

Deterministic generation of large Fock states in coupled optical-optomechanical cavities

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The preparation of Fock states, as a fundamental problem in the research of quantum optics and quantum information, still faces challenges in achieving both unconditional preparation and large photon numbers. Here, we study the deterministic generation of large Fock states through the nonlinear interaction in coupled optical-optomechanical cavities. Under the condition that the mechanical frequency is twice the optical coupling strength, and much larger than the optomechanical coupling strength, there occurs a nonlinear interaction among two optical eigenmodes and the mechanical mode, i.e., a photon is annihilated, accompanied by the creation of another photon together with a phonon, and vice versa. It is analytically proved that, through this interaction, Fock states $|n\rangle$ and $|1\rangle$ will generate $|n+1\rangle$ at specific evolution times if the mechanical mode is initially on the ground state. In addition, numerical results show that Fock states $|n\rangle$ and $|p\rangle$ can also generate $|n+p\rangle$ with the fidelities larger than 94% ($1 < p \leq n \leq 40$). The proposed mechanism achieves deterministic generation of large Fock states, which can be applied to quantum metrology and quantum simulation.

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I. INTRODUCTION

Fock states are fundamental quantum states containing a definite number of indistinguishable photons. As the basis of quantum optics, Fock states have been widely applied to quantum metrology [1], quantum simulation [2], quantum algorithm [3], and the preparation of other quantum states [4]. So far, many schemes have been presented to generate Fock states. Nonlinear optics is one of the most rapidly developing platforms [5]. For example, using coherent photon conversion through four-wave-mixing interaction, a several-photon Fock state can be generated conditionally from a coherent state [6,7]. Besides, by collecting signal photons from parametric down conversion into a storage cavity, a desired Fock state can be obtained after several heralded measurements [8–12]. Unfortunately, their success rates both decrease rapidly with the increase of photon number. Recently, a theoretical scheme based on Kerr effect is able to generate Fock states without measurement [13]. However, it is difficult to realize the adiabatic variation of system parameters required in this scheme.

Therefore, it is still challenging to deterministically generate a large Fock state by nonlinear optics.

Traditional cavity quantum electrodynamics (CQED), which studies the interaction of optical cavities and two-level systems, is a possible means to overcome the difficulty of deterministic generation [14]. Through the strong coupling between Rydberg atoms and a micromaser cavity [15–18], or between a superconducting qubit and a microwave resonator [19,20], Fock states are deterministically generated. For one or two photons, the experimental fidelity can easily reach over 95% [17]. However, for larger photon number the experimental fidelity becomes much lower (e.g. $F < 50\%$ for $n > 5$ [19]). Even in theory the fidelity of a large Fock state is not very high (e.g. $F = 81\%$ for $n = 20$ [21]). Moreover, the above schemes are only applicable to the microwave band. For optical band, only single photon states can be prepared by traditional CQED, i.e., the single photon source [22,23]. Recently, a study of non-Hermitian coupled waveguides presents that Fock states can be generated in Parity-Time-broken regime [24], while the fidelity is very low. In addition, waveguide quantum electrodynamics may achieve efficient Fock states generation on the optical band, but it still needs heralded measurement for a large photon number [25]. Therefore, we hope to propose a mechanism that may achieve the deterministic generation of large optical Fock states.

As a special type of CQED systems, cavity optomechanical systems [26], which study the interaction between mechanical motion and cavity optical modes, exactly meet our needs. Cavity optomechanical systems are widely applied to mechanical oscillators cooling [27], slow light [28], quantum

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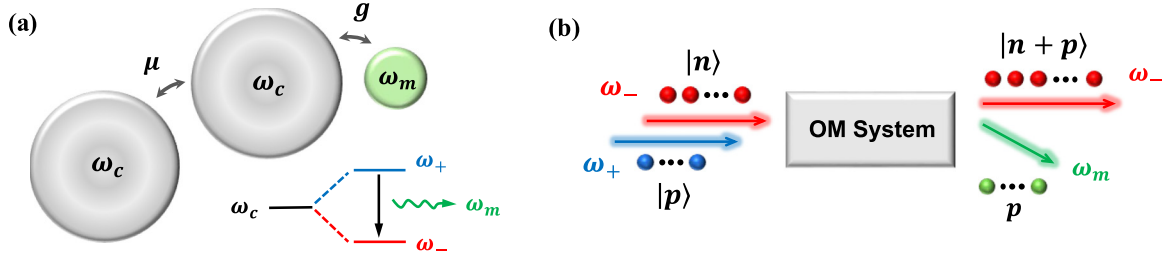


