

Evidencing quantum gravity with thermodynamical observables

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Proposed experiments for obtaining empirical evidence for a quantum description of gravity in a tabletop setting focus on detecting quantum information signatures, such as entanglement or non-Gaussianity production, in gravitationally interacting quantum systems. Here, we explore an alternative approach where the quantization of gravity could be inferred through measurements of macroscopic, thermodynamical quantities, without the need for addressability of individual quantum systems. To demonstrate the idea, we take as a case study a gravitationally self-interacting Bose gas and consider its heat capacity. We find a clear-cut distinction between the predictions of a classical gravitational interaction and a quantum gravitational interaction in the heat capacity of the Bose gas.

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A consensus has yet to emerge on how the two fundamental frameworks of physics, quantum mechanics (QM) and general relativity (GR), combine. Since the standard approach for combining the other fundamental interactions with QM fails with gravity, approaches have been developed, such as string theory, loop quantum gravity, and theories where gravity is not quantized and remains fundamentally classical. Recently, tabletop experiments have been proposed that can probe whether gravity is fundamentally quantum or classical [1]. While these experiments operate in the nonrelativistic, Newtonian regime and are thus unlikely to discriminate between leading quantum gravity theories, they can still, remarkably, provide the first empirical evidence of how QM and GR combine. Examples include seeking entanglement between two massive systems prepared in Schrödinger cat states [2,3] and detecting quantum non-Gaussianity in a squeezed Bose-Einstein condensate [4] as evidence of quantum gravity. These proposals for tabletop tests of quantum gravity involve preparing a sufficiently massive system in a quantum state and then analyzing its quantum information properties to evidence nonclassicality of states of its gravitational field. This general methodology presents a substantial technical challenge due to the need for preparing and isolating the system from the environment and detecting such highly nonclassical quantum states.

Here, we explore whether an alternative approach might be possible. Inspired by ideas of verifying entanglement via

macroscopic thermodynamic quantities, such as magnetic susceptibility and heat capacity [5–9], we consider whether a quantum gravitational effect could be detected through thermodynamic observables, thereby avoiding the need for coherent control or the application of quantum operations and measurements.

As a case study, we consider the contributions from self-gravitation to the heat capacity of a Bose-Einstein condensate (BEC). Assuming a classical linearized gravitational field in the spirit of “semiclassical gravity” [10,11], where the self-gravitational field of the BEC is treated classically and given by the expectation of the quantum state of the BEC, we find that the energy spectrum and the heat capacity remain unaffected by the self-gravity interaction in the Newtonian regime. Intuitively, this is due to the interaction being “averaged” over the state of the BEC, which results in the energy spectrum simply being offset by a constant, and has no effect on the heat capacity as it only depends on the spacing of the energy eigenvalues. In contrast, in linearized quantum gravity, where the gravitational field of the BEC is treated quantum mechanically, correlations are induced between the atoms that cause a nontrivial modification of the energy spectrum in the Newtonian regime, resulting in a dependence of the heat capacity on Newton’s constant. Moreover, since the gravitational field cannot be shielded, the self-gravity contribution to the energies and the heat capacity is not restricted to the neighboring atoms as in the case of the electromagnetic interaction, but extends to all atoms in the sample. The measurement of heat capacity or internal energy can, therefore, provide a clear-cut criterion for distinguishing semiclassical gravity and linearized quantum gravity.

I. SETUP

We now consider how a BEC self-interacts through gravity, within the standard framework of gaseous BECs. The

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grand-canonical Hamiltonian of a Bose gas with two-body electromagnetic interactions reads [12]

$$\hat{K} = \int d^3\mathbf{r} \hat{\Psi}^\dagger(\mathbf{r}) \left(-\frac{\hbar^2 \nabla^2}{2m} - \mu \right) \hat{\Psi}(\mathbf{r}) + \hat{H}_I^{\text{EM}}, \quad (1)$$

where μ denotes the chemical potential, $\hat{\Psi}^\dagger(\mathbf{r})$ is the nonrelativistic quantum field of the Bose gas, the first term represents the kinetic energy of the gas, and $\hat{H}_I^{\text{EM}}$ is due to the electromagnetic interactions:

