

Preparation of conditionally squeezed states in qubit-oscillator systems

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(Received 9 April 2025; accepted 21 January 2026; published 26 February 2026)

Inspired by recent advances in the manipulation of superconducting circuits coupled to mechanical modes in the quantum regime, we propose a protocol for generating superpositions of orthogonally squeezed states in a quantum harmonic oscillator. The protocol relies on a quadratic coupling between the oscillator and a qubit, and is conceptually similar to methods used for preparing cat states in qubit-oscillator systems. We numerically evaluate the robustness of the state-preparation scheme in the presence of decoherence, considering environmental coupling for both the harmonic oscillator and the qubit. As a potential application, we consider a quantum error-correcting code based on conditionally squeezed states and analyze its error correction properties.

DOI: [10.1103/jrrb-hymw](https://doi.org/10.1103/jrrb-hymw)

Introduction. Quantum technologies rely on the ability to design and manipulate systems that are fundamentally quantum mechanical [1]. By harnessing quantum effects such as superposition and entanglement, these technologies are expected to profoundly impact various fields, including sensing [2], information processing and communication [3], and the simulation of quantum systems [4].

A broad class of quantum systems relevant to quantum technologies can be described by a prototypical model consisting of one or more two-level systems coupled to one or more bosonic modes, under the influence of external drives. In its versatility, this model encompasses a range of physical scenarios, from matter-radiation interaction [5] to, more generally, cavity QED experiments [6], trapped ions [7], and superconducting circuit QED [8] setups. While different platforms share a common general framework, each physical implementation exhibits unique features that make it not only scientifically interesting on its own, but also potentially relevant for technological applications.

In the ongoing effort to achieve fault-tolerant quantum computation, a promising path is through the use of bosonic quantum error correction (bQEC), where logical qubits are encoded into bosonic modes [9]. A notable recent example of bQEC is the work of Sivak *et al.* [10], which demonstrated that quantum error correction can surpass the break-even point. This achievement hinges on the ability to generate specific states—Gottesman-Kitaev-Preskill (GKP) states—that form the foundation of the error-correction protocol. This exemplifies how the efficient preparation of high-

fidelity quantum states is of both practical and fundamental significance.

While a large body of work has focused on the bosonic modes implemented as microwave cavity modes, considerable research has also focused on coupling optical/microwave cavities and superconducting qubits to mechanical modes [11–18]. Depending on the mechanical mode's characteristic frequency, researchers distinguish between quantum optomechanics [19] and quantum acoustics [20]. In optomechanics, early breakthroughs included ground-state sideband cooling [12], as well as the squeezing [13,14] and entanglement [15,16] of mechanical modes. More recently, experiments have succeeded in preparing Fock [17] and cat states [18] in high-overtone bulk acoustic resonators.

From a theoretical perspective, early work [21,22] established the possibility of preparing any quantum state of a bosonic mode coupled to a qubit, under slightly different control conditions. Specifically, Law and Eberly [21] demonstrated that for a resonantly coupled qubit-oscillator system, arbitrary oscillator states could be prepared by controlling the coupling strength and applying a qubit drive. Their protocol was later used to generate arbitrary states in a circuit QED architecture [23]. Similarly, Krastanov *et al.* [22] and Eickbusch *et al.* [24] developed protocols enabling universal control of an oscillator via dispersive coupling to a qubit; these protocols have subsequently been used to prepare cat states [25], as well as GKP [10,24] and binomial code states [24] in superconducting cavities. Other protocols target specific quantum states, such as the preparation of cat states via conditional displacement and qubit measurement [26] or transient disentanglement of a qubit and an oscillator [18,27]. Beyond deterministic approaches, there is growing interest in leveraging machine learning (ML) techniques for quantum-state preparation and control [28]. Early results have demonstrated the preparation of Schrödinger cat states in a microwave circuit using neural networks [29].

In this Letter, we present a deterministic protocol for generating squeezed vacuum states with orthogonal squeezing axes,

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as well as their superpositions, in a system consisting of a bosonic mode coupled quadratically to a two-level system.

Two-component superpositions of squeezed vacuum states were studied theoretically already in the 1980s [30], while more recently they have attracted attention in the context of quantum sensing [31] and quantum information processing [32–34]. In terms of state preparation, there have been proposals for realizing two-component superpositions of orthogonally squeezed states in trapped ion [31,35] and circuit QED [33,34] systems, and they were recently demonstrated with trapped ions [36].

