed line illustrates the baseline 2×10^{-11} . The red line represents the values obtained from the Born approximation. The red squares and blue dots are experimental values with and without additional $\bar{N}_2$ data, respectively. (b) The dependence of β_{loss} on the magnetic field. The red line is the Born approximation values. The green dashed line and orange dot-dashed line are theoretical predictions based on model from Ref. [46] (see Appendixes). The green dashed curve corresponds to the values of $a_{33} = 200$, $a_{34} = 110$ (given in Bohr radii) and the orange dot-dashed curve corresponds to $a_{33} = -200$, $a_{34} = 110$. Red squares and blue dots have the same meaning as in panel (a).

10^{-11} , and thus the two-body collision model does describe the experiment in a self-consistent way.

Applied to the relaxation cross section, the Born approximation (see Appendixes) gives values lower than most experimental ones [the red line in Fig. 3(b)]. Including the interatomic potentials by applying the Fermi's golden rule requires to adjust the scattering lengths a_{33} and a_{34} ; the value $a_{44} = 144a_B$ was measured experimentally and fixed. $a_{33} = 200a_B$, $a_{34} = 110a_B$ (green dashed curve) qualitatively give the dip in the magnetic field 0.90 G, but fail to reproduce large magnitudes of β_{loss} . Larger β_{loss} was achieved for $a_{33} = -200a_B$, $a_{34} = 110a_B$ (orange dot-dashed line), but there are no values of a_{33} , a_{34} for which theoretical curve fits all the experimental data points.

For depolarization collisions, the Born approximation gives cross section independent of the magnetic field [the red line in Fig. 3(a)]. In contrast to this, in the experiment we see that depolarization has clear nonmonotonic behavior with several resonances and up to 4 orders of magnitude higher values. Thus, we conclude that perturbational approaches are insufficient to describe observed collisional rates. In fact, it is immediately evident from strong correlation between depolarization and relaxation rates that one needs to account for coupling between different states. This requires carrying out

coupled-channel calculations. For the present dipolar interactions, they were done for hydrogen [55] and in general can be carried out for alkali metals using packages such as MOLSCAT [56,57]. Such calculations require intricate knowledge of the interatomic potential curves making them hard to realize for lanthanides, because they have complex angular momentum structure leading to large amount of Born-Oppenheimer potentials [58]. This leads to the dense spectrum of Feshbach resonances that occur when energy of the energetically closed channel coincides with the energy of the atomic pair in the entrance channel and subsequent chaotic dependence of the scattering on interaction parameters [20]. The vicinity of Feshbach resonances can both enhance [59] and suppress [60] two-body inelastic collision rates, which probably explains the observed nonmonotonic behavior.

IV. CONCLUSION

Thus, the developed experimental procedure can be exploited to populate specific Zeeman sublevels and investigate spin dynamics. Particularly, in ultracold thulium at the magnetic field of 0.9 G, both the depolarization rate and the loss rate for the $|4, -3\rangle$ state are significantly suppressed by a factor of 1000 and 50, respectively, providing considerable

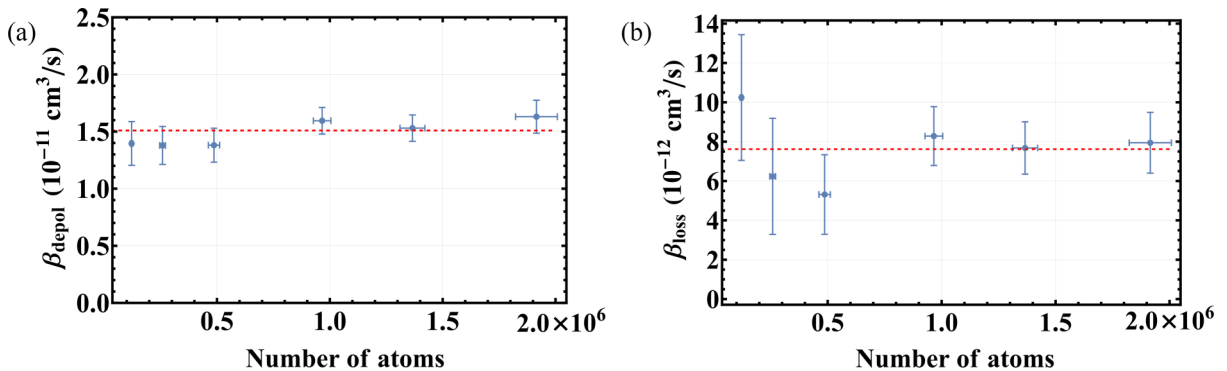


FIG. 4. The depolarization rate β_{depol} [panel (a)] and loss rate β_{loss} [panel (b)] as functions of the initial atomic numbers in the magnetic field $B = 0.66$ G.