$$\hat{H}_I^{\text{EM}} = \frac{1}{2} \int d^3\mathbf{r}' d^3\mathbf{r} \hat{\Psi}^\dagger(\mathbf{r}) \hat{\Psi}^\dagger(\mathbf{r}') V_I(\mathbf{r} - \mathbf{r}') \hat{\Psi}(\mathbf{r}') \hat{\Psi}(\mathbf{r}), \quad (2)$$

with $V_I(\mathbf{r} - \mathbf{r}')$ the two-body electromagnetic potential. For simplicity, we consider a Bose gas trapped in a box of length L with periodic boundary conditions for the field; see, for instance, experiments [13–16] and standard theoretical work [12].

For a dilute Bose gas made up of neutral atoms, the electromagnetic interactions fall off significantly faster than $1/r$. It is, therefore, a good approximation to treat the electromagnetic self-interaction as a contact interaction $V_I(\mathbf{r}) = g^{\text{EM}} \delta^{(3)}(\mathbf{r})$, where $g^{\text{EM}} = 4\pi \hbar^2 a_s/m$, with m the particle mass and a_s the s -wave scattering length. This is the standard textbook treatment of electromagnetic interactions in a BEC [12,17]. However, since gravity cannot be shielded, to study the gravitational self-interaction of the Bose gas, it is crucial to keep explicit the long-range nature of gravity.

We will study two different types of gravity self-interaction terms. These correspond to a linearized quantized metric perturbation and to a classical gravitational field sourced by quantum matter, the BEC. In the nonrelativistic limit of linearized quantum gravity (meaning zeroth order in the speed of light, but G and $\hbar$ are relevant), the interaction term due to gravity is of the form (2) with $V_I(\mathbf{r}) = -Gm^2/|\mathbf{r}|$,

$$\hat{H}_I^{\text{QG}} = -\frac{1}{2} Gm^2 \int d^3\mathbf{r}' \int d^3\mathbf{r} \hat{\Psi}^\dagger(\mathbf{r}) \hat{\Psi}^\dagger(\mathbf{r}') \times \frac{1}{|\mathbf{r} - \mathbf{r}'|} \hat{\Psi}(\mathbf{r}') \hat{\Psi}(\mathbf{r}), \quad (3)$$

which is derived from first principles in the Supplemental Material [20]. We note that all quantum gravity theories are expected to approximate linear quantum gravity in the regime of the experiment [21] and thus Eq. (3) models how a BEC self-interacts through gravity in all standard quantum gravity theories.

Considering a classical metric perturbation in the spirit of “semiclassical gravity” [10,11], where gravity is sourced by the expectation of quantum matter, the interaction Hamiltonian is of the form

$$\hat{H}_I^{\text{CG}} = -Gm^2 \int d^3\mathbf{r}' \int d^3\mathbf{r} \frac{\langle \hat{\Psi}^\dagger(\mathbf{r}') \hat{\Psi}(\mathbf{r}') \rangle}{|\mathbf{r} - \mathbf{r}'|} \hat{\Psi}^\dagger(\mathbf{r}) \hat{\Psi}(\mathbf{r}), \quad (4)$$

where the expectation value is taken over the quantum state of the Bose gas [22]. The reason the two cases of self-gravitation will give very different contributions to the heat capacity is that $\hat{H}_I^{\text{CG}}$ is of *quadratic* order in the annihilation and creation operators, while $\hat{H}_I^{\text{QG}}$ is of *quartic* order.

II. BOGOLYUBOV METHOD APPLIED TO LONG-RANGE INTERACTION

The Bogolyubov method is the standard textbook method to analyze a homogeneous, isotropic gaseous BEC with contact electromagnetic interactions [12], which is the type of BEC we assume for simplicity; see, for instance, experiments [13–16]. We adapt and apply this method to a $1/r$ (long range) interaction term, to arrive at a diagonalized Hamiltonian from which the heat capacity can be obtained. The Bogolyubov method consists principally of three major steps [12]: (1) the Bogolyubov approximation; (2) a Fourier transform to momentum space and (3) a linear symplectic transformation (Bogolyubov transformation) that diagonalizes the Hamiltonian. Finally, we use the gapless condition to determine the chemical potential [23].

