

Probing the topological protection of edge states in multilayer tungsten ditelluride with the superconducting proximity effect

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The topology of WTe_2 , a transition-metal dichalcogenide with large spin-orbit interactions, is thought to combine type II Weyl semimetal and second-order topological insulator (SOTI) character. The SOTI character should endow WTe_2 multilayer crystals with topologically protected helical states at their hinges, and, indeed, one-dimensional states have been detected thanks to Josephson interferometry. However, the immunity to backscattering conferred to those states by their helical nature has so far not been tested. To probe the topological protection of WTe_2 edge states, we have fabricated superconducting quantum interference devices (SQUIDs) in which the supercurrent through a junction on the crystal edge interferes with the supercurrent through a junction in the bulk of the crystal. We find behaviors ranging from a symmetric SQUID to asymmetric SQUID patterns, including one in which the modulation by magnetic field reveals a sawtoothlike supercurrent versus phase relation for the edge junction, demonstrating that the supercurrent at the edge is carried by ballistic channels over 600 nm, a telltale sign of the SOTI character of multilayer WTe_2 .

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I. INTRODUCTION: DETECTING HELICAL STATES IN WTe_2

Three-dimensional (3D) second-order topological insulators (SOTIs) are a relatively newly discovered phase of matter, which possesses the interesting property of hosting topologically protected one-dimensional (1D) states. These states stem from a double band inversion in the material, and spatial symmetries in the bulk that are broken at the crystal's surfaces, turning them insulating, with surface gaps that can alternate between being inverted and not. The topological one-dimensional states are expected at the hinges between such surfaces. One of the most interesting properties of these states is their ballistic conduction over long distances even in the presence of disorder. This is due to the protection offered

by the spin-momentum locking characteristic of the helical nature of topological quantum spin Hall states and contrasts with the Anderson localization by disorder of 1D states that have no such topological protection. Their identification is not straightforward, however, in part because more numerous nontopological states often coexist with the 1D helical states.

Among the proposed 3D SOTIs, WTe_2 is a transition-metal dichalcogenide (TMD) whose monolayer has been shown to possess a two-dimensional (2D) topological insulator (TI) character, with a large insulating bulk gap (in the tens of meV range), and quantum spin Hall edge conduction [1–6]. The 2D TI character stems from inversion symmetry combined with band inversion at the gamma point. The topology of multilayer WTe_2 is less clear. Its T_d phase (symmetry group $Pmn2_1$) is characterized by a double band inversion at the gamma point. Combined with the screw symmetry along the c axis, this can lead to a bulk 3D topological state in which helical modes can exist at surface domain walls and hinges. The precise configuration of these helical modes depends on a combination of surface band inversion and disorder. In practice, like in Bi_4Br_4 [7–10], helical states are likely to appear along the a and b directions on alternating terraces or step edges (see Figs. 1 and 30 and Refs. [11–14]). Observing these helical modes can, however, be made difficult by the (type II Weyl) semimetallic nature of WTe_2 : Electron and hole pockets at the Fermi level provide a high density of trivial bulk carriers that may obscure the signatures of the fewer topological modes. Nevertheless, signatures of topological hinge states have been reported: Optical Kerr rotation microscopy detected spin polarization confined to hinges of WTe_2 [15]. In mesoscopic samples with superconducting contacts, proximity-induced superconductivity enhances the relative contribution of topological states

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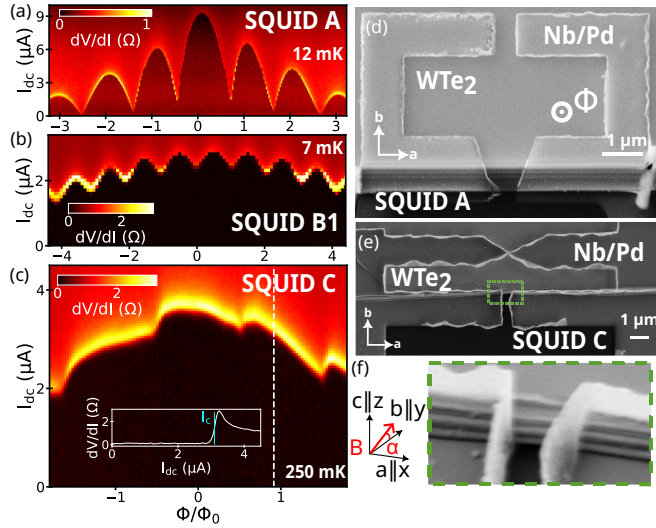


FIG. 1. WTe₂-based superconducting quantum interference devices (SQUIDs) A, B1, and C and their interference patterns. (a)–(c) Color-coded differential resistance dV/dI as a function of the dc current I_{dc} through the bulk and edge junctions in parallel, and the magnetic flux Φ through the SQUID loop, in units of the flux quantum $\Phi_0 = h/2e$. The critical current I_c corresponds to the boundary between the zero (black) and finite (red) resistance regions. Dilution refrigerator temperatures are indicated in the graphs. (a) SQUID A displays a practically full modulation of I_c , indicative of a symmetric SQUID. (b) SQUID B1 displays a small sinusoidal-like modulation of I_c . (c) SQUID C's I_c (defined in inset line cut at fixed Φ , indicated by white dashed line) has a sawtoothlike modulation, atop a smooth background, a telltale sign of ballistic transport. The leads resistance has been subtracted. (d), (e) Scanning electron micrograph (SEM) of SQUIDs A and C, with (f) a close-up of SQUID C's edge junction, and a sketch of the magnetic field B 's orientation relative to the crystal axes. The magnetic field B is in the (b, c) plane, at a small angle α with the b axis. Thus, $\Phi = B_z S \simeq \alpha BS$, with $S \simeq 40 \mu\text{m}^2$ the SQUID loop area, which includes focusing effects (see Appendix D). $\alpha \simeq 0.02$ rad is deduced from the oscillation period B_0 via $\alpha = \Phi_0/B_0 S$.

with respect to trivial states [16]. The critical current (maximal induced supercurrent) dependence on magnetic field, also called Josephson interferometry, has been exploited to reveal supercurrents flowing at the edges of a sample rather than in the bulk [17–21]. In addition, the critical current's decay with field depends on the number and width of the supercurrent-carrying modes: Nanometer-size 1D channels can carry supercurrent over micrometers up to fields of several teslas, as was demonstrated in bismuth nanowires [16,17]. In WTe₂, Josephson interferometry suggests that the supercurrent is carried preferentially along the a -direction crystal edges [21,22]. However, Josephson interferometry cannot measure the degree of protection against backscattering, a key signature of helical transport. Instead, the measurement of a supercurrent-versus-phase relation (CPR) can provide such a signature, because its shape is exquisitely sensitive to the transport regime: CPRs of junctions with no backscattering display a jump at π phase difference, in stark contrast to the smooth, sinusoidal-like CPRs of tunnel or diffusive junctions. A sawtooth-shaped CPR is the signature of a long

ballistic Josephson junction and has been found in bismuth- and BiSb-based Josephson junctions [16,19]. Interestingly, CPRs with jumps at π are robust to the case of an imperfect interface between a superconductor and a non-superconducting, or “normal” metal (an NS interface) only if there is topological protection [23,24] (see Appendix A). In addition, CPRs can probe the dynamics of Andreev states, providing a test of their helical character [25].

II. DETERMINATION OF THE SUPERCURRENT VERSUS PHASE RELATION USING ASYMMETRIC SQUIDS

To determine the CPR, we fabricate SQUIDs in which the critical current results from interference between one Josephson junction on an a -oriented crystal edge and another Josephson junction in the bulk of the crystal's (001) surface. This configuration is somewhat similar to Ref. [26], except that we design devices with supercurrents small enough that distortions of the critical current by screening effects are negligible (see Appendix E). We find different behaviors, ranging from symmetric to asymmetric SQUID patterns. The interference pattern of one asymmetric SQUID, SQUID C, with the shortest edge junction, reveals a sawtooth-shaped CPR. The temperature and field dependences of SQUID C's critical current, and notably its harmonic content, demonstrate that the supercurrent at the edge is carried quasiballistically over 600 nm, a telltale sign of the SOTI character of WTe₂. From the decay with field of the supercurrent, we determine that the edge states extend less than 2 nm in the c direction, and 20 nm in the (a, b) plane. The main text focuses on this SQUID C, while in the Appendix D we present the other SQUIDs and offer explanations for the range of behaviors observed.

The SQUIDs are formed by sputter deposition of an (8-nm/80-nm)-thick Pd/Nb superconducting film onto 30- to 300-nm-thick WTe₂ flakes obtained by exfoliation in a glove box of high-quality crystals with a residual resistivity ratio exceeding 1000 [27]. The use of Pd favors good contacts to the WTe₂, thanks to the superconducting PdTe_x compound that forms by interdiffusion of Pd in WTe₂ [22,28,29]. The SQUIDs are designed such that one junction, formed in the bulk of the crystal surface, acts as a reference junction with a higher critical current I_c^{ref} , while the other junction, on the crystal edge, is expected to have a smaller critical current I_c^{edge} , predominantly carried by the edge states (see Fig. 1). For such an asymmetric SQUID in a magnetic field B with B_z projection perpendicular to the SQUID loop plane, the critical current reads [30]

$$I_c(B) \simeq I_c^{\text{ref}}(B) + I_0(B)f\left(\varphi_0 - 2\pi \frac{B_z S}{\Phi_0}\right),$$

with $I^{\text{edge}}(B, \varphi) = I_0(B)f(\varphi)$ the edge junction's CPR, S the area defined by the SQUID loop, $\Phi_0 = h/2e$ the superconducting flux quantum, and φ_0 the phase at which the reference junction's CPR is maximal. For such an asymmetric SQUID, I_c is then practically a direct measurement of the edge junction's CPR. In contrast, if the SQUID is symmetric, $I_c^{\text{edge}} \simeq I_c^{\text{bulk}}$, a different, even-in-field interference pattern emerges, with full periodic cancellation of I_c in the case of a sinusoidal CPR.

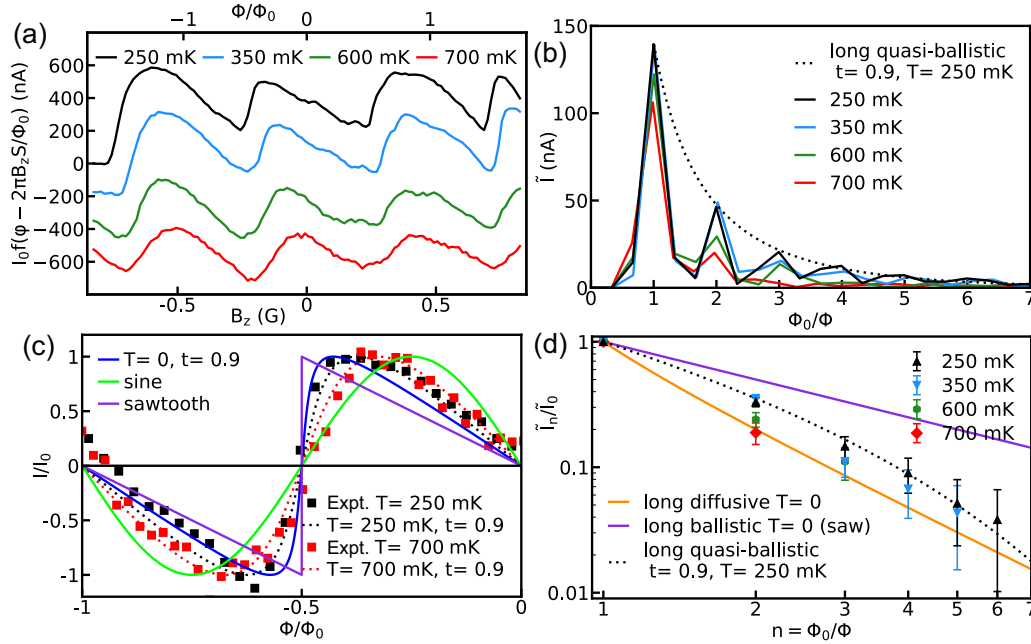


FIG. 2. Temperature dependence and harmonic content of SQUID C's edge junction CPR. (a) CPR at 250, 350, 600, and 700 mK, obtained by high-pass filtering the SQUID's critical current. Curves are shifted vertically for clarity. (b) Fast Fourier transform (FFT) of curves in panel (a), and (dashed black line) quasiballistic dependence given by Eq. (2), $\tilde{I}_n(x) \propto e^{-a(250 \text{ mK})x}/x$, $x = \Phi_0/\Phi \geq 1$, where $a(250 \text{ mK}) = 0.384$. Six harmonics are visible at 250 mK, three at 600 mK, and two at 700 mK. (c) Symbols: CPR over one period, at 250 and 700 mK, compared to the quasiballistic expressions (1) and (2) with $t = 0.9$ and $E_{\text{Th}}^{\text{edge}} = 6.5$ K (full lines). The quasiballistic result at $T = 0$, rounded solely by $t = 0.9$, is plotted as a blue line. The sawtooth CPR, corresponding to $t = 1$ and zero temperature, is plotted in purple. The rounding of the CPR by temperature in the experiment is clear, but the CPR is nonetheless much more skewed than a sinus, shown in green. (d) Relative amplitude of the FFT peaks, symbols, on a log-log scale, compared to the zero-temperature predictions of the long ballistic CPR FFT (purple line), Eqs. (1) and (2) for $T = 250$ mK, using $t = 0.9$ and $E_{\text{Th}}^{\text{edge}} = 560 \mu\text{eV} \simeq 6.5$ K (black dotted line) [16,31]. The decay with field of the bulk junction's critical current limits the FFT resolution. We estimate a 3 nA uncertainty for $\tilde{I}_n$ (see Appendix C).

Out of the six bulk/edge SQUIDs we have measured, SQUID A displays such a full modulation of I_c [Fig. 1(a)]. SQUIDs B1, B2, D1, and D2 display a small sinusoidal modulation of I_c (see Fig. 1(b) and Fig. 27), which disappears in fields on the order of 100 G (see Appendix D). This suggests that the junctions with the smaller current have a sinusoidal CPR. SQUID C, with the shortest edge junction, displays a sawtooth-shaped modulation (Fig. 2), revealing that the edge junction's CPR is that of a ballistic junction. In contrast to I_c^{bulk} , which is divided by 2 on a B_z field scale of 2 G and a temperature scale of 300 mK, I_c^{edge} is practically unchanged up to hundreds of gauss, and decreases by only 25% between 20 and 700 mK (Figs. 2, 3, and 19), a robustness that we assign to topological protection.

