

Entanglement and dynamical scaling laws in quantum superabsorption

Juan David Álvarez-Cuartas^{✉*} and John H. Reina^{✉†}

Departamento de Física and Centre for Bioinformatics and Photonics—CIBioFi, Universidad del Valle, Cali 760032, Colombia and Department of Physics and Center for Computing in Science Education, University of Oslo, N-0316 Oslo, Norway



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Quantum batteries (QBs) exploit collective quantum resources to surpass the limits of classical energy storage and power delivery. We analyze N -qubit cavity-coupled QBs governed by Dicke and Tavis-Cummings models under Gaussian driving and open-system dynamics. Finite-size scaling laws $\mathcal{O}(N) \sim N^\alpha$ demonstrate an optimal region of relaxation and dephasing where coherent driving stabilizes entanglement entropy growth for thermodynamic observables (maximum energy $E_{\max}$, charging time τ , and maximum power $\bar{P}_{\max}$) and for qubit and cavity entanglement entropies. The Dicke model exhibits entropy-suppressed extensive behavior, while the Tavis-Cummings model achieves superextensive scaling with $\alpha_{E_{\max}} \in [1.08, 1.26]$, $\alpha_\tau \approx -0.49$, and $\alpha_{\bar{P}_{\max}} \in [1.57, 1.73]$, supported by qubit-cavity entanglement. We demonstrate that dissipation can act as a stabilizer source, yielding scaling benchmarks that are relevant to several experimental platforms. Our findings connect entanglement, dissipation-enhanced scaling laws, and superabsorption, outlining a pathway toward scalable quantum batteries offering practical quantum advantage.

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I. INTRODUCTION

Superabsorption is the cooperative enhancement of light absorption in an ensemble of quantum emitters, arising from the same collective coherence that gives rise to superradiance, but operating in reverse. In a superabsorbing system, the rate at which excitations are absorbed increases faster than linearly with the number of emitters, leading to faster energy transfer than any incoherent sum of independent units [1]. This phenomenon was theoretically predicted as the time-reversed analog of Dicke's superradiance [2], in which a collection of emitters coupled to a common electromagnetic mode undergoes collectively enhanced spontaneous emission, releasing radiation with an intensity that scales as N^2 . Both effects originate from the formation of collectively coupled Dicke states, whose shared coherence gives rise to nonclassical scaling in the emission and absorption rates. Understanding how such cooperative effects persist or degrade in open quantum environments is a central question for quantum technology.

Quantum batteries (QBs) are quantum devices designed to store and deliver energy by exploiting resources such as coherence and entanglement [3]. These uniquely quantum features raise the prospect of outperforming classical batteries in charging and power delivery. While classical thermodynamics establishes strict limits on extractable work from any system

[4–6], it does not constrain the timescales over which energy transfer occurs. This distinction has motivated intense interest in seeking a quantum advantage in out-of-equilibrium processes [7–9]. Although entanglement is not strictly necessary for optimal work extraction [10,11], it can significantly impact the rate and efficiency of charging, making it a key resource in quantum superabsorption technology [12,13].

Like conventional batteries, QBs are temporary storage units subject to physical constraints such as finite energy storage capacity and bounds on power density [14,15]. Strategies to enhance their performance have therefore been explored, including collective charging schemes that boost capacity [12,16], stabilization mechanisms that mitigate losses to the environment [17–19], and the use of decoherence-free subspaces [20–22]. Since the concept of a QB was introduced [23], several key theoretical advances have followed. In particular, Campaioli *et al.* showed that the choice of charging protocol can lead to superextensive behavior: When N identical qubits are charged collectively, the total charging power can scale faster than N , resulting in a $\sqrt{N}$ speedup compared to parallel charging [11]. Further progress came with the realization that the Dicke QB model could be implemented in solid-state architectures [12,24]. Quach *et al.* [25] achieved superabsorption at room temperature in a molecular QB and compared their measurements with results from the Tavis-Cummings (TC) model, demonstrating a quantum charging advantage in a physical system.

These findings highlight a key point: Quantum batteries are inherently open systems. They interact with their environment through processes such as spontaneous emission, dephasing, and photon leakage, which weaken quantum correlations and reduce stored energy. Improved energy retention and faster charging have also been reported under non-Markovian dynamics, where system-environment correlations persist over

*Contact author: j.d.a.cuartas@fys.uio.no

†Contact author: john.reina@correounivalle.edu.co

time [26,27]. However, in this work, we focus our attention on the Markovian regime, as discussed below.

A second subtle yet significant aspect relates to the nature of light-matter interactions in quantum batteries. Many studies use the rotating-wave approximation (RWA)—which neglects counterrotating (CR) terms—leading to the Tavis-Cummings model [28–30]. While this approximation remains valid in the weak-coupling regime ($g \ll \omega_q, \omega_c$, where g is the coupling strength and ω_q and ω_c are the qubit and the cavity transition frequencies, respectively), it breaks down in the ultrastrong regime [31]. The full Dicke model, which includes counterrotating terms, predicts phenomena such as vacuum-induced excitations, entangled ground states, and superradiant phase transitions [32]. However, the implications for charging dynamics in quantum batteries are not fully understood.

In this work, we present a comprehensive analysis of a QB comprising N identical two-level systems (TLSs), which we refer to as “qubits,” that are collectively coupled to a single-mode cavity and driven by a Gaussian pulse. We compare two interaction regimes: the Tavis-Cummings model and the Dicke model, which includes counterrotating processes. Dissipation is incorporated via three channels: (1) TLS relaxation through spontaneous emission, (2) TLS pure dephasing, and (3) cavity photon loss. Through a systematic analysis of these models, we quantify how the energy storage capacity, charging power, charging time, and entanglement entropy scale with the size of the system. This clarifies the role of dissipation in the thermodynamic performance of quantum batteries and the corresponding scaling laws.

The remainder of this paper is organized as follows. Section II introduces the QB models, the Tavis-Cummings and Dicke Hamiltonians, the Gaussian driving protocol, and the Lindblad dissipation channels. Section III describes the numerical implementation, which exploits permutation symmetry to access system sizes up to $N \sim 50$. Section IV presents the results: quasi-ideal dynamics, open-system behavior under cavity loss, relaxation and dephasing, and finite-size scaling laws $\mathcal{O}(N) \sim N^\alpha$ for the thermodynamic observables. Section V maps the operating regimes onto experimentally realistic parameters in leading platforms. Section VI provides a unified discussion and outlook, highlighting the microscopic mechanism of dissipation-enhanced superabsorption, the stabilizing role of coherent driving, and perspectives for optimized control and scalable implementations. Finally, Sec. VII summarizes the main findings of this work.

II. THEORETICAL FRAMEWORK

A primary goal of QB research is to identify the microscopic mechanisms that enable efficient energy storage and to understand how collective quantum phenomena enhance performance. One widely studied architecture is the cavity-mediated battery [24,25,33,34], where an ensemble of qubits interacts with a single electromagnetic mode. This setup incorporates both collective effects, which can enhance charging performance, and unavoidable sources of irreversibility, such as dissipation channels and entanglement with the cavity. To study these competing effects, we compare two complementary light-matter models: the Dicke

Hamiltonian (non-RWA), commonly adopted in theoretical analyses [2,31,35–37], and the Tavis-Cummings Hamiltonian (RWA), often employed to benchmark experimental results [28,29].

As illustrated in Fig. 1, the quantum battery consists of an ensemble of N qubits, which are collectively coupled to a single mode of a cavity. The qubits act as storage units, while the cavity mediates the transfer of collective excitation and serves as an intermediary charger. Charging can occur by initializing the qubits in their excited state or by applying a resonant Gaussian pulse to inject energy into the cavity. Dissipation is included through photon leakage from the cavity at rate κ , as well as through qubit dephasing and relaxation at rates γ^z and γ^- , respectively. Together, these processes define the open-system dynamics analyzed in this work.

Each qubit has transition frequency ω_q between ground $|g\rangle$ and excited $|e\rangle$ states, and the cavity has frequency ω_c , with qubit-cavity detuning $\Delta_c = \omega_q - \omega_c$. All qubits couple identically to the cavity mode with strength g . The full Hamiltonian of the driven system reads

$$\hat{H}(t) = \hat{H}_{\text{cav}} + \hat{H}_{\text{qub}} + \hat{H}_{\text{int}} + \hat{H}_{\text{drive}}(t), \quad (1)$$

where

$$\hat{H}_{\text{cav}} = \hbar \omega_c \hat{a}^\dagger \hat{a}, \quad (2)$$

$$\hat{H}_{\text{qub}} = \frac{\hbar}{2} \omega_q \sum_{j=1}^N \hat{\sigma}_j^z, \quad (3)$$

with $\hat{a}^\dagger$ ($\hat{a}$) the photon creation (annihilation) operators, and $\hat{\sigma}_j^\alpha$ the α -Pauli operator of the j th qubit, with $\alpha = x, y, z$.

To charge the system, an external Gaussian pulse is applied to the cavity mode. The corresponding external driving Hamiltonian is given by

$$\hat{H}_{\text{drive}}(t) = \hbar \eta(t) (\hat{a}^\dagger + \hat{a}), \quad (4)$$

$$\eta(t) = \eta_0 \exp\left(-\frac{(t - t_0)^2}{2\sigma^2}\right), \quad (5)$$

where η_0 is the drive amplitude, σ its width, and t_0 the pulse center. In the laboratory frame, the full expression for the drive usually includes a carrier oscillation of the type $\cos(\omega_d t)$; i.e., Eq. (5) takes the form $\eta(t) \cos(\omega_d t)$, where ω_d is the drive frequency. However, since our simulations are performed in the rotating frame of the cavity mode and the drive is resonant ($\Delta_d \equiv \omega_d - \omega_c = 0$), only the Gaussian envelope $\eta(t)$ remains in Eq. (4).

A. Light-matter interaction models

Taking into account all light-matter interaction terms leads to the Dicke Hamiltonian (non-RWA), given by

$$\hat{H}_{\text{int}}^{(\text{D})} = \hbar g \sum_{j=1}^N (\hat{a}^\dagger \hat{\sigma}_j^- + \hat{a} \hat{\sigma}_j^+ + \hat{a}^\dagger \hat{\sigma}_j^+ + \hat{a} \hat{\sigma}_j^-), \quad (6)$$

where $\hat{\sigma}_j^+ = |e\rangle_j \langle g|$ and $\hat{\sigma}_j^- = |g\rangle_j \langle e|$. Counterrotating terms $\hat{a}^\dagger \hat{\sigma}_j^+$ and $\hat{a} \hat{\sigma}_j^-$ simultaneously create or annihilate an excitation in both the qubit and the cavity. While these terms are negligible in weak coupling, they become relevant in

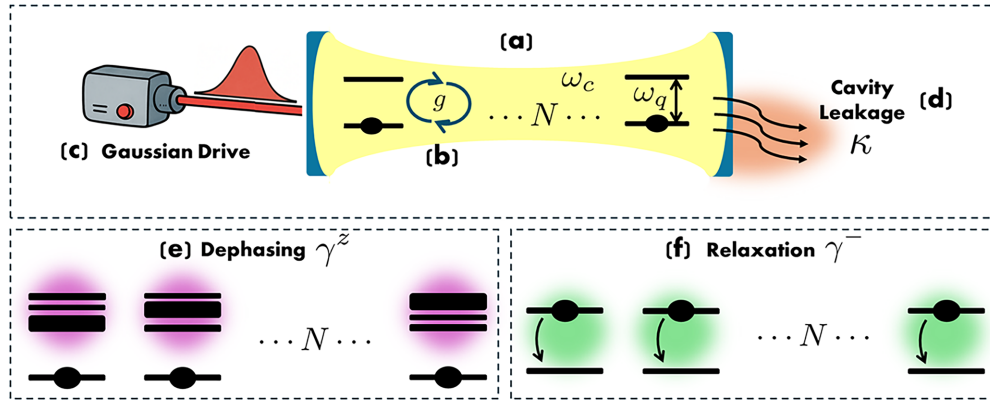


FIG. 1. Scheme of the cavity-driven QB and dissipation channels. (a) An ensemble of N identical qubits, each with a transition frequency ω_q , is (b) collectively coupled to a single cavity mode of frequency ω_c with a coupling strength g . (c) The battery is charged by a resonant Gaussian drive $\eta(t)$ that excites the cavity (see below). (d) The cavity mode loses photons through leakage at a rate κ . (e) Each qubit undergoes pure dephasing at rate γ^z , which randomizes the relative phases without exchanging energy; (f) qubit relaxation occurs at a rate γ^- , corresponding to spontaneous decay from the excited state to the ground state.

the ultrastrong coupling regime, roughly characterized by $g/\min(\omega_q, \omega_c) \sim 1$ [31,37]. This leads to effects such as virtual excitations and Bloch-Siegert shifts [35,36]. Retaining these terms allows us to investigate the effect of light-matter correlations during the charging dynamics [31,38].

In the weak-coupling regime, the counterrotating terms oscillate rapidly and can be neglected under the RWA, yielding the Tavis-Cummings Hamiltonian,

$$\hat{H}_{\text{int}}^{(\text{TC})} = \hbar g \sum_{j=1}^N (\hat{a}^\dagger \hat{\sigma}_j^- + \hat{a} \hat{\sigma}_j^+). \quad (7)$$

This Hamiltonian conserves the total number of excitations, making the dynamics integrable and supporting the emergence of coherent Rabi oscillations [28–30].

B. Thermodynamic observables and entanglement

To characterize the charging dynamics, we compute a set of observables that capture both thermodynamic behavior and quantum correlations.

The *maximum energy* stored in the N -qubit ensemble,

$$E_{\text{max}} = \max_{t_f} [\langle \hat{H}(t) \rangle - \langle \hat{H}(0) \rangle], \quad (8)$$

measures the largest energy that is stored in the battery during the charging procedure ($t = t_f$), referenced to the initial ground state $\langle \hat{H}(0) \rangle$. It coincides with the maximum increase of the internal energy of the qubit subsystem, a standard definition adopted in QB analyses [24,33,34].

The *charging time* τ is the first time at which $E_{\text{qub}}(t)$ reaches E_{max} , setting a characteristic speed for the process [24,34],

$$\tau = \inf \{ t \geq 0 : E(t) = E_{\text{max}} \}. \quad (9)$$

The *average maximum charging power*, or *maximum power* for short, P_{max} is obtained from the ratio

$$\bar{P}_{\text{max}} = \frac{E_{\text{max}}}{\tau}, \quad (10)$$

balancing the energy storage capacity against the time needed to reach it. This definition is now widely used in quantum battery studies to assess quantum advantage in collective charging [24,34].

The *entanglement entropy* provides a quantitative measure of correlations within the composite system through the von Neumann entropy of the reduced subsystems [39,40]. We denote $S_q(t)$ and $S_c(t)$ as the entanglement entropies of the N -qubit ensemble and the cavity, respectively, defined as

$$S_q(t) = -\text{Tr}[\hat{\rho}_{\text{qub}}(t) \ln \hat{\rho}_{\text{qub}}(t)], \quad (11)$$

$$S_c(t) = -\text{Tr}[\hat{\rho}_{\text{cav}}(t) \ln \hat{\rho}_{\text{cav}}(t)], \quad (12)$$

where the reduced density matrices are obtained from the total state $\hat{\rho}_{\text{tot}}(t)$ as

$$\hat{\rho}_{\text{qub}}(t) = \text{Tr}_{\text{cav}}(\hat{\rho}_{\text{tot}}(t)), \quad (13)$$

$$\hat{\rho}_{\text{cav}}(t) = \text{Tr}_{\text{qub}}(\hat{\rho}_{\text{tot}}(t)). \quad (14)$$

For closed dynamics, the global state remains pure; hence, $S_q = S_c$, and the entropy directly measures qubit-cavity entanglement [39]. Under dissipation, however, both S_q and S_c capture the combined effects of coherent qubit-cavity correlations and incoherent mixing induced by the environment, thereby providing a unified measure of entanglement and noise [7,41–43].

We characterize the information content during the charging protocol by means of two complementary entropy observables. The first is the *maximum entanglement entropy*,

$$S_q \equiv \max_t S_q(t), \quad S_c \equiv \max_t S_c(t), \quad (15)$$

which quantifies the largest buildup of correlations during the charging process. The behavior of S_q with respect to the system parameters allows us to identify the regimes in which strong collective effects and superabsorption emerge, thereby serving as a witness of quantum advantage.

The second observable is the *final entanglement entropy*,

$$S_q^f \equiv S_q(t_f), \quad S_c^f \equiv S_c(t_f), \quad (16)$$

evaluated at a time $t_f \gg \tau$ well after the charging peak, once nonstationary processes have passed. These final entropies capture the residual quantum correlations and classical mixedness remaining in the battery and cavity after the protocol has been carried out. This allows us to probe the stability and resistance to decoherence of the stored energy.