FIG. 1. Deterministic preparation of large Fock states based on an optomechanical system. (a) Schematic of an optomechanical system consisting of two optical microcavities and a mechanical mode. Inset: the nonlinear interaction, i.e., creating a photon with ω_- and a phonon with ω_m while annihilating a photon with ω_+ . (b) Fock states $|n\rangle$ and $|p\rangle$ are transformed into Fock state $|n+p\rangle$ by an optomechanical system (OM System).

information processing [29], and so on. However, almost no one tries to prepare optical large Fock states in cavity optomechanical systems except one work using the temporal modulated driving light, which can also hardly generate high Fock states [30]. Actually, a system composed of coupled optical-optomechanical cavities can support a nonlinear interaction [31], which is mainly required to generate Fock states. That is to say, it is possible to combine the advantages of nonlinear optics and traditional CQED systems.

Here, we study the deterministic generation of large Fock state in the coupled optical-optomechanical cavities. Under the condition that the frequency of the mechanical resonator is twice the coupling strength between two optical cavities, there is a nonlinear interaction among two optical eigenmodes and the mechanical mode; i.e., a photon is annihilated, accompanied by the creation of another photon together with a phonon, and vice versa. If the mechanical mode is initially on the ground state, then no phonon can be annihilated. According to the above nonlinear interaction, all photons will concentrate in one optical eigenmode, resulting in a large Fock state. It is analytically proved that, based on this mechanism, Fock states $|n\rangle$ and $|1\rangle$ can generate pure $|n+1\rangle$ at specific moments. Numerical results show that Fock states $|n\rangle$ and $|p\rangle$ can also generate $|n+p\rangle$ with the fidelities larger than 94% ($1 < p \leq n \leq 40$). In addition, we discuss the effects of cavity losses, mechanical loss, and the experimental feasibility. These results are useful in quantum metrology, quantum simulation, and the preparation of other quantum states.

II. MODEL SETUP

The proposed mechanism of high Fock states generation is based on a cavity optomechanical system, consisting of an optical cavity and an optomechanical cavity, as shown in Fig. 1(a). The Hamiltonian of this system is [26]

$$H_0 = \hbar\omega_c(a_1^\dagger a_1 + a_2^\dagger a_2) + \hbar\mu(a_1^\dagger a_2 + a_2^\dagger a_1) + \hbar\omega_m b^\dagger b - \hbar g a_2^\dagger (b + b^\dagger), \quad (1)$$

where ω_c is the optical resonance frequency of two isolated cavities and μ is the coupling strength caused by the evanescent field between two cavities. ω_m is the resonance frequency of mechanical mode, which is attached to the second optical cavity to form an optomechanical cavity, and g is the optomechanical coupling strength. $a_1(a_1^\dagger)$ and $a_2(a_2^\dagger)$ are the annihilation (creation) operators of two optical modes, and

$b(b^\dagger)$ is the annihilation (creation) operator of the mechanical mode. For now, we do not consider the effect of losses, so the evolution of the system follows the Schrödinger equation. The quantum state at any time can be formally written as $|\psi(t)\rangle = \exp(-iH_0 t/\hbar)|\psi(0)\rangle$, from which we can calculate the dynamics of all physical quantities.

However, using Hamiltonian (1) to describe the nonlinear interaction occurring in the system is not intuitive. To this end, we introduce the eigenmode annihilation operators $a_\pm = (a_1 \pm a_2)/\sqrt{2}$ of the two coupled optical cavities to rewrite the Hamiltonian. These annihilation operators and the corresponding creation operators still satisfy the bosonic commutation relation $[a_+, a_+^\dagger] = 1$, $[a_-, a_-^\dagger] = 1$, and $[a_+, a_-^\dagger] = [a_-, a_+^\dagger] = 0$. Then, the Hamiltonian (1) becomes

$$H_0 = \hbar(\omega_c + \mu)a_+^\dagger a_+ + \hbar(\omega_c - \mu)a_-^\dagger a_- + \hbar\omega_m b^\dagger b - \frac{1}{2}\hbar g(a_+^\dagger a_+ + a_-^\dagger a_- - a_+^\dagger a_- - a_-^\dagger a_+)(b + b^\dagger). \quad (2)$$

