

Fock-space fragmentation in quenches of disordered interacting fermions

Ishita Modak¹,^{*} Rajesh Narayanan,² Ferdinand Evers³, and Soumya Bera^{1,*}

¹Department of Physics, Indian Institute of Technology Bombay, Mumbai 400076, India

²Department of Physics, Indian Institute of Technology Madras, Chennai 600036, India

³Institute of Theoretical Physics and Halle-Berlin-Regensburg Cluster of Excellence CCE, University of Regensburg, D-93053 Regensburg, Germany



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Fock-space fragmentation, as it is currently investigated, primarily originates from specific kinematic constraints or emergent conservation laws in many-body systems with translation invariance. It leads to nonergodic dynamics and possible breakdown of the eigenstate thermalization hypothesis. Here, we demonstrate that also in disordered systems, such as the XXZ model with random on-site fields, fragmentation appears as a natural concept offering fresh perspectives, for example, on many-body localization. Specifically, for a given quenched initial state, we identify a potential-energy shell in Fock space, defined by its energy and energy uncertainty, which represents the energetically accessible part of the many-body configuration space. For example, the drastic sample-to-sample fluctuations observed in the weak disorder regime, $W < W_{\text{frag}}$, represent fluctuations of the mass of the shell. Upon increasing disorder, the potential-energy shell becomes increasingly fragmented. Its finite-size evolution exhibits a turnaround flow; extrapolating this flow to the disorder at which the turnaround vanishes provides a heuristic estimate of the strong-fragmentation scale W_{frag} . We conjecture that the fragmentation identified here provides a geometric mechanism contributing to the finite-time slowing down of relaxation observed in studies of disordered interacting systems.

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I. INTRODUCTION

The time evolution of charge imbalance is widely used as an indicator to probe thermalization in isolated disordered interacting systems, experimentally and numerically (e.g., see Refs. [1–4]). Starting from a product state such as the Néel state, thermalization implies that the imbalance decays to zero at long times; conversely, a finite imbalance at long times signals a breakdown of thermalization, marking the emergence of a nonergodic, i.e., many-body-localized (MBL) phase [5,6].

Despite a significant effort, experimentally and numerically, there is no consensus as to whether or not the MBL phase has indeed been observed in generic models of disordered fermions [7–19]. In the absence of rigorous analytical results, it is not clear presently in what sense MBL exists. The main detection challenge is taking the double limit of long observation times and large system sizes, which are currently still out of reach numerically and experimentally.

Still, numerical studies have revealed intriguing transient phenomena. We mention only two: (1) The relaxation behavior of quenches changes from accelerated in time to decelerated once the disorder strength W exceeds a threshold

W_c that is comparable to the bandwidth of the noninteracting reference system [8]. (2) The sample-to-sample fluctuations in the imbalance relaxation reflect this qualitative change. For a given disorder strength $W \lesssim W_c$, the ensemble consisting of all samples with size L contains sizable fractions of perfectly thermalizing as well as highly insulating samples [20–23]. Comparatively, at $W \gtrsim W_c$ the sample-to-sample fluctuations are moderate.

In this work, we identify a geometric mechanism in Fock space that underlies part of the finite-size and finite-time crossover phenomenology observed at intermediate disorder. To this end, we embark on the paradigmatic model of disordered, spinless fermions:

$$\hat{\mathcal{H}} := \hat{T} + \hat{H}', \quad (1)$$

with $\hat{H}' := \hat{V} + \hat{U}$ and

$$\hat{T} := -\frac{t_{\text{hop}}}{2} \sum_{x=1}^{L-1} (\hat{c}_x^\dagger \hat{c}_{x+1} + \text{H.c.}),$$

$$\hat{U} := \sum_{x=1}^L \epsilon_x \left(\hat{n}_x - \frac{1}{2} \right),$$

$$\hat{V} := V \sum_{x=1}^{L-1} \left(\hat{n}_x - \frac{1}{2} \right) \left(\hat{n}_{x+1} - \frac{1}{2} \right).$$

Here, x labels the lattice sites, L denotes the systems size, and $\hat{n}_x = \hat{c}_x^\dagger \hat{c}_x$ is the local density operator. We choose interaction $V = t_{\text{hop}}$ throughout, and ϵ_x denotes the onsite random

*Contact author: soumya.bera@iitb.ac.in

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potential drawn independently from a uniform distribution in $[-W, W]$.

