

Environment-induced transitions in many-body quantum teleportation

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Quantum teleportation is a phenomenon arising from entanglement, decisively distinguishing the classical and quantum worlds. The recent success of many-body quantum teleportation is even more surprising: Although input information is initially dispersed and encoded into the many-body state in a complex way, the teleportation process can refocus this highly nonlocal information at the receiver's end. This success manifests intriguing capability of many-body systems in quantum information processing. Current studies indicate that information scrambling, a generic dynamic process in many-body systems, underlies the effectiveness of many-body quantum teleportation. However, this process is known to undergo a scrambling-dissipation transition in the presence of environments. How environments affect the quantum information processing capability of many-body systems calls for further investigation. In this work, we study many-body quantum teleportation in the presence of environments. We predict two emergent critical points that hallmark the transitions of the teleportation performance from the quantum regime to the classical regime and finally to the no-signal regime as the system-environment coupling, quantified by γ , increases. In the quantum regime, teleportation can outperform its classical counterparts, while in the classical regime, it can be replaced by a classical channel. Our prediction is based on a generic argument harnessing the relationship between many-body quantum teleportation and information scrambling, corroborated by solvable Brownian Sachdev-Ye-Kitaev models.

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Introduction. Quantum teleportation harnesses entanglement, a unique nonlocal resource heralded by Schrödinger as the characteristic trait of quantum mechanics [1]. With preshared entanglement, distant parties can transfer quantum states without sending any physical objects in these states [2]. The recent surprising success of many-body quantum teleportation protocols has elevated the phenomenon to a new level [3–10]. These protocols first *scrambled* input information across the many-body system at the sender's hand under a sufficiently long unitary evolution. After teleportation, the information, which has spread nonlocally, can miraculously be *unscrambled* and refocused, available for extraction in the many-body system at the receiver's hand.

Many-body quantum teleportation manifests intriguing capability of interacting many-body systems in quantum information processing. The renowned traversable wormhole (TW) protocol, realized with the Sachdev-Ye-Kitaev (SYK)

models [11–13] in the low-temperature regime, enjoys a fascinating dual interpretation in terms of gravitational physics [14]: It involves the opening up of a traversable wormhole between the two parties, enabling information transmission [3–5]. Beyond this correspondence, the effectiveness of many-body teleportation has been further demonstrated in generic chaotic many-body systems [7–9]. Current studies point to information scrambling, a dynamic process universal in interacting many-body systems [15–18], as an underlying mechanism for the effectiveness of many-body quantum teleportation [6–9]. This process is also responsible for quantum thermalization in closed systems [19,20], and it centers on quantum many-body dynamics studied in condensed matter physics, quantum information, and quantum gravity [21,22].

On the other hand, Refs. [23,24] predict that if environments are present, information scrambling in many-body systems can undergo a dynamic transition from the scrambling phase to the dissipative phase when the system-environment coupling, quantified by γ , exceeds a critical value γ_s . In the latter phase, the scrambling is halted completely [25]. The close relation between information scrambling and many-body quantum teleportation urges us to ask how environments would affect many-body systems' capability of accomplishing the teleportation protocols.

In this work, we investigate the performance of the TW many-body quantum teleportation protocol in the presence of

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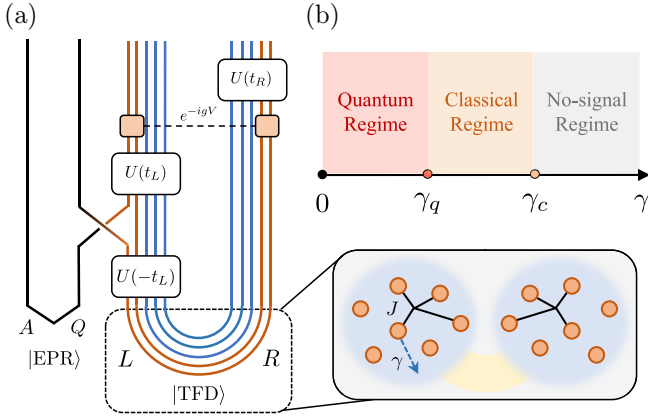


