

Tunable harmonic frequency combs via dual-beam or structured-target laser-solid interactions

Raoul Trines^{1,*}, Holger Schmitz¹, Martin King^{2,3}, Paul McKenna^{2,3} and Robert Bingham^{1,2}

¹*Central Laser Facility, STFC Rutherford Appleton Laboratory, Didcot OX11 0QX, United Kingdom*

²*Department of Physics, SUPA, University of Strathclyde, Glasgow G4 0NG, United Kingdom*

³*The Cockcroft Institute, Sci-Tech Daresbury, Warrington WA4 4AD, United Kingdom*



(Received 19 September 2025; revised 13 November 2025; accepted 14 December 2025; published 5 March 2026)

When an intense laser pulse strikes a flat solid target at normal incidence, only odd, equally spaced harmonics are typically generated. Using particle-in-cell simulations, we show that introducing transverse spatial modulation—either via two beams incident at small opposing angles or via a single beam reflecting from a corrugated surface—produces a two-dimensional harmonic spectrum. Selecting the emission through a narrow slit isolates an angle-dependent subset of harmonics, forming a tunable frequency comb controlled by both the viewing angle and the drive-laser frequency. Distinct combs arise for *s*- and *p*-polarized emission, while increasing the focal spot size reduces divergence and improves efficiency. Additional control is achieved by varying beam frequencies, target tilt, or beam number. Replacing the dual-beam configuration with a single beam and structured surface provides equivalent functionality with significantly reduced experimental complexity.

DOI: [10.1103/j531-q3tk](https://doi.org/10.1103/j531-q3tk)

In “classic” high-harmonic generation (HHG) driven by high-intensity laser beams in isotropic configurations, the emitted spectrum contains only odd integer multiples of the pump frequency, as in HHG from ionized gases [1] or from solids under normal incidence [2]. Depending on the pump wavelength, which typically ranges from the mid-infrared to the ultraviolet, the harmonic spectrum may appear as either a coherent supercontinuum or a set of narrowband peaks [3,4]. For many applications, however, greater control over the harmonic spacing is required, e.g., to tune the effective carrier frequency of attosecond pulses generated via HHG. This has motivated the development of “frequency comb” schemes, in which only a subset of harmonics is selected rather than the full series. The chosen subset can be tuned via higher-order beam (e.g., transverse shape, polarization, or phase) or target (e.g., transverse structure) properties [5–8].

A recent example is the “necklace beam” concept introduced by Rego *et al.* [7]. Two copropagating beams with parallel linear polarizations, of equal frequency but different orbital angular momenta (OAM), with integer indices ℓ_A and ℓ_B that share no common divisor and are not both odd, drive HHG in an isotropic medium (gas). When the emission is collected through an on-axis pinhole, only light with zero net OAM is transmitted. This light contains harmonics of the order $(2n + 1)|\ell_A - \ell_B|$, corresponding to odd multiples of $|\ell_A - \ell_B|\omega_0$ rather than ω_0 . The harmonic spacing is therefore tunable via the OAM indices.

In this paper, we show that the role of the OAM index in the necklace beam scheme can instead be assumed by a small transverse wave vector component, $k_\perp$. Specifically, we consider two identical, linearly *s*-polarized beams of frequency ω_0 , incident onto a plasma target or nonlinear medium, crossing at a small angle $2\alpha_0$, similar to the geometry in Hickstein *et al.* [9], and illustrated in Fig. 1(a). Rather than asking, “For a given frequency, at which angles is light emitted?”, we invert the problem and ask, “For a given emission angle, which frequencies appear?”. We demonstrate that the resulting comb spacing can be tuned via the viewing angle α_v and the ratio $\tan \alpha_0 / \tan \alpha_v$, providing greater versatility than tuning via OAM levels.