III. ANALYSIS OF THE ASYMMETRIC SQUID WHICH REVEALS BALLISTIC EDGE CONDUCTION

This SQUID C, shown in Figs. 1(e) and 1(f), is based on a 223-nm-thick WTe₂ flake. The superconducting contacts on the edge junction are 600 nm apart, with a lateral overlap on the flake that is smaller than 300 nm (see also Fig. 23). The bulk junction has an hourglass shape, a shape specifically chosen to allow the investigation of high field properties. Figures 1(c) and 3(a) show SQUID C's differential resistance as a function of dc current and field. An $I_0 \simeq 200$ nA, 2.35 mT periodic component, with a clear sawtooth

shape, is superimposed on a larger (several microamperes), more slowly varying background, which we identify as I_c^{bulk} . Despite its unconventional hourglass shape, this bulk junction behaves as a wide and long disordered Josephson junction (see Appendix C). $I_c^{\text{bulk}}(B)$ has the Fraunhofer-like dependence expected of a diffusive junction with an aspect ratio $W/L = 1.5$ [32,33]. $I_c^{\text{bulk}}(T)$ also follows the roughly exponential decay expected of a long diffusive junction, $I_c^{\text{bulk}}(T) \simeq \exp(-T/T_c)$, where the characteristic temperature T_c is related to the Thouless energy via $T_c = 3.8E_{\text{Th}}^{\text{bulk}}$ [34,35]. This yields $T_c \simeq 250$ mK, and $E_{\text{Th}}^{\text{bulk}} \simeq 65$ mK, much less than the superconducting gap, which is greater than 2 K, confirming the disordered long junction regime. Finally, combining $I_c^{\text{bulk}} \approx 7 \mu\text{A}$ with the normal state resistance $R_N \simeq 1 \Omega$ yields $eR_N I_c^{\text{bulk}} = 80$ mK, four to ten times less than the perfect interface value $eR_N I_c^{\text{bulk}} = 10.82E_{\text{Th}}^{\text{bulk}}$, indicating that the transparency of the NS interface is less than 1. More details on the bulk junction are given in Appendix C.

The edge junction has strikingly different characteristics. Its supercurrent is easier to extract above 250 mK, when I_c^{bulk} is small enough that its field dependence is not a hindrance (see Appendix C). We find that I_c^{edge} is much less sensitive to temperature and magnetic field than I_c^{bulk} . I_c^{edge} decreases by only 25% between 20 and 700 mK (see Fig. 2) and is almost unaffected by magnetic fields up to hundreds of gauss, $I_c^{\text{edge}} \simeq I_0 \simeq 200$ nA (see Appendix C).

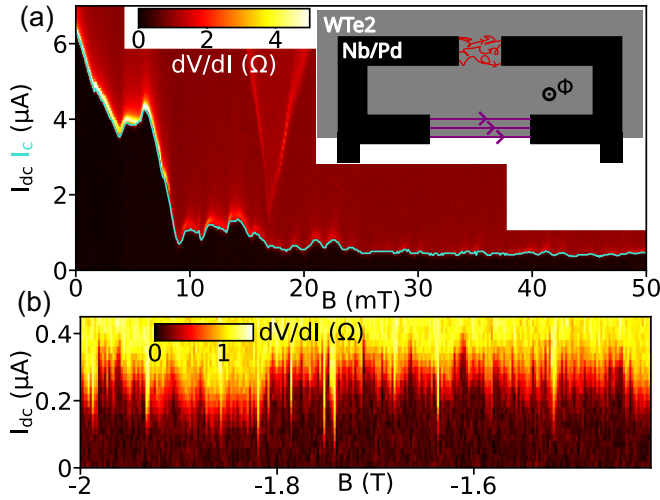


FIG. 3. Evolution with magnetic field of SQUID C's interference pattern at $T = 20$ mK. (a) Color-coded differential resistance as a function of dc current I_{dc} and magnetic field B between 0 and 50 mT, and critical current outlined by the turquoise line. The bulk junction is the reference junction in this field range. Its microampere-range critical current decays with a 10-mT-period Fraunhofer-like behavior, typical of a wide junction. The edge junction's CPR adds a 2-mT-period sawtooth-shaped modulation. B is predominantly along the crystal's b axis. B_z , the component along the c axis, is 2% of B . (b) Interference effects at higher field. The bulk junction's critical current is negligible and the SQUID's critical current is solely due to the edge junction (a 150 nA current offset has been removed; see Appendix C). Inset: Sketch of the asymmetric SQUID C, with the reference junction (diffusive trajectories in red), and the edge junction, here with three ballistic 1D edge states (in purple).

Note that above those temperatures and fields, I_c^{bulk} is too small to consider that the SQUID's I_c reflects the CPR of the edge junction, as discussed later. This very slow decrease of $I_c^{\text{edge}}(T)$ is a first indication of a much greater Thouless energy and thus a cleaner regime than the bulk junction. We compare $I_c^{\text{bulk}}(T)$ and $I_c^{\text{edge}}(T)$ in Fig. 19.

We now focus on the edge junction's CPR. It is extracted by high-pass filtering the SQUID's critical current $I_c(B)$ and plotted in Fig. 2(a) for four temperatures. A sawtooth shape is clearly visible, far different from the sinusoidal CPR of conventional Josephson junctions. Up to six harmonics are visible in the CPR's FFT at low temperature [see Fig. 2(b)]. Each harmonic is plotted at different temperatures in Fig. 2(d) and compared to the predicted zero-temperature ballistic and diffusive limits. We find that the edge junction's CPR harmonics decay more slowly than the CPR of a diffusive junction, confirming that the edge states are not in a diffusive regime. We argue that the high harmonic content and its robustness is an indication of topological protection, by contrasting the rounding (by an imperfect interface transmission) of the CPR of topologically protected conductors and nonprotected conductors. Let us first consider a Josephson junction with a perfect conductor (no backscattering) as the weak link. The junction's CPR features a jump at phase difference $\varphi = \pi$ [see Figs. 2(c) and 4]. This jump translates into a high harmonic content, with the amplitude of n th harmonic decaying

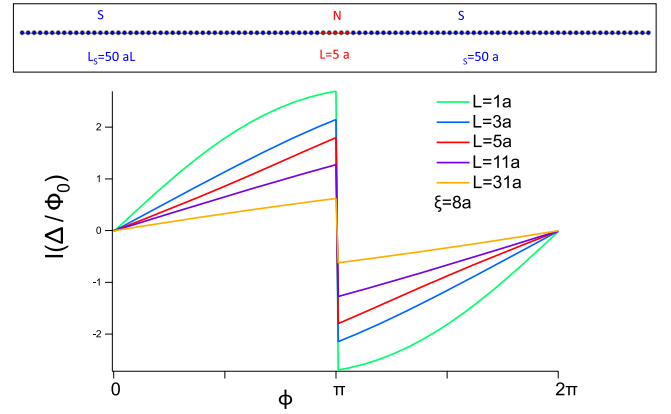


FIG. 4. Tight-binding simulation of a ballistic 1D superconductor/normal/superconductor (SNS) chain with varying number of normal sites, and its zero-temperature supercurrent-vs-phase relation. One can notice that the linear, sawtooth CPR of a so-called long ballistic junction is achieved for a normal chain length longer than three sites, well below the long junction limit of $\xi \gg L$. The CPR made of branches of sinus, the so-called short ballistic junction CPR, is only verified for a one-atom long chain. In this calculation, the hopping energy is 4, superconducting gap $\Delta = 1$, Fermi energy $E_F = 0$, and the superconducting reservoirs are each 50 sites long. The superconducting coherence length is eight sites long, $\xi = 8a$, and the disorder on on-site energy is taken to be 10^{-4} . L is the length of the normal chain, $L = 1a, 3a, 5a, 11a$, and $31a$, with a the interatomic distance in the chain.

as $1/n$ [see Fig. 2(d)]. In the nontopologically protected case, backscattering within the weak link as well as at the NS interface causes rounding of the CPR and decrease of the critical current. The harmonics decay faster (as $1/n^2$ in a diffusive junction; see Figs. 2(d) and 5). In contrast, for a topological

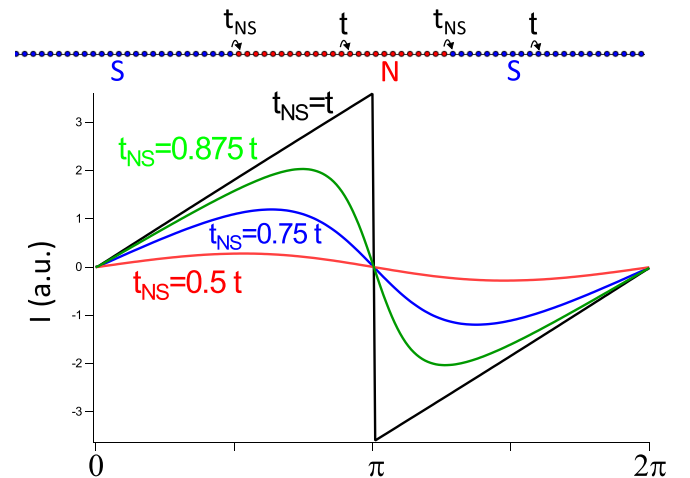


FIG. 5. Nontopological 1D Josephson junction without disorder, for varying interface transmission t_{NS} . As sketched, the tight-binding model contains two 1D, disorderless superconducting electrodes 50 sites long and one 1D disorderless N chain with 25 sites. The Fermi energy = 0, nearest-neighbor hopping $t = 4$, $\Delta = 1$, and t_{NS}/t varies between 1 and 0.5. The jump at phase difference π is rounded as soon as the interface transmission is not perfect, in contrast with the topological junction case, illustrated in Fig. 6.

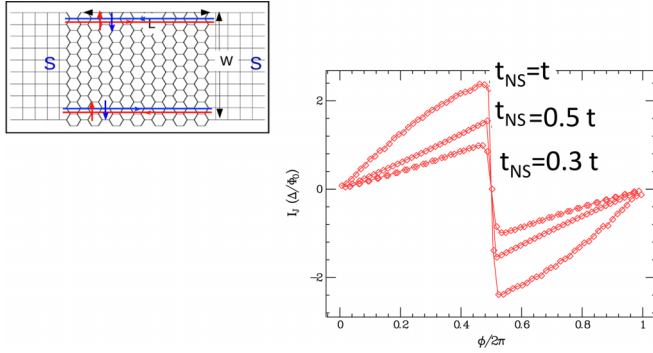


FIG. 6. Topological 2D Josephson junction with small disorder, realized with the Kane-Mele model, as presented in detail in Ref. [23]. The CPR of the quantum spin Hall 1D helical states is presented for varying interface transmission t_{NS} . We find that, in contrast to the 1D chain presented in Fig. 5, which has no disorder but no topological protection, in the quantum spin Hall case presented here, the jump at phase difference π is not rounded when the NS interface transmission decreases. Only the amplitude of the jump decreases. This is similar to what was published in Refs. [23,24].

junction, it has been shown [23,24,36,37] that an imperfect interface transmission does not round the CPR, but merely decreases the supercurrent amplitude (Fig. 6). Therefore, a jump in the CPR at π is strongly suggestive of topological states. Of course, the CPRs of topological materials can be rounded by several factors such as interactions, coupling between helical states with opposite helicities, or the junction's electromagnetic environment. We also note that a CPR with a discontinuity will also be rounded in an asymmetric SQUID measurement, as pointed out by Refs. [26,38,39], and illustrated in Fig. 7.

Quantitatively, the rounding of the CPR by temperature T , which translates into a stronger T decay of the higher harmonics, can be analyzed to estimate the edge junction's Thouless energy E_{Th}^{edge} as well as the zero-temperature, intrinsic (i.e., without contacts) transmission t through the edge state. t characterizes a small avoided crossing of the Andreev levels at π near zero energy, causing a zero-temperature rounding of the CPR. Both temperature and t contribute to the decay of the n th harmonic $\tilde{I}_n$, according to

$$I^{edge}(\varphi) = \sum_n \tilde{I}_n \sin n\varphi = I_0 \sum_n e^{-a(T)n} \frac{(-1)^n}{n} \sin n\varphi, \quad (1)$$

with the damping coefficient $a(T)$ relating to t via [16]

$$e^{-a(T)n} = t^n e^{-n2\pi k_B T / E_{Th}^{edge}}, \text{ i.e.,} \\ a = 2\pi k_B T / E_{Th}^{edge} - \ln(t). \quad (2)$$

Fitting at each T the ratio $\tilde{I}_n / \tilde{I}_1 = e^{-a(n-1)/n}$ for $n \in [1, 6]$ with a , the only fitting parameter yields $E_{Th}^{edge} = 560 \pm 75 \mu\text{eV}$ and $t = 0.89 \pm 0.04$ (see Appendix C). Thus, E_{Th}^{edge} is two orders of magnitude greater than E_{Th}^{bulk} , i.e., an order of magnitude more than expected if both junctions were in the same (long diffusive) regime (given the factor of 3 difference in the junction lengths). The high E_{Th}^{edge} is thus a clear indication that the edge junction is in a quasiballistic regime.