FIG. 1. (a) Schematics of the TW many-body quantum teleportation in the presence of environments. The close-up illustrates that within each party, the system-environment coupling is quantified by γ , while the intrasystem coupling is quantified by J . The two parties are initially prepared in the $|\text{TFD}\rangle$ state. (b) Environment-induced teleportation transitions at γ_q and γ_c hallmark the teleportation performance from the quantum regime to the classical regime and to the no-signal regime. In the quantum regime, the teleportation can outperform its classical counterparts, while in the classical regime, it is replaceable by a classical channel.

environments. We model the many-body systems and their environments using solvable Brownian Sachdev-Ye-Kitaev models [26,27]. We show that as the system-environment coupling γ increases, two consecutive teleportation transitions emerge at γ_q and γ_c , which nail down the degeneration of the teleportation effectiveness from the quantum regime to the classical regime and finally to the no-signal regime. The schematics are given in Fig. 1. In the quantum regime, the protocol manages to maintain its quantum nature and outperform its classical counterparts, while in the classical regime, the environmental influence is so strong that the protocol can be replaced by a classical channel. Beyond γ_c , the transferable information becomes negligible. Building upon the relation between many-body quantum teleportation and information scrambling, we are able to not only identify γ_c as the same as the scrambling transition point γ_s , but also provide a general argument that the two discovered teleportation transitions shall be generic to situations with all-to-all interactions, assuming that the teleportation signal varies continuously as the system-environment coupling is tuned. Finally, we find that scramblons [13,28,29], the collective modes peculiar to quantum many-body chaos, yield an anomalously large correction to γ_q in finite-size systems. Our results unveil the intriguing environmental effects on the quantum information processing capability of many-body systems.

Teleportation with environments. We generalize the widely studied TW protocol [3–10] to take into account environments. The teleportation regards information transferring between two parties, referred to as the left (L) and right (R), each of which, denoted by $\alpha \in \{L, R\}$, contains an equal number of qubits partitioned into a system S represented by N Majorana fermions $\chi_{j,\alpha}$ with $j \in \{1, 2, \dots, N\}$ and its environment E by M fermions $\psi_{a,\alpha}$ with $a \in \{1, 2, \dots, M\}$. Each pair of the fermions represents a qubit; e.g., the first system qubit

in the left (right) party L_{S_1} (R_{S_1}) contains fermions $\chi_{1,L}$, $\chi_{2,L}$ ($\chi_{1,R}$, $\chi_{2,R}$). The environments have much more degrees of freedom than the systems, i.e., $M \gg N$. We take the normalization $\chi_{j,\alpha}^2 = \psi_{a,\alpha}^2 = 1$. The Hamiltonians of the two parties, H_L and H_R , are related by a time reversal operation, i.e., $H_L = H \otimes I$ and $H_R = I \otimes H^*$, and $H = H_S[\chi] + H_E[\psi] + H_{SE}[\chi, \psi]$.

The protocol starts with the two parties being jointly prepared in the thermofield double state [30,31]

$$|\text{TFD}\rangle = \frac{1}{\sqrt{Z}} \sum_n e^{-\beta E_n/2} |E_n\rangle_L \otimes |E_n\rangle_R^*, \quad (1)$$