C. Dissipation channels and master equation

The qubit-cavity system is typically not isolated, but rather interacts with the external environment, inducing irreversible processes that degrade coherence and reduce charging efficiency. When modeling the dissipative quantum dynamics of the QB, we consider the following primary dissipation channels:

Cavity leakage. Photons escape into external modes at a rate κ , representing energy loss from the cavity. This is modeled by the collapse operator

$$\hat{C}_{\text{cav}} = \sqrt{\kappa} \hat{a}. \quad (17)$$

TLS relaxation. Describes the spontaneous decay of a qubit from $|e\rangle$ to $|g\rangle$, releasing energy into the environment at a rate γ^- . This process discharges the battery by removing stored energy [44,45]. It is modeled by

$$\hat{C}_j^- = \sqrt{\gamma^-} \hat{\sigma}_j^-, \quad j = 1, \dots, N. \quad (18)$$

TLS dephasing. Random environmental fluctuations randomize the phase between $|g\rangle$ and $|e\rangle$ without energy exchange. This suppresses coherence and qubit-cavity (bipartite) entanglement, reducing collective charging efficiency. It occurs at rate γ^z and is described by

$$\hat{C}_j^z = \sqrt{\gamma^z} \hat{\sigma}_j^z, \quad j = 1, \dots, N. \quad (19)$$

The dynamics of the driven qubit-cavity system under dissipation can be modeled using the Lindblad master equation in the Born-Markov approximation [40,46–48]:

$$\frac{d\hat{\rho}}{dt} = -\frac{i}{\hbar} [\hat{H}(t), \hat{\rho}] + \mathcal{L}[\hat{C}_{\text{cav}}] + \sum_{j=1}^N (\mathcal{L}[\hat{C}_j^-] + \mathcal{L}[\hat{C}_j^z]), \quad (20)$$

where $\hat{\rho} \equiv \hat{\rho}(t)$ is the density operator of the full system, $\hat{H}(t)$ is given in Eq. (1), and the Lindblad terms are given by $\mathcal{L}[\hat{C}] = \hat{C} \hat{\rho} \hat{C}^\dagger - \frac{1}{2} \{\hat{C}^\dagger \hat{C}, \hat{\rho}\}$, where the $\hat{C}$ operators are as defined in Eqs. (17)–(19). For the rest of the paper, we adopt natural units, setting $\hbar = 1$.

The Lindblad equation provides a rigorous Markovian description of open-system dynamics, ensuring the positivity and trace preservation of the density operator $\hat{\rho}(t)$ [40,46–48]. This framework enables a consistent comparison of RWA and non-RWA dynamics under Markovian dissipation.

III. NUMERICAL IMPLEMENTATION

Simulating the Lindblad master equation for an open system comprising many qubits is computationally prohibitive due to the exponential growth of the state space. Evolving a density matrix of size $2^N \times 2^N$ becomes numerically intractable for $N \gtrsim 10$ [40]. This challenge has also been recognized in QB studies, where exact diagonalization has been limited to small N [24,33]. More generally, as

the Hilbert space dimension grows as 2^N , the Liouvillian space has a dimension 4^N , rendering brute-force integration infeasible. Approximate approaches have therefore been explored to address this issue. Cumulant expansions (mean-field truncations) can capture low-order correlations, but miss finer quantum details [49]. Quantum trajectory methods (Monte Carlo wavefunctions) unravel the master equation into stochastic pure-state evolutions, reducing the complexity by averaging over many realizations [50,51].

In this work, we use the open-source Permutational Invariant Quantum Solver (PIQS) library [49], integrated into QuTiP [52–54], which is designed for ensembles of indistinguishable TLSs. The key idea is that, when all qubits are identical and couple symmetrically to the environment, the dynamics are confined to the symmetric Dicke manifold. Rather than having a dimension 2^N , this subspace has dimension $N + 1$ (spanned by Dicke states $|J = N/2, m\rangle$ with $m = -J, \dots, +J$), providing a dramatic reduction in complexity [28]. By exploiting this symmetry, PIQS constructs the Liouvillian directly in the Dicke basis, yielding polynomial scaling in N rather than exponential scaling [49]. The qubit ensemble is represented via the collective spin operators $\hat{J}_{x,y,z}$ and $\hat{J}_\pm$. The qubit Hamiltonian is proportional to $\hat{J}_z$, the qubit-cavity interactions couple $\hat{J}_\pm$ to a and $a^\dagger$, and dissipation channels are reformulated consistently: Collective relaxation (γ^-) and dephasing (γ^z) rates are proportional to Lindblad collapse operators, $\hat{J}_-$ and $\hat{J}_z$, respectively, while cavity leakage remains a local channel proportional to a ($\propto a$). This representation enables the efficient construction and integration of the Liouvillian while preserving full quantum correlations.

For the bosonic mode, we truncate the cavity Hilbert space to the Fock basis $\{|n\rangle\}_{n=0}^{n_{\text{max}}}$ with $n_{\text{max}} = 10$. An analytical bound and an explicit convergence test are provided in Appendix A.

We simulated QBs with qubit numbers ranging from $N = 5$ to $N = 50$ in steps of five, and coupling strengths $0 \leq g/\omega_q \leq 1$. The parallel charging protocol [11] was implemented, and dissipation was included through cavity leakage ($\kappa/\omega_q \in [10^{-3}, 1]$), qubit relaxation ($\gamma^-/\omega_q \in [10^{-3}, 1]$), and qubit pure dephasing ($\gamma^z/\omega_q \in [10^{-3}, 1]$).

We performed two complementary sets of simulations under the parallel charging protocol, where all the qubits are coupled to a single cavity mode within the same cavity, for both driven and undriven scenarios. The first set focused on quasi-ideal dynamics, where all dissipation channels were fixed at their minimum values. This enabled us to isolate the impact of coherent qubit-cavity interactions by examining how energy and entropy scaled with the coupling strength g and system size N . The second set incorporates open-system dynamics with fixed $g/\omega_q = 0.1$. Here, we distinguish two regimes: (1) cavity effects, in which we analyze the interplay between cavity loss κ and system size N , while keeping qubit relaxation and dephasing to a minimum; and (2) decoherence effects, in which we explore the combined impact of qubit relaxation γ^- and pure dephasing γ^z across all system sizes, while fixing cavity loss to its minimum value.

In the *undriven* scenario, the N -qubit ensemble is initialized in its ground state, while the cavity mode is prepared in an excited Fock state. In the *driven* setup, both the

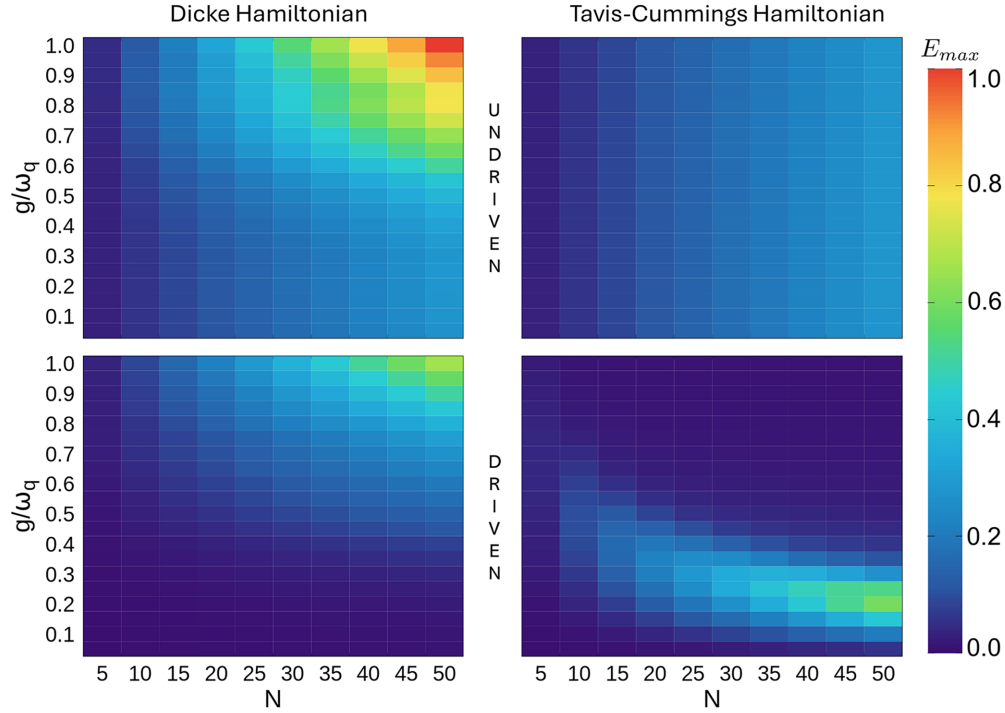


FIG. 2. Maximum stored energy $E_{\max}$ as a function of qubit number N and coupling strength g/ω_q ; $\kappa/\omega_q = \gamma^z/\omega_q = \gamma^-/\omega_q = 10^{-3}$. Results are shown for Dicke (left) and Tavis-Cummings (right) Hamiltonians in the undriven (top) and driven (bottom) setups. Color scales are row normalized: For each setup (driven or undriven), we divide by the maximum $E_{\max}$ across both scenarios and for all $(N, g/\omega_q)$ conditions.

qubits and the cavity start in their ground states. A resonant Gaussian pulse ($\Delta_d = 0$), with amplitude $\eta_0 = \omega_q$, width $\sigma = 2.0/\omega_q$, and center $t_0 = 5.0/\omega_q$, then injects energy into the system through the cavity. These pulse parameters ensure a controlled, resonant charging process: The amplitude is comparable to the light-matter coupling for efficient, balanced excitation; the width provides a smooth, broadband drive faster than the dissipative timescales ($\omega_q/\gamma^{[z,-]} \sim 10^3$); and the early pulse timing leaves an ample evolution window. This guarantees stability and highlights intrinsic differences between Dicke and Tavis-Cummings batteries rather than artifacts of the drive.

For each parameter configuration, the time evolution of the thermodynamic observables introduced in Sec. II B is computed. From these data, we extract the maximum stored energy, $E_{\max}$, the charging time, τ , the maximum power, $\bar{P}_{\max}$, and the maximum (final) entanglement entropies, $S_q(S_q^f)$ and $S_c(S_c^f)$. Heatmaps are then generated over the dissipation parameter space and scaling exponents α are obtained from log-log fits of observables versus system size.

IV. RESULTS

We analyze the dynamics of the QB across different regimes. First, we characterize the dynamics in a scenario where the dissipation channels are included but remain very weak compared to the system frequencies, i.e., $\kappa, \gamma^-, \gamma^z \ll \omega_q$, such that coherence is long lived. For this reason, we refer to this as a *quasi-ideal* cavity. This regime effectively captures near-coherent evolution while allowing a realistic description of residual losses and serves as a benchmark

for optimal performance. Next, we incorporate open-system dynamics and analyze the effects of cavity photon leakage and environmental decoherence via dephasing and relaxation. This analysis demonstrates how specific dissipation channels impact the scaling and stability of energy storage. Finally, we provide a detailed examination of the scaling laws of thermodynamic observables, extracting the scaling exponents of $E_{\max}$, τ , and $\bar{P}_{\max}$. We also identify regimes of enhanced performance and entanglement entropy suppression.

A. Quasi-ideal cavity dynamics

In the undriven scenario, the Tavis-Cummings model exhibits full energy revivals with Rabi oscillations at frequency $\omega \propto \sqrt{N}g$ [28], where the entanglement entropy oscillates periodically and nearly synchronously with the stored energy, reflecting coherent and reversible exchange. The Dicke model only reproduces this behavior for small g and N . At stronger coupling ($g \sim \omega_q$), energy transfer becomes faster but less reversible, with delayed and broadened entropy peaks that remain large even as energy decays, evidencing persistent entanglement between the qubits and the cavity field. These observations are based on the real-time evolution of energy and entanglement entropy, not shown here for brevity. However, it reproduces the standard revival dynamics that are well established in the literature [55–59].

In the Dicke model, the stored energy (see Fig. 2) exhibits growth with N when $g/\omega_q \gtrsim 0.5$, highlighting the role of counterrotating processes in enhancing capacity. In contrast, the Tavis-Cummings model displays nearly vertical contours, indicating that energy absorption remains approximately constant across different coupling strengths

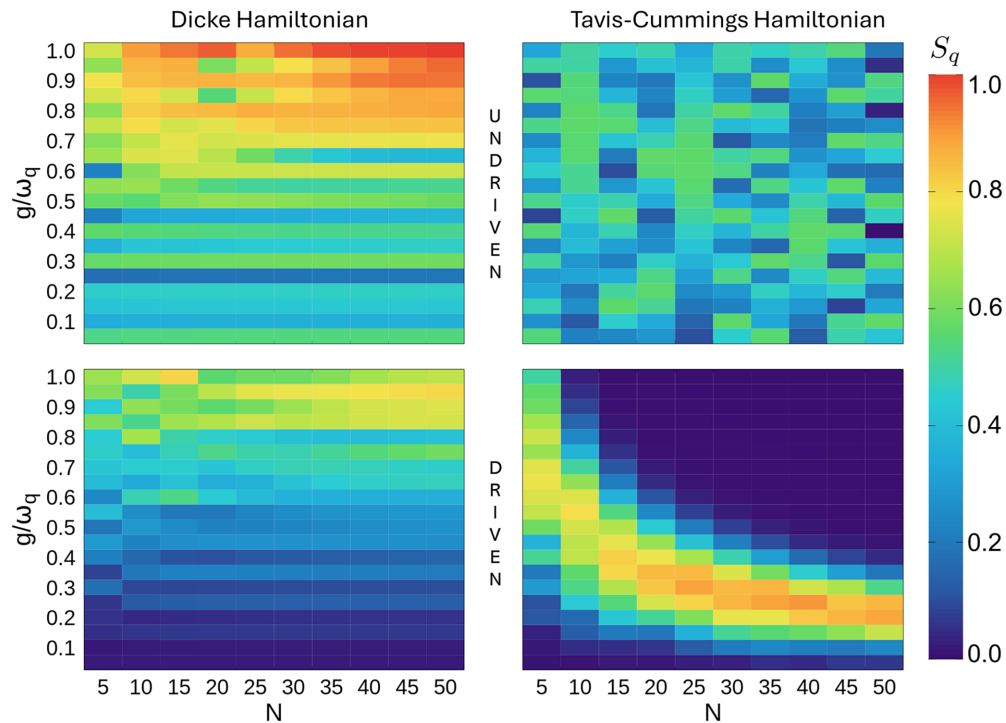


FIG. 3. Entanglement entropy S_q , as a function of qubit number N and coupling strength g/ω_q , with the same parameters as in Fig. 2. The panels compare the Dicke (left) and Tavis-Cummings (right) Hamiltonians for the undriven (top) and driven (bottom) setups. The color scales are row normalized as in Fig. 2, but using the maximum value of S_q .

and depends primarily on the system size N . When driven, the Dicke model retains its advantage, reaching maximum energy value at $g \sim \omega_q$. In contrast, the Tavis-Cummings model only shows localized enhancement around $g/\omega_q \sim 0.1$ – 0.4 for large N , where the coupling strength and driving frequency are optimally matched.

Figure 3 plots the qubit entanglement entropy as a function of the coupling strength and the system size. The Dicke model exhibits strong and structured qubit-cavity correlations for $g/\omega_q \gtrsim 0.5$, which persist as N increases. When driving is introduced, the dynamics of the Dicke model remain largely unchanged; in the Tavis-Cummings model, however, laser pumping dramatically enhances correlations within a narrow resonance band [60], where the Gaussian pulse bandwidth $\Delta\omega \approx 1/\sigma = \omega_q/2$ matches the collective Rabi frequency $g\sqrt{N}$, producing the localized entanglement enhancement observed in Fig. 3.

Thus, coherent driving introduces controlled energy injection altering the Dicke dynamics qualitatively, whereas in the Tavis-Cummings model, it selectively amplifies correlations near resonance. This establishes coherent driving as an effective mechanism for initiating collective charging and entanglement.

B. Open-system dynamics

First, we examine the effect of cavity leakage, which accounts for photon losses from the bosonic mode, reflecting the cavity's level of nonideality. Second, we examine the impact of dephasing and energy relaxation on the N -qubit ensemble. By independently varying their strengths, we can identify regimes in which moderate dissipation improves

performance, stabilizes entanglement entropy production, and enhances scalability.