This concept is further extended to a single-beam transmissive configuration using a corrugated slit, shown in Fig. 1(b), where similar tuning is achieved through variation of the corrugation spacing. As demonstrated by Yi [10], when a laser beam interacts with an aperture, the edge oscillates at the laser frequency, generating both odd and even harmonics. We show that a corrugated edge with spacing L acts as a diffraction grating, producing transverse diffraction with wave vector separation $\Delta k_\perp = 2\pi/L$. This contrasts with the case of two laser beams incident on a flat target, where nonlinear interference between the beams forms an effective diffraction grating. An illustrative example of harmonic generation from a corrugated target is presented in Figs. 1(c)–1(e), which present three-dimensional (3D) simulation results for the transverse angular profiles of the first three harmonics for a corrugation period of λ_L . Specific frequency combs can be isolated at experimentally accessible viewing angles, as shown in Fig. 1(f) for $\alpha_v = 0^\circ$ (black), $\alpha_v = 27^\circ$ (red), and $\alpha_v = 18^\circ$ (blue), each sampled within a 1° radius. The viewing angle, and therefore the resulting comb structure, can be tuned by adjusting the corrugation period. Compared with the dual-beam configuration, the corrugated-target approach relaxes laser system

*Contact author: raoul.trines@stfc.ac.uk

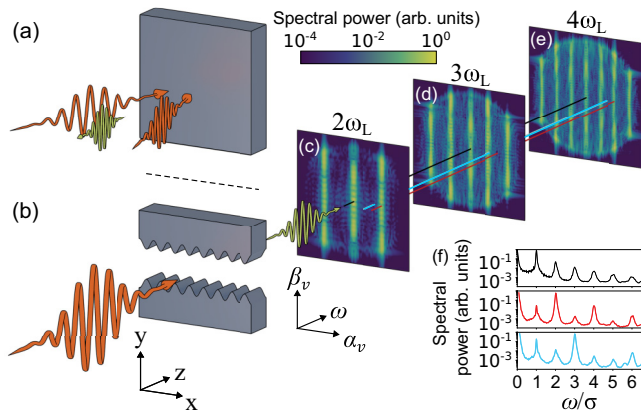


FIG. 1. (a) Dual-beam reflective setup on a planar target. (b) Single-beam transmissive setup through a corrugated slit target. Example observed angular profiles (in terms of viewing angle perpendicular, α_v , and parallel, β_v , to the laser polarization direction) of the generated harmonic light at (c) $2\omega_L$, (d) $3\omega_L$, and (e) $4\omega_L$ for the transmissive setup. (f) Corresponding frequency spectra along the laser axis (black), $\alpha_v = 27^\circ$ (red), and $\alpha_v = 18^\circ$ (blue) for a corrugation period of λ_L , each sampled within a 1° radius.

constraints, enabling the generation of selectable frequency combs using a single laser beam while minimizing the number of parameters requiring active stabilization, particularly the stringent spatial and temporal overlap of two tightly focused ultrashort pulses.

We exploit that results obtained with transverse wave vector $k_\perp$ are often analogous to those obtained with orbital angular momentum ℓ [7]. Several representative configurations illustrate this correspondence: (1) Two-dimensional (2D) harmonic spectra based on $k_\perp$ (see Fig. 1 of Ref. [11]) versus 2D spectra based on ℓ (see Fig. 4 of Ref. [12]), with related studies by Kong *et al.* [13], Heyl *et al.* [14], and Li *et al.* [15]; (2) two overlapping beams with distinct $k_\perp$ [Figs. 3(a) and 3(b) of Ref. [9], “bilinear” case] versus two overlapping beams with distinct ℓ (Fig. 3 of Ref. [16]); (3) two overlapping beams with opposite circular polarizations and opposite $k_\perp$ [Figs. 3(c), 3(d), and 5 of Ref. [9], “bicircular” case] versus HHG with vector beams, effectively two overlapping modes with opposite circular polarizations and opposite ℓ (Fig. 3 of Ref. [17]); (4) pulses with time-varying $k_\perp$, as in “noncollinear gating” or the “attosecond lighthouse” [18–21] (see Fig. 4 of Ref. [20]), versus pulses with time-varying ℓ , “self-torque” (see the abstract figure of Ref. [22] and Fig. 1 of Ref. [23]); and (5) HHG driven by a Gaussian pulse with a tilted wave vector k versus HHG driven by a Laguerre-Gaussian pulse [12,24]. In the former, harmonics obey $k_\perp \propto \omega$, while in the latter $\ell \propto \omega$, ensuring parallel harmonic wave vectors and phase fronts. Collectively, these examples highlight that although most studies address either $k_\perp$ or ℓ , or treat them separately, their combined behavior remains unexplored, underscoring the importance of developing a unified framework for their interaction.