III. BOGOLYUBOV APPROXIMATION

The first step of the Bogolyubov method is to split the field operator as $\hat{\Psi}(\mathbf{r}) = \hat{\phi}(\mathbf{r}) + \hat{\psi}(\mathbf{r})$, where $\hat{\phi}(\mathbf{r})$ and $\hat{\psi}(\mathbf{r})$ are for the condensate and excited states, respectively [12]. Taking the definition of a BEC to be the macroscopic occupation of the ground state, i.e., $N \gg 1$ and $N \approx N_0$, where N_0 and N are the total number of atoms and number of condensed atoms, respectively, the Bogolyubov approximation is that the field operator $\hat{\phi}(\mathbf{r})$ is traded for a classical variable ϕ using the substitution $\hat{\phi}(\mathbf{r}) = \sqrt{n}$, where $n := N/\mathcal{V}$ is the number density of the Bose gas, with $\mathcal{V} = L^3$ the volume of the box [24]. Inserting the above decomposition into the full grand-canonical Hamiltonian $\hat{K}$, only terms with two or more $\hat{\phi}(\mathbf{r})$ are kept since $N \gg 1$ [12]. All other terms are taken negligible. An explicit derivation is provided in the Supplemental Material [20].

IV. FOURIER TRANSFORM

Next, we Fourier expand the excitation fields as $\hat{\psi}(\mathbf{r}) = \frac{1}{\sqrt{\mathcal{V}}} \sum_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{r}} \hat{a}_{\mathbf{k}}$, where $\hat{a}_{\mathbf{k}}$ is the annihilation operator for mode $\mathbf{k}$ [12]. Similarly, the interactions are expanded as $V_I(\mathbf{r}) = \frac{1}{\mathcal{V}} \sum_{\mathbf{k}} V_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{r}}$, where $V_{\mathbf{k}} = \int_{\mathcal{V}} d^3\mathbf{r} V_I(\mathbf{r}) e^{-i\mathbf{k}\cdot\mathbf{r}}$, with $\mathbf{k} = 2\pi \mathbf{n}_{\mathbf{k}}/L$, $\mathbf{n}_{\mathbf{k}} = (n_x, n_y, n_z)$, and n_x, n_y, n_z integers ranging from $-\infty$ to $+\infty$.

For the electromagnetic interactions, which are modeled as contact interactions in standard treatments of BECs as discussed above, $V_I(\mathbf{r}) = g^{\text{EM}} \delta^{(3)}(\mathbf{r})$, we simply have $V_{\mathbf{k}} = g^{\text{EM}}$ for all momenta $\mathbf{k}$. For the long-range gravitational interaction, we must calculate

$$V_{\mathbf{k}}^G = \int d^3\mathbf{r} \frac{-Gm^2}{|\mathbf{r}|} e^{-i\mathbf{k}\cdot\mathbf{r}} \quad (5)$$

$$= \int_{-\frac{L}{2}}^{\frac{L}{2}} dx \int_{-\frac{L}{2}}^{\frac{L}{2}} dy \int_{-\frac{L}{2}}^{\frac{L}{2}} dz \frac{-Gm^2 e^{-2\pi i(n_x x + n_y y + n_z z)/L}}{\sqrt{x^2 + y^2 + z^2}}. \quad (6)$$

This is well approximated by $V_{\mathbf{k}}^G = g_{\mathbf{k}}^G$, where

$$g_{\mathbf{k}}^G = \begin{cases} g_0^G, & k = 0, \\ -4\pi Gm^2/k^2, & k > 0, \end{cases} \quad (7)$$

with $g_0^G \approx L^3 V_{\text{cube}}$, and $V_{\text{cube}} = -2.38 Gm^2/L$ the gravitational potential of a cube of side L at its center. A derivation is given in the Supplemental Material [20].