1. Cavity leakage effects

We begin the analysis by setting the coupling strength to $g/\omega_q = 0.1$ and fixing the dephasing and relaxation rates to $\gamma^z/\omega_q = \gamma^-/\omega_q = 10^{-3}$. As the dominant channel of energy loss in cavity-mediated QBs, leakage competes directly with the charging dynamics by enabling excitations to escape into external modes. By varying κ over a broad range, we can determine the effect of cavity imperfections on energy storage capacity and qubit-cavity correlations. Figure 4 shows that the Tavis-Cummings Hamiltonian maintains its characteristic linear scaling with N for the maximum stored energy, across all values of cavity leakage κ , with nearly linear growth even at large dissipation rates. In contrast, the Dicke model quickly saturates; energy increases with N only for very small κ , and decreases as leakage increases, indicating strong susceptibility to photon loss. Under driving, this asymmetry becomes more pronounced. The Tavis-Cummings model retains its linear scaling, while the Dicke model essentially flattens at low energy once appreciable cavity leakage occurs. These results demonstrate that counterrotating processes, which in the quasi-ideal limit enhanced capacity, here accelerate the degradation of energy storage under cavity losses.

In the Dicke model, the maximum entanglement entropy (see Fig. 5) displays a pronounced minimum in the region $\kappa/\omega_q \approx 0.2$ – 0.4 , in both undriven and driven scenarios. For leakage rates that are smaller or larger, the entropy rises again. This shows that moderate photon losses briefly suppress qubit-cavity correlations before stronger dissipation

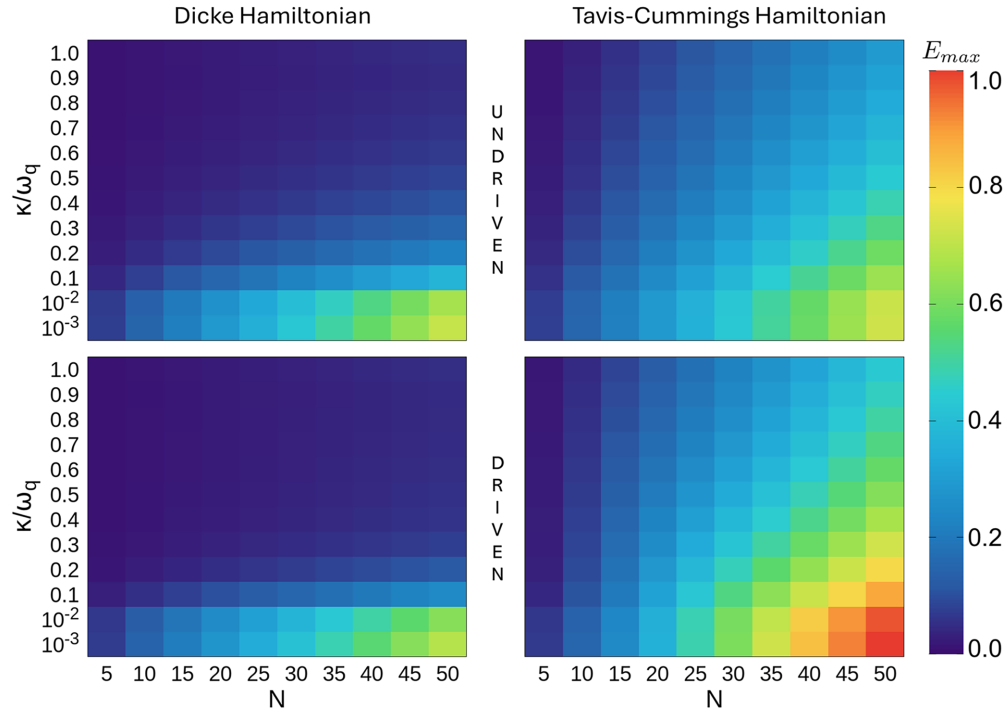


FIG. 4. Maximum stored energy E_{max} as a function of κ and qubit number N . $g/\omega_q = 0.1$ and $\gamma^z/\omega_q = \gamma^-/\omega_q = 10^{-3}$. The color scale is row normalized, as in Fig. 2, using the maximum value of E_{max} . The κ rates are varied over the discrete set of values $\{10^{-3}, 10^{-2}, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0\}$.

reinduces mixedness through residual cavity coupling. The trend is more evident in the undriven scenario, where the entropy peaks at low κ , while driving smooths the contrast and

slightly enhances correlations at larger N . In contrast, in the Tavis-Cummings model, the entropy is almost zero for most parameters, with small oscillatory bands near $\kappa/\omega_q \approx 0.1$ – 0.2

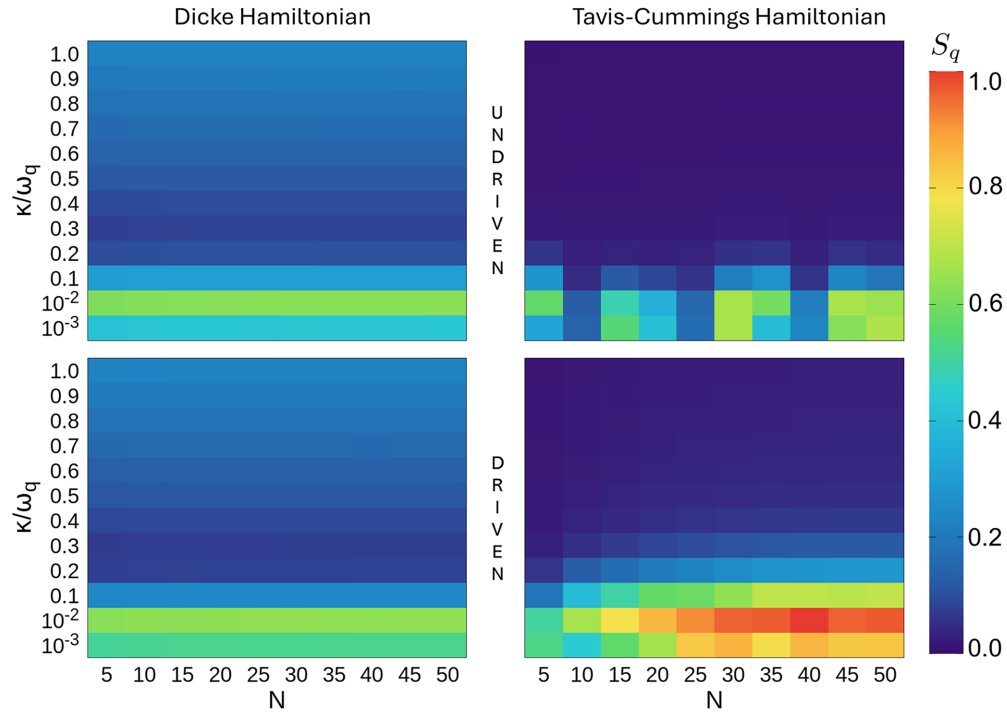


FIG. 5. Maximum entanglement entropy S_q as a function of κ and qubit number N , with same parameters and κ values as in Fig. 4. Results are shown for Dicke (left) and Tavis-Cummings (right) Hamiltonians under undriven (top) and driven (bottom) conditions. Color scale is row normalized, as in Fig. 2, using the maximum value of S_q .

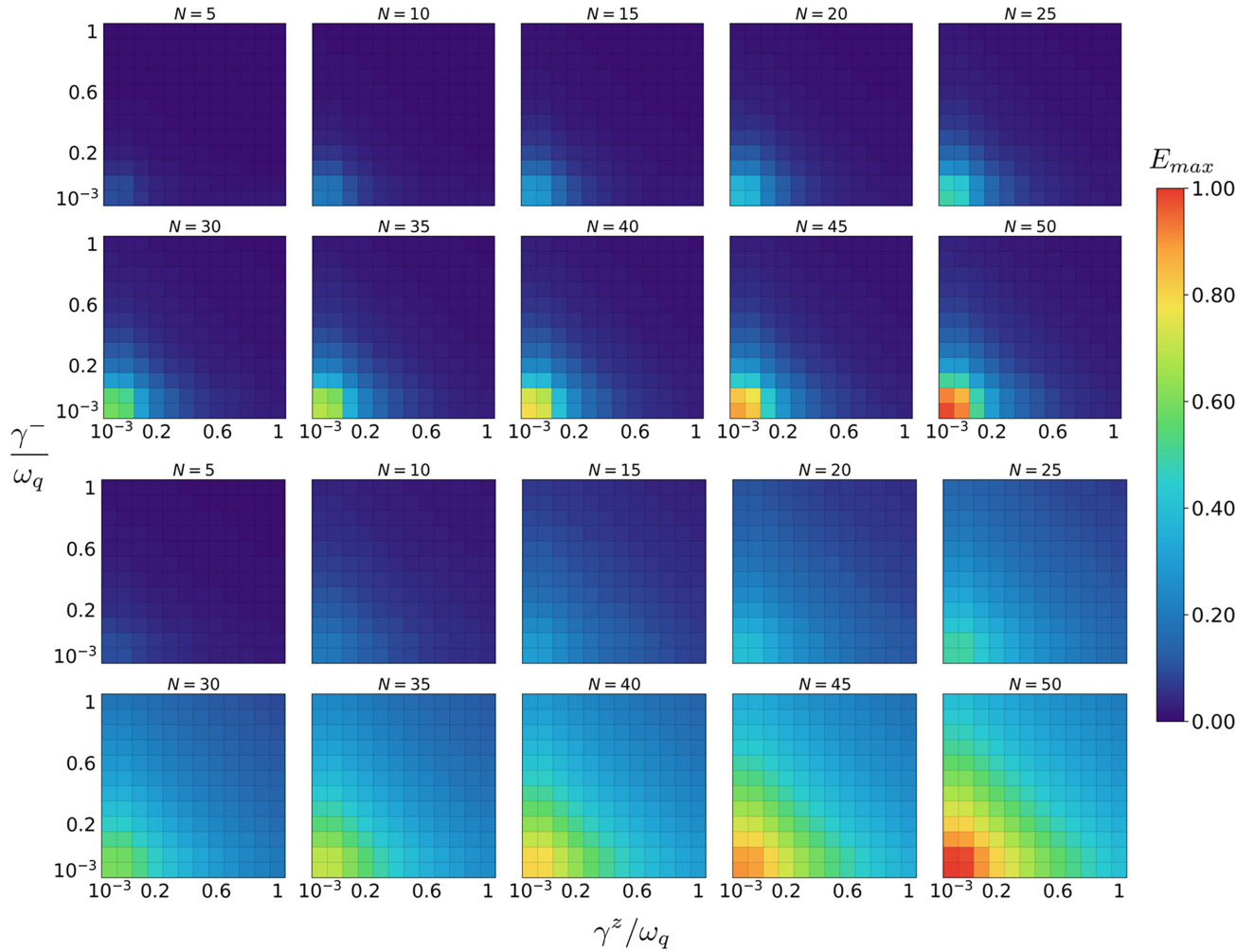


FIG. 6. Heatmaps of the maximum stored energy $E_{\max}$ as a function of γ^- and γ^z , for different qubit number N ; Dicke (top panels) and Tavis-Cummings (bottom panels) models. $g/\omega_q = 0.1$ and $\kappa/\omega_q = 10^{-3}$. Both the relaxation and dephasing rates, γ^-/ω_q and γ^z/ω_q , are varied over the same discrete set of values $\{10^{-3}, 10^{-2}, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0\}$. Color scale is model normalized, using the maximum $E_{\max}$ within the corresponding panel set.

that disappear under driving. The applied drive then produces a monotonic increase in S_q with system size for decreasing κ , resulting in maximal correlations at large N and low leakage.

2. Dephasing and relaxation interplay

We now turn to the combined influence of qubit dephasing and relaxation. For this analysis, we set the coupling strength to $g/\omega_q = 0.1$ and fix the cavity leakage rate to $\kappa/\omega_q = 10^{-3}$, allowing us to isolate the interplay between γ^- and γ^z . Both the relaxation and dephasing rates, γ^-/ω_q and γ^z/ω_q , are varied over the same discrete set of values $\{10^{-3}, 10^{-2}, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0\}$.

For maximum stored energy [see Fig. 6 (upper panels)], the Dicke model shows a clear region of enhanced energy storage extending up to approximately $\gamma^-/\omega_q, \gamma^z/\omega_q \approx 0.4$ as system size increases, showing that collective coupling amplifies the tolerance to moderate noise. Within this range, both dissipation channels contribute constructively without strongly

degrading energy storage capacity. Beyond this threshold, energy storage rapidly decays as losses dominate.

In contrast, the Tavis-Cummings model (Fig. 6, lower panels) shows a broader stability domain, with significant energy retention persisting up to $\gamma^-/\omega_q, \gamma^z/\omega_q \approx 0.8$, also depending on system size. Although the overall dependence is smoother and less sharply peaked than in the Dicke case, the RWA dynamics exhibit greater tolerance to dissipation, maintaining substantial energy storage even under relatively strong dephasing and relaxation.

Figure 7 plots the cavity entanglement entropy; this exhibits markedly different behavior for the two models. In the Dicke model (top panels), S_c exhibits a broad diagonal gradient: Entropy remains low when either relaxation or dephasing is small, but increases significantly once both channels act together, roughly along the diagonal $\gamma^- \approx \gamma^z$; a banded pattern emerges, with S_c oscillating between a suppression band $\gamma^-/\omega_q, \gamma^z/\omega_q \approx 0.2-0.6$ and growth as the dissipation rates increase. This nonmonotonic behavior reflects the competition between the collective coherent exchange frequency $\sqrt{N}g$

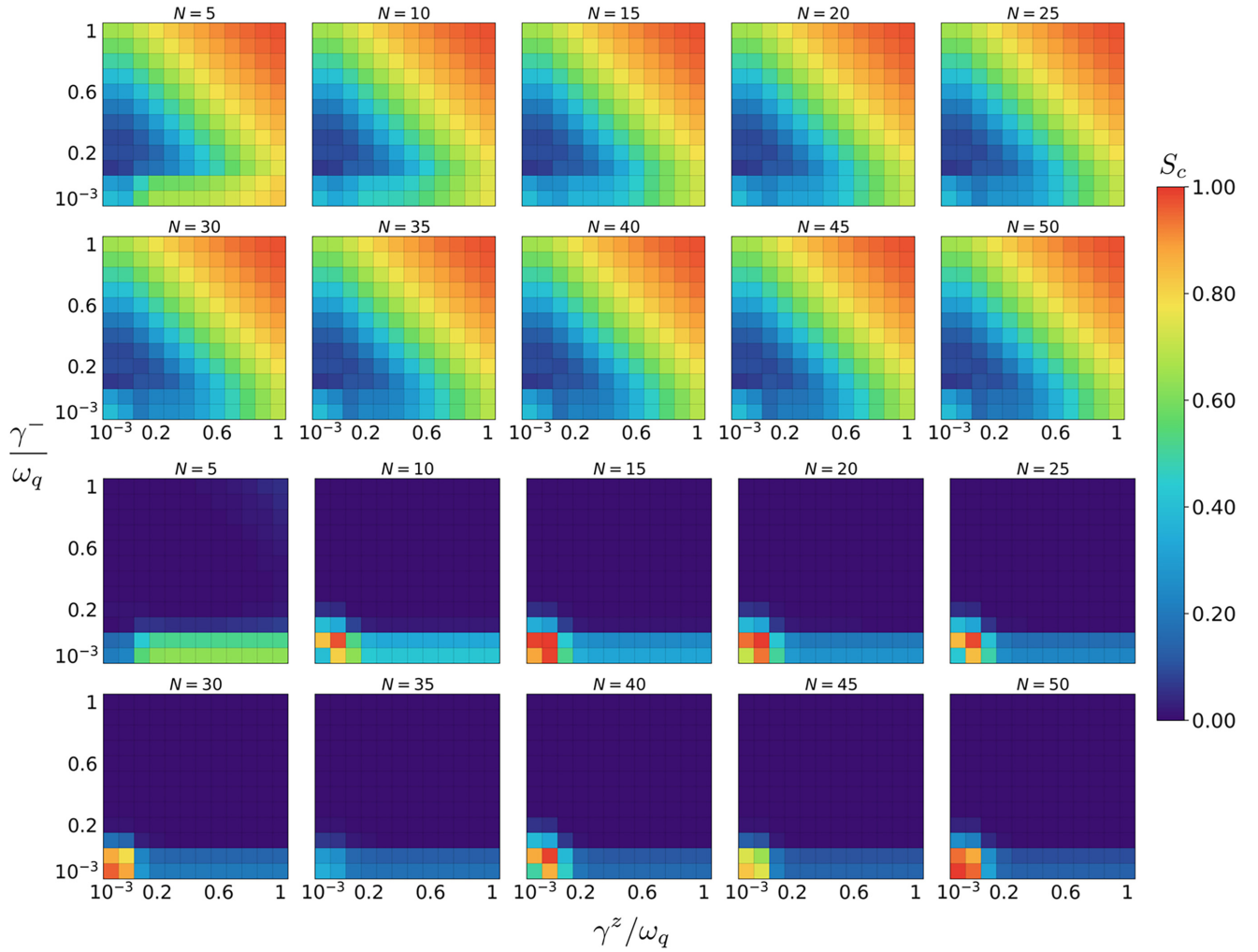


FIG. 7. Heatmaps of the cavity entanglement entropy S_c as a function of γ^- and γ^z , for different qubit number N . $g/\omega_q = 0.1$ and $\kappa/\omega_q = 10^{-3}$. The Dicke model (top) exhibits broad regions of finite entropy, while the Tavis-Cummings model (bottom) strongly suppresses entropy generation. Color scale is model normalized, as in Fig. 6, using the maximum value of S_c or S_c^{RWA} , with the relaxation and dephasing rates varied over the same range as in that figure.