HHG with noncollinear laser beams has been explored previously [9,11,12,14,15,25–27], often using beams of different wavelengths (e.g., 800 nm fundamental and its 400 nm second harmonic). In such studies, the two-dimensional $(\omega, k_\perp)$

spectrum is typically presented, with emphasis on angular diffraction patterns of individual harmonic frequencies. Here, in contrast, we begin with a fixed diffraction angle and identify the subset of harmonic orders that contribute at that angle. We also distinguish between *s*- and *p*-polarized emission, which generate different harmonic sequences and thus offer distinct routes to frequency comb formation.

Additionally, we consider the divergence of the harmonic modes. For Gaussian pump beams, the effective radius of the emission region decreases with harmonic order, since only the central high-intensity region contributes, leading to increasing divergence at higher orders. In contrast, necklace beams produce harmonics in a ring-shaped region of fixed radius R_0 , so the Rayleigh length scales as $z_q = qz_0$, yielding decreasing divergence with order [7]. We employ super-Gaussian driving pulses to ensure that the effective emission radius does not shrink too rapidly with harmonic order, thereby mitigating excessive divergence and preserving comb quality.

As in our previous work [8], we decompose the light into pure circular polarization modes with frequency $\omega > 0$, wave vector $\mathbf{k}$, OAM ℓ , and spin $\sigma = \pm 1$. Harmonic progressions are then analyzed in $(\omega/\sigma, \mathbf{k}/\sigma, \ell/\sigma)$ space, where they always form regular grids, typically with one or two degrees of freedom. Since all our harmonic frequencies will be integer multiples of the pump laser frequency ω_0 , we will use the implicit scaling ω/ω_0 for the remainder of this paper.

For the original necklace beam with $|\ell_A| \neq |\ell_B|$ [7], the pump configuration corresponds to the four points $\pm(1, \ell_{A,B})$ in $(\omega/\sigma, \ell/\sigma)$ space. The harmonic progression has two degrees of freedom and is given by

$$(\omega/\sigma, \ell/\sigma)_{mn} = (1, \ell_A) + m(2, \ell_A + \ell_B) - n(0, \ell_A - \ell_B),$$

i.e., diagonal rows of points in $(\omega/\sigma, \ell/\sigma)$. On axis, zero-OAM modes peak in intensity, while nonzero-OAM modes have an intensity minimum. Selecting the on-axis light via a pinhole thus preferentially selects the zero-OAM subset, found from $\ell_A + m(\ell_A + \ell_B) - n(\ell_A - \ell_B) = 0$. For $\ell_A = +2$, $\ell_B = -3$ (so $|\ell_A + \ell_B| = 1$), this gives $2 - m - 5n = 0$, or $\omega/\sigma = 1 + 2(2 - 5n) = 5(1 - 2n)$ with $n \in \mathbb{Z}$. Recombination then yields odd multiples of $5\omega_0$.

For two identical *s*-polarized beams crossing at a shallow angle $2\alpha_0$ (we use $\alpha_0 = 15^\circ$ in our simulations below), as in Ref. [9], the pump vector potential $\mathbf{A}_0 \propto \mathbf{e}_s$ is given by the four points $(\pm 1, \pm k_{\perp 0})$ in $(\omega/\sigma, k_\perp/\sigma)$ space, with $k_{\perp 0} = k_0 \sin \alpha_0$. The lowest-order term driving the *s*-polarized harmonics is $A_0^2 \mathbf{A}_0$, giving the progression, with two degrees of freedom:

$$(\omega/\sigma, k_\perp/(\sigma k_{\perp 0}))_{mn} = (1, 1) + m(2, 0) + n(0, 2), \quad (1)$$

i.e., horizontal rows, with no harmonic light in the on-axis direction. However, the necklace beam behavior [7] is reproduced by observing slightly off axis.

The pump crossing half angle is $\tan \alpha_0 = k_{\perp 0}/k_{\parallel 0}$, while the emission angle of modes in Eq. (1) is given by $\tan \alpha_{mn} = (2n + 1)k_{\perp 0}/[(2m + 1)k_{\parallel 0}]$. A viewing angle α_v given by $\tan \alpha_v = k_{\perp 0}/(Nk_{\parallel 0})$, where N is odd, preferentially selects harmonics $q\omega_0$ with $q = 2m + 1 = (2n + 1)N$, $n \in \mathbb{N}$. This mirrors the role of $N = |\ell_A + \ell_B|$ in Ref. [7], but here N is tuned by the viewing angle α_v , which is easier than tuning via the pump beam OAM levels $\ell_{A,B}$.