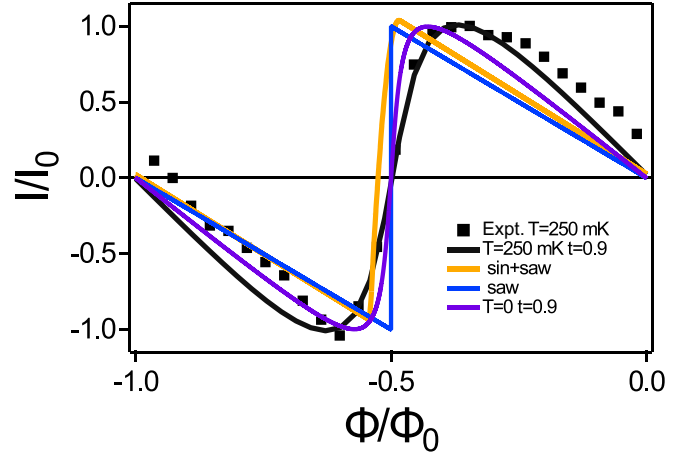


FIG. 7. Different causes for the CPR appearing rounded. This figure compares (1) the sharp sawtooth result, in blue, which is the CPR of a long ballistic junction at $T = 0$; (2) the quasiballistic CPR in which the rounding is given by the coefficient $t = 0.9$ (see the main text). The CPRs are shown at $T = 0$ in purple and $T = 250 \text{ mK}$ in black, for an $E_{Th} = 6.5 \text{ K}$, along with the $T = 250 \text{ mK}$ experimental CPR shown as black squares; and (3) in orange the critical current expected of an asymmetric SQUID in which the reference junction has a sinusoidal CPR and the weak link has a sawtooth CPR with an amplitude 30 times smaller than the reference junction, as in the experiment. The critical current reflects the sawtooth CPR of the weak link over most of the flux region, but in a (small) flux region around which the sawtooth jumps, the critical current varies like the CPR of the reference junction rather than the CPR of the weak link junction.

In fact, E_{Th}^{edge} determined this way agrees with the ballistic expression $E_{Th,ball}^{edge} = \hbar v_F / L^{edge} \simeq 520 \mu\text{eV}$, given the edge junction length $L^{edge} = 600 \text{ nm}$ and the estimated Fermi velocity of 1D states $v_F \simeq 5 \times 10^5 \text{ m s}^{-1}$ [40]. The high value of t is also suggestive of a topological protection. The fact that $t \simeq 0.9$ rather than 1 may be attributed to a small coupling between helical states a distance W apart, which causes an avoided crossing of Andreev levels at π that is proportional to $\exp(-W/\xi)$, with ξ the superconducting coherence length [23]. Evaluating ξ in the ballistic junction regime yields $\xi = \frac{\hbar v_F}{\pi \Delta} \approx 200\text{--}600 \text{ nm}$, for $\Delta \simeq 170\text{--}500 \mu\text{eV}$ (the precise value of Δ at the contact is hard to determine; see Appendix C). Thus, a coupling between edge states can be significant if they are less than 100 nm apart, as is the case for some of the step edges [see Figs. 1(f) and 23]. The coupling of helical states to trivial (nonhelical) states would also round the CPR and thus decrease t .

We now discuss the number of helical states that carry the edge supercurrent. The expected critical current of a single helical state [41] in the long or short junction regime is

$$i_{ch}^{long} = \frac{ev_F}{2L^{edge}} = \frac{eE_{Th}^{edge}}{2\hbar} \approx 68 \pm 9 \text{ nA}, \\ i_{ch}^{short} = \frac{e\pi\Delta}{h} \approx 20 - 60 \pm 10 \text{ nA}$$

for $\Delta \simeq 170\text{--}500 \mu\text{eV}$. Thus, the total supercurrent through the edge junction, 200 nA, must be carried by three or more

helical states, depending on the NS interface quality. Such states could be located at the step edges visible in the sample images of Figs. 1 and 23. We note that the edge junction length $L^{\text{edge}} = 600 \text{ nm}$ is between one and three times ξ , corresponding to an intermediate regime, between short and long, that is characterized by a sawtooth-shaped CPR (see Appendix A). The exact number of channels as well as the interface transmission distribution is difficult to determine. More helical channels are required to carry the supercurrent if the NS interface transmission t_{NS} is smaller than 1, since the supercurrent carried by each helical channel is reduced (roughly as the transmission amplitude $\sqrt{t_{\text{NS}}}$, in contrast to the stronger reduction of nonhelical channels, which varies as t_{NS} [37,42]). We can, however, posit that the number of channels is not too large, given the interference pattern we measure: A large number of channels would lead to scrambled interference at relatively low magnetic field, in contrast to the measurement, as we now discuss.

With increasing field, the bulk and edge junctions swap roles (Fig. 3). At low field, the bulk junction acts as the reference junction, due to its order-of-magnitude larger critical current, and the modulation of the SQUID's I_c is related to the edge junction's CPR. However, a relatively low field of a few tens of millitesla causes I_c^{bulk} to decay in an oscillatory, Fraunhofer-like manner, because of interference between diffusive trajectories within the bulk junction [33]. In contrast, I_c^{edge} is unaffected by field and becomes the largest current above 100 mT. The SQUID's I_c is then in fact dominated by I_c^{edge} , and the CPR of the bulk junction is expected to modulate I_c , a small modulation that is not visible. One can notice an aperiodic modulation on the scale of a few hundred milliteslas at high field [see Fig. 3(b)]. Such modulations must be due to interference between edge channels. Since two channels would cause more regular interference, we believe that at least three channels are likely (see Appendix C for a discussion of interference patterns in different edge state configurations). Such interference could occur due to orbital dephasing by the in-plane field, generating a flux between states on the edges of different atomic steps. It could also be due to Zeeman dephasing by different g factors, given the large in-plane field.

Finally, the lateral extension of the edge states can be estimated by noting that the critical current decay due to orbital dephasing by the magnetic field occurs on a scale of a flux quantum through the edge state area. The 3 T scale in plane and the 0.1 T scale along the c axis suggest that the hinge states extend over 2 nm in the ac plane and 30 nm in the ab plane, demonstrating the 1D character of the hinge states.

IV. CONCLUSION

To conclude, we have probed six junctions at the edges of high-quality multilayer WTe_2 flakes. The shortest, 600-nm-long junction displays a strongly nonsinusoidal CPR, with a steep slope at π that is characteristic of quasiballistic channels. At least three channels carry the supercurrent, in magnetic fields up to 3 T, pointing to narrow edge states that could be situated at the atomic step edges of the flake. These results support the hypothesis of a SOTI character of WTe_2 , as expected given its double band inversion and screw symmetry. The sinusoidal CPR found for some SQUIDs can be explained

by several factors: Edge states may simply not systematically be contacted by the superconducting contacts, so that the sinusoidal CPR would be that of disordered trivial bulk states. Even if contacted, the edge states in those other, longer edge junctions would carry smaller critical currents, which could cause a sizable rounding of their CPR by a dissipative environment such as the trivial bulk states of WTe_2 . The fact that helical edge states can be found in multilayer samples, which are easier to fabricate than WTe_2 monolayers, makes multilayer WTe_2 interesting candidates for the exploration of 1D helical states, for instance, via microwave spectroscopy.

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DATA AVAILABILITY

The data that support the findings of this article are openly available [43].

APPENDIX A: CONSIDERATIONS ON THE SHAPE OF THE CPR IN DIFFERENT SITUATIONS

1. Transition from short to sawtooth-shaped CPR

As displayed in Fig. 4, the sawtooth-shaped CPR characteristic of a long ballistic junction in fact develops in rather short junctions, even shorter than the superconducting coherence length, and is not restricted to the regimes $\xi \gg L$. In fact, it is the so-called short-junction CPR that is rarest, and restricted to junctions just a few atoms long.

2. Contrasting the effect of an imperfect NS interface on the CPR of a ballistic junction in the topological versus nontopological case

In this Appendix, we illustrate how disorder affects differently the CPR of a ballistic junction depending on whether or not topological protection causes the ballistic behavior. As demonstrated in Refs. [23,24], disorder at the NS interface affects differently the harmonics content of the CPR for a trivial or a topological junction. Because backscattering by nonmagnetic impurities in the topological wire is forbidden [36], topological (i.e., spin-momentum-locked, helical) channels are expected to exhibit a perfect Andreev reflection at the NS interface even if the interface is not perfectly transparent, $t_{\text{NS}} < 1$. One therefore expects a robust discontinuity at phase

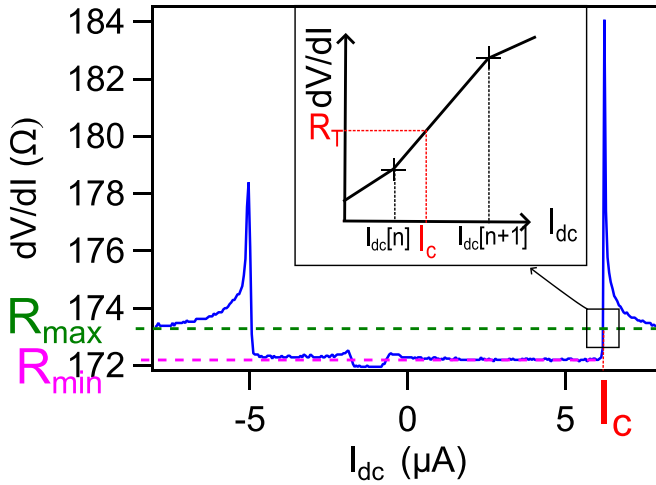


FIG. 8. Differential resistance dV/dI as a function of dc current for an upward and a downward sweep at $B = 0$ and at the lowest temperature, around 20 mK.

π in the CPR at zero temperature (i.e., same as for $t_{NS} = 1$) whose amplitude, however, decreases as t_{NS} decreases. This contrasts with the case of ballistic trivial (nontopological) junctions, whose harmonics content is very sensitive to the NS interface transmission: In such junctions, the discontinuity at π gets rounded by an imperfect interface, and the amplitude of the critical current decreases faster with t_{NS}^2 . Figures 5 and 6 compare the effect of decreasing NS interface transmission on two ballistic SNS junctions, a topologically protected one and a nonprotected one. It is clear that the sawtooth shape is preserved in the case of the topologically protected junction, but not in the other junction, in which rounding of the CPR appears as soon as t_{NS} is not 1.

APPENDIX B: ROUNDING CAUSED BY THE ASYMMETRIC SQUID CONFIGURATION

As pointed out by Refs. [26,38,39], a CPR with a discontinuity will appear rounded in an asymmetric SQUID measurement, because the phase of the reference junction adjusts to avoid the discontinuous decrease in the supercurrent of the weak junction. This causes the SQUID's critical current to reflect the CPR of the reference junction over a (small) portion of the flux variation, as illustrated in Fig. 7. Because this effect is smaller than the intrinsic CPR rounding given by the $t = 0.9$ coefficient, also plotted in the figure, even at zero temperature, we can safely attribute the rounding in the experiment to the edge junction's intrinsic small rounding of the sawtooth CPR.

APPENDIX C: ADDITIONAL DATA AND INFORMATION ON DATA PROCESSING FOR SQUID C

1. Extraction of the current-phase relation

The current-phase relation is obtained from the measurement of $\frac{dV}{dI}(B, I_{dc})$ as a function of magnetic field B and dc bias current I_{dc} , measured using a standard lock-in measurement technique. One obtains typically curves like the one in Fig. 8.

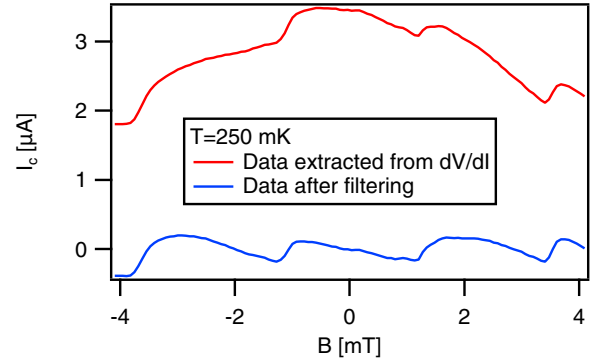


FIG. 9. Red: Variations with field of the “raw” critical current extracted from the differential resistance as a function of dc current, according to the procedure described in the text. Blue: CPR obtained after filtering of the raw critical current curve. These curves correspond to the 250 mK data.

We note a feature in the dV/dI measurement around $I_{dc} \approx -1.3 \mu A$. We attribute it to the switching to the normal state of a junction in series with the measured SQUID. This junction is most likely formed between this SQUID and a second, nearby SQUID, which happened to be connected to a line grounded halfway up the dilution refrigerator. This causes an unbalance in the thermoelectric voltages of the in and out measurement lines, generating a net voltage, and explaining why the feature is not centered around $I_{dc} = 0$.

We also find a small offset current that is mostly visible in the high-field data. This current may be due to a supercurrent along edges of the junction outside the junction area. Its magnitude can be estimated to vary between 100 and 200 nA depending on temperature, by checking for time-reversal symmetry; see the paragraph on the superconducting diode effect. We have therefore shifted the zero current line in Fig. 3(b), as indicated in the caption.

To treat the dV/dI data, we first subtract the background resistance $R_{min} \approx 172 \Omega$, corresponding to the wiring resistance (these are two wire measurements). We then define $R_N = R_{max} - R_{min}$ the normal state resistance. The SQUID's critical current is then extracted as the bias current at which the differential resistance reaches a threshold defined as $R_T = R_N/2$. If the threshold falls between two discrete measurement points, we interpolate I_c as described in the inset of Fig. 8 to reduce discretization.

An example of the extracted critical current I_c is plotted in red in Fig. 9.

The current-phase relation is visible on top of a modulation on a large (several milliteslas) field scale, originating from the Fraunhofer-like pattern due to interference across the reference junction. The critical current versus field curve is numerically low-pass filtered to remove this contribution. To this end, the Fourier transform is computed over the full data range ($B_{tot} = 8$ mT, which is slightly larger than three periods of the current-phase relation) and the three first frequencies are cut ($f = 0$, $f = \frac{1}{B_{tot}}$, and $f = \frac{2}{B_{tot}}$). The CPR is then computed as the inverse Fourier transform, yielding the data presented in the main text, and in blue in Fig. 9. The extraction for the 20 mK data, the lowest temperature, is complicated by

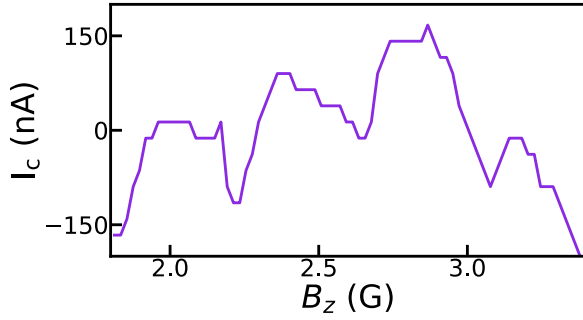


FIG. 10. Extracted current-phase relation at 20 mK, the lowest temperature. The extraction of this curve is complicated by instabilities due to superconducting vortices around zero field, so the CPR was extracted from the SQUID's critical current at higher field for which the critical current of the reference junction is much smaller. The quality of these data is much worse than, yet consistent with the higher-temperature data.

instabilities around zero field due to superconducting vortices, but is presented for slightly higher fields in Fig. 10.