(amplified by counterrotating processes) and the dissipation rates γ^- , γ^z .

In the Tavis-Cummings model [see Fig. 7 (lower panels)], the cavity entanglement entropy remains strongly suppressed across most of the dissipation plane, with nontrivial values confined to a narrow horizontal band for $\gamma^-/\omega_q \lesssim 0.2$ and $\gamma^z/\omega_q \lesssim 1.0$. As the system size increases, small localized hot spots emerge within this band around $\gamma^-/\omega_q, \gamma^z/\omega_q \lesssim 0.2$, showing strong cavity mixedness. Here, dephasing acts as a tunable channel that modulates entropy generation within the cavity.

In the Dicke model, the nontrivial values of the qubit entanglement entropy are confined to a narrow band extending up to $\gamma^-/\omega_q \lesssim 0.2$ and $\gamma^z/\omega_q \lesssim 1.0$ (see Fig. 8). Within this sector, a crossover region for $\gamma^-/\omega_q, \gamma^z/\omega_q \lesssim 0.2$, indicating the coexistence of low and moderate entanglement entropy production when both decay channels are comparable. Along the dephasing-dominated axis ($0.2 \lesssim \gamma^z/\omega_q \lesssim 1.0$), the hot ridge observed at small system sizes progressively cools down as N increases, showing that collective coupling suppresses excessive mixing. Moreover, for $\gamma^-/\omega_q, \gamma^z/\omega_q \in [0.2, 0.6]$,

a distinct colder stripe emerges as system size increases, indicating the formation of a regime of moderate and stabilized entanglement entropy production. This behavior suggests that increasing the number of qubits enhances the resistance of the ensemble to both relaxation and dephasing, leading to controlled entanglement production.

In the Tavis-Cummings model (see Fig. 8), the overall trend is similar: Entropy values remain low across most of the dissipation plane and are confined to a narrow horizontal band for $\gamma^-/\omega_q \lesssim 0.2$. As with the Dicke model, dependence on system size is less pronounced, confirming that, even in the presence of dephasing, excitation-number conservation within the RWA limits entanglement generation.

As shown in Fig. 9, charging time shows a homogeneous behavior in the Dicke model (upper panels), remaining small and nearly independent of the dissipation rates. This implies that the battery charges efficiently, even in the presence of decoherence channels. In contrast, the Tavis-Cummings (lower panels) model predicts significant delays: For $\gamma^-/\omega_q \lesssim 0.2$ and $0.6 \lesssim \gamma^z/\omega_q$, the charging time increases noticeably, for

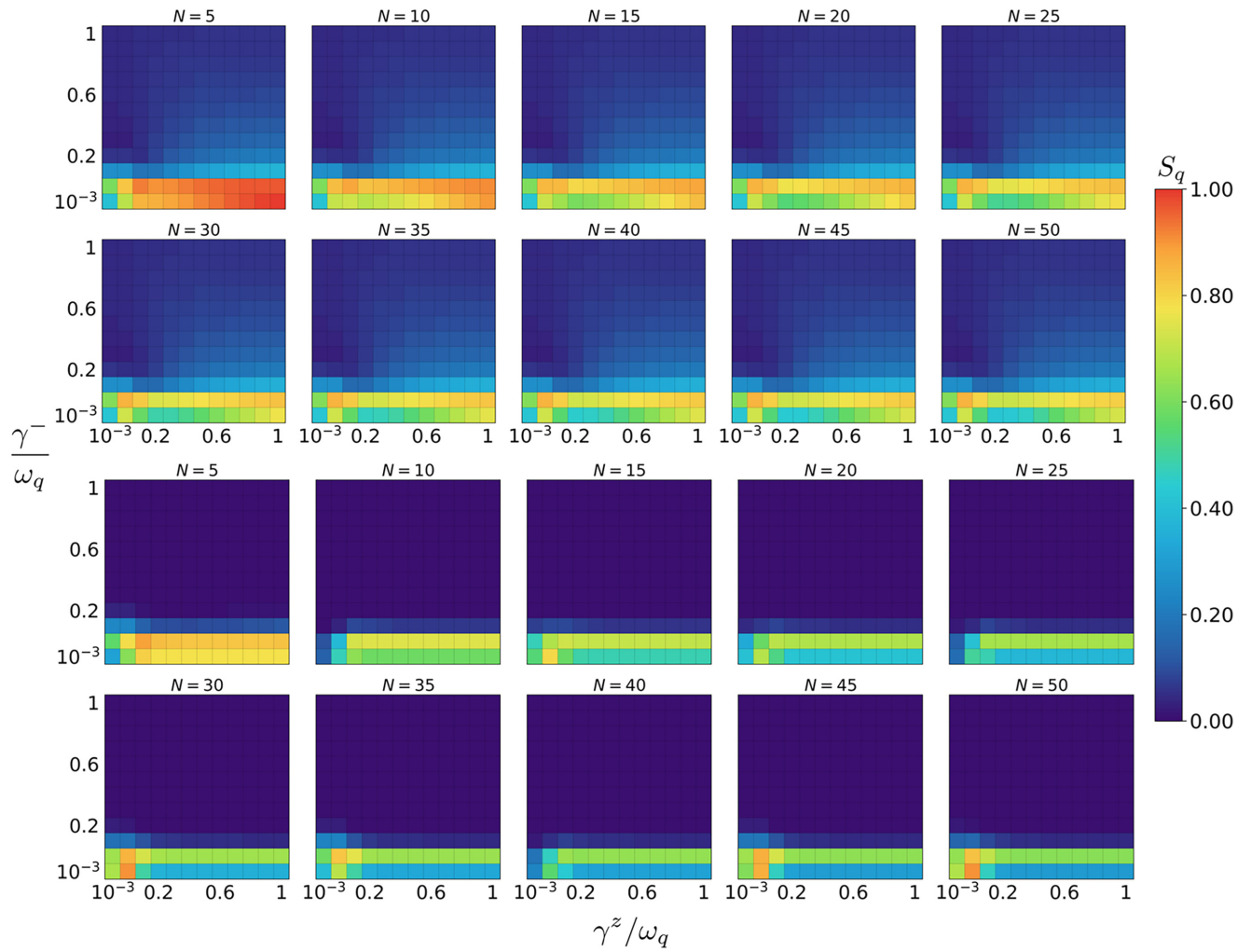


FIG. 8. Heatmaps of the qubit entanglement entropy S_q as a function of γ^- and γ^z , for different qubit number N . Parameters used are as in Fig. 6. In the Dicke model (top), entropy is sustained under low relaxation and moderate dephasing, whereas the Tavis-Cummings model (bottom) predicts negligible entropy across almost all regimes. Color scale is model normalized, using the maximum value of S_q or S_q^{RWA} , with the relaxation and dephasing rates varied over the same range as in that figure.

relatively small number of qubits ($N \lesssim 25$), suggesting that dissipation slows down the charging process. However, as system size increases ($N \gtrsim 30$), this dissipation-induced delay tends to disappear.

Figure 10 shows that the maximum power closely follows the trends in stored energy because τ is nearly homogeneous in the Dicke case (see Fig. 9). In the Dicke model (upper panels), $\bar{P}_{\text{max}}$ is only appreciable within a narrow window of relaxation and dephasing ($\gamma^-/\omega_q, \gamma^z/\omega_q \lesssim 0.2$); outside this region, power is strongly suppressed as dissipation reduces E_{max} . In contrast, the Tavis-Cummings model (lower panels) exhibits good performance over a broader domain as the system size increases ($N \gtrsim 30$), extending up to $\gamma^-/\omega_q, \gamma^z/\omega_q \sim 0.8$. RWA dynamics maintain scalable performance under dephasing and relaxation. Therefore, achieving high power in the non-RWA regime requires minimizing relaxation while operating at moderate dephasing. In the RWA regime, however, a broader sector yields better overall performance.

The results presented in Figs. 6–10 demonstrate that better overall performance is associated with a moderate buildup of qubit entanglement entropy, which is degraded by cavity mixedness: E_{max} and $\bar{P}_{\text{max}}$ are optimized when the qubit ensemble yields an intermediate value of S_q within a dephasing-dominated corridor. However, a large value of S_{cav} consistently anticorrelates with energy retention, power, and charging speed.

In the Dicke case, this favorable window is narrow and sensitive, approximately $\gamma^-/\omega_q, \gamma^z/\omega_q \lesssim 0.2$, and power mirrors E_{max} because τ remains nearly constant. With increasing N , the hot band in S_q cools, indicating stabilized, controlled qubit entanglement. In contrast, under the Tavis-Cummings dynamics, the optimal sector is broader, extending up to $\gamma^-/\omega_q, \gamma^z/\omega_q \sim 0.8$, and the weak delay band in τ only mildly degrades the power maps, yielding more size-resilient performance.

From a practical point of view, optimal operation requires keeping the cavity at low S_{cav} , minimizing relaxation, and

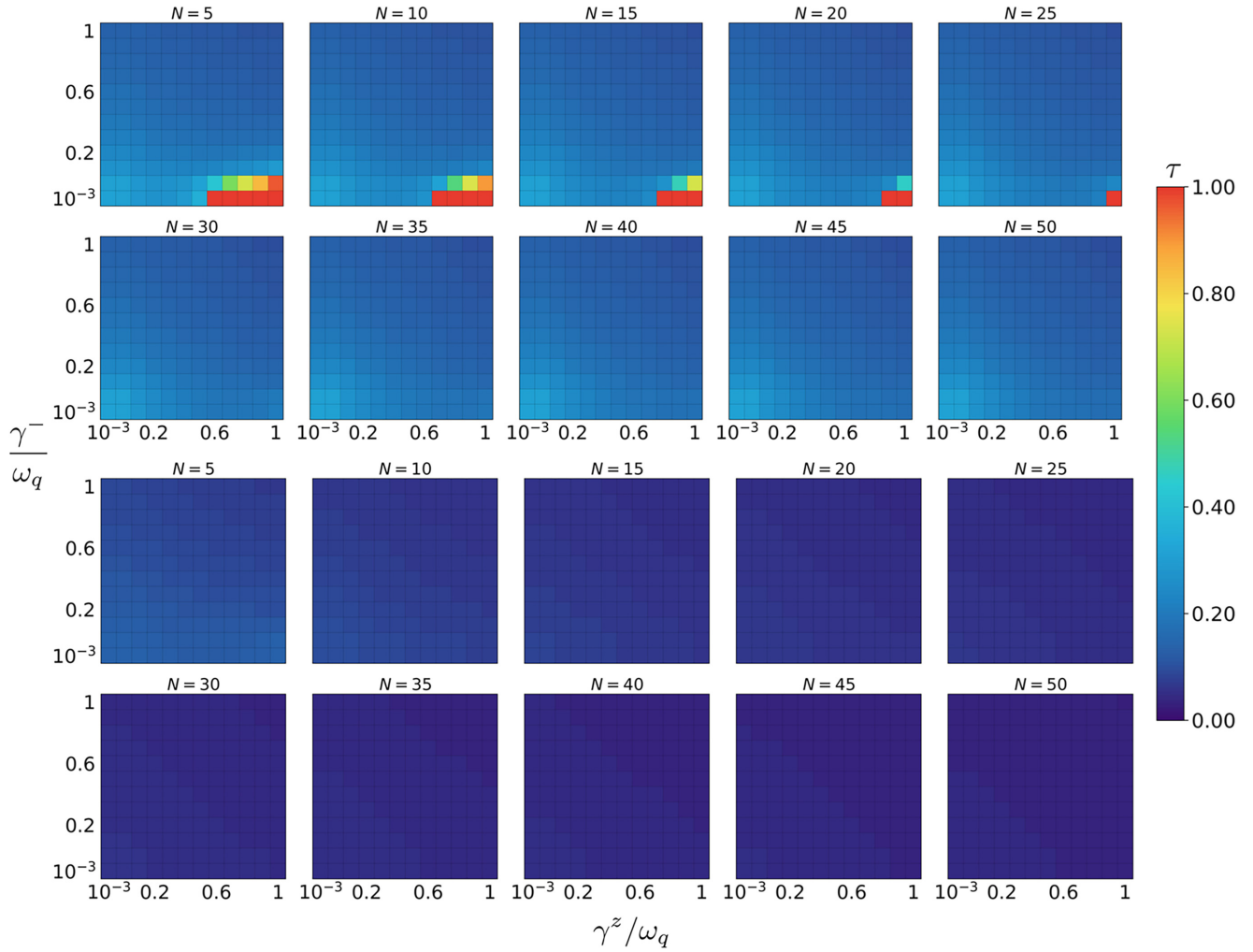


FIG. 9. Heatmaps of the charging time τ as a function of γ^- and γ^z , for different qubit number N . The Dicke model (top) shows nearly constant and short charging times, while the Tavis-Cummings model (bottom) overestimates dissipation-induced delays. Color scale is model normalized, using the maximum value of τ or τ^{RWA} , with the relaxation and dephasing rates varied over the same range as in that figure.

tuning dephasing to regulate S_q . This strategy enables collective absorption while avoiding excessive mixing. This leaves us with an optimal operating window of $\gamma^-/\omega_q \lesssim 0.20$ and $\gamma^z/\omega_q \lesssim 0.50$. The coherent drive enlarges these favorable regions without altering this hierarchy, primarily acting as a stabilizer of entanglement entropy growth rather than a source of additional scaling advantage.

C. Dynamical scaling laws of entanglement and thermodynamic observables

To systematically determine whether cavity-coupled QBs provide a collective advantage, it is also important to compare performance scales as the number of qubits increases. Finite-size scaling laws show whether collective effects, such as superextensivity, persist under dissipation. In this context, we model the dependence of an observable, $\mathcal{O}(N)$, on the system size by a power law

$$\mathcal{O}(N) \sim N^\alpha, \quad (21)$$

and extract the exponent α from log-log fits. Throughout, g is kept fixed as N increases; therefore, α characterizes

dynamical scaling laws, while quantum collective effects manifest as deviations from linearity [24,34]. Hence, α serves as a compact figure of merit: $\alpha > 1$ signals superextensive enhancement (quantum advantage), $\alpha \approx 1$ corresponds to classical extensivity, and $\alpha < 1$ indicates degraded scaling. The following analysis focuses on the optimal regime of low relaxation $\gamma^-/\omega_q \in [10^{-3}, 0.10]$ and moderate dephasing $\gamma^z/\omega_q \in [10^{-3}, 0.50]$. The fitted values of the scaling exponents can be found in Appendix B.

Three characteristic domains emerge: (1) tuned-decay relaxation and dephasing ($\gamma^-/\omega_q, \gamma^z/\omega_q \in [10^{-3}, 10^{-2}]$), (2) intermediate regime ($\gamma^-/\omega_q = \{10^{-2}\}$ and $\gamma^z/\omega_q \in [0.10 - 0.50]$), and (3) dephasing-dominated regime ($\gamma^-/\omega_q = \{10^{-3}\}$ and $\gamma^z/\omega_q \in [0.10, 0.50]$).