In this form, the optical coupling term $\hbar\mu(a_1^\dagger a_2 + a_2^\dagger a_1)$ has been absorbed into different eigenmode terms, implying that the eigenfrequencies of two optical cavities are $\omega_\pm = \omega_c \pm \mu$, rather than the original frequency ω_c . Specifically, if $g \ll \omega_m$ and $\omega_m = 2\mu$, the rotating wave approximation will be available, i.e., omitting the terms that do not satisfy energy conservation, then we obtain

$$H_1 = \hbar(\omega_c + \mu)a_+^\dagger a_+ + \hbar(\omega_c - \mu)a_-^\dagger a_- + \hbar\omega_m b^\dagger b + \frac{1}{2}\hbar g(a_+^\dagger a_- b + a_+ a_-^\dagger b^\dagger). \quad (3)$$

So this cavity optomechanical system is composed of two optical eigenmodes and a mechanical mode. In the following, we will use the notation $|\psi\rangle = |p\rangle_+ \otimes |n\rangle_- \otimes |q\rangle_m = |p, n, q\rangle$ to represent that there are p (n) photons in eigenmode ω_+ (ω_-) and q phonons in the mechanical mode. Importantly, there is a nonlinear interaction among these three modes: A photon with frequency ω_+ is annihilated, accompanied by the creation of another photon with frequency ω_- , and a phonon together with frequency ω_m (forward process), and vice versa (backward process). In general, forward process and backward process usually occur simultaneously, which has been studied in the literature [31–33]. However, if the mechanical mode is initially on the ground state, the backward process will be suppressed because it must be accompanied by the annihilation of a phonon. Therefore, the forward process dominates the optomechanical system at the beginning, providing

an opportunity for us to achieve the deterministic generation of a large Fock state.

III. DETERMINISTIC GENERATION OF LARGE FOCK STATES

The mechanism of deterministic generation of large Fock states in coupled optical-optomechanical cavities is as follows. First, the mechanical mode should be prepared on the ground state, and then Fock states $|n\rangle_-$ with frequency ω_- and $|p\rangle_+$ with frequency ω_+ are input into the optomechanical system. According to Eq. (3), the nonlinear interaction has two parts. Photons with frequency ω_+ can be transferred to frequency ω_- , accompanied by the creation of phonons, i.e., $a_+a_-^\dagger b^\dagger$, called the forward process. And its inverse process $a_+^\dagger a_- b$ is called the backward process. However, the backward process cannot occur at the beginning since there are no initial phonons. So if $p = 1$, after a certain evolution time, this photon with frequency ω_+ will be deterministically transferred to frequency ω_- , which becomes indistinguishable from the original n photons. However, if $p > 1$, things are different. First, some photons with frequency ω_+ are transferred to photons with frequency ω_- , together generating some phonons. Then, these phonons trigger the backward process. As a result, the fidelity is no longer 100%. Fortunately, the speed of the backward process is much lower than the forward process, so that most of the photons still concentrate in the eigenmode with ω_- . Then we can choose an appropriate time to obtain the optimal result. Therefore, two smaller Fock states $|n\rangle_-$ and $|p\rangle_+$ deterministically generate a larger Fock state $|n+p\rangle_-$ in the coupled optical-optomechanical cavities [Fig. 1(b)].

A. Generation of $|n+1\rangle$ from $|1\rangle$ and $|n\rangle$

Actually, such a trilinear Hamiltonian as Eq. (3) has been studied [34], without a brief analytical solution. However, we find that in some specific cases such as adding one photon to the Fock state $|n\rangle_-$, it can be analytically solved. The result is that Fock states $|n\rangle_-$ and $|1\rangle_+$ can deterministically generate Fock state $|n+1\rangle_-$ with fidelity $F = 100\%$. Suppose the initial state is

$$|\psi(t=0)\rangle = |1\rangle_+ \otimes |n\rangle_- \otimes |0\rangle_m. \quad (4)$$

Under the rotating wave approximation, the quantum state at any time is

$$|\psi(t)\rangle \approx e^{-iH_1 t/\hbar} |\psi(t=0)\rangle. \quad (5)$$

It can be seen that when H_1 acts on two bases $|1, n, 0\rangle$ and $|0, n+1, 1\rangle$, the result is still the superposition of these two bases. That is to say, when $e^{-iH_1 t/\hbar}$ acts on these two states, the result must also be their superposition, so Eq. (5) can be written as

$$|\psi(t)\rangle = \alpha_n(t)|1, n, 0\rangle + \beta_n(t)|0, n+1, 1\rangle. \quad (6)$$

Substituting it into the Schrödinger equation, we obtain

$$i \frac{d}{dt} \begin{pmatrix} \alpha_n(t) \\ \beta_n(t) \end{pmatrix} = \begin{pmatrix} n\omega_- + \omega_+ & \frac{1}{2}g\sqrt{n+1} \\ \frac{1}{2}g\sqrt{n+1} & n\omega_- + \omega_+ \end{pmatrix} \begin{pmatrix} \alpha_n(t) \\ \beta_n(t) \end{pmatrix}. \quad (7)$$