While sharing similarities, the implementation of a quadratic coupling in circuit QED, electromechanical, and ion-trap systems can differ in multiple aspects, owing to the fundamentally distinct physical realizations. For ion-trap systems, the quadratic coupling is proportional to the intensity of an external drive at a specific frequency (“second-order sideband” drive) and can therefore be controlled by the experimentalist. In circuit QED, the qubit-oscillator coupling is often fixed, and an effective quadratic coupling naturally arises as a perturbative effect under a dispersive interaction between the qubit and oscillator [37]. Additionally, recent work has demonstrated that it is possible to design a resonant (nonperturbative) quadratic coupling between qubit and bosonic mode [33], exploiting the symmetry of the underlying superconducting circuit.

In our work, we focus on a protocol adapted for the large-detuning regime between qubit and bosonic mode, which offers potential advantages with respect to realizations based on resonant coupling schemes owing to the increased level of isolation from decoherence of the bosonic mode through its coupling with the qubit. The protocol is thus relevant for qubits dispersively coupled to microwave cavities and for electromechanical implementations where a strong effectively quadratic coupling can be achieved in a similar manner, as was recently demonstrated in Ref. [38]. In addition, the protocol is primarily motivated by the electromechanical setup introduced in Ref. [39], for which the quadratic coupling can be seen as “native”—i.e., not induced by an external drive nor the result of a perturbative expansion in the dispersive regime.

Beyond the description of the state preparation protocol, we consider the bosonic qubit encoding based on the even and odd superpositions of orthogonally squeezed vacuum states, as was recently proposed in Refs. [33,34], and we analyze its error-correcting properties. We first use the standard Knill-Laflamme (KL) conditions [40] to show that the code can correct for single-boson loss and dephasing in the limit of large squeezing. An equivalent analysis is then performed using the framework of bosonic rotation codes, in which this code is conveniently classified, and where different codes can be quantitatively compared by their inherent phase uncertainty under realistic conditions [41].

Finally, we investigate the impact of dissipation and environmental coupling, focusing our parameter choices on experimentally realistic conditions relevant for emerging platforms in quantum optomechanics [38,39]. While all-electric circuit QED platforms presents some advantages (e.g., speed of operation), the exceptionally small decoherence rates of mechanical elements make the electromechanical

implementations attractive candidates when the preservation of quantum information is crucial (e.g., quantum memories).

System. We consider a harmonic oscillator with a strong quadratic coupling to an externally driven qubit. The system is described by the Hamiltonian ($\hbar = 1$)

$$H(t) = \frac{\omega_q}{2}\sigma_z + \omega_m b^\dagger b + g(b + b^\dagger)^2 \sigma_z + H_d(t), \quad (1)$$

where ω_q (ω_m) is the qubit (oscillator) energy, g is the coupling strength, $b^\dagger$ (b) is the bosonic raising (lowering) operator, and σ_z is the Pauli z operator. The qubit drive is described by the term $H_d(t)$.

Although our treatment is, in principle, device independent, we keep in mind that a strong quadratic coupling in the form of Eq. (1) was recently reported in an electromechanical setup, coupling a Cooper-pair box to a mechanical oscillator [38], and proposed [39] for a nanopillar coupled to a superconducting circuit.

Our goal is to use the qubit drive to generate strong and controllable squeezing of the oscillator. It is possible to show [42] that this can be achieved by a harmonic drive of the qubit at $\omega_d = \omega_m$ in a frame rotating at ω_q . In the laboratory frame, the drive is of the form

$$H_d(t) = A \cos(\omega_d t) [\cos(\omega_q t) \sigma_x + \sin(\omega_q t) \sigma_y], \quad (2)$$

where the amplitude A and frequency ω_d are assumed to be freely adjustable parameters. In the laboratory frame, the relevant terms of Eq. (2) oscillate at $\omega_q \pm \omega_d$ and therefore can be construed as inducing sideband transitions between the qubit and the oscillator for $\omega_d \simeq \omega_m$. Importantly, our analysis focuses on a regime of large detuning between the qubit and the bosonic mode. In the absence of driving, this detuning renders the two subsystems effectively decoupled and off-resonant, thereby suppressing unwanted hybridization and energy exchange. Notably, this configuration reduces the induced mechanical decoherence through the qubit once the external drive is turned off, offering an additional level of experimental control [42].