2. Fourier transform analysis

To analyze the harmonic content of the CPR, we compute its Fourier transform. To this end, we prepare the data by selecting an integer number of periods (here three) and interpolating the data in order to get a table of $2^8 = 256$ points that exactly match the three periods. With data acquired on such a small number of periods, this operation is crucial to visualize properly the lowest and higher harmonics (frequencies). Indeed, we recall that the resolution of the calculated Fourier transform is given by $\frac{1}{B_{\text{tot}}} = \frac{1}{3B_0}$, with B_{tot} the total measurement range and B_0 the lowest B period, implying that the amplitude is calculated for all multiples of $\frac{1}{3B_0}$. Consequently, if the only frequencies present in the signal are B_0^{-1} and its harmonics, they should be captured by the calculation.

We then compute the absolute value of the FFT calculation using the `fft.rfft` function of the `numpy` library in Python, without windowing (windowing would be useful if we did not select an integer number of periods), and renormalize by multiplying by $2/N$ (N is the number of points of the input data). Finally, we numerically filter out the lowest two frequencies; namely, the peaks at 0 and $\frac{1}{3B_0}$ are set to zero. This leads to the curves presented in Fig. 2.

We estimate that the uncertainty on the harmonics' amplitude is on the order of the noise at high frequency, corresponding to 3 nA. This is an uncertainty at 95%, meaning that there is 95% chance that the "real" value is in the interval $[x_{\text{mes}} - dx, x_{\text{mes}} + dx]$. The next step is to fit the harmonics' amplitude by the function $\tilde{I}(n) = \tilde{I}_0 \frac{e^{-an}}{n}$, with $a = 2\pi k_B T / E_{\text{Th}}^{\text{edge}} - \ln(t)$. To get rid of the parameter $\tilde{I}_0$, we instead fit $\frac{\tilde{I}_n}{\tilde{I}_1} = \frac{e^{-a(n-1)}}{n}$, with a the only fitting parameter. Figure 11 displays the extracted values of a for four temperatures, and the fit to a linear temperature dependence, from which we extract $t = 0.89 \pm 0.04$ and $E_{\text{Th}}^{\text{edge}} = 560 \pm 75 \mu\text{eV} \simeq 6.5 \text{ K}$. To determine the uncertainty on the parameter a , given the uncertainty on $\tilde{I}(n)$, we use a Monte Carlo method. The idea is to generate numerically a large number (here $N = 1000$)

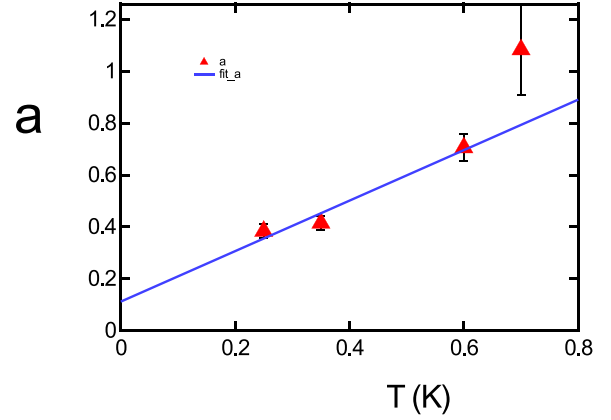


FIG. 11. Extracting SQUID C's edge junction Thouless energy and intrinsic transmission factor t , from the decay of the CPR's harmonics' relative amplitudes expressed as a function of the decay coefficient a via $\frac{\tilde{I}_n}{\tilde{I}_1} = \frac{e^{-a(n-1)}}{n}$. a depends on temperature, and the linear fit to $a = 2\pi k_B T / E_{\text{Th}}^{\text{edge}} - \ln(t)$ yields the edge's Thouless energy $E_{\text{Th}}^{\text{edge}} = 560 \pm 75 \mu\text{eV} \simeq 6.5 \text{ K}$ and the intrinsic transmission coefficient accounting for the CPR rounding $t = 0.89 \pm 0.04$.

of new datasets that are consistent with uncertainty. Each of these dataset is fitted and provides a value of a . An estimate of the uncertainty at 95% on a is given by twice the standard deviation of the N values of a obtained. The uncertainties on t and $E_{\text{Th}}^{\text{edge}}$ are also estimated using the Monte Carlo method.

3. Short versus long ballistic junction: estimate of the superconducting coherence length in the ballistic regime using an estimate for the superconducting gap at the interface

We have seen that the CPR of a ballistic junction takes the shape of a sawtooth, i.e., the characteristic "long-junction" CPR, as soon as the junction is more than a few atomic sites long. The difference between long and short ballistic regimes is also hard to determine from the decay with temperature of the critical current in both regimes, since the decay has a similar close-to-exponential-like dependence, with a characteristic energy given by either the Thouless energy or the contacts' superconducting gap. To identify the junction regime, we therefore estimate the superconducting coherence length using the junction parameters and determine whether the junction length is smaller (short-junction regime) or greater (long-junction regime) than this coherence length. The superconducting coherence length in a ballistic junction is $\xi = \frac{\hbar v_F}{\pi \Delta} \approx 200 - 600 \text{ nm}$ (using the helical state Fermi velocity $v_F = 5 \times 10^5 \text{ m s}^{-1}$ estimated in Ref. [1]) and $\Delta \approx 170 - 500 \mu\text{eV}$. The lower estimate for Δ is the value reported in Refs. [44,45] for a Pd(7 nm)/Nb(20 nm)/Al(40 nm), AlOx, and Al(120 nm) multilayer contacting carbon nanotubes. The gap in the present experiment's Pd (8 nm)/Nb(80 nm) bilayer contact may be higher because of the thickener Nb, and the fact that Pd has been shown to form a superconducting compound with Te. We therefore consider that the 600-nm-long edge junction is in the intermediate regime between short and long. In such a regime, the CPR is expected to have a sawtooth shape.

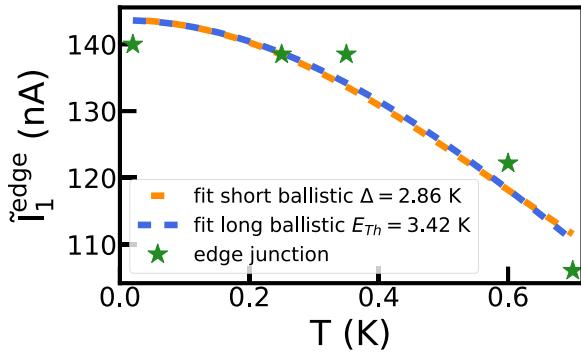


FIG. 12. Analysis of the temperature dependence of the critical current of SQUID C's edge junction. Data points are the amplitude of the CPR's first harmonic at different temperatures I_1^{edge} . We find that the data can be fitted equally well by the temperature dependence of the first harmonic of the ballistic short junction, $I_{\text{short}}^{\text{ball}}(\varphi, T) \propto \Delta \sin(\frac{\varphi}{2}) \tanh(\frac{\Delta}{2T} \cos(\frac{\varphi}{2}))$, or the first harmonic of the ballistic long junction $I_{\text{long}}^{\text{ball}}(\varphi, T) \propto \sum_{k=1}^{\infty} \sin(k\varphi) \frac{4(-1)^{k+1}T}{\sinh(2\pi k \frac{T}{E_{\text{Th}}})}$ [46,47]. The gap energy extracted of the fit to the short-junction expression is $\Delta \simeq 2.9$ K, whereas the Thouless energy extracted from the fit to the long-junction expression is $E_{\text{Th}} \simeq 3.4$ K. These results are compatible with the Thouless energy extracted from the analysis of the sawtooth harmonics discussed in the main text and confirm that the edge junction is in a ballistic regime that is intermediate between short and long.

4. Bulk junction

In this Appendix, we give a more detailed view of the bulk junction, argue that due to its geometry it must act as the reference junction, and show that this hourglass-shaped junction on the bulk of the SQUID C's surface behaves equivalently to a rectangular-shaped junction, whose parameters we estimate.

a. Details of the bulk junction geometry. The bulk junction has the form of an hourglass, with two triangular superconducting electrodes with a small separation of less than 100 nm, as can be seen from the scanning electron micrographs of Fig. 13. The hourglass junction layout is sketched in Fig. 14, defining the junction's parameters $y_{\text{max}} = 0.73 \mu\text{m}$, $t = 223$ nm the thickness of the flake, $d = 2.21 \mu\text{m}$ the length of the diagonal of the hourglass (used to calculate the angle), and $l(y_{\text{max}}) = 2.06 \mu\text{m}$.

b. Why this junction acts as the reference junction in the asymmetric SQUID. The hourglass geometry, with its very closely spaced superconducting electrodes, was specifically chosen to create a reference junction with as small a surface

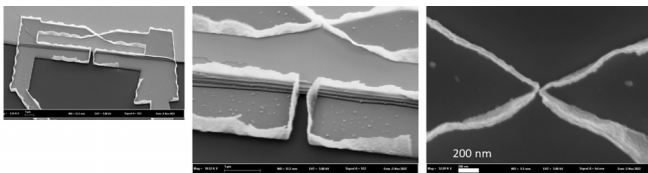


FIG. 13. Scanning electron micrograph of SQUID C, including enlarged views of the bow-tie-shaped bulk junction. The white contours of the electrodes are a (common) consequence of the sputter deposition through a resist mask.

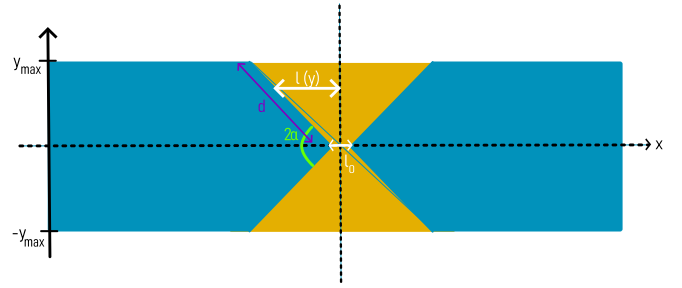


FIG. 14. Sketch of the hourglass-shaped junction, with parameters used to determine the junction's conductance. The blue part represents the superconducting leads and the yellow part the WTe_2 flake in the junction. $y_{\text{max}} = 0.73 \mu\text{m}$, $d = 2.21 \mu\text{m}$, $t = 223$ nm the thickness of the flake, and $l(y_{\text{max}}) = 2.06 \mu\text{m}$.

as possible. This ensures that the junction's critical current is quite large and also enables to follow the magnetic field dependence up to the high fields (indeed, a junction's critical current dies out at fields that vary inversely proportionally to the junction area). We would like to insist on the fact that such an unconventional shape is by no means a hindrance to this junction acting as a reference junction, since the only criterion for a junction in an asymmetric SQUID to play the role of the reference junction is that its critical current be much larger than the critical current of the other junction (whose CPR is sought). Note that it is possible that diffusion of Pd from the electrodes into WTe_2 could occur and generate a superconducting $\text{Pd}_{1-x}\text{Te}_x$ compound that would connect the two electrodes. We believe that even if the weak link in the bulk junction is a PdTe_x superconducting filament rather than (or in parallel with) the proximitized bulk WTe_2 , that would be a valid weak link with which to measure the CPR of the edge junction. In the present case, as we present in Fig. 15, it is clear that the critical current of the bulk junction must be

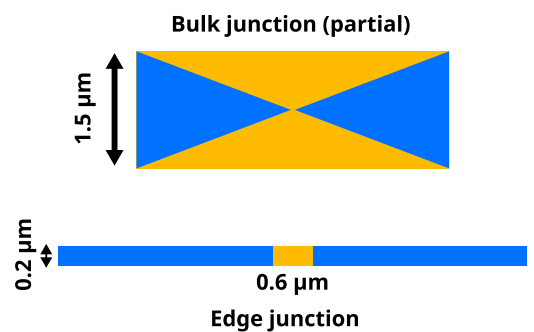


FIG. 15. Comparison between the bulk and edge junction geometries, which illustrates why the bulk junction carries the largest supercurrent and therefore must be the reference junction (superconducting Pd/Nb in blue, WTe_2 in yellow): The edge junction's superconducting electrodes are 600 nm apart, with a narrow region of WTe_2 as the weak link (bottom sketch). The bulk junction (top sketch) has electrodes with a separation much smaller than 600 nm, and may also have a central PdTe_x superconducting filament in parallel: The critical current of this bulk junction must be much larger than the edge junction's. Consequently, the bulk junction must act as the reference junction, and the small, 150 nA oscillation must be due to the edge junction.

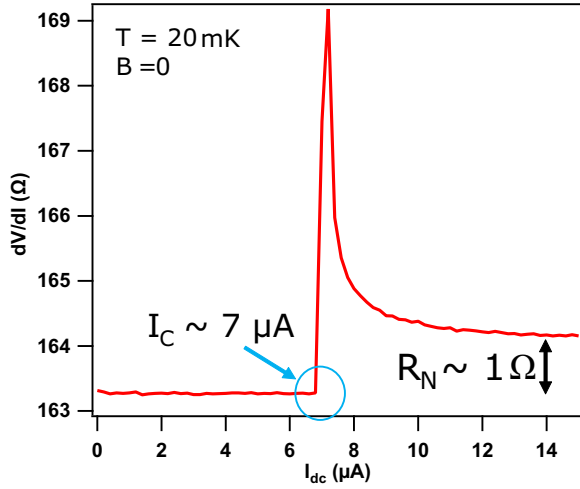


FIG. 16. Differential resistance curve at 0 field and $T = 20$ mK. The critical current, highlighted by the blue circle, is about $7 \mu\text{A}$. The jump in resistance, i.e., the difference between the low-bias and high-bias regions, is the sample resistance, about 1Ω . The 161.3Ω resistance corresponds to the measurement lines leading to the sample from room temperature.

much larger than that of the edge junction: The edge junction's superconducting electrodes are 600 nm apart, whereas the bulk junction's electrodes are much closer (as seen in the SEM image), and may also have a parallel conduction via the PdTe_x filament. Consequently, the small (150 nA) oscillation must be due to the junction with the electrodes furthest apart, i.e., the edge junction.

c. Equivalent model for the hourglass-shaped junction. In this Appendix, we show that the hourglass-shaped junction on the bulk of the SQUID C's surface behaves equivalently to a rectangular-shaped junction, whose parameters we estimate. To this end, we evaluate the junction's resistance and find the dimensions of the rectangular junction with the same resistance.