Figure 11 summarizes the scaling of the final subsystem entropies, S_q^f and S_c^f , across dissipation regimes for both interaction models. These residual entropies quantify the correlations and mixedness that remain once the charging dynamics have settled (around $t_f \sim 50/\omega_q$). The scaling exponent α indicates whether larger devices end up more correlated ($\alpha > 0$) or increasingly purified ($\alpha < 0$) at the end

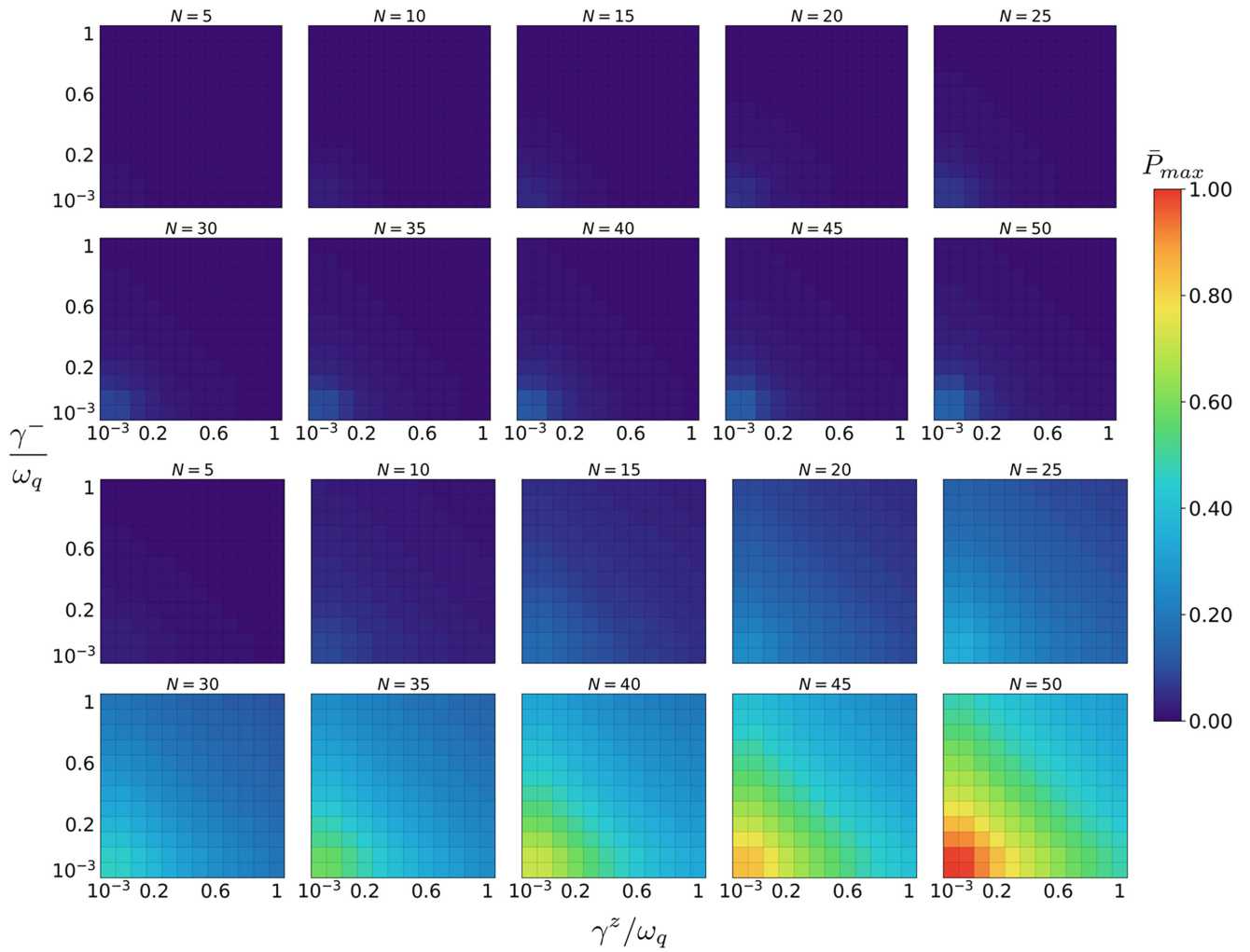


FIG. 10. Heatmaps of the maximum power $\bar{P}_{\max}$ as a function of γ^- and γ^z , for different qubit number N . The Dicke model (top) shows enhancement only in very restricted regions, while the Tavis-Cummings model (bottom) predicts more robust and broadly distributed power scaling. Color scale is model normalized, using the maximum value of $\bar{P}_{\max}$ or $\bar{P}_{\max}^{\text{RWA}}$, with the relaxation and dephasing rates varied over the same range as in that figure.

of the protocol. The chosen dissipation parameters target low cavity mixedness while allowing for moderate qubit entanglement growth. For completeness, supplementary fitted plots of $\alpha_{S_q^f}$ and $\alpha_{S_c^f}$ are provided in Figs. 14–16.

For the Dicke model, under both driven and undriven protocols, the entropy curves in Fig. 11 display nearly flat log-log slopes ($\alpha_{S_q^f}, \alpha_{S_c^f} \approx 0$), showing that long-time correlations saturate rather than scale with N . As the dephasing starts dominating over relaxation both entropies exhibit negative exponents— $\alpha_{S_c^f} \in [-0.20, -0.31]$ for the qubit ensemble and $\alpha_{S_q^f} \in [-0.39, -0.57]$ for the cavity—indicating a partial purification of the steady state as the system size increases. This suppression of entanglement confirms that, in the Dicke model, dephasing stabilizes the long-time dynamics: Entanglement generation remains bounded, leading to entanglement-clean operation that preserves coherence while preventing uncontrolled mixing. The drive does not qualitatively modify this behavior, reinforcing the interpretation

that counterrotating processes act as internal stabilizers of coherence in the presence of moderate noise.

In contrast, the Tavis-Cummings model shows a more intricate dependence on dissipation. At low dephasing (tuned-decay regime), the cavity entropy presents weak positive slopes ($\alpha_{S_c^f} \approx 0.2-0.4$) while the qubit entanglement entropy remains almost size independent ($\alpha_{S_q^f} \approx 0$). As dephasing increases, these trends invert: Both entropies develop bounded negative exponents— $\alpha_{S_c^f} \in [-0.39, -0.68]$ and $\alpha_{S_q^f} \in [-0.24, -0.43]$ —showing that decoherence ultimately limits entanglement growth. This transition from positive to negative scaling marks the point where dephasing ceases to act as a destructive mechanism and instead regulates the entanglement buildup. Hence, moderate noise proves beneficial: It allows correlations to emerge sufficiently to enhance collective response, but then constrains them, yielding a steady operating regime in which coherence and stability coexist.

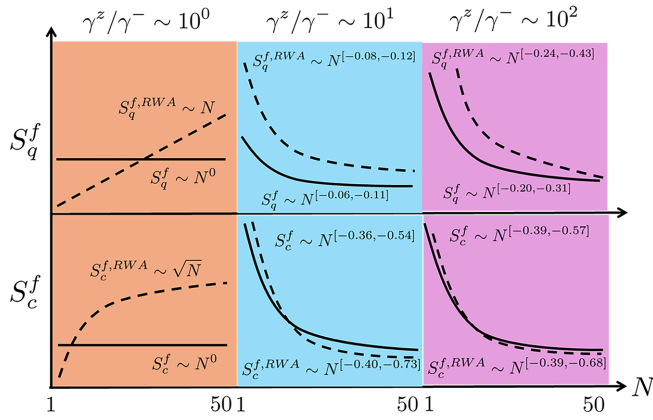


FIG. 11. Scaling behavior of final entropies across dissipation regimes. The vertical panels indicate the different dissipation regimes (see the legend), and each graph compares the two interaction models: the Tavis-Cummings model (dashed, RWA) and the Dicke model (solid, non-RWA). The curves show the log-log scaling of the final qubit and cavity entropies, S_q^f and S_c^f , from which the exponents $\alpha_{S_q^f}$ and $\alpha_{S_c^f}$ are extracted: $\alpha > 0$ denotes growth of residual correlations with N , $\alpha \approx 0$ saturation, and $\alpha < 0$ suppression.

Building on these entropy results, Fig. 12 presents the scaling of thermodynamic observables—maximum stored energy $E_{\max}$, charging time τ , and maximum power $\bar{P}_{\max}$ —as functions of N and noise regimes.

In the tuned-decay regime ($\gamma_z/\gamma^- \sim 10^0$), the Dicke model exhibits purely extensive thermodynamic behavior, with linear energy scaling ($\alpha_{E_{\max}} = 1.00$), negligible dependence of the charging time on system size ($\alpha_\tau = 0$), and correspondingly

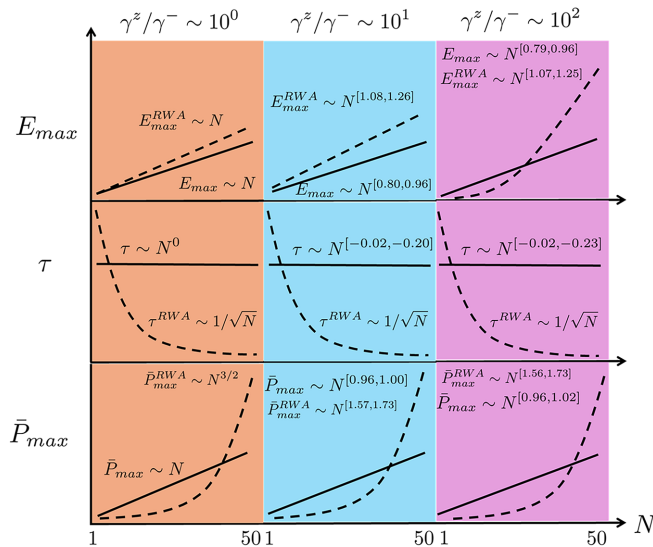


FIG. 12. Dynamical scaling behavior of thermodynamic observables across dissipation regimes. The vertical panels indicate the different dissipation regimes and interaction models, as in Fig. 11. The curves show the log-log scaling of the maximum stored energy $E_{\max}$, charging time τ , and maximum power $\bar{P}_{\max}$ with qubit number N , from which the exponents $\alpha_{E_{\max}}$, α_τ , and $\alpha_{\bar{P}_{\max}}$ are obtained. Values $\alpha > 1$ signal superextensive (quantum-enhanced) scaling, $\alpha \approx 1$ classical extensivity, and $\alpha < 1$ degraded performance.

linear power growth ($\alpha_{\bar{P}_{\max}} = 1.00$). In contrast, the Tavis-Cummings model already shows a collective enhancement: the power scales superextensively ($\alpha_{\bar{P}_{\max}} \approx 1.50$) as a consequence of faster charging times ($\alpha_\tau \approx -0.50$), while energy storage remains approximately linear ($\alpha_{E_{\max}} \approx 1$). These results highlight the emergence of a weak quantum advantage in the RWA dynamics, which correlates with the moderate growth of entanglement entropy observed in the same regime.

As dephasing begins to dominate ($\gamma_z/\gamma^- \sim 10^1$), the Dicke model remains close to extensive scaling, with $\alpha_{E_{\max}} \in [0.80, 0.96]$, $\alpha_\tau \in [-0.02, -0.20]$, and $\alpha_{\bar{P}_{\max}} \in [0.96, 1.00]$. The slight improvement in τ partially compensates for the small reduction in energy scaling, resulting in stable yet only mildly accelerated charging. In the same regime, the Tavis-Cummings model consolidates its advantage: The energy scaling becomes superextensive ($\alpha_{E_{\max}} \in [1.08, 1.26]$), the power exponent increases up to $\alpha_{\bar{P}_{\max}} \in [1.57, 1.73]$, and the charging time keeps its favorable contribution to the thermodynamic performance enhancement ($\alpha_\tau \approx -0.48$). These trends demonstrate that the emergence of entanglement, as quantified by the fitted scaling exponents of the final entropies (see Fig. 11), directly supports the observed enhancement in collective performance.

In the dephasing-dominated regime ($\gamma_z/\gamma^- \sim 10^2$), the Dicke model sustains nearly extensive dynamical scaling, with $\alpha_{E_{\max}} \in [0.79, 0.96]$, $\alpha_{\bar{P}_{\max}} \in [0.96, 1.02]$, and $\alpha_\tau \in [-0.02, -0.23]$. Conversely, the Tavis-Cummings model reaches its strongest quantum advantage: Both energy and power scale superextensively ($\alpha_{E_{\max}} \in [1.07, 1.25]$, $\alpha_{\bar{P}_{\max}} \in [1.56, 1.73]$), with accelerated charging times ($\alpha_\tau \approx -0.49$). The concurrent bounded suppression of entanglement entropies ($\alpha_{S_q^f}, \alpha_{S_c^f} < 0$) confirms that moderate dephasing enhances performance by stabilizing coherence while maintaining cooperative energy transfer. Overall, dephasing plays a dual role: It mitigates uncontrolled entanglement entropy growth and simultaneously reinforces superextensive dynamical scaling, enabling a steady and scalable quantum advantage in the open Tavis-Cummings battery.

Together, Figs. 11 and 12 demonstrate the intrinsic connection between entanglement scaling and thermodynamic enhancement in cavity-driven QBs. The Tavis-Cummings model achieves genuine quantum advantage in energy and power scaling, but only in regimes where qubit and cavity entanglement entropies increase with N , indicating the buildup of entanglement. Conversely, the Dicke model achieves cleaner, entropy-suppressed charging at the expense of reduced collective scaling. The coherent drive stabilizes entanglement entropy production within the optimal window, maintaining a balance in which controlled entanglement supports superextensive performance without excessive noise. This interplay links the observed scaling laws directly to the phenomenon of superabsorption, in which cooperative quantum correlations accelerate energy transfer. Bipartite entanglement, therefore, emerges as the key resource underlying the collective enhancement of quantum battery performance.

V. QUANTUM HARDWARE IMPLEMENTATION

In solid-state and circuit-QED platforms, the coherence and relaxation properties are typically characterized by the

longitudinal relaxation time T_1 and the transverse coherence time T_2 . The relaxation rate $\gamma^- = 2\pi/T_1$ quantifies energy loss through spontaneous decay, whereas the dephasing rate arises from fluctuations in the qubit transition frequency that randomize phase coherence without exchanging energy. These quantities are related through the standard expression

$$\frac{1}{T_2} = \frac{1}{2T_1} + \frac{1}{T_\phi}, \quad (22)$$

where T_ϕ is the pure-dephasing time associated exclusively with phase noise. Therefore, $\gamma^z = 2\pi/T_\phi$, and the ratio γ^z/γ^- provides a convenient metric for identifying the dominant decoherence mechanism across different experimental platforms. We identify three main regimes:

- (1) Dephasing dominated: $\gamma^z/\gamma^- \sim 10^2$.
- (2) Intermediate: $\gamma^z/\gamma^- \sim 10^1$.
- (3) Tuned-decay channels: $\gamma^z/\gamma^- \sim 10^0$.

In the optimal operating window that yields the best performance, $\gamma^-/\omega_q \in [10^{-3}, 2 \times 10^{-1}]$ and $\gamma^z/\omega_q \in [0.1, 0.4]$, i.e., $\gamma^z/\gamma^- \sim 10^0$ – 10^2 . There are several experimentally relevant platforms that naturally realize the required balance of relaxation and dephasing, and that are compatible with Dicke/Tavis-Cummings QBs.

We have made the following hardware-matching identification, although this is not exhaustive.

Dephasing-dominated regime. Solid-state defect ensembles, such as NV centers in diamond [61,62], SiC divacancies [63], and rare-earth ions in crystals [64], typically exhibit very long T_1 (milliseconds to seconds) but shorter coherence due to dephasing (microseconds to milliseconds). Trapped-ion qubits [65] likewise feature negligible relaxation ($T_1 \sim 10^3$ s) with coherence limited by technical dephasing ($T_2 \sim 10$ s). Organic microcavities with molecular dyes [25] have demonstrated QB-like superabsorption at room temperature, with typical $T_1 \sim 1$ ns and $T_2 \sim 100$ fs.

Intermediate regime. Self-assembled In(Ga)As quantum-dot excitons in microcavities [66,67] commonly exhibit picosecond-to-hundreds-of-picoseconds pure dephasing with nanosecond radiative lifetimes, providing a standard Tavis-Cummings platform with tunable dissipation. Electrons trapped on superfluid helium surfaces and coupled to high-impedance superconducting resonators ($\gamma^z/\gamma^- \sim 10^1$) have recently reached the strong-coupling regime, achieving vacuum Rabi splittings by combining long electron coherence times with large dipole moments [68], offering a promising platform for implementing collectively coupled quantum batteries within the optimal relaxation-dephasing window identified in this work.

Tuned-decay channels regime. Superconducting transmons in circuit QED typically have comparable $T_1 \sim T_2 \sim$ tens to hundreds of microseconds and already host multiqubit QB protocols and demonstrations [69,70].

Collectively, these systems span the entire ratio range that we probe, and each one meets the key requirement identified by our maps—low S_c with moderate S_q —either natively (NV/SiC/REI/ions and organic materials) or with standard engineering (quantum dots and transmons) to tune toward the desired $(\gamma^-/\omega_q, \gamma^z/\omega_q)$ sector.