Constraints and conditions for optimal squeezing are determined by the form of the qubit-mechanics coupling Hamiltonian in the interaction picture [42]. For $\omega_d \simeq \omega_m$ and $\omega_m \gg g$, we can perform a rotating-wave approximation (RWA) and obtain

$$\tilde{H}_{\text{RWA}} = gJ_0(\bar{A})(2b^\dagger b + 1)\sigma_z + gJ_2(\bar{A})(b^2 + b^{\dagger 2})\sigma_z, \quad (3)$$

where $\bar{A} \equiv 2A/\omega_d$ and J_n is the n th-order Bessel function of the first kind. The two terms in $\tilde{H}_{\text{RWA}}$ can be thought of as a qubit splitting dependent on the occupation number of the oscillator and a conditional squeezing term where the squeezing parameter depends on the state of the qubit. When the amplitude is such that $J_0(\bar{A}) = 0$, the first term vanishes and $\tilde{H}_{\text{RWA}}$ reduces to a simple conditional squeezing Hamiltonian

$$H_{\text{cs}} = g_{\text{cs}}(b^2 + b^{\dagger 2})\sigma_z, \quad (4)$$

where $g_{\text{cs}} \equiv gJ_2(\bar{A})$. Choosing the strongest possible squeezing results in the condition $\bar{A} \approx 2.405$, i.e., the first root of J_0 .

Preparation of squeezed states. We now consider the dynamics generated by H_{cs} . If the qubit is in the $+1$

eigenstate of σ_z , it is straightforward to show that the time-evolution operator for the oscillator is equivalent to the single-mode squeeze operator $S(\xi) = \exp[\frac{1}{2}(\xi^* b^2 - \xi b^{\dagger 2})]$ with squeezing parameter $\xi \equiv 2ig_{cs}t$. Similarly, when the qubit is in the -1 eigenstate, the time evolution of the oscillator is governed by $S(-\xi)$. In the phase space of the oscillator, these two cases correspond to squeezing along orthogonal axes.

Building on these results, we now describe a protocol to generate symmetric and antisymmetric superpositions of orthogonally squeezed states [33,34], similar to how even and odd cat states can be generated by conditional displacement in qubit-oscillator systems with linear coupling [26]. We consider an initial product state with a ground-state oscillator and a qubit in one of the σ_x eigenstates. Without loss of generality, we take the initial state to be $|\psi(0)\rangle \propto |0\rangle \otimes [|e\rangle + |g\rangle]$ (where $|e\rangle$ and $|g\rangle$ are the σ_z eigenstates). When the system evolves according to H_{cs} , the qubit and oscillator become entangled, and the state after time t can be written as $|\psi(t)\rangle \propto [S(\xi)|0\rangle \otimes |e\rangle + S(-\xi)|0\rangle \otimes |g\rangle]$. By expressing $|e\rangle$ and $|g\rangle$ in terms of the σ_x eigenstates, one can verify that a projective measurement of the qubit in the σ_x basis leaves the oscillator in either the symmetric or antisymmetric superposition of the squeezed vacuum states, depending on the measurement outcome. Figure 1 depicts the Wigner functions of the symmetric and antisymmetric superposition states with a squeezing parameter $\xi = i$, corresponding to a time $t = 1/(2g_{cs})$ under the evolution generated by H_{cs} .

If the driving is subsequently turned off, the dynamics in the rotating frame is described by the first term in $\tilde{H}_{RWA}$, leading to a dephasing of the bosonic superposition state. As a consequence, a measurement of σ_z immediately after σ_x can in this case improve the stability of the bosonic state [42], effectively suppressing the mechanical state dephasing induced by the qubit.

To analyze the sensitivity of the state preparation to the RWA and the amplitude condition in Eq. (4), we plot in Fig. 2 the fidelity between the symmetric superposition generated by H_{cs} and the state generated by the full Hamiltonian in Eq. (1) for different coupling strengths and drive amplitudes.

Application as a quantum code. As previously mentioned, the states in Fig. 1 are, in some sense, the squeezing analog of the even and odd cat states. It is therefore natural to consider them as potential logical qubits (see, e.g., Refs. [33,34]) and define a QEC code analogous to the two-component cat code [43] in the following way

$$|0_L\rangle = \frac{1}{\sqrt{\mathcal{N}_+}} [S(\xi) + S(-\xi)]|0\rangle, \quad (5a)$$

$$|1_L\rangle = \frac{1}{\sqrt{\mathcal{N}_-}} [S(\xi) - S(-\xi)]|0\rangle, \quad (5b)$$

where $S(\xi)|0\rangle = \frac{1}{\cosh r} \sum_{n=0}^{\infty} (-1)^n \frac{\sqrt{(2n)!}}{2^n n!} e^{in\varphi} \tanh^n r |2n\rangle$ is the squeezed vacuum state with complex squeezing parameter $\xi = re^{i\varphi}$. The normalization constants are given by $\mathcal{N}_{\pm} = 2[1 \pm \cosh^{-1/2}(2r)]$.