Following Lichters *et al.* [2] and Yi [28], this geometry also produces p -polarized “even” harmonics driven by the nonlinear term $A_0^2 \tan(\alpha_0) \mathbf{e}_p$, with the progression given by

$$(\omega/\sigma, k_\perp/(\sigma k_{\perp 0}))_{mn} = m(2, 0) + n(0, 2). \quad (2)$$

Here, the emission half angle of any harmonic mode is given by $\tan \alpha_{mn} = (2n)k_{\perp 0}/[(2m)k_{\parallel 0}] = (n/m) \tan \alpha_0$. For example, a viewing angle α_v given by $\tan \alpha_v = k_{\perp 0}/(Nk_{\parallel 0})$, with N any integer, preferentially selects modes $q\omega_0$ with $q = 2m = (2n)N$.

For the interaction of a single beam with a corrugated slit target, corrugation period L_{corr} , and corresponding $k_{\text{corr}} = 2\pi/L_{\text{corr}}$, the harmonic spectrum is given by

$$(\omega/\sigma, k_\perp/(\sigma k_{\text{corr}}))_{mn} = (m, n). \quad (3)$$

Again, the emission half angle of any harmonic mode is given by $\tan \alpha_{mn} = (n)k_{\perp 0}/[(m)k_{\parallel 0}] = (n/m) \tan \alpha_0$. A smaller corrugation period results in a larger value for $k_{\perp 0}$ and thus a larger angular separation for the harmonics. This renders it easier to obtain frequency combs with a large spacing, which emerge at small angles with respect to the central axis.

As noted in Ref. [8], every 2D harmonic progression contains the possibility of a frequency comb; the key is to choose the appropriate viewing angle.

We tested these ideas with 2D particle-in-cell simulations using the EPOCH code [29–31]. Two identical s -polarized beams strike a flat target at $\pm\alpha_0 = 15^\circ$. This involved a box size $16 \times 64 \mu\text{m}^2$, grid 4096×8192 , 500 particles per cell, and total simulation time of 250 fs. Each laser was defined with a wavelength $\lambda_L = 1 \mu\text{m}$, a peak intensity $I_L = 3 \times 10^{19} \text{ W/cm}^2$, a beam waist $w_0 = 16 \mu\text{m}$, and a Gaussian longitudinal temporal envelope $\tau_L = 50$ fs. The laser transverse profile was either Gaussian ($\exp[-r^2]$) or super-Gaussian ($\exp[-r^{16}]$). The plasma slab was defined with a density $8n_{\text{cr}}$ with a $1 \mu\text{m}$ preplasma on the front surface.

Two-dimensional simulations suffice here: They reduce cost, permit higher resolution, and improve the quality of Fourier spectra. A wide spot is required, $k_0 R \tan 15^\circ > 1$, otherwise (1) too few “grating” periods form within the spot, weakening harmonic amplitudes; (2) strong divergence causes overlap of harmonics from different angles, preventing angle-resolved selection; and (3) higher-order harmonics, which demand a threshold intensity for nonperturbative generation, would emerge from too small an area, yielding excessive divergence. For these reasons, all simulations shown here employ super-Gaussian envelopes, whose harmonics have divergence properties similar to ring-shaped envelopes [7].

The generation of the odd-valued s -polarized harmonics of Eq. (1) does not require the presence of a dc mode [2,8], only a cubic nonlinear term in the wave equation, and they are also observed in experiments on HHG in noble gases [1]. In contrast, the p -polarized harmonics of Eq. (2) are specific to the two-beam, solid-target configuration, as their production does require a dc mode. The results for both s - and p -polarized harmonics for this configuration are summarized below.