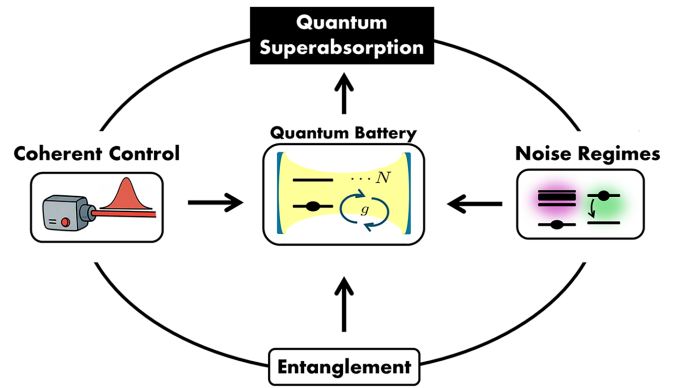


FIG. 13. Mechanism of dissipation-enhanced quantum superabsorption. Qubit-cavity entanglement generates the cooperative correlations necessary for collective energy superabsorption, resulting in superextensive thermodynamic performance. A noise-enhanced regime forms the basis of dissipation-assisted superabsorption, where scaling remains favorable. Coherent driving acts as a stabilizer of entanglement entropy growth. The trade-off between quantum advantage and controlled noise enables stable superabsorption.

VI. DISCUSSION AND OUTLOOK

Our study shows a fundamental mechanism for understanding quantum superabsorption in open, cavity-coupled QBs. While previous studies have identified superabsorption [1,21] and suggested a link to qubit-cavity entanglement [11,18,21,24], they have not provided a quantitative description of how correlations scale with system size or persist under noise. In this work, we explicitly quantify these correlations through the scaling laws of subsystem entropies, demonstrating that bipartite entanglement partly underlies the observed superextensive energy and power scaling. We demonstrate a distinct noise-enhanced regime in which moderate dephasing and low relaxation stabilize cooperative faster charging, leading to dissipation-enhanced superabsorption. Within this window, we also show that external coherent driving can act as an entanglement entropy growth stabilizer, maintaining favorable scaling and enabling stable quantum advantage.

The interplay between these processes is summarized schematically in Fig. 13, which highlights the self-consistent cycle connecting qubit-cavity entanglement, thermodynamic enhancement, and noise-assisted scaling. These processes are balanced by a fundamental trade-off between performance and entanglement entropy generation.

To our knowledge, this work provides the first quantitative framework that (1) extracts finite-size scaling exponents for qubit and cavity entropies under Markovian dynamics, (2) correlates those exponents with superextensive finite-size scaling in order to diagnose superabsorption, and (3) identifies a low-relaxation, moderate-dephasing window in which coherent driving stabilizes entanglement entropy growth while sustaining collective enhancement.

Even though systems with tens of qubits already demonstrate remarkable collective effects and performance enhancements, increasing the number of qubits to larger ensembles is expected to improve charging efficiency further and yield more favorable scaling laws. Recent experimental detection of

superabsorption in organic microcavities [25] has shown that collective coupling can enhance energy uptake even for modest ensemble sizes. This study, however, did not quantify the entanglement responsible for the cooperative behavior. Our results indicate that collective effects, including superextensive power and enhanced energy scaling, become increasingly pronounced with system size. We show that these advantages originate from bipartite entanglement between the qubit ensemble and the cavity mode. By deriving and quantifying the corresponding scaling laws, our work clarifies the microscopic mechanism behind the emergent collective advantage observed in both theoretical and experimental QBs. Improving QBs' performance lies in optimizing the coherent drive used for charging. In our model, we considered Gaussian pulses mimicking ultrafast laser control as a minimal protocol; however, more refined control of the driving field could enhance both efficiency and stability. Tailored pulse shaping and optimal control techniques could be employed to maximize energy transfer while minimizing entanglement entropy growth. Such coherent control strategies are particularly relevant for laser-driven or circuit-based implementations [60,71–73], where the drive determines how excitations are distributed. Recent theoretical studies have shown that properly designed driving fields can accelerate charging, suppress decoherence, and maintain quantum correlations during the process [72,74,75]. Thus, engineered dissipation provides a means for stabilizing many-body states and dynamics in platforms ranging from atoms and ions to superconducting circuits [76–78]. Developing laser control protocols that exploit these principles offers a promising pathway toward stable, high-efficiency collective charging of QBs.

Although our analysis focused on Markovian noise, several physical environments exhibit memory effects that could be exploited to preserve coherence. Structured reservoirs, such as photonic crystals, can lead to the local density of states to slow, enhance, or even inhibit spontaneous emission, thereby prolonging coherence [79]. Correlated spin baths underlying central-spin decoherence generate non-Markovian dynamics [80]. This is just one of many scenarios in which non-Markovian effects dictate quantum dynamics [81–83]. In this context, data-driven approaches are emerging, with machine-learning models able to detect, classify, and quantify non-Markovian processes, as well as inferring bath parameters directly from qubit dynamics [84–87].

Future work will focus on the collective quantum enhancement effects associated with non-Markovian qubit dynamics in the QB system, as well as the corresponding superabsorption and scaling-law behavior of entanglement and thermodynamic observables.

VII. CONCLUSIONS

A comparative analysis of cavity-coupled quantum batteries under Markovian dissipation shows that there is a fundamental balance between thermodynamic advantage and entanglement entropy generation. In the Tavis-Cummings model, collective coupling sustains superextensive scaling of stored energy and power despite decoherence, enabled by bipartite entanglement, which grows with system size. In contrast,

the Dicke model exhibits cleaner, entropy-suppressed charging, albeit with extensive or sublinear scaling.

By quantifying the scaling of qubit and cavity entropies, we demonstrate that long-time mixedness follows a power law with N , delineating two regimes. The first is a balanced relaxation-dephasing regime where $\alpha_S > 0$ and collective superabsorption emerges. The second is a dephasing-dominated regime where $\alpha_S < 0$ and the dynamics purify. These results show that dephasing can generate and control quantum correlations, highlighting entanglement as the essential underlying resource of dissipation-assisted superextensive charging.

Remarkably, the strongest quantum advantage emerges within the dephasing-dominated window. Here, moderate dephasing suppresses uncontrolled oscillations and stabilizes collective energy transfer. This sustains superextensive scaling while limiting entanglement entropy growth, demonstrating that controlled decoherence can act as a functional resource rather than a hindrance. Furthermore, the dissipation ratios associated with this optimal regime align with state-of-the-art experimental cavity and circuit QED platforms, including transmon qubits and NV-center ensembles. This highlights the practical relevance of our findings.

In this context, coherent driving acts as a regulatory mechanism rather than a direct source of additional energy. Introducing a resonant drive to the cavity does not significantly alter the energy or power scaling exponents beyond what collective interactions already provide. Its primary function is to stabilize the charging dynamics and control entanglement growth. Coherent driving prevents excessive entanglement entropy growth while maintaining multipartite correlations across the ensemble. In this sense, the drive acts as a coherence stabilizer, extending the operational lifetime of the quantum advantage without compromising scalability.

In summary, this work establishes an understanding of the performance and stability of open QBs. We identify the regime of low relaxation and moderate dephasing as the optimal operational window in which collective effects, controlled entanglement entropy growth, and quantum advantage can coexist. Deriving entropy scaling laws provides a framework for quantifying the evolution of quantum correlations with system size in the presence of noise. This bridges the gap between thermodynamic performance and information-theoretic stability. Our findings demonstrate that dissipation, when balanced with coherent control, can enable scalable quantum advantage rather than being purely destructive.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The database and Python codes used to develop this work are available upon reasonable request to the authors.

APPENDIX A: CAVITY PHOTON-NUMBER CUTOFF AND NUMERICAL CONVERGENCE

We derive analytical bounds on the cavity photon number to justify the Fock-space truncation used in the numerical simulations. We also performed explicit convergence checks as $n_{\max}$ increased.

1. Tavis-Cummings model

a. Undriven case

The total excitation operator $\hat{\mathcal{N}} = \hat{a}^\dagger \hat{a} + \sum_{j=1}^N \hat{\sigma}_j^+ \hat{\sigma}_j^-$ satisfies $[\hat{H}_{\text{TC}}, \hat{\mathcal{N}}] = 0$, i.e., $\hat{\mathcal{N}}$ is conserved. Therefore, if the system is initialized in the one-excitation sector (e.g., $|1\rangle_{\text{cav}} \otimes |G\rangle$), the dynamics remains in $\mathcal{N} = 1$ and Fock states $|n \geq 2\rangle$ are never populated. Hence, a cutoff with $n_{\max} = 1$ is exact for the undriven TC dynamics under this initialization.

b. Driven case

When a coherent drive $\hat{H}_{\text{drive}}(t) = \eta(t)(\hat{a}^\dagger + \hat{a})$ is present, $\hat{\mathcal{N}}$ is no longer conserved, but its change is fully controlled by the drive, as follows:

$$\frac{d\langle \hat{\mathcal{N}} \rangle}{dt} = -i\langle [\hat{\mathcal{N}}, \hat{H}_{\text{drive}}(t)] \rangle = -i\eta(t)\langle \hat{a}^\dagger - \hat{a} \rangle. \quad (\text{A1})$$

Taking absolute values and integrating over the pulse yields the bound

$$\Delta \mathcal{N} \lesssim \int_{-\infty}^{\infty} |\eta(t)| dt, \quad (\text{A2})$$

up to an order-unity prefactor set by $|\langle \hat{a}^\dagger - \hat{a} \rangle|$. For the Gaussian envelope $\eta(t) = \eta_0 \exp[-(t - t_0)^2 / (2\sigma^2)]$,

$$\mathcal{N}_{\max} \sim \eta_0 \sigma \sqrt{2\pi}, \quad (\text{A3})$$

so for our parameters ($\eta_0 = \omega_q$, $\sigma = 2/\omega_q$), we obtain $\mathcal{N}_{\max} \approx 2\sqrt{2\pi} \approx 5$ injected excitations.

2. Dicke model

In the Dicke model, the (CR interaction $\hat{V}_{\text{CR}} = g \sum_{j=1}^N (\hat{a}^\dagger \hat{\sigma}_j^+ + \hat{a} \hat{\sigma}_j^-)$ does not conserve $\hat{\mathcal{N}}$ and can generate virtual photon-qubit pairs. In the regime $g/\omega_c \ll 1$ (here, $g/\omega_c = 0.1$), the photon population induced solely by counterrotating processes can be estimated perturbatively. We split the Hamiltonian into $\hat{H} = \hat{H}_0 + \hat{V}_{\text{CR}}$, with $\hat{H}_0 = \omega_c \hat{a}^\dagger \hat{a} + (\omega_q/2) \sum_j \hat{\sigma}_j^z$, where V_{CR} is treated as a perturbation. The relevant counterrotating process starting from the vacuum sector creates one photon and excites one qubit, as follows:

$$\hat{a}^\dagger \hat{\sigma}_j^+ |0\rangle_{\text{cav}} \otimes |G\rangle = |1\rangle_{\text{cav}} \otimes |e_j\rangle, \quad (\text{A4})$$

with an energy cost $\Delta E \simeq \omega_c + \omega_q$. Thus, the amplitude of this virtual admixture is of order $g/(\omega_c + \omega_q)$ for each qubit.

TABLE I. Convergence with respect to the cavity photon-number cutoff $n_{\max}$. We report the maximum stored energy $E_{\max}$ and the cavity entropy S_c for the Dicke (D) and Tavis-Cummings (TC) models for $g/\omega_c = 0.1$ and the lowest dissipation rates (γ^-/ω_q , γ^z/ω_q , $\kappa/\omega_q = 10^{-3}$) used in the article. The results quickly saturate and are effectively converged by $n_{\max} = 10$.

$n_{\max}$	$E_{\max}^{(\text{D})}$	$E_{\max}^{(\text{TC})}$	$S_c^{(\text{D})}$	$S_c^{(\text{TC})}$
2	0.034 59	0.158 14	0.061 71	0.063 31
5	0.231 27	0.806 04	0.103 32	0.256 76
10	0.493 74	0.990 73	0.082 94	0.626 38
15	0.494 52	0.992 94	0.083 93	0.630 75
20	0.494 52	0.992 94	0.083 93	0.630 75
25	0.494 52	0.992 94	0.083 93	0.630 75

Since the N states $|1\rangle_{\text{cav}} \otimes |e_j\rangle$ are orthogonal, their contributions add in probability, yielding a mean virtual photon number

$$\Delta n_{\text{virtual}} \equiv \langle \hat{a}^\dagger \hat{a} \rangle_{\text{CR}} \sim N \left(\frac{g}{\omega_c + \omega_q} \right)^2 \lesssim N \left(\frac{g}{\omega_c} \right)^2, \quad (\text{A5})$$

where the last inequality corresponds to the resonance condition $\omega_c = \omega_q$. For our parameters $g/\omega_c = 0.1$ and $N = 50$, Eq. (A5) gives $\Delta n_{\text{virtual}} \lesssim 0.5$, i.e., only a fraction of one photon on top of the excitation-number bound given by Eq. (A3).

3. Numerical convergence test

In addition to the analytical estimates above, we performed an explicit convergence test with respect to the cavity photon-number cutoff $n_{\max}$. We selected a set of parameters, $g/\omega_c = 0.1$ and γ^-/ω_q , γ^z/ω_q , $\kappa/\omega_q = 10^{-3}$, such that we maximize photon retention and therefore provide the strictest requirement on the Fock-space truncation. Table I shows representative observables for both the Dicke and Tavis-Cummings models as $n_{\max}$ is increased.

All quantities rapidly saturate; already at $n_{\max} = 10$, the values agree with the $n_{\max} = 25$ reference within $\lesssim 1\%$ (worst case, $\approx 1.2\%$ for S_c in the Dicke model), which is acceptable. Accordingly, we use $n_{\max} = 10$ as the cavity cutoff throughout the article.

APPENDIX B: SCALING LAW EXPONENTS

This Appendix supplements the main text by presenting the detailed log-log fits that were used to obtain the scaling exponents supporting the results of Sec. IV. Figures 14–16 display the system-size dependence of the cavity S_c^f and qubit S_q^f entanglement entropies, as well as the thermodynamic quantities $E_{\max}$, τ , and $\bar{P}_{\max}$ for the Dicke (non-RWA) and Tavis-Cummings (RWA) models. Each curve represents a power-law fit of Eq. (21) given by $\log_{10}(O(N)) \sim \alpha \log_{10}(N)$, from which the corresponding α exponents are extracted. These fits demonstrate how collective effects and dissipation jointly determine the scaling behavior of entanglement and thermodynamic observables across different regimes.