In the following, we provide an analysis of the error-correction properties of the code using the standard KL conditions [40], and we classify the code within the

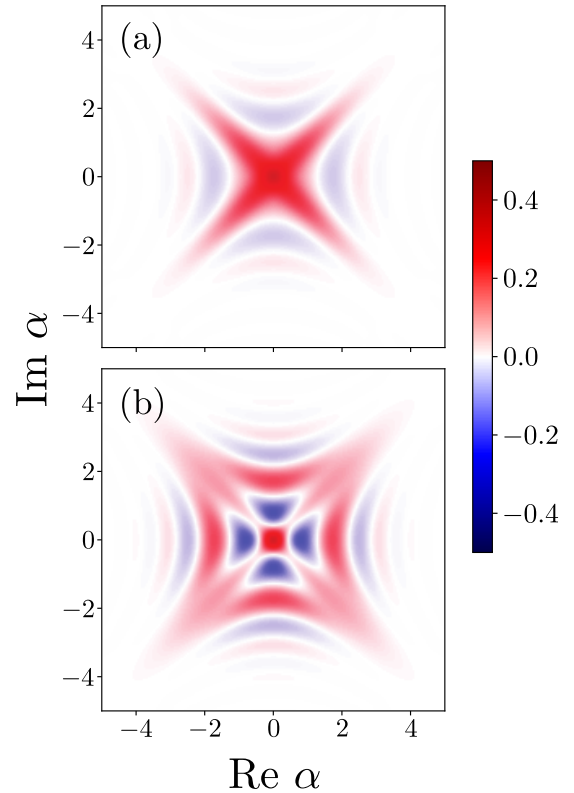


FIG. 1. Wigner functions $W(\alpha)$ of (a) the symmetric and (b) the antisymmetric superposition of orthogonally squeezed states, with squeezing parameter $\xi = i$. The states can be prepared by a conditional squeezing protocol: An initial state $|0\rangle \otimes [|e\rangle + |g\rangle]$ evolves according to H_{cs} , followed by a projective measurement of the qubit in the σ_x basis. The symmetric (antisymmetric) superposition state is obtained in the oscillator when the qubit is measured to be in the state $|e\rangle + |g\rangle$ ($|e\rangle - |g\rangle$). More generally, if the qubit is initially prepared in one of the σ_x eigenstates, the symmetric (antisymmetric) superposition is obtained in the oscillator when the qubit is later measured to be in the same (different) state.

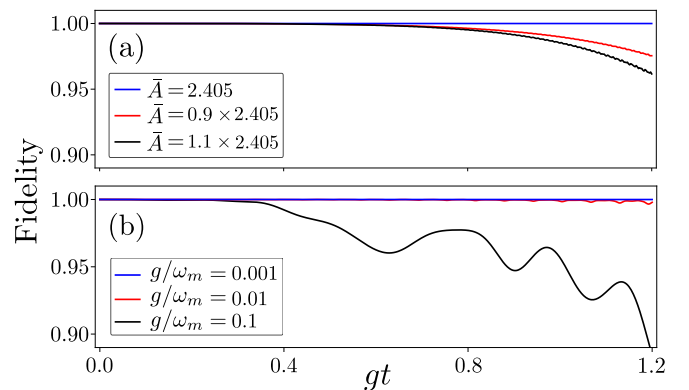


FIG. 2. Sensitivity of the state preparation protocol to the conditions on (a) driving amplitude $\bar{A}$ and (b) relative coupling strength g/ω_m . The fidelity is taken between the symmetric superposition state generated by H_{cs} and the state generated by the full Hamiltonian in Eq. (1). The end time is chosen such that the magnitude of the squeezing parameter ξ approaches 1 at the end.