Following previous authors [24] (see also Ref. [47] for further discussion), we adopt a Fock-space perspective of the many-body dynamics. Our focus will be on samples in the regime of moderate to strong disorder, i.e., $W \gtrsim t_{\text{hop}}$, comparable to the single-particle bandwidth in Eq. (1). In this regime, the potential energy, $\hat{H}'$, begins to dominate and therefore the site-occupation basis is a natural choice for the Fock-space construction: Every basis state, $|b\rangle$, is an eigenstate of the local occupations, $\hat{n}_x$, and hence also an eigenstate of $\hat{H}'$ with corresponding eigenvalue E_b .

The relaxation dynamics we are after is observed in quench protocols performed in isolated finite-size systems. The corresponding time evolution conserves the initial state's energy E_{in} and, in fact, all moments of the Hamiltonian $\hat{H}$. To the extent that the potential energy dominates, this implies that the time evolution involves all Fock-space sites $|b\rangle$ with energies E_b near E_{in} within a window given by the energy variance of the initial state. We refer to this set of Fock-space sites as “potential-energy shell”; it provides an effective “dynamically active region” of Fock space.

We will show that the concept of the potential-energy shell in Fock space turns out to be fruitful in multiple ways. For example, the sample-to-sample fluctuations at moderate disorder turn out to have a geometric interpretation: They reflect fluctuations of the effective size of the potential-energy shell. Moreover, the concept also sheds light on the somewhat counterintuitive observation that sample-to-sample fluctuations appear weaker at stronger disorder. The underlying mechanism is a “shell fragmentation” once (W) increases above twice the bare bandwidth ($W \approx 3.5-4$), the potential-energy shell falls into “fragments”; the distribution of fragments that host the initial state happens to be narrow (within a window of system sizes), and therefore sample-to-sample fluctuations are suppressed [47].

Fragmentation strongly restricts the directly connected region available to the initial state. Relaxation beyond this region then requires processes that couple different neighboring fragments, leading naturally to slow dynamics. We stipulate that this slowing down has been observed in many studies, numerical and experimental, as “creep” and gave rise to introducing accelerating and decelerating regions in phase space in Ref. [8].

Finally, we present an analysis of the cluster statistics suggesting that, unlike percolation, the fragmentation transition exhibits a regime of turnaround flow whose floating toward strong coupling is just transient; only at very large disorder values, the attractive realm of the strong-coupling fixed point is ultimately reached. Relating to earlier work on MBL, an intermediate flow toward the insulator has been reported [48]; since the turnaround is seen already at moderate disorder $W \approx 3.5$, we believe that it reflects fragmentation occurring in the numerical reference data. We mention that there is growing evidence that also in the MBL context, the strong-coupling fixed point attains stability—if at all—only at extremely strong disorder.

We emphasize that shell fragmentation is a geometric property of the Fock-space graph and is not, by itself, a diagnostic of MBL. Quantum tunneling between frag-

ments and interference effects remain outside the present construction.

II. METHOD

The relaxation of the initial state is described by the Hamiltonian (1). For time propagation, we rely upon the standard Chebyshev expansion technique; the technical details are given in Refs. [4,23].

Our primary observable for relaxation processes is the sublattice imbalance,

$$I(t) := \frac{2}{L} \sum_{x=1}^L (-1)^x \langle \Psi(t) | \hat{n}_x | \Psi(t) \rangle, \quad (2)$$

where $|\Psi(t)\rangle := e^{-i\hat{H}t} |\text{Néel}\rangle$ with the initial state at $t = 0$ being the Néel state, represented as $|\text{Néel}\rangle := |1010\dots\rangle$ in the site-occupation basis. We fix the particle number to be $L/2$, which defines the dimension of the FS $N_{\text{FS}} = \binom{L}{L/2}$. In the Supplemental Material [47], we tested the sensitivity to initial conditions by starting from other initial states.