Inserting the above steps into the quantized gravity interaction Hamiltonian [Eq. (3)], we obtain

$$\hat{H}_I^{\text{QG}} = \frac{1}{2}n \left(N g_0^G + \sum_{k \neq 0} g_k^G \hat{A}_k \right), \quad (8)$$

with $\hat{A}_k = 2\hat{a}_k^\dagger \hat{a}_k + \hat{a}_k^\dagger \hat{a}_{-k}^\dagger + \hat{a}_k \hat{a}_{-k}$. Compare this to the electromagnetic interaction Hamiltonian, which in the same standard BEC approximation reads

$$\hat{H}_I^{\text{EM}} = \frac{1}{2}n g^{\text{EM}} \left(N + \sum_{k \neq 0} \hat{A}_k \right). \quad (9)$$

The difference between the long-range nature of the quantum gravitational case (8) and the short-range nature of the (also quantum) electromagnetic case [Eq. (9)] is manifested in the second term (excitations) of the above expressions: In the gravity case we have a k -dependent coupling, while in the electromagnetic case we have a constant coupling as in standard treatments of BECs [12].

Finally, in sharp contrast to the quantum gravity case, for classical gravity we find that the chemical potential is updated

to $\mu \rightarrow \mu - n g_0^G$ and the interaction Hamiltonian (4) can be written as just

$$\hat{H}_I^{\text{CG}} = n_T N_T g_0^G, \quad (10)$$

where $N_T := \sum_{k \neq 0} \langle \hat{a}_k^\dagger \hat{a}_k \rangle$ is the number of noncondensed atoms and $n_T := N_T/V$ is their density. That is, the interaction Hamiltonian is simply a constant term. The radically different behavior of the interaction between the classical and quantum gravity cases we consider is what allows us to discern the two cases. In particular, as we will see in a moment, the difference can be read from the energy spectrum and heat capacity.

V. BOGOLYUBOV TRANSFORMATION

We next apply a linear symplectic transformation on phase space to diagonalize the canonical Hamiltonians [12]: $\hat{b}_k = u_k \hat{a}_k + v_{-k}^* \hat{a}_{-k}^\dagger$ and $\hat{b}_k^\dagger = u_k^* \hat{a}_k^\dagger + v_{-k} \hat{a}_{-k}$, where $|u_k|^2 - |v_{-k}|^2 = 1$. For the quantum gravity case [Eq. (8)], this results in the grand-canonical Hamiltonian

$$\hat{K}^{\text{QG}} = E_0^{\text{QG}} + \sum_{k \neq 0} \epsilon_k^{\text{QG}} \hat{b}_k^\dagger \hat{b}_k, \quad (11)$$

with excitation energies

$$\epsilon_k^{\text{QG}} = \sqrt{\left(\frac{\hbar^2 k^2}{2m} - \mu + n(2g^{\text{EM}} + g_k^0 + g_k^G) \right)^2 - (n(g^{\text{EM}} + g_k^G))^2} \quad (12)$$

and ground state energy

$$E_0^{\text{QG}} = E_0 + \frac{1}{2}nN g_0^G + \frac{1}{2} \sum_{k \neq 0} [\epsilon_k^G - n g_k^G], \quad (13)$$

with

$$E_0 = \frac{1}{2}nN g^{\text{EM}} + \frac{1}{2} \sum_{k \neq 0} \left[\mu - \frac{\hbar^2 k^2}{2m} - 2n g^{\text{EM}} - n g_0^G \right]. \quad (14)$$

For the classical gravity case [Eq. (4)], we find

$$\hat{K}^{\text{CG}} = E_0^{\text{CG}} + \sum_{k \neq 0} \epsilon_k^{\text{CG}} \hat{b}_k^\dagger \hat{b}_k, \quad (15)$$

where

$$\epsilon_k^{\text{CG}} = \sqrt{\left(\frac{\hbar^2 k^2}{2m} - \mu + 2n g^{\text{EM}} + n g_0^G \right)^2 - (n g^{\text{EM}})^2} \quad (16)$$

and

$$E_0^{\text{CG}} = E_0 + n_T N_T g_0^G + \frac{1}{2} \sum_{k \neq 0} \epsilon_k^{\text{CG}}. \quad (17)$$

VI. ENERGY SPECTRUM

In order to obtain the final form of the energy spectrum, we apply the condition that the excitations must be gapless:

$$\lim_{k \rightarrow 0} \epsilon_k^{\text{QG}} = 0 \quad \text{and} \quad \lim_{k \rightarrow 0} \epsilon_k^{\text{CG}} = 0. \quad (18)$$