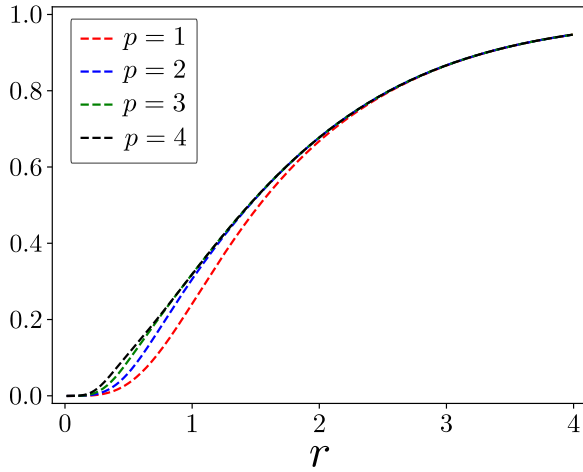


FIG. 3. Ratio of the expectation values $\langle (b^\dagger b)^p \rangle$ in the two logical states of the squeezed vacuum code, i.e., $\langle 0_L | (b^\dagger b)^p | 0_L \rangle / \langle 1_L | (b^\dagger b)^p | 1_L \rangle$, for $p \in \{1, 2, 3, 4\}$ as a function of the squeezing strength r .

framework of bosonic rotation codes [41]. Due to the symmetry of the logical states, it is easily verified that $|0\rangle_L, |1\rangle_L$ only contains terms of the form $|4n\rangle, |4n+2\rangle$ in the Fock basis. This corresponds to a distance 2 in Fock space, meaning that the code can at most detect a single-boson loss event. To the leading order in the decoherence rates, the single loss and dephasing channel correspond to the error set $\mathcal{E} = \{I, b, b^\dagger b, (b^\dagger b)^2\}$, where I is the identity operator [41,44]. For a code to be able to correct the set $\mathcal{E}$, the KL conditions require that the logical states remain orthogonal after the action of any operator in $\mathcal{E}$ and that the expectation value $\langle (b^\dagger b)^p \rangle$ with $p \in \{1, 2, 3, 4\}$ is the same in both logical states. Due to the separation between the logical states in Fock space, the orthogonality condition is identically satisfied for all the operators in $\mathcal{E}$. For the same reason, the expectation values are never exactly equal. However, the logical states become indistinguishable in the limit of large squeezing, analogous to the cat codes where the KL conditions are approximately satisfied for large displacements [43]. In Fig. 3, we have plotted the ratio of the expectation values in the different logical states as a function of squeezing strength.

The separation in Fock space between the logical states is a consequence of their rotational symmetry in phase space. By definition, a bosonic rotation code has an N -fold symmetry if a phase space rotation by $2\pi/N$ acts as the identity operator, while a rotation by π/N acts as a logical Pauli z operator [41]. It is readily verified that the squeezed vacuum code satisfies these properties with a twofold symmetry. Furthermore, it can be shown in general that an N -fold symmetric code only has support on Fock states $|2nN\rangle$ and $|(2n+1)N\rangle$, corresponding to a distance N in Fock space and a rotational distance π/N in phase space [41].

Although the squeezed vacuum code was motivated by the conceptual similarity with the two-component cat code, its symmetry in phase space is similar to a four-component code. Indeed, it can be shown that the code is equivalent to a four-component squeezed cat code with zero displacement [41]. In the limit of large squeezing, the phase properties of a squeezed vacuum state approach those of a superposition

of two phase states with opposite phases [45]. Following the definition in Ref. [41], the squeezed vacuum code is therefore an instance of the approximate number-phase codes, approaching the ideal code in the limit of infinite squeezing, where, in principle, it can detect a number shift smaller than 2 and a phase rotation smaller than $\pi/2$. With the assumption that loss is the dominant number shift and there is unbiased Gaussian phase noise, the code can approximately correct for single-boson loss and phase errors smaller than $\pi/4$ [41].

While the distance in Fock space only depends on the symmetry of the code, the ability to detect phase errors strongly depends on the energy of the bosonic code words (i.e., the squeezing strength r in the current context). The practical performance of a code therefore depends on its ability to detect phase errors under experimentally relevant conditions [41]. In the Supplemental Material [42], we have compared the phase uncertainty of the squeezed vacuum code with that of other rotation codes, including the cat code [43,46] and the binomial code [47]. We find that the squeezed vacuum code exhibits a larger phase uncertainty under moderate excitation values, representing a disadvantage of the current approach. On the other hand, the fact that squeezing can be generated rather straightforwardly in many experimental settings could present an advantage in state preparation and control, compared to codes that are based on more complex superpositions of Fock states.