III. POTENTIAL-ENERGY SHELL CONCEPT

We represent Fock space in the occupation basis, $|b\rangle$, which consists of all states that are eigenstates of the occupation number operators $\{\hat{n}_x, x = 1, \dots, L\}$; by construction, $\hat{H}'|b\rangle = E_b|b\rangle$ and $E_b = \langle b | \hat{H} | b \rangle$ because $\langle b | \hat{T} | b \rangle = 0$. The eigenvalues are given by $E_b = \frac{1}{2} \sum_{x=1}^L \epsilon_x \sigma_x + \frac{V}{4} \sum_{x=1}^{L-1} \sigma_{b,x} \sigma_{b,x+1}$, where $\sigma_{b,x} = 2(n_x - 1/2)$ denote a sequence of L “bits” characteristic for every basis vector $|b\rangle$. The notation emphasizes the close relation between the statistics of the potential energy content of $|b\rangle$ and the thermodynamics of the one-dimensional Ising model.

Toward the flat-band limit, $t_{\text{hop}} \ll V, W$, one expects that the relaxation dynamics is approximately confined to the subset of all basis states with

$$|E_{\text{in}} - E_b| \lesssim \alpha \Delta_{\text{in}}, \quad (3)$$

where $\Delta_{\text{in}} := (\langle \Psi(0) | \hat{H}^2 - E_{\text{in}}^2 | \Psi(0) \rangle)^{1/2}$ and α is a phenomenological parameter of order unity, which we take as $\alpha = 1/2$. The parameter α sets the width of the shell; the qualitative fragmentation behavior is robust against moderate variations of α , as shown in the Supplemental Material [47]. The set of all basis states that satisfy condition (3) we refer to as the potential-energy shell. The number of states contained in this set, N_{sh} , we refer to as the “mass” of the shell.

In the following, we focus on initial states, $|\Psi(0)\rangle$, that have been chosen as one of the basis vectors $|b\rangle$, e.g., $|\Psi(0)\rangle = |\text{Néel}\rangle$. The corresponding variance Δ_{in} scales with the kinetic-energy content of $|\Psi(0)\rangle$. Specifically, for the Néel state $\Delta_{\text{in}} = \Delta_{\text{Néel}}$ with $\Delta_{\text{Néel}} = (t_{\text{hop}}/2)\sqrt{L-1}$; we notice that $\Delta_{\text{Néel}}$ represents an upper boundary for the kinetic-energy content of all basis states, $\Delta_{\text{Néel}} \geq \langle b | \hat{T}^2 | b \rangle^{1/2}$.

IV. ILLUSTRATION

Figure 1 illustrates the concept. It displays three examples for potential-energy shells (red circles) embedded into the full Fock space (red and gray circles) taken at moderate