For s -polarized light, we find the following: (1) The peak locations in $(\omega/\sigma, k_\perp/(\sigma k_{\perp 0}))$ agree with Eq. (1), as shown in Fig. 2(a), and can be transformed to show their locations in terms of α_v in Fig. 2(b). (2) Gaussian pulses (not shown) yielded less defined spectral peaks, consistent

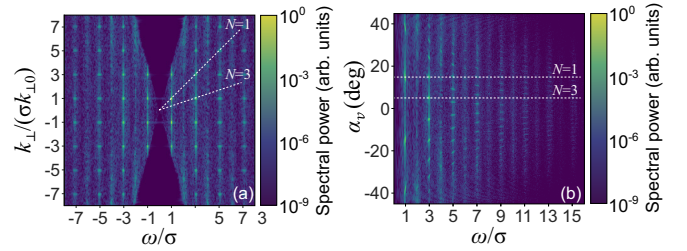


FIG. 2. (a) $(\omega/\sigma, k_\perp/(\sigma k_{\perp 0}))$ spectrum of s -polarized harmonic radiation from the dual-beam reflective setup, with peak positions as predicted by Eq. (1). (b) Spectrum as a function of α_v for s -polarized light. Only positive ω is shown in panel (b) due to inherent symmetry. Dashed white lines indicate specific frequency combs at $N=1$ and $N=3$.

with earlier work [32,33], while super-Gaussian pulses gave sharply improved peak structure. For both super-Gaussian and annular laser intensity envelopes, the contour radius is nearly intensity-independent, giving divergence $\lambda_q/R = \lambda_0/(qR)$, which decreases with harmonic order. (3) The spread in ω arises from the short pump duration and could be reduced with longer pulses. (4) Cross-cuts at $\tan(\alpha_v) = ck_\perp/\omega = \tan(15^\circ)/N$, with N odd-valued, produce harmonic combs at $(2n+1)N\omega_0$, all extracted from the same simulation, shown in orange for $N=1$ and $N=3$ in Figs. 3(a) and 3(b), respectively. (5) Peaks emitted at angles between those of the pump beams dominate over peaks emitted at larger angles, consistent with the “sum” versus “difference” processes of Ellis *et al.* [27], where the latter are higher order and weaker.

The reflected p -polarized light contains “even” harmonics with spectrum given by Eq. (2). For viewing angles $\tan \alpha_v = k_{\perp 0}/(Nk_{\parallel 0}) = \tan(15^\circ)/N$ ($N \in \mathbb{Z}$), one preferentially selects modes $q\omega_0$ with $q = 2m = (2n)N$, $n \in \mathbb{N}$. These results, shown in blue for $N=1$ and $N=2$ in Figs. 3(a) and 3(b), respectively, come from the same dual-beam simulation above (separate configurations are not needed to produce the different frequency combs).

We also demonstrate an alternative, single-beam transmissive scheme using a corrugated slit target. These require full 3D simulations. Here, a linearly polarized beam with a Gaussian transverse profile ($\lambda_L = 1 \mu\text{m}$, $I_L = 5 \times 10^{19} \text{ W/cm}^2$, $w_0 = 6.8 \mu\text{m}$, and $\tau_L = 50$ fs) propagates along z with polarization along y , incident on a $1 \mu\text{m}$ Al^{13+} slab of density $40n_{\text{cr}}$ at $z = 0$. The target includes a rectangular slit

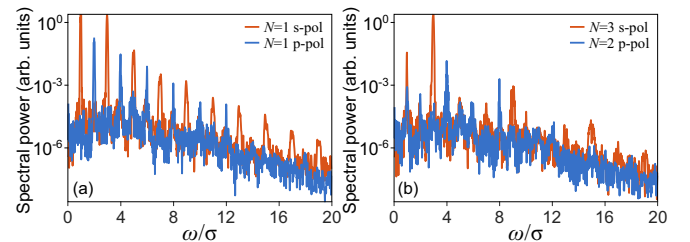


FIG. 3. Lineouts of s - and p -polarized spectra at viewing angles (a) $\alpha_v \approx 15^\circ$ ($N=1$ E_z orange, E_y blue) and (b) $\alpha_v \approx 5^\circ$ ($N=3$ E_z orange) and $\alpha_v \approx 7.5^\circ$ ($N=2$ E_y blue). $N=1$ and $N=3$ correspond to the dashed white lines in Fig. 2.