Outlier removal was applied to all figures below by computing the residuals with respect to a preliminary fit and estimating their dispersion through the median absolute de-

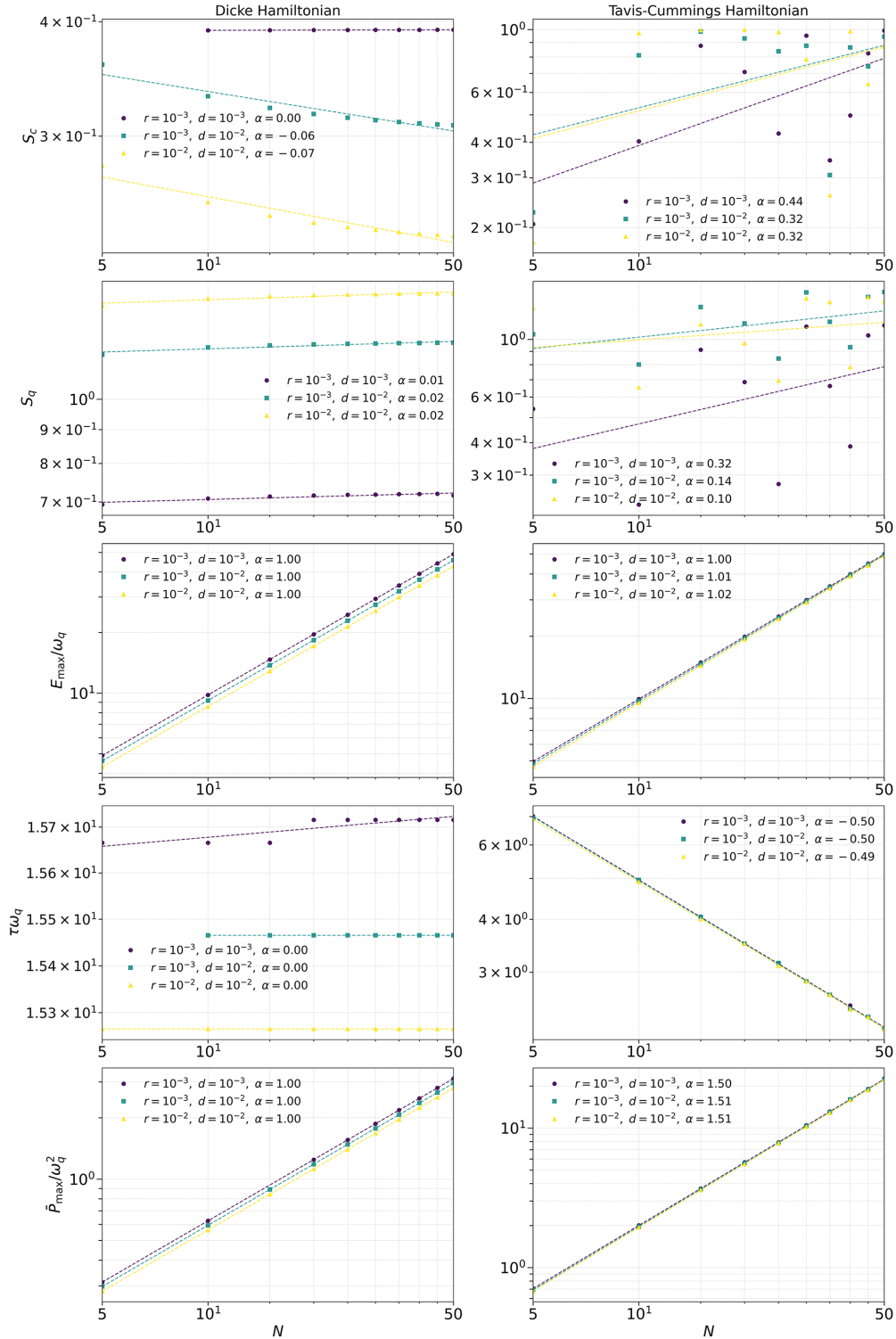


FIG. 14. Tuned-decay regime $\gamma^z/\gamma^- \sim 10^0$. Entanglement entropies and thermodynamic observables for the Dicke (left) and Tavis-Cummings (right) models are shown as a function of system size N , for $\gamma^-/\omega_q, \gamma^z/\omega_q = \{10^{-3}, 10^{-2}\}$. In the legend of the figures, we have set $\gamma^-/\omega_q \equiv r$ and $\gamma^z/\omega_q \equiv d$; $r \leq d$. The system parameters were fixed at $g/\omega_q = 0.1$ and $\kappa/\omega_q = 10^{-3}$.

viation. Points whose normalized residual $|z| > 2$ were then discarded [88,89].

Figure 14 shows the log-log fits for the tuned-decay regime, where relaxation and dephasing act on comparable timescales, $\gamma_z/\gamma^- \sim 10^0$. Under this dissipation condition,

the Dicke model exhibits extensive thermodynamic behavior, i.e., linear scaling in both energy storage capacity and average maximum power ($\alpha_E, \alpha_{\bar{P}} = 1$), due to the absence of favorable scaling in charging time ($\alpha_\tau = 0$). In contrast, the Tavis-Cummings model exhibits linear scaling in energy

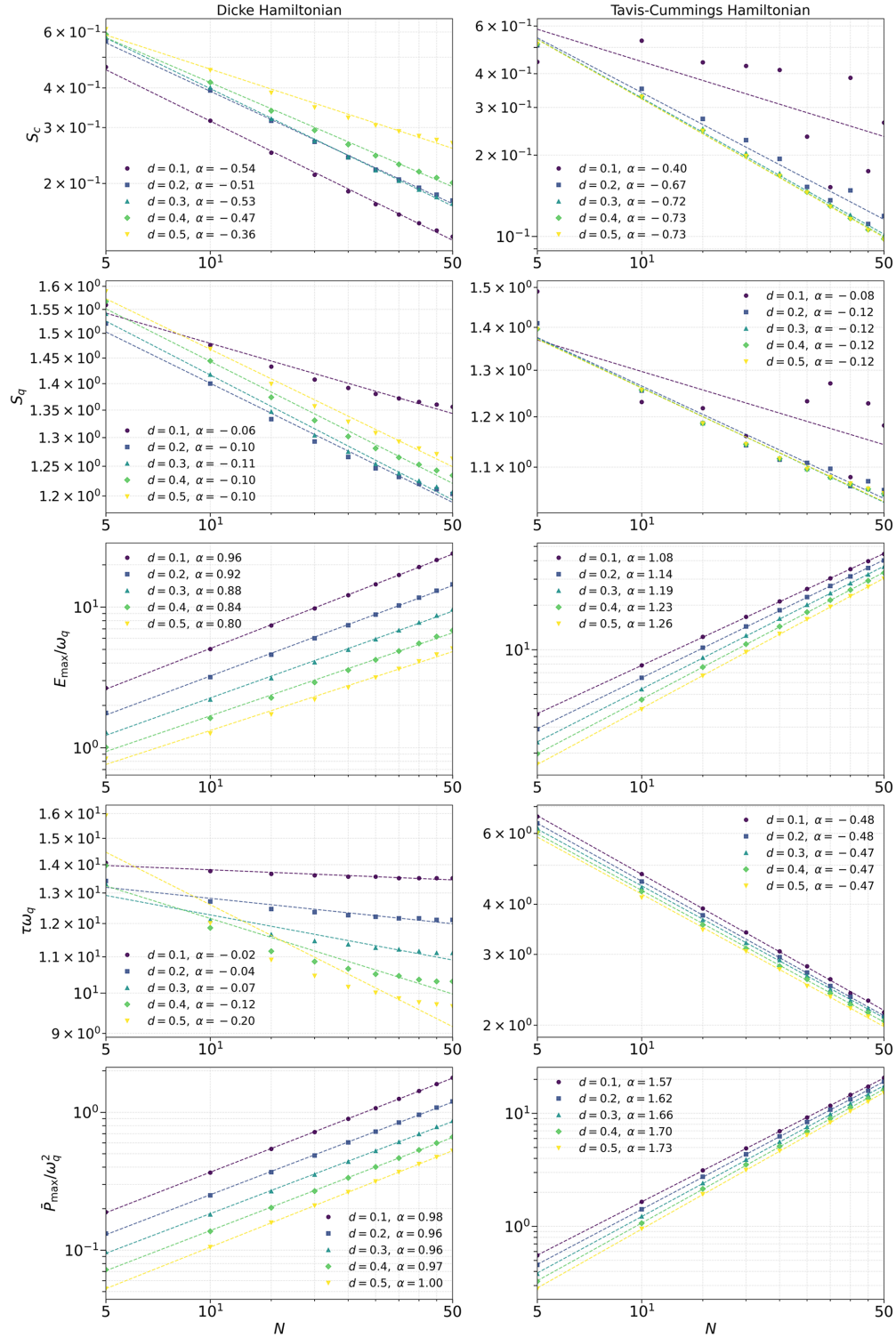


FIG. 15. Intermediate regime $\gamma^z/\gamma^- \sim 10^1$. Entanglement entropies and thermodynamic observables for the Dicke (left) and Tavis-Cummings (right) models are shown as a function of system size N , for $\gamma^-/\omega_q = 10^{-2}$ and $\gamma^z/\omega_q = \{0.1, 0.2, 0.3, 0.4, 0.5\}$. The system parameters were fixed as in Fig. 14.

capacity ($\alpha_E \approx 1$) and a quantum advantage in maximum power ($\alpha_{\bar{P}} \approx 1.5$), due to faster charging times ($\alpha_\tau \approx -0.5$).

For entanglement entropies, the Dicke Hamiltonian exhibits nearly flat log-log trends ($\alpha_{S_q^f}, \alpha_{S_q^c} \approx 0$), confirming that extensive thermodynamic behavior is accompanied by the absence of entanglement. In contrast, the Tavis-Cummings

model displays non-null entropy slopes, albeit with less regular fits. While the quality of these fits suggests that scaling is still developing in this balanced dissipation regime, moderate entanglement entropy growth appears to correlate with the emerging quantum advantage in power. As the noise regime shifts toward stronger dephasing, correlations between

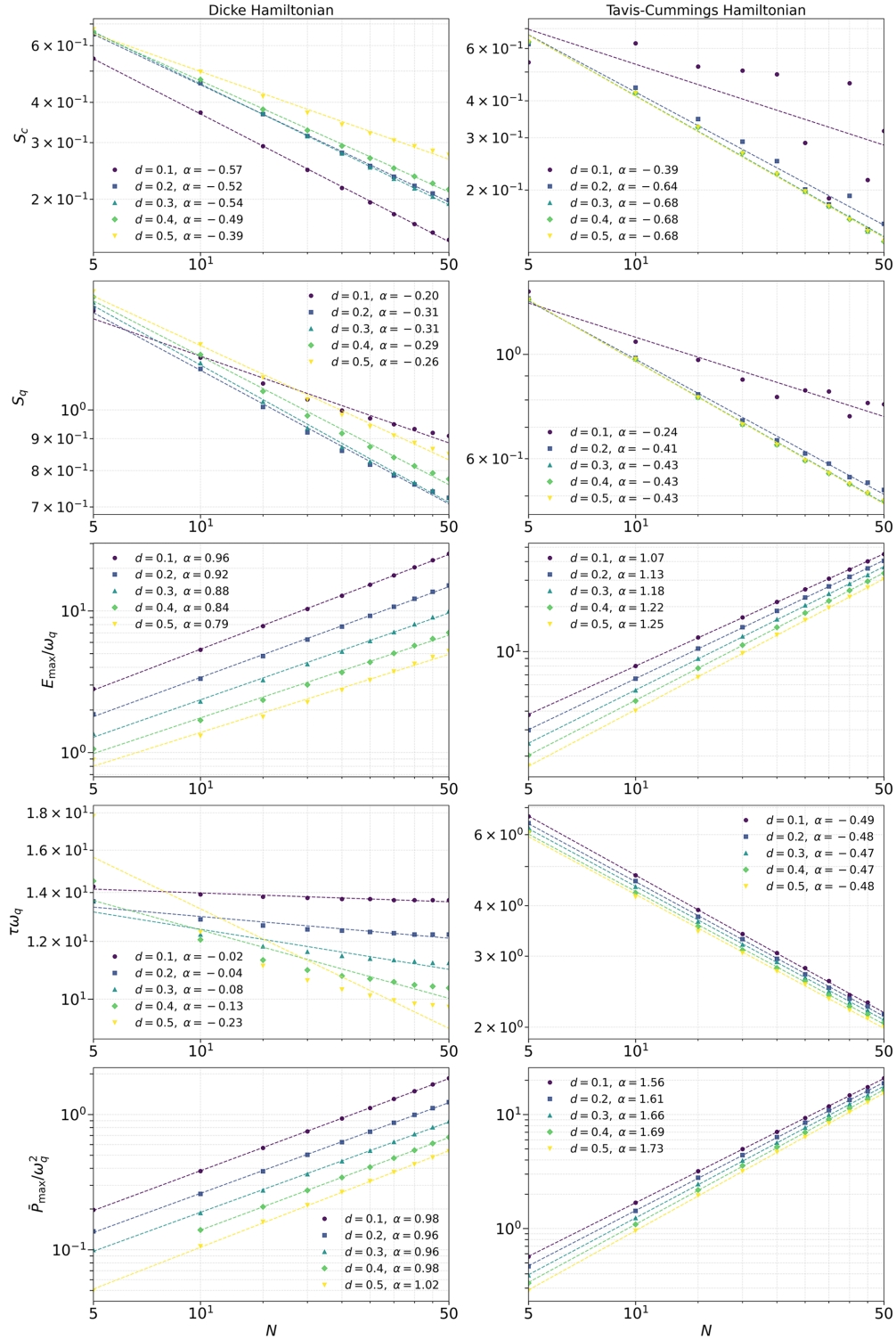


FIG. 16. Dephasing-dominated regime $\gamma^z/\gamma^- \sim 10^2$. Entanglement entropies and thermodynamic observables for the Dicke (left) and Tavis-Cummings (right) models are shown as a function of system size N , for $\gamma^-/\omega_q = 10^{-3}$ and $\gamma^z/\omega_q = \{0.1, 0.2, 0.3, 0.4, 0.5\}$. The system parameters were fixed as in Fig. 14.

entropy and performance become clearer and more consistent, as shown in the following regimes.

Figure 15 shows the log-log fits for the intermediate dissipation regime, where dephasing begins to dominate relaxation, $\gamma_z/\gamma^- \sim 10^1$. In this case, the Dicke model

shows significant entanglement decay in the cavity ($\alpha_{S_c^f} \in [-0.36, -0.54]$), accompanied by slight suppression of correlations in the N -qubit ensemble ($\alpha_{S_q^f} \in [-0.06, -0.11]$). The thermodynamic performance remains almost extensive, with a slight reduction in the energy-scaling exponent ($\alpha_E \in$

[0.80, 0.96]) being offset by a modest improvement in charging times ($\alpha_\tau \in [-0.02, -0.20]$), resulting in almost extensive power scaling ($\alpha_P \in [0.96, 1.00]$). In contrast, the Tavis-Cummings model enters a clearer quantum-advantage regime: The energy scaling becomes slightly superextensive ($\alpha_E \approx 1.08$ – 1.26), while the power exponent rises to $\alpha_P \in [1.57, 1.73]$, supported by sublinear charging times ($\alpha_\tau \approx -0.48$). The entanglement entropy fits in this regime exhibit bounded negative slopes: $\alpha_{S_c^f} \in [-0.40, -0.73]$ and $\alpha_{S_q^f} \in [-0.08, -0.12]$, suggesting that controlled entanglement decay accompanies the enhancement of collective performance.

Figure 16 shows the log-log fits for the dephasing-dominated regime ($\gamma_z/\gamma^- \sim 10^2$). This defines the optimal operational window discussed in Sec. IV. In this limit, dephasing suppresses residual oscillations while preserving collective energy exchange, resulting in stable and reproducible scaling behavior. For the Dicke model, both the cavity and qubit entanglement entropies exhibit negative

exponents ($\alpha_{S_c^f} \in [-0.39, -0.57]$, $\alpha_{S_q^f} \in [-0.20, -0.31]$), indicating partial purification of the steady state as the system size increases. This entropy suppression correlates with nearly extensive behavior, where the energy and power remain close to linear scaling ($\alpha_E \in [0.79, 0.96]$, $\alpha_P \in [0.96, 1.02]$), and the charging time shows only weak and inconsistent size dependence ($\alpha_\tau \in [-0.02, -0.23]$). In contrast, the Tavis-Cummings model presents energy and power scaling $\alpha_E \in [1.07, 1.25]$, $\alpha_P \in [1.56, 1.73]$ alongside faster charging times ($\alpha_\tau \approx -0.49$). This is accompanied by bounded yet strong entropy suppression in both the cavity ($\alpha_{S_c} \in [-0.39, -0.68]$) and the N -qubit ensemble ($\alpha_{S_q} \in [-0.24, -0.43]$). These trends show that moderate dephasing not only stabilizes correlations but also enhances collective performance, enabling superextensive finite-size scaling with controlled entanglement growth. This regime therefore realizes the coexistence of moderate noise, stabilized coherence, and scalable quantum advantage, demonstrating that dissipation can act as a constructive resource in open quantum batteries.