This is typically considered a necessary step in order to have a BEC according to the Nambu-Goldstone theorem [25,26] and allows one to infer the chemical potential μ [23]. Assuming that the electromagnetic interactions are repulsive $g^{\text{EM}} > 0$, for the classical gravity interaction Hamiltonian (4) we obtain $\mu = n(g^{\text{EM}} + g_0^G)$, leaving us with

$$\epsilon_k^{\text{CG}} = \hbar k \sqrt{\frac{\hbar^2 k^2}{4m^2} + \frac{n g^{\text{EM}}}{m}}. \quad (19)$$

This result is just the standard excitation energies for a Bose gas with electromagnetic interactions in the approximation of the Bogolyubov method [12].

For the quantum gravity interaction Hamiltonian (8) and assuming again $g^{\text{EM}} > 0$, the situation is more subtle. We will discuss separately the case when the electromagnetic coupling is larger than the gravitational coupling and vice versa. With $g^{\text{EM}} > |g_0^G|$, we find $\mu = n(g^{\text{EM}} + g_0^G)$ and obtain

$$\epsilon_k^{\text{QG}} = \hbar k \sqrt{\frac{\hbar^2 k^2}{4m^2} + \frac{n g^{\text{EM}}}{m} + \frac{n g_k^G}{m}}. \quad (20)$$

However, if $g^{\text{EM}} < |g_0^G|$, then $\mu = 3n(g^{\text{EM}} + g_0^G)$, in which case we obtain

$$\epsilon_k^{\text{QG}} = \sqrt{\left(\frac{\hbar^2 k^2}{2m} - 2n(g^{\text{EM}} + g_0^G)\right)\left(\frac{\hbar^2 k^2}{2m} - 2n(g_0^G - g_k^G)\right)}. \quad (21)$$

Note that the expression under the root never becomes negative. This is because g^G is always negative and $|g_0^G| > g^{\text{EM}}$ and $|g_0^G| > |g_k^G|$. Also, with $g_0^G = 0 = g_k^G$, both Eqs. (20) and (21) become, as expected, the standard quasiparticle excitations of a uniform BEC with, respectively, repulsive and attractive electromagnetic interactions [12].

Note that in Eq. (20), there will be phononic excitations when $\hbar k \ll mc_s$ and $n_0 g_k^G/m \ll c_s^2$, where we define $c_s^2 = n_0 g^{\text{EM}}/m$. In contrast, in Eq. (21), it is never possible to obtain phononic excitations given the form of g_k^G in Eq. (7). This means that if gravity becomes greater than the electromagnetic interactions, instead of phonons (type-A Nambu-Goldstone bosons) [27], one should see the emergence of type-B Nambu-Goldstone bosons (NGBs) sourced by the quadratic dispersion relation. Additionally, the fact that quantum gravity produces type-B NGBs also presents a clear-cut criterion to tell it apart from an EM-sourced dispersion, as the latter only ever produces type-A NGBs (phonons).

VII. HEAT CAPACITY

Specific heat capacity is defined by $c_V = \partial E / \partial T$, where $E = \langle \hat{K}^G \rangle + \mu N$ is the internal energy and $\hat{K}^G$ is $\hat{K}^{\text{QG}}$ or $\hat{K}^{\text{CG}}$ in our case. Given that $\langle b_k^\dagger b_k \rangle = \int dk \, 1/(\exp(\beta \epsilon_k) - 1)$, the specific heat capacity c_V of a BEC is [12]

$$c_V = \frac{1}{k_B T^2} \sum_{k \neq 0} \frac{\epsilon_k^2 e^{\beta \epsilon_k}}{(e^{\beta \epsilon_k} - 1)^2}, \quad (22)$$

where $\beta := 1/(k_B T)$ and ϵ_k in our case is ϵ_k^{QG} for quantum gravity and ϵ_k^{CG} for classical gravity, assuming sufficiently low temperatures. Since the form of Eq. (22) is the same for both quantum and classical gravity, any difference comes entirely from the respective ϵ_k calculated above. In fact, since ϵ_k^{CG} is the same as that for a non-self-gravitating BEC with electromagnetic interactions, the heat capacity of a BEC is unaffected by a classical gravity interaction of the form (4). Therefore, we see that the heat capacity, a macroscopic thermodynamical observable, will in principle cleanly distinguish between the two cases we consider: In the classical gravity case, the heat capacity will be unaffected and will be the same as if we had neglected the self-gravitational interaction, which is not the case for the quantum gravitational self-interaction. This is due to the spacing of the energy eigenvalues being only affected by quantum gravity: they follow Eqs. (20) or (21) rather than Eq. (19).