where $|E_n\rangle$ represents an eigenstate of the Hamiltonian H with eigenenergy E_n and $Z = \sum_n e^{-\beta E_n}$ is the thermal partition function. In the infinite temperature limit ($\beta \rightarrow 0$), $|\text{TFD}\rangle$ becomes the familiar maximally entangled Einstein-Podolsky-Rosen state $|\text{EPR}\rangle$, which can be constructed to have a very useful property $(\chi_{j,L} + i\chi_{j,R})|\text{EPR}\rangle = (\psi_{a,L} + i\psi_{a,R})|\text{EPR}\rangle = 0$ [32]. For arbitrary β , $|\text{TFD}\rangle = \rho_L^{1/2}|\text{EPR}\rangle = \rho_R^{1/2}|\text{EPR}\rangle$ with $\rho_\alpha = e^{-\beta H_\alpha}/Z$. The extensive entanglement of $|\text{TFD}\rangle$ inherited from $|\text{EPR}\rangle$ provides a resource for quantum teleportation [1]. A similar setup consisting of systems and environments has been used to study the information paradox in eternal black holes [33].

Our aim is to teleport an arbitrary single-qubit state denoted as $|\psi\rangle_Q$ from the left party to the right. We maintain a record of $|\psi\rangle_Q$ by introducing an auxiliary qubit A , which is prepared in the EPR state with Q ; the qubit A will be used as the reference to test the success of teleportation at the end. As illustrated in Fig. 1(a), the protocol consists of three steps in the timeline operating on the initial state $|\text{TFD}\rangle$ [3–9]:

(1) Evolve backward to time $t = -t_L$, and input the unknown state $|\psi\rangle_Q$ via a swap operation between Q and L_{S_1} .

(2) Evolve to time $t = 0$, and instantaneously couple the two systems in the two parties, L_S and R_S , via $\mathcal{U} = e^{-igV}$ with $V = -i \sum_j \chi_{j,L} \chi_{j,R}/2$.

(3) At the final time $t = t_R$, examine whether R_{S_1} has acquired information of $|\psi\rangle_Q$ via the reduced density matrix $\rho_{AR_{S_1}}$.

Since we want teleportation embodying quantum many-body systems' intrinsic information processing capability, we take $|g| \ll 1$. Otherwise, significant entanglement may be trivially established in $\rho_{A,R_{S_1}}$ due to the external unitary coupling $\mathcal{U}$. Physically, the role of $\mathcal{U}$ is equivalent to local operations and classical communication [2,4,34]. It is worth emphasizing that in implementing teleportation, except the initial preparation of the $|\text{TFD}\rangle$ state, we have access only to the system fermions $\chi_{j,\alpha}$; i.e., $\mathcal{U}$ does not involve $\psi_{a,\alpha}$.

The effectiveness of our protocol in the presence of environments can be evaluated by the response function in the Kubo formula [4,5,35]

$$\mathcal{K}(t_L, t_R) = \frac{1}{2} \langle \text{TFD} | \{ \chi_{1,L}(-t_L), \mathcal{U}^\dagger \chi_{1,R}(t_R) \mathcal{U} \} | \text{TFD} \rangle, \quad (2)$$

which quantifies how the perturbation of $\chi_{1,L}$ at $t = -t_L$ affects the expectation of $\chi_{1,R}$ at $t = t_R$. Here, the time evolution is under $H_{\text{tot}} \equiv H_L + H_R$. For Hamiltonians H with all-to-all interactions [36], such as the SYK models [11–14], $\mathcal{K}(t_L, t_R)$ completely determines the reduced density matrix $\rho_{AR_{S_1}}$ in

the thermodynamic limit $N \rightarrow \infty$ [5]; its nonvanishing matrix elements read

$$\begin{aligned}\rho_{AR_{S_1}}^{11} &= \rho_{AR_{S_1}}^{44} = \frac{1}{4}(1 + \mathcal{K}^2), & \rho_{AR_{S_1}}^{14} &= \rho_{AR_{S_1}}^{41} = \frac{1}{2}\mathcal{K}, \\ \rho_{AR_{S_1}}^{22} &= \rho_{AR_{S_1}}^{33} = \frac{1}{4}(1 - \mathcal{K}^2).\end{aligned}\quad (3)$$