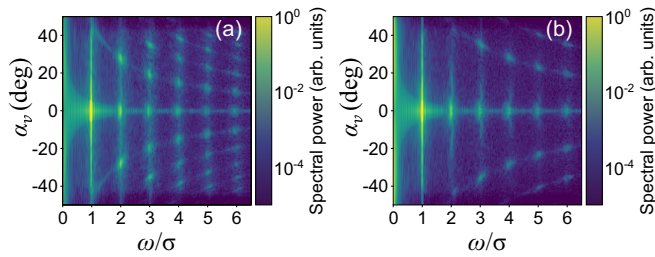


FIG. 4. Spectrum of s -polarized harmonic radiation as a function of viewing angles, α_v , from the transmissive corrugated slit target with corrugation periods of (a) λ_L and (b) $0.5\lambda_L$.

($-4\text{ }\mu\text{m} \leq y \leq 4\text{ }\mu\text{m}$) with a sinusoidal corrugation (depth $1\text{ }\mu\text{m}$, period $0.5\text{--}1\text{ }\mu\text{m}$). Simulations used a box $-10\text{ }\mu\text{m} \leq x, y \leq 10\text{ }\mu\text{m}$, $-5\text{ }\mu\text{m} \leq z \leq 15\text{ }\mu\text{m}$ with $720 \times 720 \times 1000$ cells.

Spectra of the s -polarized radiation of the transmissive scheme as a function of α_v at $z = 6\text{ }\mu\text{m}$ ($y = 0$ plane) show regular structures: for a $1\text{ }\mu\text{m}$ corrugation period, as shown in Fig. 4(a), each harmonic frequency exhibits peaks in the $k_\perp$ spectrum at integer multiples of the corrugation wave number $k_{\text{corr}} = k_L$, yielding frequency combs at predictable viewing angles. These results are in line with the predictions of Eq. (3). Reducing the period to $0.5\text{ }\mu\text{m}$ (i.e., doubling k_{corr}) broadens the angular spread of the harmonic structure, as shown in Fig. 4(b). Note that in this transmissive case, there is no difference in the harmonic structure of the s - and p -polarization.

In general, frequency combs with integer spacing n_{harm} are predicted based on the α_v and k_{corr} , according to $\tan \alpha_v = k_{\text{corr}}/(n_{\text{harm}}k_L)$. For smaller corrugation periods (i.e., larger k_{corr}), the required α_v for lower n_{harm} values may exceed the harmonic emission cone. Conversely, for larger corrugation periods, the required α_v decreases, but adjacent harmonics may begin to overlap, albeit at reduced power. This overlap effect is already observable in Fig. 1(f) for $k_{\text{corr}} = k_L$.

In terms of conversion efficiency, for the configuration where $k_{\text{corr}} = k_L$, the total emission at $2\omega_L$ corresponds to 10.3% of the incident laser energy. The intensity of higher harmonic orders decays approximately as $\propto e^{-2}$. However, this efficiency is highly sensitive to various laser and target parameters [34], including focal distribution, target thickness, slit separation, and the depth and period of the corrugation. A comprehensive characterization of these dependencies lies beyond the scope of the present study and will be addressed in future work.

Many extensions of our scheme are possible. (1) A tilted plane target makes the angles of incidence unequal, shifting

the $k_\perp/\sigma$ grid asymmetrically about the origin. (2) HHG in a nonlinear gas with negligible reflection, as in Ref. [9], should follow the same principle but be experimentally simpler. (3) Using beams of different frequencies changes the minimal grid cell from a rectangle to a parallelogram. While non-collinear ω - 2ω HHG has been studied [11,12,14,15], it has not been exploited for direction-dependent combs. (4) Introducing the second transverse dimension with four beams at slightly different crossing angles in x and y allows selection via pin-hole or slit, yielding one-dimensional (1D) or 2D subsets of a 3D harmonic progression. (5) Beams preconditioned by a “necklace” HHG process could be crossed again to generate combs from an already sparse spectrum.

In summary, we have shown that HHG driven by two identical high-power beams incident at small, opposing angles on a solid target produces harmonic emission spanning multiple frequencies and transverse wave vectors $k_\perp$. For a given viewing angle α_v , only harmonics satisfying $\tan \alpha \equiv k_\perp/k_\parallel = \tan(\alpha_v)$ are observed, enabling directional selection of frequency combs whose spacing depends on α_v . Multiple combs can thus be extracted from a single experiment or simulation, with s - and p -polarized components yielding distinct series of odd and even multiples of ω_0 .