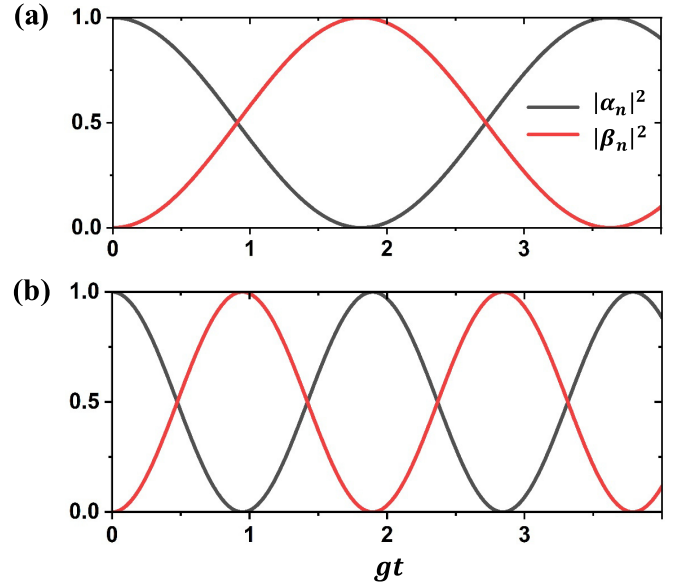


FIG. 2. Analytical solutions of adding one photon to generate Fock states $|n+1\rangle$. The evolutions of $|\alpha_n|^2$ and $|\beta_n|^2$ for (a) $n = 2$ or (b) $n = 10$ are plotted from Eq. (9).

The eigenvalues and eigenstates of the above matrix are

$$\lambda_{1,2} = n\omega_- + \omega_+ \pm \frac{1}{2}g\sqrt{n+1}, \quad u_{1,2} = \begin{pmatrix} 1 \\ \pm 1 \end{pmatrix}. \quad (8)$$

Then using the initial condition $\alpha_n(0) = 1$, $\beta_n(0) = 0$, the final solution can be obtained,

$$\begin{aligned} \alpha_n(t) &= e^{-i(n\omega_- + \omega_+)t} \cos \frac{\sqrt{n+1}}{2} gt, \\ \beta_n(t) &= -ie^{-i(n\omega_- + \omega_+)t} \sin \frac{\sqrt{n+1}}{2} gt. \end{aligned} \quad (9)$$

The modular squares of these superposition coefficients are plotted in Fig. 2. If the time is $t = (2k\pi + \pi)/(g\sqrt{n+1})$, $k = 0, 1, \dots$, i.e., the condition $|\alpha_n(t)|^2 = 0$, $|\beta_n(t)|^2 = 1$ is satisfied, then the quantum state will be $|\psi(t)\rangle = |0, n+1, 1\rangle$. In summary, by directly solving the Schrödinger equation, we prove that a Fock state $|n+1\rangle_-$ is deterministically generated from two Fock states $|n\rangle_-$ and $|1\rangle_+$. The required evolution time gt becomes shorter for an increasing value of n .

B. Generation of $|n+p\rangle$ from $|p\rangle$ and $|n\rangle$

In practical applications, in order to generate high Fock states, adding photons one by one is inefficient. So it is important to implement adding multiple photons, in which case we perform numerical calculations using Quantum Toolbox in Python (QuTiP). For adding multiple photons, the initial state is

$$|\psi(t=0)\rangle = |p\rangle_+ \otimes |n\rangle_- \otimes |0\rangle_m, \quad (10)$$

with $p > 1$. The target state is

$$|\psi_{\text{target}}\rangle = |0\rangle_+ \otimes |n+p\rangle_- \otimes |p\rangle_m. \quad (11)$$

Instead of a perfect target state, a highly similar state is generated at some specific moments through the nonlinear