In general, $|\mathcal{K}| \leq 1$, bounded by the norm of the Majorana operators. The protocol maintains its quantum nature only if entanglement establishes in $\rho_{AR_{S_1}}$; i.e., the bipartite $\rho_{AR_{S_1}}$ becomes nonseparable. This nonseparability can be probed by the nonvanishing entanglement negativity $\mathcal{N}_{AR_{S_1}} \equiv (||\rho_{AR_{S_1}}^{T_A}||_1 - 1)/2$ [37–41], which requires $|\mathcal{K}| > \mathcal{K}_q = \sqrt{2} - 1$. Here, T_A represents the partial transpose in A and $||\cdot||_1$ denotes the trace norm. If $\mathcal{N}_{AR_{S_1}} = 0$, but $\mathcal{K}$ is nonzero, and so is the mutual information $I_{AR_{S_1}} \equiv S_A + S_{R_{S_1}} - S_{AR_{S_1}}$ with S_C denoting the von Neumann entropy of a subsystem C , there is still information transmission, though replaceable by a classical channel. Alternative criteria regarding fidelity would lead to the same conditions on $\mathcal{K}$ (see the Supplemental Material [34]). Thus, our task is to find out how environments affect $\mathcal{K}$.

General analysis. Since information scrambling underlies many-body quantum teleportation, we are able to provide a generic argument for our prediction of the teleportation transition as depicted in Fig. 1(b), based on characteristics of information scrambling in the presence of environments [23,24].

In our protocol, unitary time evolution under H_{tot} results in the operator expansion

$$\sqrt{\rho_L} \chi_{1,L}(t) = \sum_S c_S(t) S \quad (4)$$

in terms of the complete orthonormal basis $\mathcal{S} \in \{\chi_{1,L}^{p_1} \cdots \chi_{N,L}^{p_N} \psi_{1,L}^{q_1} \cdots \psi_{M,L}^{q_M}\}$ with $p_j, q_a \in \{0, 1\}$; the operator wave function $c_S(t)$ encapsulates complexity evolution due to interactions. An expression similar to Eq. (4) also holds for $\sqrt{\rho_R} \chi_{1,R}(t)$.

For $|g| \ll 1$, only the out-of-time-order correlations (OTOCs) in $\mathcal{K}(t_L, t_R)$ remain non-negligible [4], as in the OTOCs, the combination $\sim g e^{\kappa t}$ emerges as a hallmark of quantum chaos, rendering perturbation expansions in g problematic if $\kappa > 0$. Here, κ is the Lyapunov exponent. The effect of $\mathcal{U}$ is to count the number of system fermions, i.e., $\mathcal{U}S|\text{EPR}\rangle = e^{ig(n_S - N/2)}S|\text{EPR}\rangle$ with the operator size $n_S \equiv \sum_j p_j$ regardless of $\psi_{a,\alpha}$ [24]. Correspondingly, combined with the time reversal between H_L and H_R , the response function becomes [6,8,34]

$$\mathcal{K}(t_L, t_R) = -\text{Im} \sum_{S(\tilde{n}_S > 0)} e^{-ig\tilde{n}_S} c_S(-t_L) c_S(-t_R), \quad (5)$$

with the shifted operator size $\tilde{n}_S \equiv n_S - n_0$ and the zero point $n_0 \equiv N/2 - \langle \text{TFD} | V | \text{TFD} \rangle$ coming from the size of $\rho_L^{1/2}$, which vanishes at $\beta = 0$. It has been shown that typical basis appearing in Eq. (4) has $\tilde{n}_S \in [0, N/2 - n_0]$ [42,43]. However, only those with $\tilde{n}_S > 0$ contribute in Eq. (5) [34].