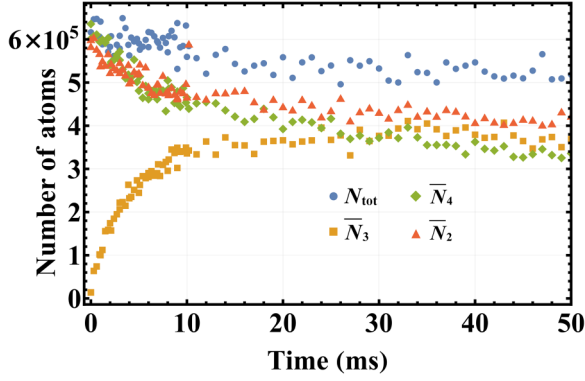


FIG. 5. The number of atoms vs time in the magnetic field $B = 0.588$ G.

lifetime of this state. Such features pave the way for utilizing the Zeeman manifold of the thulium atom to explore the properties of strongly correlated matter.

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DATA AVAILABILITY

The data that support the findings of this article are openly available [61].

APPENDIX A: ATOM DETECTION DETAILS

To account for the imaging detection efficiency dependence on magnetic field, atomic Zeeman state, and probe beam detuning, we performed the following calibration procedure. We prepared atomic ensembles under the same conditions in the ODT and then detected them in two ways: under previously calibrated conditions (magnetic field aligned with resonance probe beam with linear polarization; all atoms are in the state $|4, -4\rangle$ [18]) or under the experimental conditions. Considering atomic ensembles to contain the same numbers of atoms, the detection efficiencies were found as ratios between numbers of atoms in two described cases.

APPENDIX B: MF = -2 DETECTION

To observe atoms in the state $|4, -2\rangle$, we applied $|4, -2\rangle \rightarrow |3, -1\rangle$ π pulse to the atoms that were initially

in the $|4, -3\rangle$ state and detected them as described above (Fig. 5). One can observe $\bar{N}_2$ data points for the number of atoms in all the sublevels except $|4, -2\rangle$. They lie lower than N_{tot} , which corresponds to the appearance of the $|4, -2\rangle$ state.

APPENDIX C: EXTENDED KINETIC EQUATIONS

At the long times, system (1) can be appended with next-order processes as follows:

$$\begin{aligned}\dot{N}_3 &= -\alpha\beta_{\text{loss}}N_3^2 - \alpha\beta_{\text{depol}}(N_3^2 - N_2N_4) \\ &\quad - \frac{\alpha}{2}(\beta_l^{32} + \beta_d^{32})N_2N_3 - \frac{\alpha}{2}\beta_l^{34}N_3N_4, \\ \dot{N}_4 &= \frac{\alpha}{2}\beta_{\text{depol}}(N_3^2 - N_2N_4) - \frac{\alpha}{2}\beta_l^{34}N_3N_4 + \frac{\alpha}{2}\beta_d^{32}N_2N_3, \\ \dot{N}_2 &= \frac{\alpha}{2}\beta_{\text{depol}}(N_3^2 - N_2N_4) - \frac{\alpha}{2}(\beta_l^{32} + \beta_d^{32})N_2N_3, \quad (\text{C1})\end{aligned}$$

where $\beta_l^{ij}, \beta_d^{ij}$ are loss and depolarization rates for additional channels involving collisions of two atoms in $|4, -i\rangle$ and $|4, -j\rangle$ states.

While the large number of additional parameters does not allow to extract all these additional parameters, one could see contribution of additional channels by fitting data at long times with system (C1) [see Fig. 2(b)]. Yet, due to the complication of Eq. (C1) only Eq. (1) was used for further analysis.