VIII. SIZE OF THE EFFECT

We now consider what sort of experimental parameters could be required to evidence quantum gravity through measurements of the heat capacity. Since any differences come through the energy eigenvalues, as discussed above, we first

discuss how measurements of just these eigenvalues could also be used to distinguish quantum and classical gravity. For example, we could consider a 0.1% deviation from the standard values ϵ_k^{CG} as the minimum threshold for a measurable result and evidence of quantum gravity. That is, we can consider what experimental parameters are required for $\Delta \epsilon_k / \epsilon_k^{\text{CG}} \times 100\% \geq 0.1\%$, where $\Delta \epsilon_k := |\epsilon_k^{\text{QG}} - \epsilon_k^{\text{CG}}|$. For now, we assume that the electromagnetic interactions can be set negligible compared to quantum gravity using Feshbach resonances, and return to this later. Then, from Eq. (21), if we ignore g_k^G for simplicity, the condition for the energy eigenvalues to be distinguishable is satisfied when

$$100 n g_0^G \gtrsim 0.1 \hbar^2 k^2 / (4m^2) \quad (23)$$

$$\Rightarrow NL \gtrsim \pi^2 \hbar^2 |\mathbf{n}_k|^2 / (2380 G m^3). \quad (24)$$

Taking a ^{174}Yb Bose gas, this sets the approximate condition $NL \gtrsim 2.9 \times 10^{14} |\mathbf{n}_k|^2 \text{ m}^{-3}$. Clearly, it is preferable to measure the $|\mathbf{n}_k| = 1$ mode. Then, an approximate 0.1% deviation would require, for example, $N \approx 10^{15}$ and $L \approx 1 \text{ cm}$. This is above what has been achieved in experiments—with $N = 10^9$ hydrogen atoms being the greatest [28], with the condensate being 0.5 cm in length and 15 μm in diameter, and 10^8 sodium atoms [29] in a BEC with axial and radial sizes of 1 mm and 20 μm . It also requires a relatively high density compared to typical experiments, resulting in a theoretical half-life of around 1 ms due to three-body decay, where the rate of change in density is $dn(t)/dt = -Ln^3(t)$, with L the rate coefficient, which is of order $10^{-41} \text{ m}^6 \text{ s}^{-1}$ for ^{174}Yb [30].

Now we consider what experimental parameters might be required to distinguish quantum and classical gravity in measurements of the *thermodynamic* quantity c_V . We again consider a 0.1% deviation in these values as evidence of quantum gravity, i.e., $100\% \times \Delta c_V / c_V^{\text{CG}} \geq 0.1\%$, where $\Delta c_V := |c_V^{\text{QG}} - c_V^{\text{CG}}|$, with c_V^{QG} and c_V^{CG} defined in Eq. (22) for the respective energy eigenvalues. In this case, similar but slightly more demanding parameters are found to be required than in the measurement of the lowest-energy mode, with, for example, $N \approx 10^{16}$, $L \approx 1 \text{ cm}$, and a temperature $T \approx 0.01 \text{ pK}$ providing a 0.1% deviation from a standard (classical gravity) heat capacity of $c_V^{\text{CG}}/k_B \approx 3.164$ [31]. Again this is very much pushing current experiments, with a temperature of order 10 pK, the smallest observed so far in a Bose gas [32]. However, although the required numbers appear slightly more demanding, it may be experimentally preferable to measure the heat capacity, a macroscopic thermodynamic quantity, rather than attempting to single out only the first quasiparticle mode and measure its energy.