In the absence of environments, the average operator size $\tilde{n}(t) \equiv \sum_S \tilde{n}_S |c_S(t)|^2$ grows exponentially at early times and saturates to $\sim N/2 - n_0$ at late times. Simultaneously, $|\mathcal{K}|_{\text{max}}$, the maximum value of $|\mathcal{K}|$, can exceed $\mathcal{K}_q$ (as in the SYK models [5]), and teleportation operates in the quantum

regime. In stark contrast, the presence of environments can completely crush information scrambling [23,24]: When the system-environment coupling γ exceeds a critical value γ_s , $\tilde{n}(t)$ is found to decay exponentially in time [24], indicating that quickly each individual c_S is exponentially small for $\tilde{n}_S > 0$ and so is anticipated for $|\mathcal{K}|_{\text{max}}$. In this dissipative phase, information loaded into L_{S_1} rapidly dissipates into the environment, rendering the expansion, Eq. (4), essentially dominated by basis $\mathcal{S}$ of $\tilde{n}_S = 0$ (consisting of only environment fermions $\psi_{a,L}$). Due to the suppression of $|\mathcal{K}|_{\text{max}}$, the protocol cannot transfer any information, i.e., not only $\mathcal{N}_{AR_{S_1}}$ but also $I_{AR_{S_1}}$ vanishes. Thus, the transition point γ_c that bounds the no-signal regime shall be identical to γ_s .

When γ decreases from γ_s toward zero, information scrambling is found to be established, and both the early time growth rate and the saturation value of $\tilde{n}_S(t)$ gradually increase [24]. If we assume that $|\mathcal{K}|_{\text{max}}$ simultaneously increases from being exponentially small toward its value at $\gamma = 0$ continuously, and given $|\mathcal{K}|_{\text{max}} > \mathcal{K}_q$ at $\gamma = 0$, there shall be another teleportation transition at a new critical point $\gamma = \gamma_q (< \gamma_s)$, across which $\mathcal{N}_{AR_{S_1}}$ changes from zero to nonzero. The two transition points γ_q and γ_c separate the quantum regime, the classical regime and the no-signal regime for the teleportation. We emphasize that the above assumption of a continuous variation in $|\mathcal{K}|_{\text{max}}$ is needed to guarantee the separation between the quantum and classical regimes. The validity of the assumption is demonstrated explicitly in the following solvable models, and awaits examinations in more general systems.

Solvable SYK models. We demonstrate our prediction of the teleportation transitions via solvable Brownian SYK models [24,26,27]. The Hamiltonian of the left party reads

$$H_L = \sum_{j < k < l, a} J_{jkl a}(t) \chi_{j,L} \chi_{k,L} \chi_{l,L} \psi_{a,L} + \sum_{j,a} iV_{ja}(t) \chi_{j,L} \psi_{a,L}, \quad (6)$$

where coupling constants $J_{jabc}(t)$ and $V_{ja}(t)$ with different indices are independent Brownian variables satisfying

$$\begin{aligned}\overline{J_{jkl a}(t_1) J_{jkl a}(t_2)} &= 2J\delta(t_1 - t_2)/MN^2 \\ \overline{V_{ja}(t_1) V_{ja}(t_2)} &= V\delta(t_1 - t_2)/M.\end{aligned}\quad (7)$$

The right party Hamiltonian H_R is generated from H_L by replacing $(\chi_{j,L}, \psi_{a,L})$ with $(\chi_{j,R}, \psi_{a,R})$ and changing $V_{ja}(t)$ to $-V_{ja}(t)$.