We first point out that the change in resistance above the SQUID's critical current is the parallel combination of the bulk and edge junction resistances, but we can check *a posteriori* that the bulk junction resistance is smaller than the edge junction resistance and therefore that the change in resistance is practically equal to the bulk junction resistance. This resistance is roughly $R_N \simeq 1 \Omega$; see the $dV/dI(I_{dc})$ curve in Fig. 16.

The hourglass junction, sketched in Figs. 14, 15 and 17, is characterized by the following parameters: $y_{\max} = 0.73 \mu\text{m}$,

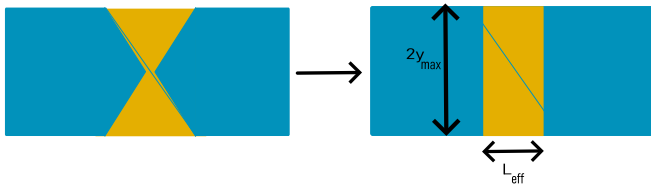


FIG. 17. The hourglass junction can be recast as an equivalent rectangular junction of width $w = 2y_{\max} \simeq 1.46 \mu\text{m}$ and length $L_{\text{eff}} \simeq 0.94 \mu\text{m}$, much simpler to study.

$t = 223 \text{ nm}$ the thickness of the flake, $d = 2.21 \mu\text{m}$ the length of the diagonal of the hourglass (used to calculate the angle), and $l(y_{\max}) = 2.06 \mu\text{m}$. We assume that the junction is symmetric and for the calculations we thus only consider the top, triangular-shaped half of the hourglass, starting at $y = 0$ up to $y = y_{\max}$. We define an elementary conductance $g(y)$ as

$$g(y) = \sigma \frac{S}{2l(y) + l_0}, \quad (\text{C1})$$

with $S = t \times dy$, σ the conductivity, and $l_e \simeq 50 \text{ nm}$ the mean free path. The angle α is defined as

$$\tan(\alpha) = \frac{y}{l(y)} = \frac{y_{\max}}{l(y_{\max})} = 0.35. \quad (\text{C2})$$

This allows to write the elementary conductance as

$$g(y) = \frac{\sigma t dy}{2 \frac{y}{\tan(\alpha)} + l_e}, \quad (\text{C3})$$

so that the conductance of the junction is

$$G = 2 \times \int_0^{y_{\max}} \frac{\sigma t}{2y/\tan(\alpha) + l_e} dy = \sigma t \tan(\alpha) \ln \left(\frac{2y_{\max}}{\tan(\alpha)l_e} + 1 \right). \quad (\text{C4})$$

Using $\sigma = 2 \times 10^6 \Omega^{-1} \text{m}^{-1}$ [21], we obtain

$$R = \frac{1}{G} = 1-1.5 \Omega, \quad (\text{C5})$$

close to the value found in the experiment for both bulk and edge junctions in parallel, confirming that the bulk junction's resistance is smaller than that of the edge junction. With the expression of G obtained in Eq. (C4), we can compute an effective length for the hourglass junction using

$$G = \frac{\sigma S}{L_{\text{eff}}}, \quad (\text{C6})$$

with

$$L_{\text{eff}} = \frac{2y_{\max}}{\ln \left(\frac{2y_{\max}}{l_0 \tan(\alpha)} + 1 \right)} \times \frac{1}{\tan(\alpha)} = 0.94 \mu\text{m} \quad (\text{C7})$$

and the hourglass junction is equivalent, as a first approximation, to a rectangular junction of width $w = 2y_{\max}$ and length L_{eff} , as illustrated in Fig. 17. In the next Appendix, we show that the Thouless energy describing the bulk junction's experimental behavior agrees with the Thouless energy computed for the equivalent rectangular junction.

5. Extraction of the Thouless energy $E_{\text{Th}}^{\text{bulk}}$ from the temperature dependence of the critical current of SQUID C's bulk junction, and comparison with $E_{\text{Th}}^{\text{bulk,eff}}$ of the equivalent rectangular junction

In the main text, we use an approximate expression for the temperature dependence of a long diffusive junction, $I_c^{\text{bulk}}(T) \simeq \exp(-\frac{T}{T_c})$, with a characteristic temperature T_c related to the Thouless energy via $T_c = 3.8 E_{\text{Th}}^{\text{bulk}}$ [34,35]. This yields $T_c \simeq 250 \text{ mK}$, and $E_{\text{Th}}^{\text{bulk}} \simeq 65 \text{ mK} \approx 6 \mu\text{eV}$, much less

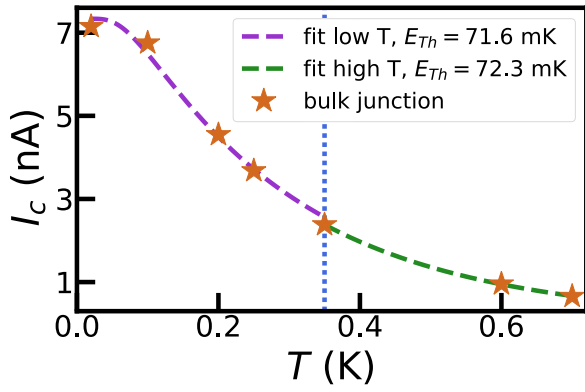


FIG. 18. The critical current of SQUID C's bulk junction can be fitted in two parts, using the expression at low temperature ($T < 5E_{\text{Th}}$) and the one at high temperature ($T > 5E_{\text{Th}}$), yielding the same $E_{\text{Th}} \approx 70$ mK and a cutoff at $5E_{\text{Th}} \approx 350$ mK.

than the superconducting gap, which is greater than 2 K, confirming the disordered long-junction regime. In fact, there are more appropriate expressions that should be used to deduce $E_{\text{Th}}^{\text{bulk}}$ from $I_c^{\text{bulk}}(T)$, as described in the following.

The critical current extracted from the differential resistance curves such as presented in Fig. 16 is plotted as a function of temperature in Fig. 19. Extracting the Thouless energy from the temperature dependence relies on fitting to the appropriate expression, which depends on the junction and temperature regime.

Long diffusive junction. The long diffusive regime corresponds to $\Delta > 100E_{\text{Th}}$. Depending on the temperature regime, two different expressions should be used, as described in Ref. [48]: At low temperature ($T < 5E_{\text{Th}}$), the decrease of the critical current is fit with

$$eR_N I_c(T) = 10.82 E_{\text{Th}} \left(1 - 1.3e^{-\frac{10.82 E_{\text{Th}}}{3.2k_B T}}\right), \quad (\text{C8})$$

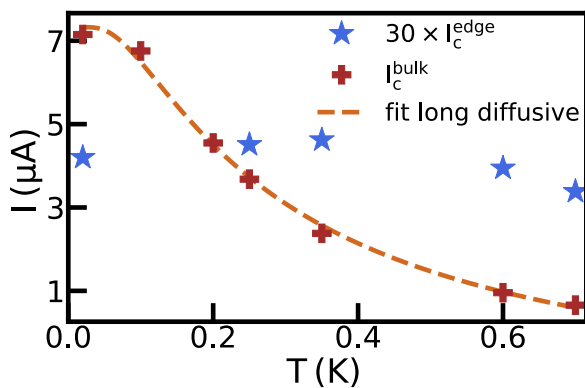


FIG. 19. Temperature dependence of the critical current of SQUID C (I_c^{bulk}), compared to the edge junction's CPR I_c^{edge} (multiplied by 30 for visibility). The amplitude of the critical current for the temperatures of 250, 350, 600, and 700 mK is obtained by reconstructing the signal from all the harmonics extracted from Fig. 2 and then taking the maximum of this curve. The shape of the CPR being dominated by the first harmonic, the dependence is extremely close to the one of the first harmonic alone (Fig. 12). The greater resilience to temperature of the edge supercurrent is clear.

whereas at higher temperature the expression is

$$eR_N I_c(T) = \frac{32}{3 + 2\sqrt{2}} E_{\text{Th}} \left(\frac{2\pi k_B T}{E_{\text{Th}}}\right)^{\frac{3}{2}} \times \sum_{n=0}^{\infty} \sqrt{2n+1} e^{-\sqrt{2n+1} \sqrt{\frac{2\pi k_B T}{E_{\text{Th}}}}}. \quad (\text{C9})$$

As, shown in Fig. 18, we find that the entire dataset for SQUID C's bulk junction, from 10 to 700 mK, can be fit with both expressions, and yields the same Thouless energy, $E_{\text{Th}} = 72$ mK, with $5E_{\text{Th}} \approx 350$ mK.

Long to short intermediate behavior. Since $\Delta/E_{\text{Th}} \approx 30 < 100$, the junction is not fully in the long diffusive regime, and the decrease in temperature of the critical current is governed by a different decay factor. The general low-temperature behavior is given by an expression similar to the long-junction expression, but with a factor b replacing the factor 10.82 [48]:

$$eR_N I_c(T) = b E_{\text{Th}} \left(1 - 1.3e^{-\frac{b E_{\text{Th}}}{3.2k_B T}}\right). \quad (\text{C10})$$

b depends on $\frac{\Delta}{E_{\text{Th}}}$; $b = 10.82$ for a very long junction but $b = 7.7$ for $\frac{\Delta}{E_{\text{Th}}} = 20$. The previous fits are unchanged but now the value of E_{Th} is $\frac{10.82}{7.7} = 1.4$ times larger. These considerations therefore give an estimate of the uncertainty on the values of E_{Th} determined.

The result for SQUID C's bulk junction $E_{\text{Th}}^{\text{bulk}} = 72$ mK is consistent with an estimate of the Thouless energy of the rectangular junction that approximates the bow-tie bulk junction:

$$\begin{aligned} E_{\text{Th}}^{\text{bulk,eff}} &= \frac{\hbar D}{L_{\text{eff}}^2} \\ &= \frac{\hbar v_F l_e}{3L_{\text{eff}}^2} \\ &\simeq 30 \text{ mK}, \end{aligned} \quad (\text{C11})$$

using $L_{\text{eff}} \simeq 0.94 \mu\text{m}$, $l_e \simeq 50 \text{ nm}$ [49] and $v_F \simeq 2 \times 10^5 \text{ m s}^{-1}$, a Fermi velocity typical of the bulk electron and hole pockets, averaged over a , b , and c directions [50].

Another indication that the rectangular junction may be a good approximation of the bow-tie-shaped junction is the Fraunhofer-like pattern observed for $I_c^{\text{bulk}}(B)$, which is the pattern expected of a wide rectangular-shaped junction [32,33].

These considerations confirm the diffusive nature of transport in the hourglass-shaped bulk junction and the validity of approximating the bow-tie junction with an equivalent rectangular junction. Of course, these estimates suffer from the fact that the expressions considered are those for perfect NS interfaces, which is not the case in the experiment, as discussed in the main text.

6. Illustration of the very different energy scales characterizing the edge and bulk junctions of SQUID C

Figure 19 plots on the same graph the temperature dependence of the critical current of SQUID C's bulk junction at zero field, I_c^{bulk} , and that of the edge junction's critical current I_c^{edge} (multiplied by 30 for visibility). The greater resilience to temperature of the edge critical current is clear, and translated into two very different Thouless energies, $E_{\text{Th}}^{\text{bulk}} \simeq 70$ mK and

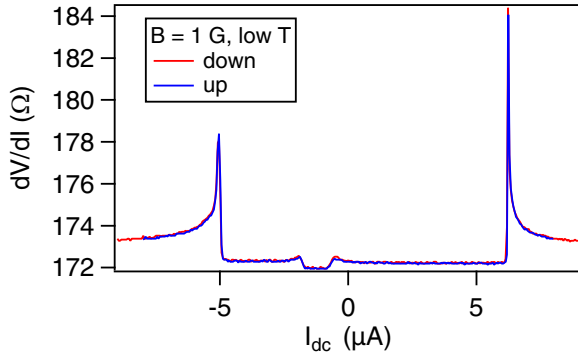


FIG. 20. Differential conductance as a function of dc bias current at $B \approx 1$ G and at the lowest temperature, around 20 mK. The critical current is extracted as the current at which the differential resistance jump exceeds a threshold of a fraction of the normal state resistance.

$E_{\text{Th}}^{\text{edge}} \simeq 6.5$ K or $E_{\text{Th}}^{\text{edge}} \simeq 2.8$ K, depending on whether $E_{\text{Th}}^{\text{edge}}$ is estimated using the decay of the CPR's harmonics (as in

the main text and Fig. 11) or by fitting the temperature dependence of the CPR's first harmonic, as in Fig. 12.

7. Superconducting diode effect in SQUID C

To examine the superconducting diodelike behavior, we compare the magnetic field dependence of the critical current measured for positive current, $I_{c,+}(B)$, with the one measured with negative current, $I_{c,-}(B)$. Specifically, $I_{c,+}$ is the switching current measured when the current is ramped up from the superconducting state and $I_{c,-}$ the absolute value of the switching current when the current is ramped down from the superconducting state. Measuring both quantities as a function of B would be quite time consuming. Instead, we start by comparing a measurement of the differential conductance as a function of bias current at a given magnetic field for both ramping up and down of the current, represented in Fig. 20.