APPENDIX D: BORN APPROXIMATION

Magnetic dipole-dipole interaction of two atoms with total moments $\vec{F}_1$ and $\vec{F}_2$ is described by the potential

$$\hat{V}_{dd} = d^2 \frac{\vec{F}_1 \cdot \vec{F}_2 - 3(\vec{F}_1 \cdot \vec{r})(\vec{F}_2 \cdot \vec{r})}{r^3}, \quad (\text{D1})$$

where $\vec{r}$ is the unit vector parallel to the line joining two atoms, $d^2 = \frac{\mu_0(g_F\mu_B)^2}{4\pi}$, g_F is the Lande g factor, μ_0 is vacuum magnetic permeability, and μ_B is the Bohr magneton. Considering the collision of two atoms initially in the state $|4, -3\rangle \otimes |4, -3\rangle$, there is an elastic channel

$$|\text{in}\rangle = |4, -3\rangle \otimes |4, -3\rangle \rightarrow |4, -3\rangle \otimes |4, -3\rangle, \quad (\text{D2})$$

and three inelastic channels

$$\begin{aligned}|\text{in}\rangle &\rightarrow |\text{out}_0\rangle = \frac{1}{\sqrt{2}}(|4, -2\rangle \otimes |4, -4\rangle + |4, -4\rangle \otimes |4, -2\rangle), \\ |\text{in}\rangle &\rightarrow |\text{out}_1\rangle = \frac{1}{\sqrt{2}}(|4, -3\rangle \otimes |4, -4\rangle + |4, -4\rangle \otimes |4, -3\rangle) + \Delta E, \\ |\text{in}\rangle &= |4, -3\rangle \otimes |4, -3\rangle \rightarrow |\text{out}_2\rangle = |4, -4\rangle \otimes |4, -4\rangle + 2\Delta E, \quad (\text{D3})\end{aligned}$$

where $\Delta E = g_F \mu_B B$. To calculate cross sections and collision rates in Born approximation, we expand Eq. (D1) in a spherical basis:

$$\vec{F}_1 \cdot \vec{F}_2 - 3(\vec{F}_1 \cdot \vec{r})(\vec{F}_2 \cdot \vec{r}) = \hat{F}_{1z}\hat{F}_{2z} + \frac{1}{2}(\hat{F}_{1+}\hat{F}_{2-} + \hat{F}_{1-}\hat{F}_{2+}) - \frac{3}{4}(2\hat{z} \cdot \hat{F}_{1z} + \hat{r}_- \hat{F}_{1+} + \hat{r}_+ \hat{F}_{1-})(2\hat{z} \cdot \hat{F}_{2z} + \hat{r}_- \hat{F}_{2+} + \hat{r}_+ \hat{F}_{2-}) \quad (\text{D4})$$

and evaluate the corresponding matrix elements

$$\langle \text{in} | \vec{F}_1 \cdot \vec{F}_2 - 3(\vec{F}_1 \cdot \vec{r})(\vec{F}_2 \cdot \vec{r}) | \text{out}_0 \rangle = -\sqrt{14}(1 - 3\hat{z}^2), \quad (\text{D5})$$

$$\langle \text{in} | \vec{F}_1 \cdot \vec{F}_2 - 3(\vec{F}_1 \cdot \vec{r})(\vec{F}_2 \cdot \vec{r}) | \text{out}_1 \rangle = 18\hat{z}\hat{r}_-, \quad (\text{D6})$$

$$\langle \text{in} | \vec{F}_1 \cdot \vec{F}_2 - 3(\vec{F}_1 \cdot \vec{r})(\vec{F}_2 \cdot \vec{r}) | \text{out}_2 \rangle = -6\hat{r}_-^2. \quad (\text{D7})$$