Our model can be solved analytically in the limit of $1 \ll N \ll M$ using the $1/N$ expansion at $\beta = 0$ [24]. For convenience, we set $J = 1/4$ as the energy unit and introduce $\gamma \equiv V/J$. The information scrambling transition has been found to occur at the critical point $\gamma_s = 1$ and the quantum Lyapunov exponent is $\kappa = 1 - \gamma$. The central function $\mathcal{K}(t_L, t_R)$ can be computed via the Wightman function

$$G^W(t_L, t_R) = -i \langle \text{TFD} | \chi_{1,L}(-t_L) \mathcal{U}^\dagger \chi_{1,R}(t_R) \mathcal{U} | \text{TFD} \rangle \quad (8)$$

as $\mathcal{K}(t_L, t_R) = \text{Im} G^W(t_L, t_R)$. The Schwinger-Dyson equation gives [34,44]

$$\begin{aligned}\left(\partial_{t_L} + \frac{\Gamma}{2}\right) \left(\partial_{t_R} + \frac{\Gamma}{2}\right) G^W(t_L, t_R) &= \delta(t_L - t_R) \Sigma^W(t_L), \\ \Sigma^W(t_L) &= G^W(t_L, t_L)^2 + \gamma - (1 - e^{-ig})\delta(t_L).\end{aligned}\quad (9)$$

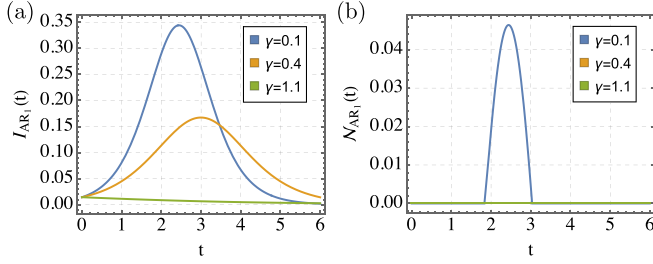


FIG. 2. Plots for the mutual information $I_{AR_{S_1}}$ (measured in the unit of $\ln 2$) and the negativity $N_{AR_{S_1}}$ with the teleportation coupling $g = 0.01$ and the system-environment coupling $\gamma \in \{0.1, 0.4, 1.1\}$. The mutual information has a maximum value $\sim O(1)$ for $\gamma = 0.1, 0.4$ and $\sim O(g)$ for $\gamma = 1.1$. The negativity can be nonzero only for $\gamma = 0.1$.

Here, the last term in the self-energy Σ^W arises from the coupling $\mathcal{U}$.

In the limit $|g| \ll 1$, the solution to Eq. (9) is given by $G^W(t_L, t_R) = e^{-\Gamma|t_L - t_R|/2} \mathcal{G}(\min\{t_L, t_R\})$ with $\Gamma = 4(J + V)$ and

$$\mathcal{G}(t) = \gamma + \frac{(1 - \gamma)^2}{1 - \gamma + i g e^{(1-\gamma)t}} \quad (10)$$

for $\gamma \neq 1$ [34]. Since the norm of $G^W(t_L, t_R)$ decays when $|t_L - t_R|$ increases, $\mathcal{K}$ acquires its maximal magnitude when $t_L = t_R$, and we have

$$\mathcal{K}(t, t) = -\frac{g(1 - \gamma)^2 e^{(1-\gamma)t}}{g^2 e^{2(1-\gamma)t} + (1 - \gamma)^2}. \quad (11)$$

When $\gamma > 1$, $|\mathcal{K}(t, t)|$ decays monotonically with t as the information scrambling process is halted. We have $|\mathcal{K}|_{\max} = |\mathcal{K}(0, 0)| \sim |g| (\ll 1)$, and even $I_{AR_{S_1}}$ is vanishingly small, not to mention $N_{AR_{S_1}} = 0$. This failure to establish notable correlations is because any initial information quickly dissipates into the environments, and cannot be rescued by manipulating the systems only. On the other hand, when $\gamma < 1$, the scrambling process is maintained, and $|\mathcal{K}(t, t)|$ shows a peak at finite t_* which is determined as $|g|e^{(1-\gamma)t_*} = (1 - \gamma)$, and $|\mathcal{K}|_{\max} = (1 - \gamma)/2$. Thus, $I_{AR_{S_1}} \sim O(1)$; $\gamma_c = \gamma_s$ separates the no-signal and classical regimes. Finally, by requiring $|\mathcal{K}|_{\max} = \sqrt{2} - 1$, we find the quantum-classical transition at $\gamma_q = 3 - 2\sqrt{2} \approx 0.17$. Back at the transition point γ_c , we instead find $\mathcal{K}(t, t) = -g/(1 + g^2 t^2)$, indicating that $I_{AR_{S_1}}$ is still $\sim O(|g|)$ as in the no-signal regime. In Fig. 2, we plot the negativity and the mutual information as functions of time t for various values of γ using the full solution of $\mathcal{K}(t_L, t_R)$ [34]; the distinctions between the quantum, classical, and no-signal regimes are illustrated.