Surprisingly, we find that both curves are exactly the same. This was not *a priori* expected, since the retrapping current (at which the junction switches from normal to superconducting when decreasing the current) can be very different from the switching current (at which the junction switches from superconducting to normal) if the junction is in an

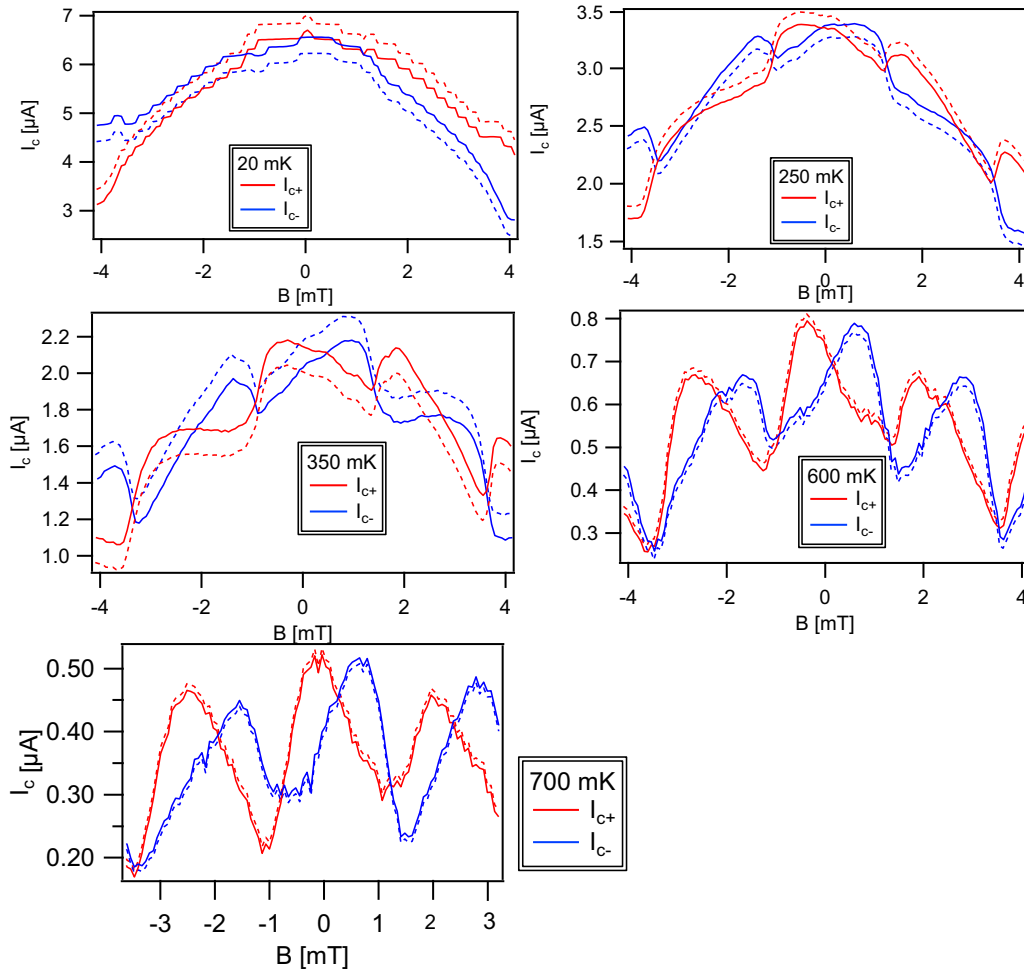


FIG. 21. SQUIDS' positive ($I_{c,+}$, red) and negative ($I_{c,-}$, blue) critical currents as a function of magnetic field B for the five measured temperatures. If time-reversal symmetry is preserved, one should have $I_{c,+}(B) = I_{c,-}(-B)$. This equality is roughly (though not exactly) verified if one corrects for the small offset current (between 100 and 200 nA, depending on temperature) mentioned in the text (see dashed lines).

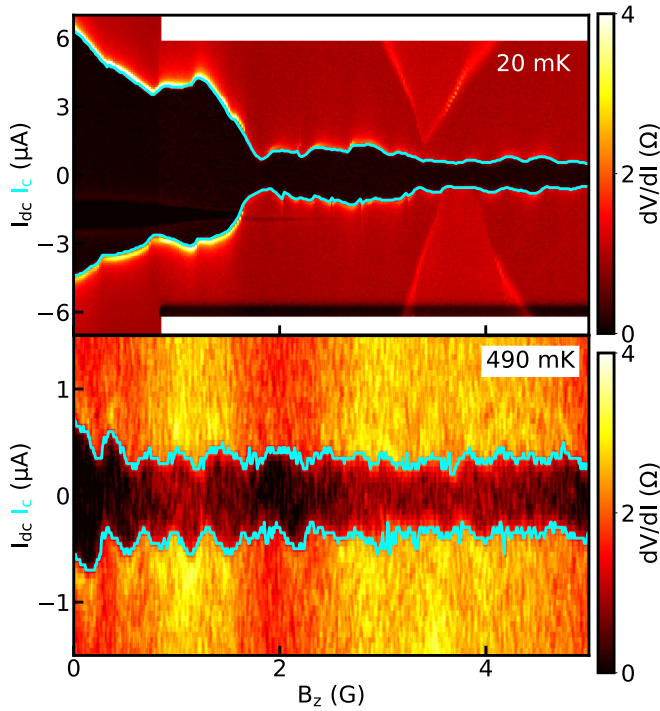


FIG. 22. Differential conductance as a function of current and perpendicular field B_z for SQUID C, at two temperature. Top: $T = 20$ mK; bottom: $T = 490$ mK. B_z is 2% of the applied magnetic field, as explained in the text. The extracted critical current is plotted in cyan. The ac current is between 100 and 20 nA depending on the field range for the 20 mK plot and is 5 nA for the 490 mK plot.

underdamped regime or in the presence of heating effects. Here, the retrapping and the switching are exactly the same (and we checked that this was the case in different experimental conditions), corresponding to an overdamped regime. This is actually very convenient, since it implies that a single up-sweep measurement of $dV/dI(I_{dc})$ yields both $I_{c,+}$ (given by the switching current) and $I_{c,-}$ (given by the retrapping current). We can then compare $I_{c,+}$ and $I_{c,-}$ for all temperatures (see Fig. 21).

The large diode effect found here is not surprising, since it is expected of an asymmetric SQUID in a magnetic field in which the weak link has a nonsinusoidal CPR.

8. SQUID C's critical current at two different temperatures

Figure 22 compares SQUID C's field dependence between 0 and 25 mT in B , i.e., 0 and 5 G in B_z , at two temperatures: the 20 mK plot, which is over a smaller range than shown in Fig. 3, and a 490 mK plot. Both plots display more than ten clear oscillations with the period of 0.46 G corresponding to the SQUID geometry.

9. Calculated interference patterns for different configurations of 1D channels in the edge junction of SQUID C

Although it is probably vain to hope to extract from the measured interference pattern the exact number and spatial distribution of 1D states in the edge junction of SQUID C, we can nonetheless simulate the interference patterns

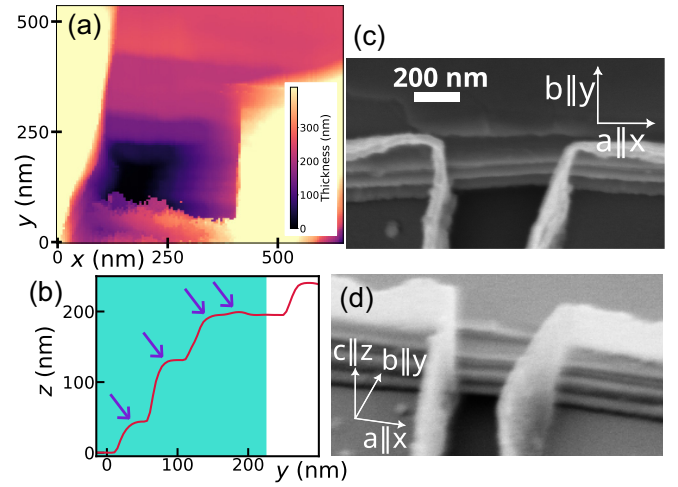


FIG. 23. Edge junction of SQUID C, as seen with atomic force microscopy (AFM) and scanning electron microscopy. (a) AFM topographic image of the junction area, with a line cut along the b axis, showing the giving the height profile of the junction. (c), (d), SEM images at a small and a larger tilt angle so that the topography is clearer in panel (d). Five steps are visible, with height ranging from less than 10 to 90 nm, and terrace width varying between 20 and 100 nm. Only the first four steps are covered by the superconducting contacts (hatched in blue).

corresponding to slightly different numbers and distributions of hinge states, taking as an input the actual layout of the edge junction of SQUID C, as pictured in Fig. 26.

We simulate the expected critical current versus magnetic field with the parameters for SQUID C (see Fig. 24). The model consists of

- (1) a reference bulk junction with a critical current $I_{c,bulk} = 7 \mu A$ and a Fraunhofer (sinc) pattern with 6 Gauss-size side-lobes;
- (2) a SQUID loop area $S_{SQUID} = 12 \mu m \times 1.2 \mu m$;
- (3) ballistic channels on the edge that do not decay in magnetic field.

The total critical current is computed as

$$I_c(B) = \max_{\varphi} \left[I_{c,bulk} f_{diff} \left(\frac{BS_{SQUID}}{\Phi_0} + \varphi \right) + \sum_{i=1}^{N_{channels}} I_{c,i} f_{ball} \left(\frac{BS_i}{\Phi_0} + \varphi \right) \right], \quad (C12)$$

where S_i is the area between channel i and channel 1. The critical current depends on the spatial separation between the different channels. We consider several cases, for which the total critical current, 210 nA, is evenly split between the different channels:

- (1) one channel with a 210 nA critical current;
- (2) two channels, each with a 105 nA critical current, whose separation defines an area that is 2% of the SQUID area;
- (3) three channels, each with a 70 nA critical current, regularly spaced, defining an area that is 2% of the SQUID area;
- (4) three channels, each with a 70 nA critical current, whose separation defines an area that is, respectively, 2% and 3% of the SQUID area;

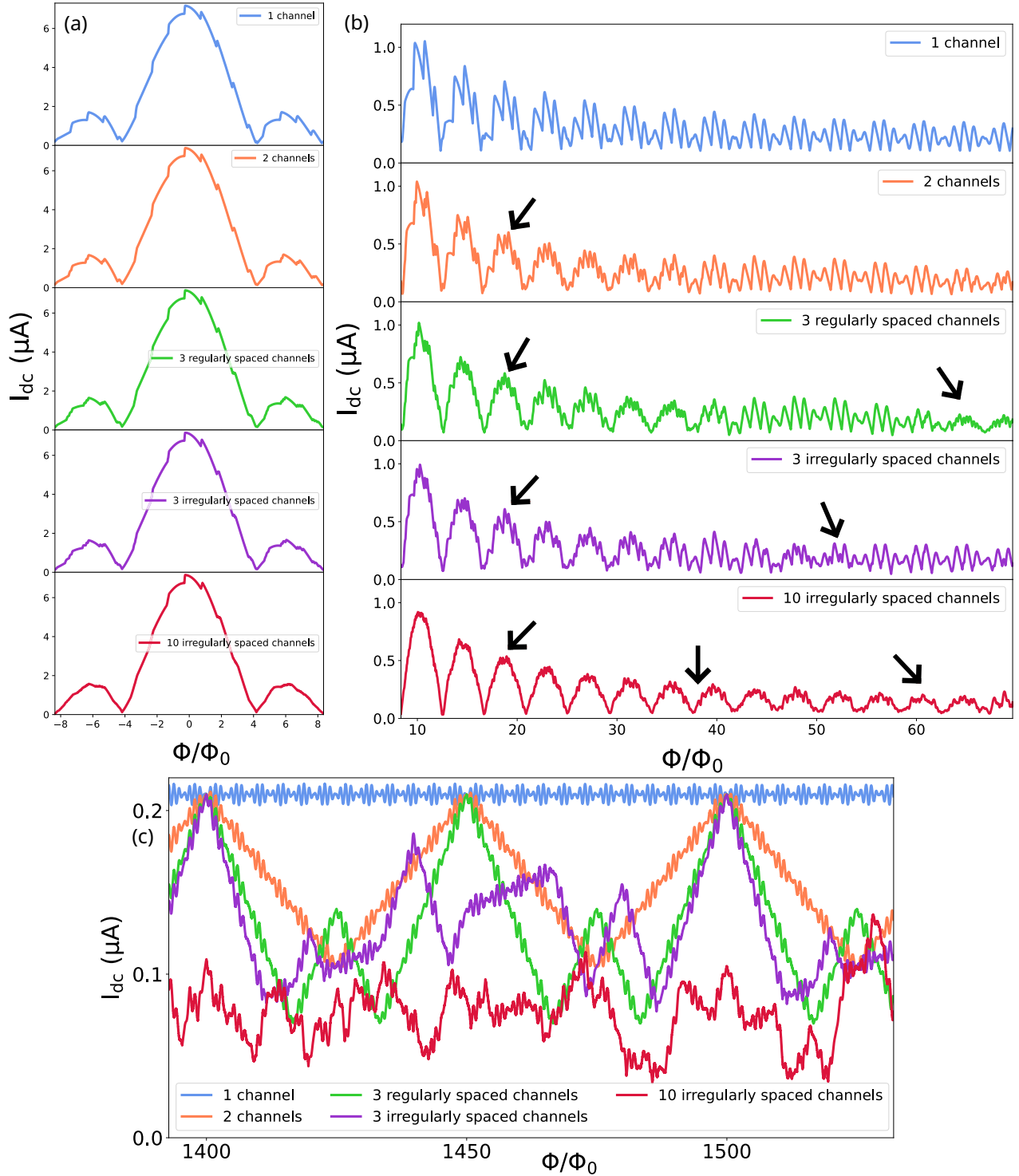


FIG. 24. Simulation of interference patterns for different number and configurations of ballistic channels in the edge junction, assuming that the reference bulk junction is wide and decays with a fast Fraunhofer pattern. (a) The sawtooth pattern appears as a modulation in all cases in the first lobe. (b) At intermediate magnetic field, the sawtooth modulation splits into several smaller sawtooths if there are several ballistic channels, blurring the current-phase relation. Black arrows show such areas up to tens of $\frac{\Phi}{\Phi_0}$. The magnetic flux at which it happens depends on the separation between ballistic channels. (c) At very high magnetic field, the current in the diffusive bulk junction has become smaller than the supercurrent through the edge junction, so the modulation of period Φ_0 is due to the (sinuslike) bulk junction's CPR. The envelope corresponds to the sum of the ballistic channels and depends on their number and separation; it is triangular for two channels (orange on the figure). Regularly spaced channels lead to a periodic pattern with a period related to the surface area separating two of those ballistic channels. However, if there are more than two channels with different separations, the pattern will look erratic on the same field scale. A higher number of irregularly positioned channels leads to an average critical current given by the square root of the number of channels times the critical current of one channel.