In the Born approximation, the scattering amplitude is

$$f(\vec{k}, \vec{k}') = -\frac{m}{4\pi\hbar^2} \int e^{i\vec{q}\vec{r}} U(\vec{r}) d^3\vec{r}, \quad (\text{D8})$$

where $\vec{k}$ and $\vec{k}'$ are wave vectors of the colliding pair before and after collision, respectively, and $\vec{q} = \vec{k} - \vec{k}'$. For two bosons, we need to take amplitude in the form $f(\vec{k}, \vec{k}') + f(\vec{k}, -\vec{k}')$, and then the cross section can be found by integration:

$$\sigma(\vec{k}) = \frac{1}{k_i k_f} \int \{|f(\vec{k}, \vec{k}')|^2 + f(\vec{k}, -\vec{k}')^* f(\vec{k}, \vec{k}')\} \times \delta(|\vec{k}'| - k_f) d^3\vec{k}', \quad (\text{D9})$$

$$\sigma = \frac{1}{4\pi} \int \sigma(\vec{k}) d\Omega_{\vec{k}}, \quad (\text{D10})$$

where k_i and k_f are magnitudes of $\vec{k}$ and $\vec{k}'$. After calculations, we arrive at the result [24]:

$$\sigma_{\text{in} \rightarrow \text{out}_0} = \frac{112\pi}{45} \left(\frac{md^2}{\hbar^2} \right)^2, \quad (\text{D11})$$

$$\sigma_{\text{in} \rightarrow \text{out}_1} = \frac{96\pi}{5} \left(\frac{md^2}{\hbar^2} \right)^2 f\left(\frac{k_f^{(1)}}{k_i}\right) \frac{k_f^{(1)}}{k_i}, \quad (\text{D12})$$

$$\sigma_{\text{in} \rightarrow \text{out}_2} = \frac{128\pi}{15} \left(\frac{md^2}{\hbar^2} \right)^2 f\left(\frac{k_f^{(2)}}{k_i}\right) \frac{k_f^{(2)}}{k_i}, \quad (\text{D13})$$

where $f(x) = (1 - \frac{1}{2} - \frac{3}{8} \frac{(1-x^2)^2}{x(1+x)^2} \ln \frac{(1-x)^2}{(1+x)^2})$ and $k_f^{(i)} = \sqrt{k_i^2 + i \frac{m}{\hbar^2} \Delta E}$. Collision rates are found from Eqs. (D11)–(D13) by taking thermal average

$$\beta_{\text{dep}} = \langle \sigma_{\text{in} \rightarrow \text{out}_0} v_{\text{rel}} \rangle_{\text{thermal}}, \quad (\text{D14})$$

$$\beta_{\text{loss}} = \langle (\sigma_{\text{in} \rightarrow \text{out}_1} + \sigma_{\text{in} \rightarrow \text{out}_2}) v_{\text{rel}} \rangle_{\text{thermal}}. \quad (\text{D15})$$

APPENDIX E: FERMI'S GOLDEN RULE FOR DIPOLAR RELAXATION

Expression (D1) for the dipole-dipole interaction potential can be written in terms of irreducible tensor operators with

definitions taken from Ref. [62]:

$$\hat{V}_{dd} = -\frac{d^2}{R^3} \sqrt{\frac{24\pi}{5}} \sum_{m=-2}^{m=2} (-1)^m \{\vec{F}_1 \otimes \vec{F}_2\}_m^2 Y_2^{-m}. \quad (\text{E1})$$

From this expression, matrix elements can be calculated in the molecular basis $|(F_1, F_2), F, m_F, l, m_l\rangle$, where $\vec{F} = \vec{F}_1 + \vec{F}_2$ is the total momentum of two atoms, m_F is its projection on the magnetic field axis, and l, m_l are the momentum of relative orbital atomic motion and its projection on the magnetic field axis:

$$\begin{aligned} & \langle (F_1, F_2), F, m_F, l, m_l | \hat{V}_{dd} | (F_1, F_2), F', m_F', l', m_l' \rangle \\ &= -\frac{d^2}{R^3} \sqrt{6} C_{F, F', F_1, F_2} \times \sum_{m=-2}^{m=2} (-1)^{m-m_l-m_F'} \\ & \times \begin{pmatrix} 1 & 2 & 1' \\ -m_l & -m & m_l' \end{pmatrix} \begin{pmatrix} F & 2 & F' \\ -m_F & m & m_F' \end{pmatrix} \\ & \times \sqrt{(2l+1)(2l'+1)} \begin{pmatrix} 1 & 2 & 1' \\ 0 & 0 & 0 \end{pmatrix}, \end{aligned} \quad (\text{E2})$$

where

$$\begin{aligned} C_{F, F', F_1, F_2} &= (-1)^F \sqrt{5(2F+1)(2F'+1)} \\ & \times \sqrt{F_1(F_1+1)(2F_1+1)F_2(F_2+1)(2F_2+1)} \\ & \times \begin{Bmatrix} F_1 & F_1 & 1 \\ F_2 & F_2 & 1 \\ F & F' & 2 \end{Bmatrix}. \end{aligned} \quad (\text{E3})$$