Anomalous finite-size effects. The above model calculation is exact in the thermodynamic limit $N \rightarrow \infty$. Recent attempts to realize the TW protocol of closed systems in experiments and numerical simulations of random quantum circuits inevitably employ a finite number of “qubits” [45]. Here, we generalize the Brownian SYK model calculation to the situations that N is large but finite while $M \rightarrow \infty$, and show that the finite size can result in an *anomalously* large enhancement to the value of γ_q .

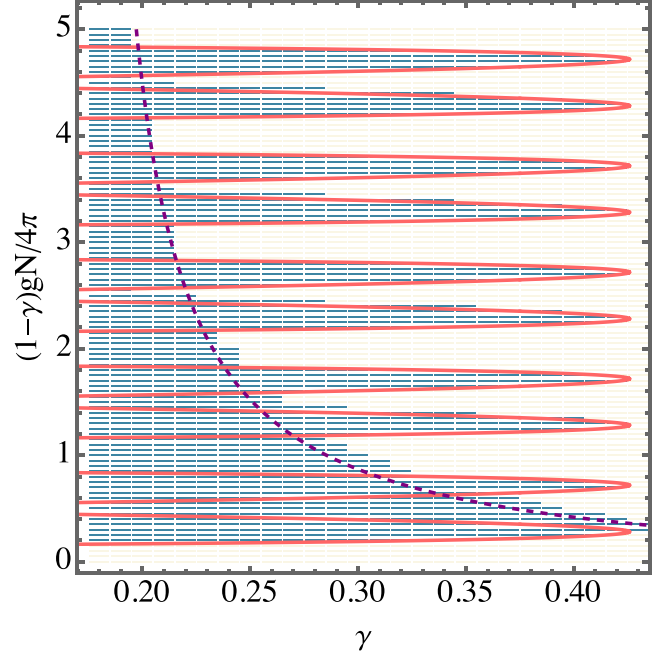


FIG. 3. Corrected γ_q for systems with large but finite N . By numerical calculation [34], the maximum of the negativity $N_{AR_{S_1}}$ is determined to be nonzero in the blue shaded region and zero elsewhere. The purple dashed curve is an extrapolation derived from Eq. (12) and the red solid curve is the theoretical prediction in the long-time limit based on Eq. (13).

For $\gamma < \gamma_c$, predominant finite-size corrections originate from out-of-time-order correlations mediated by scramblons [13, 28, 29, 35, 43, 46, 47]; as collective modes characterized by a soft action [48], the scramblons have their propagators growing exponentially over time $\sim e^{\lambda t}$ due to quantum many-body chaos. While other typical finite-size corrections are suppressed in the $1/N$ expansion and relatively negligible, the scramblon contribution is exponentially amplified and shall be substantial at time $t \approx t_* \sim -\ln |g| \gg 1$ when optimal teleportation signal is anticipated.

Away from the thermodynamic limit $N \rightarrow \infty$, we generalize the computation of $\rho_{AR_{S_1}}$ [cf. Eq. (3)] to account for contributions from all orders of scramblons [34]. Numerical results indicate that $N_{AR_{S_1}}$ still first turns nonzero at equal t_L and t_R . Figure 3 shows the region calculated numerically where $N_{AR_{S_1}} > 0$ at any time $t = t_L = t_R$ with gN and γ varying. The enhanced critical value $\tilde{\gamma}_q$ for the quantum regime can be as large as 0.43, much larger than $\gamma_q \approx 0.17$.