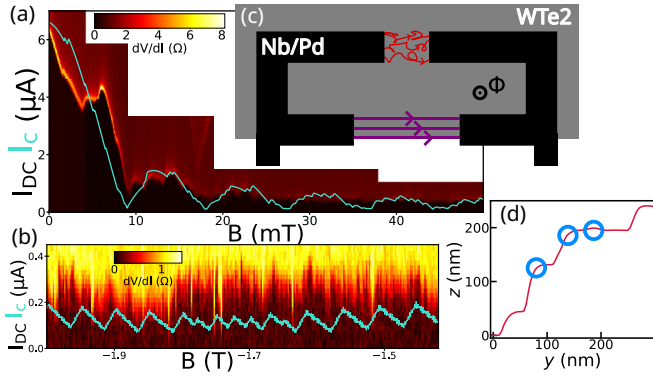


FIG. 25. (a), (b) Simulated critical current (cyan) laid over experimental data of SQUID C. (c) Sketch of a SQUID with a diffusive bulk junction and an edge junction with three ballistic channels. We suppose that no diffusive current runs through the edge junction. Blue line in panels (a) and (b): Calculated critical current of a SQUID in which one junction has a sinusoidal CPR with a zero field critical current of $6.5 \mu\text{A}$, decaying as a Fraunhofer (sinc) function, and the other junction is made of three 1D channels, each with a 70 nA sawtooth CPR, which decays on a 10 T scale. (d) For the simulation on SQUID C, we use three channels, at the positions indicated by the blue circles laid over the sample profile measured by atomic force microscopy. However, we do not claim that those are the actual ones that carry the supercurrent in the experiment.

(5) ten channels of critical current 21 nA each, separated by a random fraction of the SQUID area between 1% and 3%.

Figure 24 displays the interference patterns computed around zero field, at low field and high field for the different edge state distributions. (a) The sawtooth pattern appears as a modulation in all cases in the first lobe. (b) At intermediate magnetic field, the sawtooth modulation splits into several smaller sawtooths if there are several ballistic channels, blurring the current-phase relation. (c) At high magnetic field, the current in the diffusive bulk junction has become smaller than the supercurrent through the edge junction, so the modulation is due to the (sinuslike) bulk junction's CPR. The envelope corresponds to the sum of the ballistic channels and depends on their number and separation. If the channels are regularly spaced, we see a periodic pattern with a period related to the surface area separating two of those ballistic channels. However, if there are more than two channels with different separations, the pattern will look erratic on the same field scale. The greater the distance between two channels, the faster they contribute differently to the critical current and the faster the periodic pattern becomes blurred. A higher number of irregularly positioned channels leads to an average critical current given by the square root of the number of channels times the critical current of one channel.

Figure 25 displays a simulated critical current curve, overlaid on the experimental field dependence of SQUID C's critical current. The interference pattern is computed using three channels positioned as determined using atomic force microscopy, see Figs. 23 and 25(d), even if we have no way of knowing if those are the channels which actually carry the supercurrent in the experiment. Note that since the magnetic field is mostly in plane, the vertical distance between steps may be more relevant than the horizontal distance.

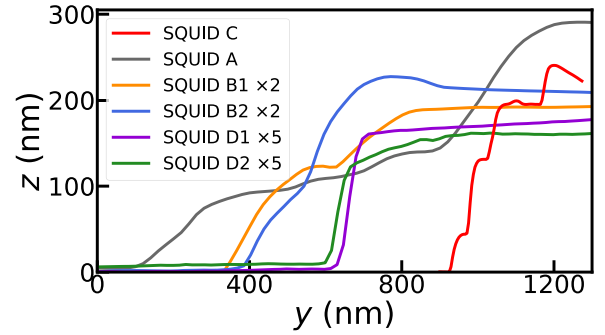


FIG. 26. Atomic force microscopy measurements of the profiles of all WTe_2 crystals of SQUIDs A to D, in the region of the edge junctions. The profile in the perpendicular axis z has been multiplied by 2 for SQUIDs B1 and B2, and by 5 for SQUIDs D1 and D2. The thicknesses of all crystals at the edge junctions are listed in Table II, as well as the length and width of the junctions, the width being defined by the lateral extension of the superconductor. The edge junctions other than SQUID C are spread over a much great lateral area, which means that a large portion of diffusive transport via nontopological bulk states is probed by the SQUIDs other than C, providing a possible explanation for the observed sinusoidal CPR.

APPENDIX D: COMPARISON OF ALL SQUIDS

1. Layout and SQUID patterns

We have fabricated a total of six SQUID samples, which all consist of one junction on the bulk of the WTe_2 crystal surface and one overlapping the crystal a edge. The WTe_2 crystals have the same origin, and their thicknesses range between 30 and 300 nm (see Table I). As shown in Fig. 1 and Fig. 27, in contrast to the sawtooth modulation of SQUID C discussed extensively in the main text, two other behaviors are found: Four other SQUIDs display a critical current of the reference junction of several microamperes and a small, 200 nA modulation, but the modulation is sinusoidal, not sawtoothlike as in SQUID C. Finally, SQUID A displays a symmetric SQUID-like pattern, with a practically full modulation of the critical current. There are several possible reasons that can explain sinusoidal rather than sawtooth behaviors for these SQUIDs. A first possibility is if in fact no edge states are connected by the superconducting electrodes on the edge junction. Then, the SQUID patterns reflect the interference of two bulk junctions in parallel, i.e., the interference of two rather wide diffusive junctions. If the areas of both junctions are similar, as in the case of SQUID A (see image in Fig. 1 and Table II), and if the interface quality is also similar, the SQUID pattern is that of a symmetric SQUID, i.e., $I_c \propto |\cos(\pi \Phi / \Phi_0)|$. The decay of the pattern is governed by the flux through the junction area, on the order of 20 G (taking into account field focusing), as observed in the experiments (see Table II). Similarly, the small sinusoidal modulations of SQUIDs B1, B2, D1, and D2 can be explained by an absence of contact to edge states and a weak contact to the diffusive states at the surface in the junction. Figure 26 displays the topography of all SQUID profiles at the edge junctions and is complemented by Table II, giving the geometry of all junctions. It is clear that the lateral extension of the edge junctions other than that of SQUID C is much greater, meaning that a large portion of

TABLE I. Thouless energy E_{Th} , diffusion constant D , and ratio $\frac{10.82E_{\text{Th}}}{eR_N I_c}$, where relevant, for all SQUID junctions.

SQUID		E_{Th} (mK)	$D = \frac{L^2}{h} E_{\text{Th}}$ ($\text{m}^2 \text{s}^{-1}$)	$\frac{10.82E_{\text{Th}}}{eR_N I_c}$
C	I_{bulk} (from CPR measurement)	72	0.0084	10
C	I_{edge} (see the main text)	2–6 K		
A (2022-10)	I_c	34	0.0011	16
B1 (2023-01)	I_{bulk} (from CPR measurement)	51	0.0024	16
	I_{edge} (from CPR measurement)	39	0.0051	
B2 (2023-01)	I_{bulk} (from CPR measurement)	78	0.0037	36
	I_{edge} (from CPR measurement)	82	0.0108	
D1 (2023-09)	I_{bulk} (from T measurement)	82.1	0.00270	51
	I_{bulk} (from CPR measurement)	72	0.0024	45
	I_{edge} (from CPR measurement)	24	0.0124	
D2 (2023-09)	I_{bulk} (from T measurement)	83.8	0.00275	52
	I_{bulk} (from CPR measurement)	79	0.0026	49
	I_{edge} (from CPR measurement)	17	0.0089	

diffusive transport via nontopological bulk states is probed by the SQUIDS, leading to the observed sinusoidal CPR. Table II also shows that SQUID C's edge junction is the shortest of all, so that in fact the 1D hinge states may not be robust and phase coherent over much more than 600 nm, explaining why a sizable supercurrent cannot travel through edge junctions that are longer than 600 nm. In addition, a dissipative environment may round a sawtooth-shaped supercurrent that is too small, transforming it into a sinusoidal-shaped CPR. These environments may depend on the aspect ratio of the SQUID loops. The loop of SQUID C is $1.2 \times 12 \mu\text{m}^2$, whereas the loop of SQUID B1 (and B2, D1, and D2) has a $2 \times 5 \mu\text{m}^2$ area. Since the SQUID loops are not empty but are filled with WTe_2 , the two junctions are in fact in parallel with bulk WTe_2 , whose normal resistance acts as a dissipative environment for the junctions. Such a dissipative environment is known to modify the behavior of Josephson junctions, especially those with small supercurrents [52], decreasing the supercurrent and rounding the CPR via finite-temperature broadening effects [30,53], leading to sinusoidal CPRs.

Finally, we also note that the sinusoidal modulations in samples with the same reference and edge supercurrents as the sawtooth-shaped modulation are a useful confirmation that the sawtooth CPR of SQUID C is not attributable to an inductive effect, as in Ref. [22].

2. Comparison of the temperature dependence of the six SQUIDS' bulk and edge junctions

Comparing the decay with temperature of the critical current of the bulk and edge junctions of all measured SQUIDS (see Fig. 28) reveals that only the edge junction displaying a sawtooth behavior is more robust (i.e., has a greater Thouless energy) than the bulk junction. The respective identification of bulk and edge junctions in a given SQUID is possible thanks to the interference pattern in a magnetic field: At low field, the SQUID's critical current is the sum of the bulk junction's critical current and a smaller modulation due to the edge junction's CPR, whose amplitude is the edge junction's critical current. Fourier transforms of those relations display a large zero-frequency peak and a smaller finite-frequency peak

TABLE II. Crystal thickness, length, and width of the junctions, expected decay field, computed as Φ_0 divided by the bulk junction area, and $R_N I_c$ product of all SQUIDS. The “measured Φ_0/S_{bulk} ” value is the perpendicular field at which the critical current is minimum, corresponding to the first zero of a Fraunhofer pattern. We use this minimum critical current criterion even for SQUIDS B and D, which do not display an exact Fraunhofer dependence, but still present a close to zero critical current followed by a rebound. We include the magnetic focusing by the superconducting electrodes in the estimate of S_{bulk} . For all SQUIDS except C, with L the length of the junction and W its width, $S_{\text{bulk}} = L \times W + \frac{W^2}{2}$ [51]. For SQUID C, we compute the area the same way but by taking the equivalent rectangular junction described in Appendix C. Values with * are not measured directly but estimated; indeed, for SQUIDS C and A, the measurements were done with a large parallel magnetic field and a small perpendicular contribution. The relevant magnetic field here is the perpendicular component; the ratio between the perpendicular component and the total magnetic field is estimated by comparing the SQUID oscillation period with the expected period given the area defined by the SQUID loop.

SQUID	Thickness (nm)	Length $\times$ width ($\mu\text{m} \times \mu\text{m}$)		Φ_0/S_{bulk} (G)		$R_N I_c$ (μV)
		Edge junction	Bulk junction	Expected	Measured	
C	223	0.6×0.15	0.94×1.46 (see Fig. 17)	8	2*	7
A (2022-10)	282	$0.7\text{--}1.5 \times 1.3$	0.6×1	19	10*	2
B1 (2023-01)	108	$0.95\text{--}1.5 \times 0.8$	0.6×1	19	17	3
B2 (2023-01)	116	$1\text{--}1.5 \times 0.7$	0.6×1	19	17	2
D1 (2023-09)	32	2×0.55	0.5×1	21	20	1.5
D2 (2023-09)	30	2×0.5	0.5×1	21	20	1.5

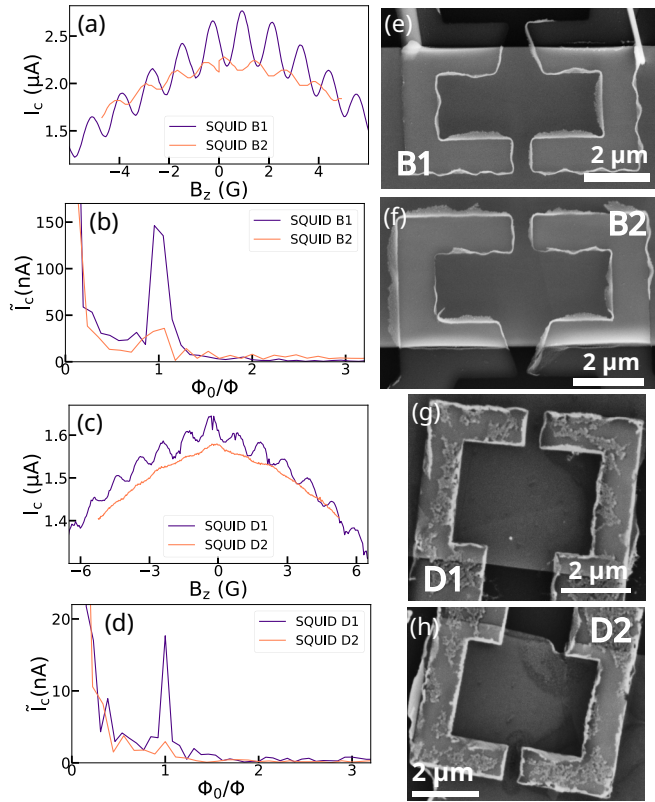


FIG. 27. Sinusoidal CPR of SQUID B and SQUID D's edge junctions. B1 [panel (e)] and B2 [panel (f)], on the one hand, and D1 [panel (g)] and D2 [panel (h)], on the other hand, are formed on opposite *a* edges of WTe₂ flakes. Like the other SQUIDs shown in Fig. 1, the superconducting ring is made of an 8-nm-thick layer of palladium covered by an 80-nm-thick layer of niobium. Those four SQUIDs show small sinusoidal oscillations on top of the bigger contribution of the bulk junction. The curves for B1 and B2 [panel (a)] are taken at 7 mK, while the ones for D1 and D2 [panel (c)] are taken at 200 mK. In both cases, the Fourier transform shows only one clearly visible peak [panels (b) and (d)], unlike the high harmonic content of SQUID C's edge junction (Fig. 2). Possible explanations for the absence of sawtooth-shaped oscillations (and more harmonics in the FFT) include the greater length of the edge junctions; a different, more dissipative environment; or simply the absence of an electrical contact between the superconductors and the 1D states.

related to the small oscillations with a period given by the SQUID loop area. Those peaks yield the amplitudes of both contributions, which we plot as a function of temperature in Fig. 19. Except for the edge junction of SQUID C, the critical current versus temperature curves are then fitted with the dependence expected for a long diffusive junction, yielding the Thouless energy for both edge and bulk junctions, given in Table I. To this end, we fit to one of those two expressions [48]:

$$eR_N I_c(T) = \frac{32}{3 + 2\sqrt{2}} E_{\text{Th}} \left(\frac{2\pi k_B T}{E_{\text{Th}}} \right)^{\frac{3}{2}} \times \sum_{n=0}^{\infty} \sqrt{2n+1} e^{-\sqrt{2n+1} \sqrt{\frac{2\pi k_B T}{E_{\text{Th}}}}} \quad (\text{D1})$$

valid for a long diffusive junction at high temperature $T > 5E_{\text{Th}}$ or

$$eR_N I_c(T) = 10.82 E_{\text{Th}} \left(1 - 1.3 e^{-\frac{10.82 E_{\text{Th}}}{3.2 k_B T}} \right) \quad (\text{D2})$$

valid for a long diffusive junction at low temperature $T < 5E_{\text{Th}}$.