We now discuss the electromagnetic self-interaction contributions to the above measurements. Being long range, the gravitational interaction scales very differently to the electromagnetic one: Since the latter is assumed to be a contact potential, g^{EM} is independent of L , whereas g^G scales with L^2 from Eq. (7). This difference makes it possible to distinguish the two cases by measuring the functional dependence of the heat capacity on L . Specifically, a spatially large condensate is preferable (with the number of atoms making no difference). Taking a ^{174}Yb BEC with $L \approx 1 \text{ cm}$ results in $g^{\text{EM}}/g_0^G \approx 10^{11}$. However, another aspect of BECs can be utilized: Feshbach

resonances such that an appropriate magnetic (or optical in the case of strontium) field applied to a BEC can reduce the strength of the electromagnetic interactions. The challenge would be to use a precise enough magnetic field such that fluctuations in its field strength are minimal [4].

IX. CONCLUSIONS

We have studied how the gravitational self-interaction of a uniform BEC affects its heat capacity within the standard approximation of Bogolyubov theory of BECs [12] and the nonrelativistic approximation of tabletop tests of quantum gravity [2,3]. We find that this thermodynamic quantity is unaffected when the gravitational interaction is classical in the spirit of Schrödinger-Newton theory [see Eq. (4)]. In contrast, if the gravitational interaction is quantized, a gravitational term that is momentum dependent enters the excitation energies [see Eqs. (20) and (21)], which also modifies the heat capacity. The two cases of classical and quantum gravity can thus be distinguished in principle by measuring these modifications. This is our main result.

In practice, observing these modifications would require substantial improvements in the state of the art of current BEC experiments. For example, while further studies are needed to assess the full feasibility of our idea, our estimates suggest that a gain of at least ~ 7 orders of the number of atoms in the condensate is required. Possible approaches to significantly increase state-of-the-art BEC atom numbers include using multifrequency traps to trap more atoms in a magneto-optical trap (MOT) [33–35] and then optical barriers to cool the atoms to a BEC, avoiding evaporative cooling, which typically results in orders of magnitude losses in atom numbers from MOTs [36–38]. Furthermore, operating BEC experiments in space can lead to increased atom numbers [35,39,40]. We also note that proposals based on measurements of quantum-information quantities, such as entanglement, also require substantial gains compared to the state of the art. For example, the Bose *et al.* proposal [2] requires Schrödinger catlike states of matter that are around 8 orders greater in mass than the current record for such states [41], and gravitationally induced entanglement with cold atom interferometers requires similar levels of atom numbers when the interferometers are not quantum squeezed [35]. In contrast to such proposals, the proposal considered here does not necessitate coherent control or the application of quantum operations and measurements.

Since high numbers of atoms are needed to detect the quantum gravity signal, it is possible that denser macroscopic

quantum systems, such as liquid helium, could be more suited to our proposed test. This is further supported by the straightforward application of our derived condition $NL \approx 10^{14}$ for quantum gravity to have a significant effect in a BEC—When applied to liquid helium, this condition is easily satisfied [42], highlighting its potential as a promising platform for such investigations. Extending our analysis to a superfluid may be nontrivial since the microscopic description of a superfluid is still being debated [43] and there are no known Feshbach resonances. We note that neutron stars have also been proposed as systems that may undergo Bose-Einstein condensation [44], raising the intriguing possibility that gravitational wave or electromagnetic signals emitted by such astronomical objects could provide insights into properties like heat capacity and, in turn, potentially offer evidence of quantum gravity.

The approach explored here focuses on providing evidence for quantum gravity through measurements of macroscopic, thermodynamical observables. Previous works concern completely different methods for finding evidence for the nonclassicality of the gravitational interaction, looking for signatures such as entanglement [2,3] or non-Gaussianity [4]. Interestingly, there is a conceptual link to these methods that rely on quantum-information principles for demonstrating the quantum nature of gravity: In the Bogolyubov approximation, the quantum gravity Hamiltonian (8) contains off diagonal terms $\hat{a}_k \hat{a}_{-k} + \text{H.c.}$, which will generate entanglement in the atomic modes, whereas no such terms exist in the classical gravity Hamiltonian (10). This suggests a connection to previous considerations of how heat capacity can be used to evidence entanglement [8]. Investigating further these connections may reveal whether measuring the change in the heat capacity could rule out forms of classical gravity beyond semiclassical gravity, such as those based on stochasticity [45].

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DATA AVAILABILITY

No data were created or analyzed in this study.

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