The new phase boundary of the quantum regime shown in Fig. 3 can be accounted for essentially by considering two limiting situations [34]. In case $N \gg 1/g$, the behavior of $\rho_{AR_{S_1}}$ at $t \approx t_*$, where optimal teleportation signal occurs in the thermodynamic limit, is expected to be modified perturbatively in orders of $1/gN$; we find that the smaller eigenvalue of $\rho_{AR_{S_1}}^{T_A}$ at $t = t_*$ to the first order is given by

$$\epsilon_+(t_*) = -\frac{1}{16}(1 - 6\gamma + \gamma^2) + \frac{4(\gamma - 3)}{16gN}. \quad (12)$$

This leads to an enhanced phase boundary at $\tilde{\gamma}_q = \gamma_q + \frac{2}{gN}$, whose extrapolation is the purple dashed curve in Fig. 3.

A more drastic situation when N is finite is that optimal teleportation signal can emerge at time $t \gg t_*$ [34]. We examine our teleportation protocol in the long-time limit $t \rightarrow \infty$; both the eigenvalues of $\rho_{AR_{S_1}}^{T_A}$

$$\epsilon_{\pm}(\infty) = \frac{1}{8} \left[1 + (2 - \gamma)\gamma + (1 - \gamma)^2 \cos \left((1 - \gamma)g \frac{N}{2} \right) \pm (\gamma^2 - 1) \sin \left((1 - \gamma)g \frac{N}{2} \right) \right] \quad (13)$$

can contribute to the negativity. The phase boundary occurs at $\epsilon_{\pm}(\infty) = 0$, depicted as the red solid curve in Fig. 3. The largest γ_* on this curve corresponds to $1 + (2 - \gamma_*)\gamma_* = (1 - \gamma_*)\sqrt{5 + 6\gamma_* + 5\gamma_*^2}$, which gives $\gamma_* \approx 0.43$.

Thus, the full phase boundary in Fig. 3 can be viewed largely as a result of bridging the above two limiting situations [34]. Equation (13) indicates that the anomalous enhancement stems from an interference effect facilitated by the scramblons; in the presence of environments, the information processing capability of systems with finite large N is, in the above sense, enhanced by these soft collective modes.

Discussions. In this work, we unveil two critical points, γ_q and γ_c , which mark the transitions of many-body quantum teleportation from the quantum regime to the classical regime and to the no-signal regime in the presence of environments. The prediction is explicitly demonstrated by our solvable Brownian SYK model calculation. By harnessing the relation between many-body teleportation and information

scrambling, we also provide a general argument establishing the transitions, assuming a continuous variation in $|\mathcal{K}|_{\max}$ when tuning the system-environment coupling. The anomalously large correction to γ_q , brought about by the scramblons for large but finite size N , manifests the nature of many-body dynamics in the teleportation phenomenon. The generality of our discovery is further strengthened as the scrambling transition has been established in a wide class of Brownian SYK models, beyond the one employed here, with generic system-environment couplings [24].

Our prediction for the transitions invites a dual understanding from the perspective of gravitational physics, which may be potentially achievable by utilizing the setup discussed in Ref. [33]. Nevertheless, reproducing the transitions may require including stringy corrections, which are known to be necessary for reducing the quantum Lyapunov exponent [17]. It would be interesting to explore the possibility of further enhancing the quantum regime, i.e., pushing γ_q toward γ_c . For instance, one could investigate the transmission of information using multiple copies of the teleportation channel, as studied in Ref. [49].

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Data availability. Data for Figs. 2 and 3 are available from the authors upon reasonable request.

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