The two expressions give almost the same E_{Th} for all the SQUIDs, leading us to use both indiscriminately even though the fitted curves can have aberrant behavior outside of the temperature range of validity.

A second determination of the bulk junction's critical current was also used for SQUIDs D1 and D2 (diamond marks) and yielded similar values of $E_{\text{Th}}^{\text{bulk}}$. It is based not on the Fourier transforms but on the total critical current measured at zero magnetic field for different temperatures. Although this critical current is the sum of the bulk and edge junction's critical currents, since the edge junction's critical current is 10–50 times smaller than the bulk junction's critical current, it is a good approximation of the bulk junction's critical current. As seen in the figure, only the current from SQUID C's edge junction, described in detail in the main text, shows a much slower decay of the critical current with temperature.

3. Estimate of WTe₂'s bulk resistivity

The two squids B1/B2 and the two squids D1/D2 are, respectively, both on the same flakes B and D of WTe₂. The bulk junction between the two squids presents a dissipationless current vanishing quickly in field and temperature. By measuring the normal resistance R_N , the length L , the width W , and the thickness d of the junction, we can estimate the resistivity ρ .

$$\rho_B = \frac{Wd}{L} R_N = \frac{1.5 \mu\text{m} \times 105 \text{ nm}}{2 \mu\text{m}} 0.8 \Omega = 6.3 \times 10^{-8} \Omega \text{ m}$$

$$\rho_D = \frac{Wd}{L} R_N = \frac{2.1 \mu\text{m} \times 35 \text{ nm}}{1.2 \mu\text{m}} 1.8 \Omega = 1.1 \times 10^{-7} \Omega \text{ m}.$$

These values agree well with those reported in Ref. [21].

APPENDIX E: ESTIMATE OF INDUCTANCE EFFECTS

Here, we argue that it is unlikely that the sawtooth-shaped CPR we observe in SQUID C's edge junction could be a sinusoidal-like CPR that is artificially tilted by inductance effects. Indeed, inductances (both geometric L_g and kinetic L_K) are known to induce a phase drop proportional to the current $L_{g,K} I / \Phi_0$ in superconductors with a current I running through them, so that the difference in phase drop across the two Josephson junctions in a dc SQUID can differ from the value estimated via the external flux Φ_{ext} (see Fig. 29 for the sketch and notations): $\varphi_1 - \varphi_2 = 2\pi \Phi_{\text{ext}} / \Phi_0 - (L_1 I_1 - L_2 I_2) / \Phi_0$. If the inductances and currents are large enough, and/or the asymmetry of inductance and/or current between the two junctions is large enough, this can lead to tilted current-versus-external flux relations $i(\Phi_{\text{ext}})$ that can be misinterpreted as tilted current versus phase relations $i(\varphi)$ [26,39,54]. These effects become important when $L_{g,K} i / \Phi_0 \gg 1$.

An estimate of SQUID C loop's geometrical inductance is $L_g = 14 \text{ pH}$ [55], given the loop dimensions $18 \mu\text{m} \times 4 \mu\text{m}$

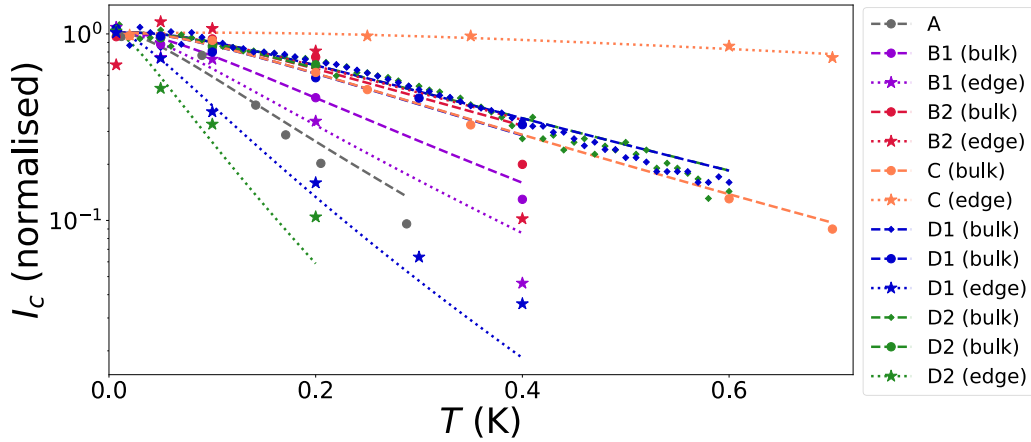


FIG. 28. Temperature dependence of the critical current of all measured SQUID junctions. The bulk and edge junctions' contributions are identified thanks to the interference patterns of each SQUID and its Fourier transform (circle and star marks). Diamond marks for D1 and D2 correspond to the the total critical current measured at zero field, which is practically equal (within a few percent) to the bulk junction's critical current. The critical current vs temperature curves are then fitted with the dependence expected for a long junction, Equation (D2), yielding the Thouless energy for both edge and bulk junctions, given in Table I. Only the current from the edge junction of SQUID C, described in detail in the main text, displays a much slower decay than the bulk junction.

and thickness 90 nm. An estimate of the loop's kinetic inductance is $L_K = 9$ pH [56], given the square resistance of the Nb film $R_\square \simeq 2 \Omega/\square$. Assuming both arms are roughly identical yields a total (kinetic and geometric) inductance: $L_1 \approx L_2 \approx 12$ pH. At worst, the difference in supercurrent in both arms can be estimated as the maximum critical current of the reference junction: $I_1 - I_2 \approx 4 \mu\text{A}$. Then, the parameter characterizing the importance of inductance effects is [54] $\beta_m = \frac{L_1 I_1 - L_2 I_2}{\Phi_0} \approx 0.03 \ll 1$, implying that inductive effects are far from sufficient to induce a deformation of the CPR such that a sinusoidal CPR would appear as a sawtooth CPR.

In Ref. [26] where a similar system is studied, the authors find an additional inductance of the order $L \approx 100$ pH, which is attributed to a PdTe compound that has formed at the interface between Pd and WTe₂. Even allowing for such a large inductance in the arm containing the edge junction, the parameters would be $L_2 = 112$ pH, $I_2 = 150$ nA, $L_1 = 12$ pH, and $I_1 = 4 \mu\text{A}$, and the importance of screening effects would be quantified by $\beta_m \approx 0.02$, which is negligible.

In addition, maybe the most convincing argument against a large effect of inductances in our measurements is the fact that we *do* measure sinusoidal CPRs in some of the SQUIDs (B1, B2, D1, and D2) that have similar contacts, geometry, and critical currents, and do not always measure sawtooth CPRs.

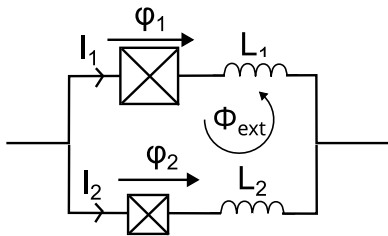


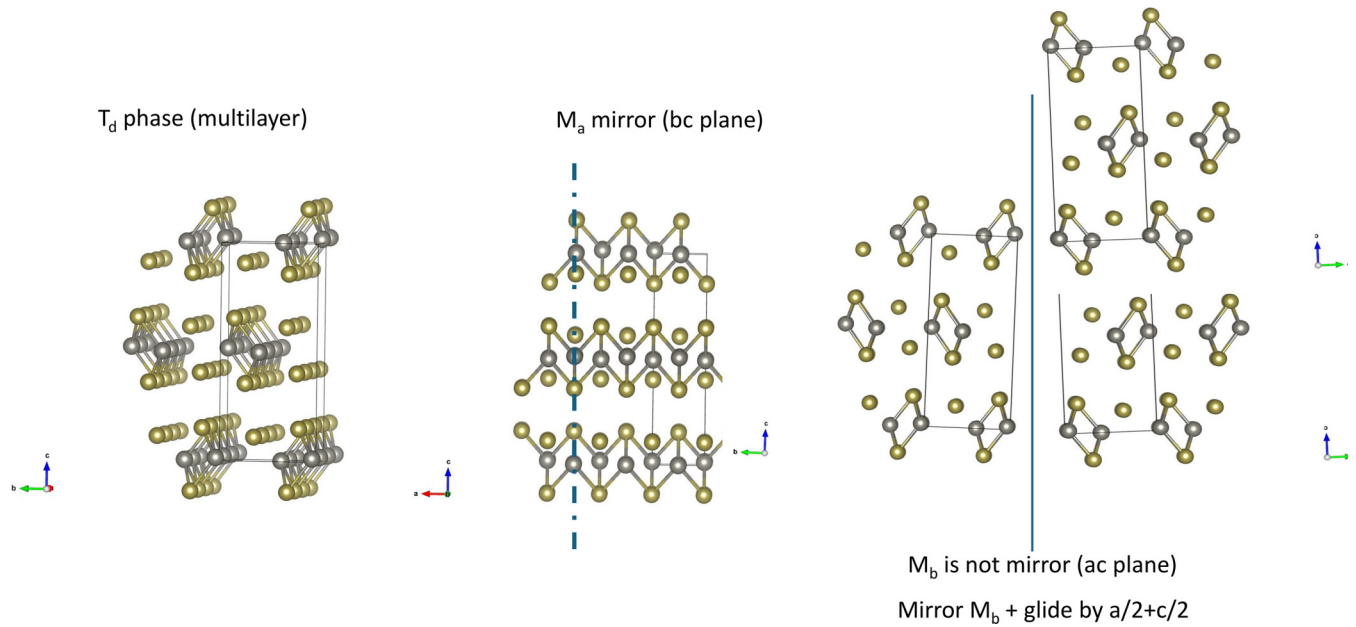
FIG. 29. Schematics of a SQUID with an inductance in each arm. The upper arm, labeled 1, contains the reference junction. The other arm, labeled 2, contains the edge junction.

APPENDIX F: SYNTHESIS OF WTe₂ CRYSTALS

The single crystals of WTe₂ were grown through the flux growth technique from a 95% Te-rich melt. High-purity W powder (Alfa Aesar, 99.999%) and Te ingots (Sigma-Aldrich, 99.999%) were used as the precursors. The obtained crystals were characterized using x-ray diffraction and energy-dispersive x-ray spectroscopy. To get a qualitative estimate of the crystallographic defect density, we measured the temperature dependence of the longitudinal resistivity of several randomly selected crystals. Within the 1.8–300 K temperature range, all WTe₂ single crystals show a residual resistivity ratio of 1000–1200, implying their high crystalline quality.

APPENDIX G: SYMMETRIES OF WTe₂

WTe₂ is a TMD material constituted of layers in the (a, b) plane stacked along the c direction. The atomic arrangement in the ab plane is characterized by tungsten atoms forming chains along the a direction, and the absence of a mirror symmetry perpendicular to the b vector [27]. Monolayer WTe₂ has been predicted to be a 2D topological insulator in the stable, inversion-symmetric 1T'-WTe₂ phase [57,58], and this prediction is supported by transport [1,2] and local probe [3–5] measurements that report insulating bulk and quantum spin Hall edge conduction. The stable 3D crystal form of WTe₂ is the T_d phase (symmetry group $Pmn2_1$), which is noncentrosymmetric. It possesses an M_a (bc -plane) mirror symmetry, an $\tilde{M}_b$ (ac -plane) glide-mirror symmetry, and a twofold screw axis along the c axis. The double band inversion at the gamma point combined with the screw symmetry along the c axis gives rise to a bulk 3D topological state for which surface domain walls and hinges can bind helical modes whose configurations are extrinsically controlled by a combination of surface band inversions and disorder [11–13]. This can be understood because double band inversion indicates that the low-energy topological physics is controlled by a massive eightfold (double) 3D Dirac fermion, for which


 FIG. 30. Representation of the multilayer WTe_2 atomic arrangement.

extrinsic mass textures bind 1D helical modes [14]. Like in BiBr, helical hinge modes in WTe_2 along the ab axes likely appear on alternating terraces/step edges, due to a combination of microscopic van der Waals coupling, the 3D bulk double band inversion, and the nontrivial 2D helical ($\mathbb{Z}_2$)

topology of monolayer WTe_2 [3]. However, the band structure of WTe_2 presents conducting electron and hole pockets with a possible type II Weyl point due to the broken inversion symmetry, which makes the observation of such 1d states in transport experiments challenging.

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