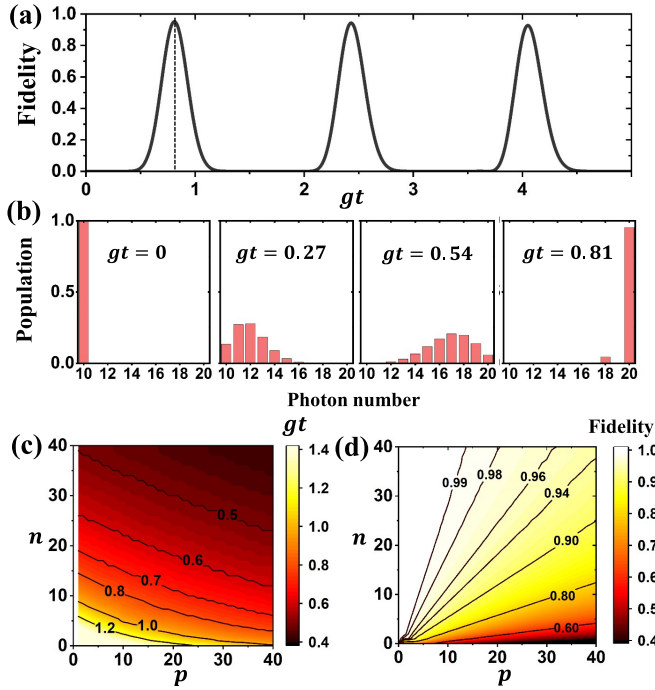


FIG. 3. Numerical results of adding multiple photons to generate Fock states $|n+p\rangle$. (a) The fidelity between reduced state on eigenmode ω_- and target state $|n+p\rangle$ during the evolution. (b) Photon number distribution on eigenmode ω_- at four specific moments $gt = 0, 0.27, 0.54$, and 0.81 . (c) The time required to firstly reach the maximum fidelity when n and p vary from 0 to 40. (d) The maximum fidelity during the evolution when n and p vary from 0 to 40. Here, the parameters are $\mu = 0.5\omega_m$, $g = 0.01\omega_m$. In both panels (a) and (b), $n = p = 10$.

interaction in optical-optomechanical cavities. As an example for $n = p = 10$ case, the fidelity can reach a maximum of 94.5% at $gt = 0.81$ [Fig. 3(a)]. The evolution is periodic, so we only need to study the period from the beginning to the maximum. During this period, the reduced photon number distribution on the lower-frequency mode is shown in Fig. 3(b). The photon number on the lower-frequency mode gradually increases, which comes from the higher-frequency mode. At $gt = 0.81$, nearly all the photons are concentrated in the lower-frequency mode. The population of $|20\rangle_-$ is 94.5%, while the remaining population is mostly distributed on $|18\rangle_-$, because of the backward process.

Then we focus on the situations where n and p take other values. For different n and p , the evolution period is different, so the time required to obtain an optimized quantum state is also different. If $n = p = 1$, then the required time will be $gt = \pi/\sqrt{2} \approx 2.2$, which is the longest. As shown in Fig. 3(c), the larger n and p , the shorter the required time. This relation is beneficial to generate a large Fock state, as a shorter evolution time implies less losses in practical operation. Moreover, for different n and p , the maximum of fidelity is also different. According to Sec. III A, if $p = 1$, then the fidelity can always reach 100%, which means a pure Fock state $|n+1\rangle_-$ is deterministically generated. However, as shown in Fig. 3(d), when p is larger, the maximum of fidelity will be lower. Specifically, in the area $n \geq p$, the

fidelity is always larger than 94%. For example, if we want to use Fock states $|10\rangle$ and $|20\rangle$ to generate $|30\rangle$, then the better choice is to let $n = 20$ and $p = 10$, so that the fidelity will be 98%. Otherwise, for $n = 10$ and $p = 20$, the fidelity will be 87%. Therefore, the fidelity of generating a large Fock state $|n+p\rangle_-$ from $|n\rangle_-$ and $|p\rangle_+$ ($n, p \leq 40$) is no less than 94%. Furthermore, from the tendency of Fig. 3(d), we can predict that fidelity will be also no less than 94% for any large n and p , which is better than those schemes based on traditional CQED [21] or nonlinear optics [6] in theory.

IV. DISCUSSIONS

A. Optical losses

In practical optomechanical systems, the optical losses always exist. Here, we consider linear optical losses and calculate the effect on the fidelity of the generated state. In our theoretical framework, optical losses are indicated by Lindblad terms in the master equation,

$$\begin{aligned} \frac{d\rho}{dt} = & -\frac{i}{\hbar}[H_0, \rho] + \gamma(2a_1\rho a_1^\dagger - \rho a_1^\dagger a_1 - a_1^\dagger a_1 \rho) \\ & + \gamma(2a_2\rho a_2^\dagger - \rho a_2^\dagger a_2 - a_2^\dagger a_2 \rho), \end{aligned} \quad (12)$$

where we assume that the loss rates γ of the two optical modes are equal. After the transformation of eigenmodes and the rotating wave approximation, it can be obtained that

$$\begin{aligned} \frac{d\rho}{dt} = & -\frac{i}{\hbar}[H_1, \rho] + \gamma(2a_+\rho a_+^\dagger - \rho a_+^\dagger a_+ - a_+^\dagger a_+ \rho) \\ & + \gamma(2a_-\rho a_-^\dagger - \rho a_-^\dagger a_- - a_-^\dagger a_- \rho). \end{aligned} \quad (13)$$