Open system dynamics. We previously showed that we can generate the logical states in Eq. (5) with a high fidelity given certain restrictions on the coupling strength and qubit drive (Fig. 2). We now assume an optimal drive and evaluate how the protocol performs in an open system where dissipation effects degrade the fidelity of the prepared states. Our objective is to estimate the experimental parameter requirements to observe the superposition states shown in Fig. 1.

We analyze the open system dynamics by numerically solving the master equation with the relevant jump operators

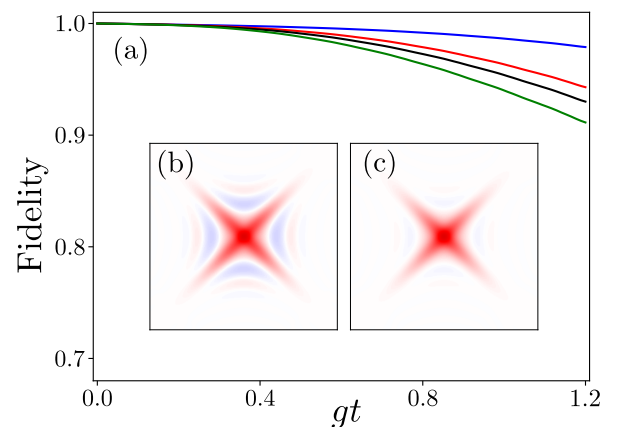


FIG. 4. (a) Fidelity of the symmetric superposition state prepared in an open system with relative qubit decoherence rates $\gamma_1/g, \gamma_\phi/g = 0.1, 0.1$ (blue); $1.0, 0.1$ (red); $0.1, 1.0$ (black); and $1.0, 1.0$ (green). In all cases, the mechanical thermal occupation is $n_{m,th} = 1$ and relative damping rate $\gamma_m/g = 0.01$. Panels (b) and (c) depicts the Wigner function at the end time $gt = 1.2$ for the blue and green curves, respectively.

[42]. In principle, the state preparation is sensitive to both decay and noise in the oscillator, as well as decoherence of the qubit. Loosely speaking, the bosonic decay rate sets a limit to the level of squeezing that can be obtained in the oscillator, while the qubit decoherence limits the coherence of the bosonic superposition state. The latter is most easily seen by the weakening of the interference fringes in the Wigner plots in Figs. 4(b) and 4(c).

As reference, we consider a mechanical oscillator with frequency $\omega_m/2\pi = 150$ MHz and a superconducting qubit with frequency $\omega_q/2\pi = 4$ GHz coupled quadratically with a coupling strength $g/2\pi = 15$ kHz. A similar setup has recently been reported in Ref. [38] and proposed in Ref. [39]. One of the benefits of using a mechanical element as the bosonic mode is a low intrinsic decay rate. Following Refs. [38,39], we will assume $\gamma_m/2\pi = 150$ Hz. When the temperature is 10 mK, the thermal occupation numbers for the oscillator and qubit baths are $n_{m,th} \approx 1$ and $n_{q,th} \approx 10^{-8}$ respectively, and the primary limitation to the state preparation is the decoherence of the qubit. In Fig. 4(a), we plot the fidelity over time of a symmetric superposition state prepared with the mentioned parameters and qubit decay ($\gamma_1/2\pi$) and dephasing ($\gamma_\phi/2\pi$) rates in the range 1.5–15 kHz. The end time corresponds to a squeezing strength $r \approx 1$ in the closed system, and we see

that successful state preparation is roughly bounded by the requirement $\gamma_1, \gamma_\phi \lesssim g = 2\pi \times 15$ kHz. This is well within what can be achieved with superconducting qubits [48,49] and around the values reported for superconducting qubits coupled to mechanical oscillators [38,50].

Conclusions. We have shown how symmetric and anti-symmetric superpositions of orthogonally squeezed vacuum states can be prepared in qubit-oscillator systems with strong quadratic coupling. The protocol uses a conditional squeezing mechanism and requires careful tuning of both frequency and amplitude of an external qubit drive. As a potential application, we considered a bosonic code based on the prepared states and showed that the code satisfies the Knill-Laflamme conditions for single-boson loss and phase errors in the limit of large squeezing.

Acknowledgments. The numerical simulations were performed using the QuantumOptics.jl numerical framework [51]. The authors thank Juuso Manninen, Jesper Lind-Olsen, and Tron Omland for fruitful discussions. F.M. acknowledges financial support from the Research Council of Norway (Grant No. 333937) through participation in the QuantERA ERA-NET Cofund in Quantum Technologies.

Data availability. The data that support the findings of this article are openly available [52].

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