Selection rules are $m_l + m_F = m_{l'} + m_{F'}$, $|l-l'| = 0, 2$ (with exception $l = l' = 0$) from Eq. (E2) and $|F-F'| = 0, 2$ from Eq. (E3).

For two identical bosons, only partial waves with even l are possible. In the ultracold limit, the main contribution to the scattering cross sections is from $l = 0 \rightarrow l' = 2$ [46]. In the molecular basis, the $|\text{in}\rangle$ and $|\text{out}\rangle$ states can be written as

$$\begin{aligned} |\text{in}\rangle &= \left(\sqrt{\frac{8}{15}} |F=8, m_F=-6, l=0, m_l=0\rangle \right. \\ & \quad \left. - \sqrt{\frac{7}{15}} |F=6, m_F=-6, l=0, m_l=0\rangle \right) \psi_{\text{in}}(r), \\ |\text{out}_1\rangle &= |F=8, m_F=-7, l=2, m_l=1\rangle \psi_{\text{out}}^1(r), \\ |\text{out}_2\rangle &= |F=8, m_F=-8, l=2, m_l=2\rangle \psi_{\text{out}}^2(r). \end{aligned} \quad (\text{E4})$$

To calculate the loss rate, we use the Fermi's golden rule

$$\Gamma_l = \frac{2\pi}{\hbar} \sum_{j=1}^2 |\langle \text{in} | \hat{V}_{dd} | \text{out}_j \rangle|^2. \quad (\text{E5})$$

Radial parts of wave functions $\psi_{\text{in}}(r)$ and $\psi_{\text{out}}^{(1,2)}(r)$ are found from the solution of radial Schrodinger's equation with

the van der Waals potential $\frac{C_6}{r^6}$ with hard wall at $r = r_0$. We used the renormalized Numerov method to numerically solve radial equations. Output channel radial wave functions $\psi_{\text{out}}^{(1,2)}(r)$ are calculated by outward propagation and energetically normalized by matching with outgoing spherical waves $\frac{1}{r} \sin(k_{\text{out}}r - \pi + \delta_l)$ at large r . Radial wave functions $\psi_{\text{in}}(r)$ are calculated by simultaneous outward and inward propagations. Temperature dependency arising from the dependence of $\psi_{\text{in}}^{(i)}(r)$ on trap levels occupation could be lifted [46] and one finally gets the relaxation rate in the form

$$\beta_l = \frac{(4\pi)^{\frac{5}{2}}}{\hbar} \frac{a_{\text{HO}}^3}{2\sqrt{2}} \sum_{j=1}^2 |\langle \text{in} | \hat{V}_{dd} | \text{out}_j \rangle|^2, \quad (\text{E6})$$

where $a_{\text{HO}} = \sqrt{\frac{\hbar}{m\omega}}$ is the harmonic oscillator length and m is the Tm atomic mass. There are no experimental data on

Tm-Tm potential curves, but the value of scattering length for two atoms colliding in $|4, -4\rangle$ states is known: $a_{44} = 144 a_0$ [20]. The C_6 value can be calculated *ab initio* using the Casimir-Polder formula

$$C_6 = -\frac{3}{\pi} \int_0^\infty \text{Re}(\alpha_{\text{sc}}(i\omega))^2 d\omega \quad (\text{E7})$$

and was found to be approximately 1628 a.u. Scalar polarizability α_{sc} was calculated using data on atomic spectra obtained from The National Institute of Standards and Technology (NIST) [63]. As fit parameters, we used wall positions r_{33} for input channels and r_{34}, r_{44} for output channels 1 and 2, respectively. Values of r_0 can be matched with the analytical expression for the scattering amplitudes [64], which we label as a_{33}, a_{34} , and a_{44} .

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