The following results are numerically solved from Eq. (13). As the loss rate increases, the fidelity will decrease [Fig. 4(a)]. Moreover, the longer the evolution time, the greater the negative effect caused by losses. So the time to reach the first peak of fidelity is the best, i.e., $gt = 1.68$ when generating $|4\rangle$ from $|2\rangle \otimes |2\rangle$. In order to maintain over 85% fidelity, the loss rate divided by optomechanical coupling strength should be less than 0.01, i.e., $\gamma/g < 0.01$. During the evolution, the photon population in the lower-frequency eigenmode gradually increases and eventually saturates at four photons. For different loss rates $\gamma/g = 0, 0.01$, and 0.02 , the final possibilities on four photons are 0.98, 0.86, and 0.75, respectively. Therefore, to better prepare a high Fock state, optomechanical system should be in strong coupling regime; i.e., optical losses should roughly satisfy $\gamma/g < 0.01$.

Then, we consider the effect of optical losses for different initial states. As shown in Fig. 5, when the photon number of initial Fock states is higher, the fidelities decrease more sharply as the loss rate increases. This is because for the same loss rate, more total photons imply a higher possibility of losing photons. For example, if we require the fidelities higher than 90%, then loss rate γ/g should be lower than 0.012, 0.006, and 0.003 for $|1\rangle \otimes |1\rangle$, $|2\rangle \otimes |2\rangle$, and $|5\rangle \otimes |5\rangle$, respectively.

B. Mechanical loss

In the above, we discussed the effect of optical losses. Here, we ignore optical losses and consider the effect of

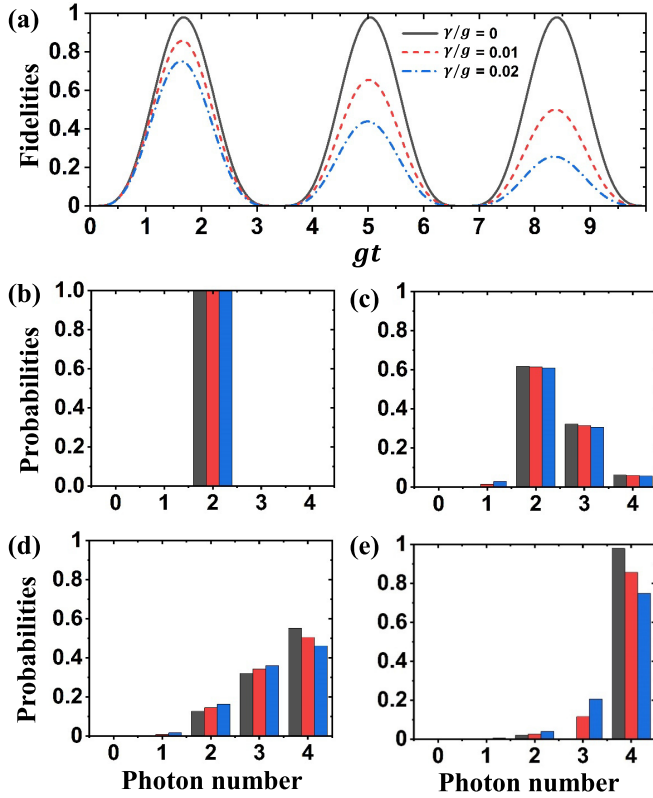


FIG. 4. Effect of optical losses on the generation of large Fock states. (a) Fidelity of generated state during the evolution for different loss rates $\gamma/g = 0, 0.01$, and 0.02 ; Photon number distribution of the eigenmode with ω_- for different loss rates $\gamma/g = 0, 0.01$, and 0.02 (black, red, blue bars, respectively) at several moments: (b) $gt = 0$, (c) $gt = 0.56$, (d) $gt = 1.12$, and (e) $gt = 1.68$. Here, the initial state is $|2\rangle \otimes |2\rangle$, the target state is $|4\rangle$, and the ratio $g/\omega_m = 0.01$.

mechanical loss. Different from optical losses, mechanical loss is not only related to the mechanical loss rate γ_m , but also dependent on the environmental temperature T . In the master equation, the mechanical loss can also be indicated by Lindblad terms,