This concept extends to a transmissive geometry in which a single high-power beam interacts with a corrugated slit; the corrugation period sets the viewing angles at which specific combs appear. The dual-beam and corrugated-slit configurations provide complementary approaches: The former enables polarization-based separation of harmonic orders, while the latter offers a simpler, more stable configuration in which the required viewing angles can be tuned via the corrugation period.

The versatility of these approaches, and their applicability to a wide range of harmonic-generation schemes, highlights a general principle: Any two-dimensional harmonic progression can form a frequency comb when observed at the appropriate angle. This makes the method broadly relevant to precision metrology [35], comb-based spectroscopy [36], and studies of quantum dynamics [37].

Acknowledgments. We thank Eva Los and Chris Murphy for valuable discussions. Simulations were performed on the SCARF cluster at STFC Rutherford Appleton Laboratory and on ARCHER2 via the Plasma Physics HEC Consortia (EP-SRC Grant No. EP/X035336/1). This work was supported by EPSRC (Grants No. EP/Z535692/1, No. EP/R006202/1, and No. EP/V049232/1). The epoch code was developed under EPSRC Grant No. EP/G054940/1.

Data availability. The data that support the findings of this article are openly available [31].

- [1] M Ferray, *et al.*, Multiple-harmonic conversion of 1064 nm radiation in rare gases, *J. Phys. B: At. Mol. Opt. Phys.* **21**, L31 (1988).
- [2] R. Lichters, *et al.*, Short-pulse laser harmonics from oscillating plasma surfaces driven at relativistic intensity, *Phys. Plasmas* **3**, 3425 (1996).

- [3] T. Popmintchev, *et al.*, Bright coherent ultrahigh harmonics in the keV x-ray regime from mid-infrared femtosecond lasers, *Science* **336**, 1287 (2012).
- [4] T. Popmintchev, *et al.*, Ultraviolet surprise: Efficient soft x-ray high-harmonic generation in multiply ionized plasmas, *Science* **350**, 1225 (2015).