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- [1] K. D. B. Higgins, B. W. Lovett, E. M. Gauger, M. M. Kohl, and S. C. Benjamin, Superabsorption of light via quantum engineering, *Nat. Commun.* **5**, 4705 (2014).
 - [2] R. H. Dicke, Coherence in spontaneous radiation processes, *Phys. Rev.* **93**, 99 (1954).
 - [3] R. Horodecki, P. Horodecki, M. Horodecki, and K. Horodecki, Quantum entanglement, *Rev. Mod. Phys.* **81**, 865 (2009).
 - [4] S. Carnot, *Reflections on the Motive Power of Fire, and on Machines Fitted to Develop That Power* (Bachelier, Paris, 1824), Vol. 108, p. 1824.
 - [5] E. Fermi, *Thermodynamics*, *Dover Books on Physics* (Dover Publications, Mineola, NY, 2012).
 - [6] M. Kardar, *Statistical Physics of Particles* (Cambridge University Press, Cambridge, 2007).
 - [7] S. Vinjanampathy and J. Anders, Quantum thermodynamics, *Contemp. Phys.* **57**, 545 (2016).
 - [8] F. Binder, L. A. Correa, C. Gogolin, J. Anders, and G. Adesso, Thermodynamics in the quantum regime, *Fundamental Theories of Physics* (Springer, Cham, Switzerland, 2018), Vol. 195.
 - [9] M. Herrera, J. H. Reina, I. D’Amico, and R. M. Serra, Correlation-boosted quantum engine: A proof-of-principle demonstration, *Phys. Rev. Res.* **5**, 043104 (2023).
 - [10] K. V. Hovhannisyan, M. Perarnau-Llobet, M. Huber, and A. Acín, Entanglement generation is not necessary for optimal work extraction, *Phys. Rev. Lett.* **111**, 240401 (2013).
 - [11] F. Campaioli, F. A. Pollock, F. C. Binder, L. Céleri, J. Goold, S. Vinjanampathy, and K. Modi, Enhancing the charging power of quantum batteries, *Phys. Rev. Lett.* **118**, 150601 (2017).
 - [12] F. Campaioli, F. A. Pollock, and K. Modi, Quantum batteries: Energy extraction, storage, and transfer beyond classical limits, *Fundamen. Theor. Phys.* **195**, 207 (2018).
 - [13] F. Campaioli, S. Gherardini, J. Q. Quach, M. Polini, and G. M. Andolina, *Colloquium: Quantum batteries*, *Rev. Mod. Phys.* **96**, 031001 (2024).
 - [14] S. Julià-Farré, T. Salamon, A. Riera, M. N. Bera, and M. Lewenstein, Bounds on the capacity and power of quantum batteries, *Phys. Rev. Res.* **2**, 023113 (2020).
 - [15] X. Yang, Y.-H. Yang, M. Alimuddin, R. Salvia, S.-M. Fei, L.-M. Zhao, S. Nimmrichter, and M.-X. Luo, Battery capacity of energy-storing quantum systems, *Phys. Rev. Lett.* **131**, 030402 (2023).
 - [16] K. Ito and G. Watanabe, Collectively enhanced high-power and high-capacity charging of quantum batteries via quantum heat engines, *arXiv:2008.07089*.
 - [17] J. Liu, D. Segal, and G. Hanna, Loss-free excitonic quantum battery, *J. Phys. Chem. C* **123**, 18303 (2019).
 - [18] S. Gherardini, F. Campaioli, F. Caruso, and F. C. Binder, Stabilizing open quantum batteries by sequential measurements, *Phys. Rev. Res.* **2**, 013095 (2020).
 - [19] K. Xu, H.-J. Zhu, G.-F. Zhang, and W.-M. Liu, Enhancing the performance of an open quantum battery via environment engineering, *Phys. Rev. E* **104**, 064143 (2021).
 - [20] J. H. Reina, L. Quiroga, and N. F. Johnson, Decoherence of quantum registers, *Phys. Rev. A* **65**, 032326 (2002).
 - [21] J. Q. Quach and W. J. Munro, Using dark states to charge and stabilize open quantum batteries, *Phys. Rev. Appl.* **14**, 024092 (2020).
 - [22] J. H. Reina, Quantum information processing in nanostructures, Ph.D. thesis, University of Oxford, 2002.
 - [23] R. Alicki and M. Fannes, Entanglement boost for extractable work from ensembles of quantum batteries, *Phys. Rev. E* **87**, 042123 (2013).
 - [24] D. Ferraro, M. Campisi, G. M. Andolina, V. Pellegrini, and M. Polini, High-power collective charging of a solid-state quantum battery, *Phys. Rev. Lett.* **120**, 117702 (2018).
 - [25] J. Q. Quach, X. L. Han, L. Ganzer, A. Bosch Aguilera, J. H. Mentink, X. Tong, J. O. Tollerud, J. A. Davis, W. J. Munro, J. Romero, and T. J. Davis, Superabsorption in an organic microcavity: Toward a quantum battery, *Sci. Adv.* **8**, eabk3160 (2022).
 - [26] F. Kamin, F. Campaioli, F. C. Binder, and S. Kurtz, Charging assisted by non-Markovianity in open quantum batteries, *Phys. Rev. E* **102**, 052109 (2020).

- [27] A. Ghosh, T. Chanda, and A. Sen (De), Enhancing the performance of quantum batteries by ordered and disordered interactions, *Phys. Rev. A* **104**, 032207 (2021).
- [28] M. Tavis and F. W. Cummings, Exact solution for an N -molecule—Radiation-field Hamiltonian, *Phys. Rev.* **170**, 379 (1968).
- [29] S. Agarwal, S. M. H. Rafsanjani, and J. H. Eberly, Tavis-Cummings model beyond the rotating wave approximation: Quasidegenerate qubits, *Phys. Rev. A* **85**, 043815 (2012).
- [30] K. Fujii, Introduction to the rotating wave approximation (RWA): Two coherent oscillations, *J. Mod. Phys.* **8**, 2042 (2017).
- [31] P. Forn-Díaz, L. Lamata, E. Rico, J. Kono, and E. Solano, Ultrastrong coupling regimes of light-matter interaction, *Rev. Mod. Phys.* **91**, 025005 (2019).
- [32] T. Jaako, Z.-L. Xiang, J. J. Garcia-Ripoll, and P. Rabl, Ultrastrong-coupling phenomena beyond the Dicke model, *Phys. Rev. A* **94**, 033850 (2016).
- [33] G. M. Andolina, M. Keck, A. Mari, V. Giovannetti, and M. Polini, Extractable work, the role of correlations, and asymptotic freedom in quantum batteries, *Phys. Rev. Lett.* **122**, 047702 (2019).
- [34] F. Campaioli, F. A. Pollock, and S. Vinjanampathy, in *Fundamental Theories of Physics*, edited by F. Binder, L. A. Correa, C. Gogolin, J. Anders, and G. Adesso (Springer, 2018), Vol. 195, pp. 207–225.
- [35] J. Hausinger and M. Grifoni, Qubit-oscillator system: Analytic treatment of the ultra-strong coupling regime, *Phys. Rev. A* **82**, 062320 (2010).
- [36] S. Ashhab and F. Nori, Two-level systems driven by large-amplitude fields, *Phys. Rev. A* **81**, 042311 (2010).
- [37] A. F. Kockum, A. Miranowicz, S. De Liberato, S. Savasta, and F. Nori, Ultrastrong coupling between light and matter, *Nat. Rev. Phys.* **1**, 19 (2019).
- [38] F. Yoshihara, T. Fuse, S. Ashhab, K. Kakuyanagi, S. Saito, and K. Semba, Superconducting qubit-oscillator circuit beyond the ultrastrong-coupling regime, *Nat. Phys.* **13**, 44 (2017).
- [39] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, Cambridge, 2010).
- [40] H.-P. Breuer and F. Petruccione, *The Theory of Open Quantum Systems* (Oxford University Press, Oxford, 2002).
- [41] J. Anders and M. Esposito, Focus on quantum thermodynamics, *New J. Phys.* **19**, 010201 (2017).
- [42] F. Binder, L. A. Correa, C. Gogolin, J. Anders, and G. Adesso, *Thermodynamics in the Quantum Regime, Fundamental Theories of Physics* (Springer, Cham, Switzerland, 2018), Vol. 195, pp. 43–73.
- [43] J. H. Reina, C. E. Susa, and F. F. Fanchini, Extracting information from qubit-environment correlations, *Sci. Rep.* **4**, 7443 (2014).
- [44] H. J. Carmichael, *An Open Systems Approach to Quantum Optics* (Springer, Berlin, 1993).
- [45] D. F. Walls and G. J. Milburn, *Quantum Optics*, 2nd ed. (Springer, Berlin, 2008).
- [46] F. Campaioli, J. H. Cole, and H. Hapuarachchi, Quantum master equations: Tips and tricks for quantum optics, quantum computing, and beyond, *PRX Quantum* **5**, 020202 (2024).
- [47] V. Gorini, A. Kossakowski, and E. C. G. Sudarshan, Completely positive dynamical semigroups of N -level systems, *J. Math. Phys.* **17**, 821 (1976).
- [48] G. Lindblad, On the generators of quantum dynamical semigroups, *Commun. Math. Phys.* **48**, 119 (1976).
- [49] N. Shammah, S. Ahmed, N. Lambert, S. D. Liberato, and F. Nori, Open quantum systems with local and collective incoherent processes: Efficient numerical simulations using permutational invariance, *Phys. Rev. A* **98**, 063815 (2018).
- [50] J. Dalibard, Y. Castin, and K. Mølmer, Wave-function approach to dissipative processes in quantum optics, *Phys. Rev. Lett.* **68**, 580 (1992).
- [51] R. Dum, P. Zoller, and H. Ritsch, Monte Carlo simulation of the atomic master equation for spontaneous emission, *Phys. Rev. A* **45**, 4879 (1992).
- [52] J. Johansson, P. Nation, and F. Nori, QuTiP: An open-source Python framework for the dynamics of open quantum systems, *Comput. Phys. Commun.* **183**, 1760 (2012).
- [53] J. Johansson, P. Nation, and F. Nori, QuTiP 2: A Python framework for the dynamics of open quantum systems, *Comput. Phys. Commun.* **184**, 1234 (2013).
- [54] N. Lambert, E. Giguère, P. Menczel, B. Li, P. Hopf, G. Suárez, M. Gali, J. Lishman, R. Gadhvi, and R. Agarwal, Others QuTiP 5: The quantum toolbox in Python, *Phys. Rep.* **1153**, 1 (2026).
- [55] J. H. Eberly, N. B. Narozhny, and J. J. Sanchez-Mondragon, Periodic spontaneous collapse and revival in a simple quantum model, *Phys. Rev. Lett.* **44**, 1323 (1980).
- [56] S. J. D. Phoenix and P. L. Knight, Collapse and revival of the entanglement between a single atom and a quantum field mode, *Phys. Rev. A* **44**, 6023 (1991).
- [57] B. W. Shore and P. L. Knight, The Jaynes–Cummings model, *J. Mod. Opt.* **40**, 1195 (1993).
- [58] B. M. Garraway, The Dicke model in quantum optics: Dicke model revisited, *Philos. Trans. R. Soc. A* **369**, 1137 (2011).
- [59] K.-M. Fan and G.-F. Zhang, Geometric quantum discord and entanglement between two atoms in Tavis-Cummings model with dipole-dipole interaction under intrinsic decoherence, *Eur. Phys. J. D* **68**, 163 (2014).
- [60] V. M. Bastidas, J. H. Reina, C. Emary, and T. Brandes, Entanglement and parametric resonance in driven quantum systems, *Phys. Rev. A* **81**, 012316 (2010).
- [61] A. Jarmola, V. M. Acosta, K. Jensen, S. Chemerisov, and D. Budker, Temperature- and magnetic-field-dependent longitudinal spin relaxation in nitrogen-vacancy ensembles in diamond, *Phys. Rev. Lett.* **108**, 197601 (2012).
- [62] P. L. Stanwix, L. M. Pham, J. R. Maze, D. Le Sage, T. K. Yeung, P. Cappellaro, P. R. Hemmer, A. Yacoby, M. D. Lukin, and R. L. Walsworth, Coherence of nitrogen-vacancy electronic spin ensembles in diamond, *Phys. Rev. B* **82**, 201201 (2010).
- [63] H. Seo, A. L. Falk, P. V. Klimov, K. C. Miao, G. Galli, and D. D. Awschalom, Quantum decoherence dynamics of divacancy spins in silicon carbide, *Nat. Commun.* **7**, 12935 (2016).
- [64] J. Alexander, G. Dold, O. W. Kennedy, M. Šimėnas, J. O’Sullivan, C. W. Zollitsch, S. Welinski, A. Ferrier, E. Lafitte-Houssat, T. Lindström, *et al.*, Coherent spin dynamics of rare-earth doped crystals in the high-cooperativity regime, *Phys. Rev. B* **106**, 245416 (2022).
- [65] J. Zhang, P. Wang, W. Chen, *et al.*, Single-ion information engine for charging a quantum battery, *Phys. Rev. Lett.* **135**, 140403 (2025).

- [66] C. Bardot, M. Schwab, M. Bayer, S. Fafard, Z. Wasilewski, and P. Hawrylak, Exciton lifetime in InAs/GaAs quantum dot molecules, *Phys. Rev. B* **72**, 035314 (2005).
- [67] N. Accanto, F. Masia, I. Moreels, Z. Hens, W. Langbein, and P. Borri, Engineering the spin-flip limited exciton dephasing in colloidal CdSe/CdS quantum dots, *ACS Nano* **6**, 5227 (2012).
- [68] G. Koolstra, E. O. Glen, N. R. Beysengulov, H. Byeon, K. E. Castoria, M. Sammon, S. A. Lyon, D. G. Rees, and J. Pollanen, Strong coupling of a microwave photon to an electron on helium, [arXiv:2509.14506](https://arxiv.org/abs/2509.14506).
- [69] C.-K. Hu *et al.*, Optimal charging of a superconducting quantum battery, *Quantum Sci. Technol.* **7**, 045018 (2022).
- [70] F.-Q. Dou and F.-M. Yang, Superconducting transmon qubit-resonator quantum battery, *Phys. Rev. A* **107**, 023725 (2023).
- [71] F.-Q. Dou, Y.-J. Wang, and J.-A. Sun, Highly efficient charging and discharging of three-level quantum batteries through shortcuts to adiabaticity, *Front. Phys.* **17**, 31503 (2022).
- [72] H. Hu, S. Qi, and J. Jing, Fast and stable charging via a shortcut to adiabaticity, [arXiv:2104.12143](https://arxiv.org/abs/2104.12143).
- [73] W. E. Salazar, O. Calderón-Losada, and J. H. Reina, Linear-nonlinear duality for circuit design on quantum computing platforms, [arXiv:2310.20416](https://arxiv.org/abs/2310.20416).
- [74] F. Mazzoncini, V. Cavina, G. M. Andolina, P. A. Erdman, and V. Giovannetti, Optimal control methods for quantum batteries, *Phys. Rev. A* **107**, 032218 (2023).
- [75] S.-F. Qi and J. Jing, Quantum recharging by shortcut to adiabaticity, *Phys. Lett. A* **530**, 130124 (2025).
- [76] S. Diehl, A. Micheli, A. Kantian, B. Kraus, H. P. Büchler, and P. Zoller, Quantum states and phases in driven open quantum systems with cold atoms, *Nat. Phys.* **4**, 878 (2008).
- [77] J. T. Barreiro, M. Müller, P. Schindler, D. Nigg, T. Monz, M. Chwalla, M. Hennrich, C. F. Roos, P. Zoller, and R. Blatt, An open-system quantum simulator with trapped ions, *Nature (London)* **470**, 486 (2011).
- [78] P. M. Harrington, E. J. Mueller, and K. W. Murch, Engineered dissipation for quantum information science, *Nat. Rev. Phys.* **4**, 660 (2022).
- [79] P. Lodahl, A. F. van Driel, I. S. Nikolaev, A. Irman, K. Overgaag, D. Vanmaekelbergh, and W. L. Vos, Controlling the dynamics of spontaneous emission from quantum dots by photonic crystals, *Nature (London)* **430**, 654 (2004).
- [80] L. Cywiński, W. M. Witzel, and S. Das Sarma, How to enhance dephasing time in superconducting qubits by dynamical decoupling, *Phys. Rev. Lett.* **102**, 057601 (2009).
- [81] J. Eckel, J. H. Reina, and M. Thorwart, Coherent control of an effective two-level system in a non-Markovian biomolecular environment, *New J. Phys.* **11**, 085001 (2009).
- [82] H.-P. Breuer, E.-M. Laine, J. Piilo, and B. Vacchini, *Colloquium: Non-Markovian dynamics in open quantum systems*, *Rev. Mod. Phys.* **88**, 021002 (2016).
- [83] I. de Vega and D. Alonso, Dynamics of non-Markovian open quantum systems, *Rev. Mod. Phys.* **89**, 015001 (2017).
- [84] I. A. Luchnikov, S. V. Vintskevich, D. A. Grigoriev, and S. N. Filippov, Machine learning non-Markovian quantum dynamics, *Phys. Rev. Lett.* **124**, 140502 (2020).
- [85] J. M. Scarpetta, J. H. Reina, and M. Hjorth-Jensen, Machine learning non-Markovian two-level quantum noise spectroscopy, *Phys. Rev. Res.* **7**, 043285 (2025).
- [86] A. Altherr and W. Yang, Quantum metrology for non-Markovian processes, *Phys. Rev. Lett.* **127**, 060501 (2021).
- [87] S. Martina, S. Gherardini, and F. Caruso, Machine learning classification of non-Markovian noise disturbing quantum dynamics, *Phys. Scr.* **98**, 035104 (2023).
- [88] E. M. Ronchetti and P. J. Huber, *Robust Statistics* (John Wiley & Sons, Hoboken, NJ, USA, 2009).
- [89] P. J. Rousseeuw and A. M. Leroy, *Robust Regression and Outlier Detection* (John Wiley & Sons, Hoboken, NJ, 2003).