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}[H_1, \rho] + \gamma_m(n_{th} + 1)(2b\rho b^\dagger - \rho b^\dagger b - b^\dagger b\rho) + \gamma_m n_{th}(2b^\dagger \rho b - \rho b b^\dagger - b b^\dagger \rho), \quad (14)$$

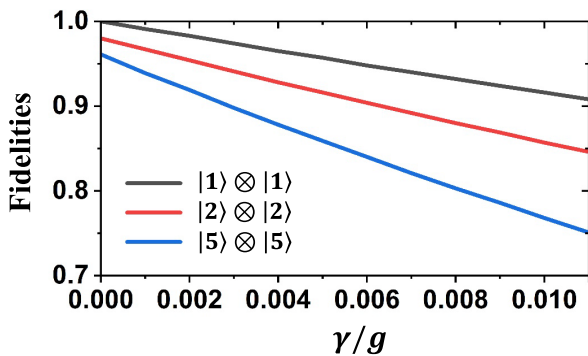


FIG. 5. Fidelities of the generated states as changing optical losses for different photon numbers. The initial state is $|1\rangle \otimes |1\rangle$ (black curve), $|2\rangle \otimes |2\rangle$ (red curve), or $|5\rangle \otimes |5\rangle$ (blue curve).

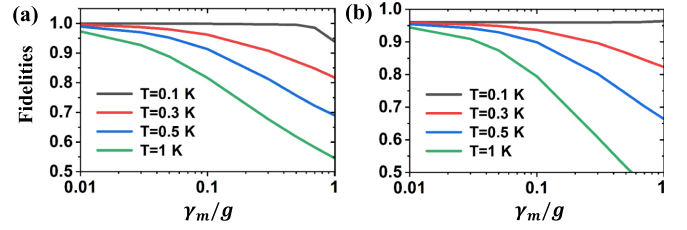


FIG. 6. Effect of mechanical loss on the generation of large Fock states. Here, the initial state is (a) $|1\rangle \otimes |1\rangle$ and (b) $|5\rangle \otimes |5\rangle$. The mechanical frequency $\omega_m = 10$ GHz and the ratio $g/\omega_m = 0.01$.

where the average phonon number in the environment is

$$n_{th} = \left[\exp\left(\frac{\hbar\omega_m}{k_B T}\right) - 1 \right]^{-1}, \quad (15)$$

where k_B is the Boltzmann constant. Assuming the mechanical frequency $\omega_m = 10$ GHz [35], the fidelities of generated states with mechanical loss can be solved from Eq. (14) and plotted in Fig. 6. As an example, if the initial state is $|5\rangle \otimes |5\rangle$ and the loss rate is $\gamma_m/g = 0.1$, reaching a temperature of 0.5 K can reduce fidelity within 10%, reaching 0.3 K can reduce fidelity within 2%, and reaching 0.1 K can result in a fidelity change of less than 0.1%, which can be ignored [Fig. 6(b)]. As a result, the requirements for a 10 GHz mechanical oscillator are $T < 0.1$ K and $\gamma_m/g < 0.1$.

C. Experimental feasibility

Optomechanical crystals are the preferred platform to realize our proposed scheme because of their high mechanical frequencies and strong optomechanical coupling strengths. Recently, an optomechanical crystal composed of pairs of coupled optical and mechanical modes has been fabricated to enhance the nonlinear measurement of mechanical motion, where $\omega_m = 4.9$ GHz, $\mu = 2.4$ GHz, and $g = 500$ kHz [36]. Another two-dimensional optomechanical crystal cavity is also fabricated to reach a high quantum cooperativity [35]. Whispering gallery microcavities serve as a secondary alternative platform because of their high quality factors ($Q \approx 10^7$) [31,37]. However, the lower mechanical frequencies and optomechanical coupling strengths ($\omega_m = 20$ MHz, $g = 500$ Hz) still need to be improved to meet our expectations [31]. Optomechanical waveguide is a one-dimensional waveguide structure with continuous spectral optical modes and continuous mechanical modes [38]. The advantage of optomechanical waveguide is that the coupling length is controllable, and the generated state can be taken at a certain time, so it may be suitable for our theoretical scheme. Optomechanical waveguides have also achieved sideband cooling technology in experiments [39]. However, our theory has not yet involved position-dependent annihilation operators, so further theoretical analysis is needed to confirm the adaptability of optomechanical waveguides. So the discussions below are all based on optomechanical crystals.