- [5] O. E. Alon, V. Averbukh, and N. Moiseyev, Selection rules for the high harmonic generation spectra, *Phys. Rev. Lett.* **80**, 3743 (1998).
- [6] D. Baykusheva, A. Chacon, D. Kim, D. E. Kim, D. A. Reis, and S. Ghimire, Strong-field physics in three-dimensional topological insulators, *Phys. Rev. A* **103**, 023101 (2021).
- [7] L. Rego, *et al.*, Necklace-structured high-harmonic generation for low-divergence, soft x-ray harmonic combs with tunable line spacing, *Sci. Adv.* **8**, eabj7380 (2022).
- [8] R. Trines, *et al.*, Laser harmonic generation with independent control of frequency and orbital angular momentum, *Nat. Commun.* **15**, 6878 (2024).
- [9] D. Hickstein, *et al.*, Non-collinear generation of angularly isolated circularly polarized high harmonics, *Nat. Photon.* **9**, 743 (2015).
- [10] L. Yi, High-harmonic generation and spin-orbit interaction of light in a relativistic oscillating window, *Phys. Rev. Lett.* **126**, 134801 (2021).
- [11] J. B. Bertrand, H. J. Worner, H. C. Bandulet, E. Bisson, M. Spanner, J. C. Kieffer, D. M. Villeneuve, and P. B. Corkum, Ultrahigh-order wave mixing in noncollinear high harmonic generation, *Phys. Rev. Lett.* **106**, 023001 (2011).
- [12] G. Gariepy, J. Leach, K. T. Kim, T. J. Hammond, E. Frumker, R. W. Boyd, and P. B. Corkum, Creating high-harmonic beams with controlled orbital angular momentum, *Phys. Rev. Lett.* **113**, 153901 (2014).
- [13] F. Kong, *et al.*, Controlling the orbital angular momentum of high harmonic vortices, *Nat. Commun.* **8**, 14970 (2017).
- [14] C. M. Heyl, P. Rudawski, F. Brizuela, S. N. Bengtsson, J. Mauritsson, and A. Lhuillier, Macroscopic effects in non-collinear high-order harmonic generation, *Phys. Rev. Lett.* **112**, 143902 (2014).
- [15] Z. Li, G. Brown, D. H. Ko, F. Kong, L. Arissian, and P. B. Corkum, Perturbative high harmonic wave front control, *Phys. Rev. Lett.* **118**, 033905 (2017).
- [16] L. Rego, J. S. Roman, A. Picon, L. Plaja, and C. Hernandez-Garcia, Nonperturbative twist in the generation of extreme-ultraviolet vortex beams, *Phys. Rev. Lett.* **117**, 163202 (2016).
- [17] A. de las Heras, *et al.*, Attosecond vortex pulse trains, *Optica* **11**, 1085 (2024).
- [18] H. Vincenti and F. Quéré, Attosecond lighthouses: How to use spatiotemporally coupled light fields to generate isolated attosecond pulses, *Phys. Rev. Lett.* **108**, 113904 (2012).
- [19] J. A. Wheeler, *et al.*, Attosecond lighthouses from plasma mirrors, *Nat. Photon.* **6**, 829 (2012).
- [20] C. M. Heyl, *et al.*, Noncollinear optical gating, *New J. Phys.* **16**, 052001 (2014).
- [21] J. P. Kennedy, B. Dromey, and M. Yeung, Isolated ultra-bright attosecond pulses via non-collinear gating, *New J. Phys.* **24**, 113004 (2022).
- [22] L. Rego, *et al.*, Generation of extreme-ultraviolet beams with time-varying orbital angular momentum, *Science* **364**, eaaw9486 (2019).
- [23] J. Wang, M. Zepf, Y. Leng, R. Li, and S. G. Rykovanov, Self-torqued harmonics and attosecond pulses driven by time-delayed relativistic vortex lasers, *Phys. Rev. A* **106**, 033515 (2022).
- [24] X. Zhang, B. Shen, Y. Shi, X. Wang, L. Zhang, W. Wang, J. Xu, L. Yi, and Z. Xu, Generation of intense high-order vortex harmonics, *Phys. Rev. Lett.* **114**, 173901 (2015).
- [25] S. V. Fomichev, P. Breger, B. Carré, P. Agostini, and D. F. Zaretsky, Non-collinear high-harmonic generation, *Laser Phys.* **12**, 383 (2002).
- [26] M. Negro, *et al.*, Non-collinear high-order harmonic generation by three interfering laser beams, *Opt. Express* **22**, 29778 (2014).
- [27] J. L. Ellis, *et al.*, Phase matching of noncollinear sum and difference frequency high harmonic generation above and below the critical ionization level, *Opt. Express* **25**, 10126 (2017).
- [28] L. Yi, Anomalous effects in single-slit diffraction of light at relativistic intensities, *Phys. Rev. Res.* **7**, 043163 (2025).
- [29] T. D. Arber, *et al.*, Contemporary particle-in-cell approach to laser-plasma, *Plasma Phys. Control. Fusion* **57**, 113001 (2015).
- [30] The epoch code is available at <https://epochpic.github.io/>.
- [31] The data that support the figures within this paper and other findings of this study are available at <https://doi.org/10.15129/78998a66-bc5c-4d60-b5aa-9d6003a58420>.
- [32] H. Wikmark, *et al.*, Spatiotemporal coupling of attosecond pulses, *Proc. Natl. Acad. Sci. USA* **116**, 4779 (2019).
- [33] L. Quintard, *et al.*, Optics-less focusing of XUV high-order harmonics, *Sci. Adv.* **5**, eaau7175 (2019).
- [34] E. F. J. Bacon, *et al.*, High order modes of intense second harmonic light produced from a plasma aperture, *Matter Radiat. Extremes* **7**, 054401 (2022).
- [35] I. Pupeza, *et al.*, Extreme-ultraviolet frequency combs for precision metrology and attosecond science, *Nat. Photon.* **15**, 175 (2021).
- [36] T. Fortier and E. Baumann, 20 years of developments in optical frequency comb technology and applications, *Commun. Phys.* **2**, 153 (2019).
- [37] S. Xun, *et al.*, Attosecond light science and its application for probing quantum materials, *J. Phys. B: At. Mol. Opt. Phys.* **53**, 184008 (2020).