We first consider the experimental feasibility of mechanical oscillators. On the one hand, a high mechanical frequency is necessary. It implies a less average phonon number in the reservoir, thereby bringing lower thermal noise that is

beneficial for quantum state generation. Assuming a temperature of 10 mK [35], the requirement of mechanical frequency then can be estimated from the formula, i.e., $n_{\text{th}} = [\exp(\hbar\omega_m/k_B T) - 1]^{-1} < 1$, obtaining that ω_m should be larger than 1 GHz. In addition, our generation mechanism is based on coupled optical-optomechanical cavities, requiring $\omega_m = 2\mu$ and $\omega_m \gg g$. All these requirements are experimentally feasible in the recent work mentioned above [36]. On the other hand, the platform requires cryogenic operation to suppress thermal noise and maintain quantum coherence. Room temperature (300 K) implies high phonon number ($n_{\text{th}} \approx 600$) in the reservoir, which introduces lots of thermal noise into the optomechanical system, thereby damaging the quantum state. According to the calculation in Sec. IV B, the temperature should be lower than 0.1 K for a mechanical frequency of 10 GHz. Moreover, our scheme requires a mechanical oscillator initially on the ground state, so it is necessary to cool the mechanical oscillator to the ground state before optomechanical coupling occurs. A common method is laser sideband cooling, but it is not recommended in our scheme. Because there is residual thermal noise ($n_{\text{th}} \approx 1$) in the cooled mechanical oscillator [27], which will cause a sharp decrease (40%) of fidelity, even if the initial state is $|1\rangle \otimes |1\rangle$. In fact, under the conditions of a mechanical oscillator frequency of 10 GHz and a temperature of 0.01 K, the phonon number in the environment is extremely low (2×10^{-22}). Then, no additional operation is required because the mechanical oscillator is naturally cooled to the ground state. Therefore, in order to suppress thermal noise and reach the mechanical ground state, a cryogenic environment is necessary.

We then consider the experimental feasibility of optical cavities; the largest difficulty is the requirement between optical loss and optomechanical coupling strength, i.e., $\gamma/g < 0.01$. Minor optical losses have a significant impact on the fidelities of generated states, which is a common phenomenon in the preparation of optical quantum states [40,41], because Fock states themselves are very sensitive to losses. For example, in a single lossy channel with the same loss and evolution time ($\gamma/g < 0.01$, $gt = 0.81$), the fidelity of Fock state $|2\rangle$ decreases to 98.3% (in our scheme, the fidelity decreases to 98.2%). Some research works that mainly focus on preparing mechanical quantum states in cavity optomechanical systems only require $\gamma/g < 1$ [42]. Other research works that focus on single photon sources and single phonon-photon converters require $\gamma/g < 1$ [29]. These different types of research focus on different issues, and therefore have different requirements for optical losses. However, at present, the parameter level in experiments is still far from reaching the condition of $\gamma/g < 0.01$, then the results of quantum state preparation will be greatly affected. Recently, in the state-of-the-art experimental

progress on two-cavity optomechanical coupling structures, a couple-mode optomechanical crystal structure with $\gamma = 600$ MHz, $g = 500$ kHz can only reach $\gamma/g = 1200$ [36], while in a single-mode optomechanical crystal, it can reach $\gamma = 170$ MHz, $g = 1.1$ MHz, i.e., $\gamma/g = 160$ [43]. Therefore, at present, the parameter level in experiments is far from reaching the condition of $\gamma/g < 0.01$. In order to bridge the gap, we may make further efforts in two directions in the future. One is to enhance the strength of optomechanical coupling or obtain a stronger efficient coupling strength [44,45], the other is to improve the optical quality factor of the optomechanical cavity, that is, to further reduce optical losses [35].

V. SUMMARY

In summary, we have proposed a mechanism of deterministically generating high Fock states based on the nonlinear interaction in optical-optomechanical cavities. If the mechanical resonator is in the ground state initially, the forward process will be promoted and the backward process will be suppressed, which is the key to our work. Discussion of optical losses indicates that the system must be in optomechanical strong coupling regime.

Our proposed mechanism has achieved deterministic generation, which can reduce the consumption of various resources in quantum information processing. Large photon number improves the precision of quantum metrology, which may also be an application of our work. The photon transfer between two modes induced by nonlinear interaction in optomechanical systems may achieve many other functions, such as a mode filter for photons, deterministically adding one photon to a squeezed state yielding a cat state. For mechanical mode, it is also a good choice to deterministically add or subtract phonons. Therefore, our work contributes to various fields, including quantum metrology, quantum optical devices, and quantum states preparation.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

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