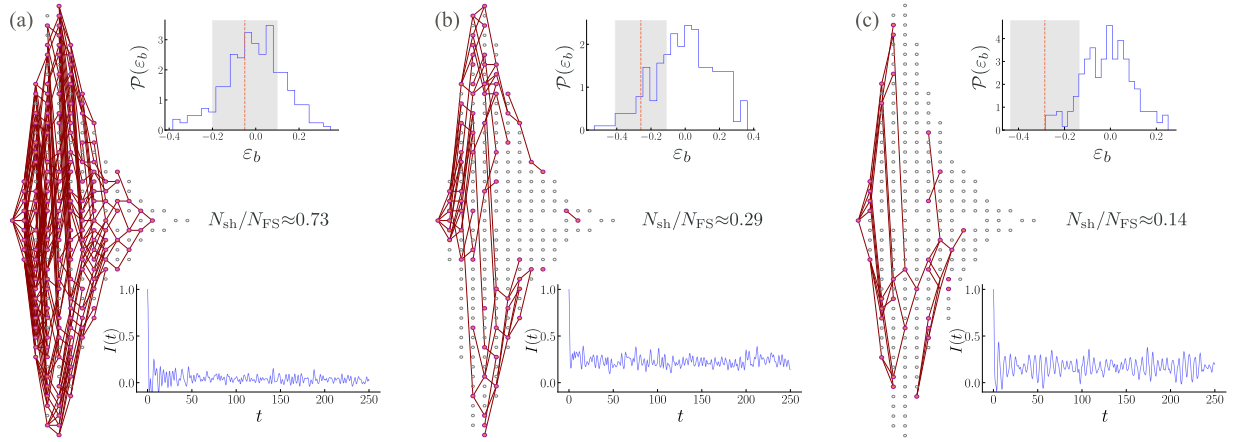


FIG. 1. Fock-space structure for a system of size $L = 10$, shown for three different disorder realizations at $W = 1.5$. The leftmost site corresponds to the Néel state, and the states to the right are ordered by one spin-flip distance from it. The vertical arrangement of the sites is arbitrary. The red dots indicate the potential-energy shell; the connectivity of the basis states introduced by the hopping term in Eq. (1) is indicated by lines. Upper inset: The distribution $\mathcal{P}(\varepsilon_b)$ of the Fock-space energy density $\varepsilon_b = E_b/L$, where E_b is defined in Eq. (3); the red vertical line marks the energy of the Néel state, and the shaded region indicates the energy variance, $\Delta_{\text{Néel}}$. Lower inset: Imbalance relaxation $I(t)$ for the corresponding sample.

disorder $W = 1.5$. The lines denote pairs of basis states, $|b\rangle, |b'\rangle$, that are linked neighbors in the sense that $\sum_{x=1}^{L-1} \langle b' | \hat{c}_x^\dagger \hat{c}_{x+1} + \text{H.c.} | b \rangle > 0$. Two basis states (“sites”) are called connected if there is a continuous path along linked neighboring sites between them; two connected sites are in the same cluster. While only a single cluster is seen in Fig. 1(a), multiple clusters exist in Figs. 1(b) and 1(c).

The potential-energy shells displayed in Fig. 1 define the active space available when quenching a Néel state. Representing the dynamically active part of the Fock space, fluctuations of the potential-energy shell manifest in sample-to-sample fluctuations of the time-evolving quenches. This is why the concept is so interesting. Furthermore, in Ref. [49], the quench dynamics for different initial conditions show a state-dependent imbalance decay, which can now be understood as the presence of distinct dynamically active parts in Fock space.

V. FLUCTUATIONS OF THE MASS OF THE POTENTIAL-ENERGY SHELL

There are three key observations to be made in Fig. 1:

(1) For the three typical moderate-disorder samples shown, the Néel state’s energy, E_{in} , fluctuates strongly with respect to the distribution $\mathcal{P}(E_b)$ of the site energies E_b (upper inset). This is qualitatively expected: First, we note that E_{in} exhibits disorder-induced fluctuations that are of order $\sqrt{4/3(W/4)^2 L}$ with box-distributed disorder. Second, within a Gaussian model, one expects $\mathcal{P}(E_b) = (2\pi\Delta_{\mathcal{P}}^2)^{-1/2} e^{-E_b^2/2\Delta_{\mathcal{P}}^2}$ with $\Delta_{\mathcal{P}} = \sqrt{(V/4)^2(L-1) + 4/3(W/4)^2 L}$ [50]. Third, since the disorder average of $\overline{E_{\text{in}}}$ is typically extensive, e.g., for the Néel state $\overline{E_{\text{Néel}}} = -(V/4)L$, one estimates that the system size shown in Fig. 1 is still at the crossover at which E_{in} begins to migrate to the tails of $\mathcal{P}$ with increasing L .

(2) The mass of the potential-energy shell, i.e., N_{sh} , is highly sensitive to its average energy. As one would expect,

for average energies near the edges of $\mathcal{P}(E_b)$, the shell volume’s fraction of the total Fock space $N_{\text{sh}}/N_{\text{FS}}$ is reduced. Typical factors are 2.5 or even 5 as extracted from the examples displayed in Fig. 1.

Shedding fresh light on the strong fluctuations seen in Fig. 1, we analyze the mass fluctuations of the potential-energy shell. Figure 2 displays the evolution of the distribution of the shell-mass fraction with disorder strength and system size. Indeed, a broad, almost flat distribution at $W = 1.5$ explains the very large sample-to-sample fluctuations seen in Fig. 1 and reported in the literature in the respective regime of W and L [17,22,23,51].

With respect to the evolution of the distribution with system size, we notice that finite-size effects are very large: Characteristic of all W values displayed in Fig. 2, the fraction of samples with $N_{\text{sh}}/N_{\text{FS}} \lesssim 0.1$ is increasing rapidly with L . We take this as a manifestation of the fact that the overlap of the distributions of E_b and E_{in} —estimated by $\mathcal{P}(\overline{E_{\text{in}}})$ —slowly decreases with increasing L ; within the Gaussian model, it is expected that $\mathcal{P}(\overline{E_{\text{in}}}) \approx e^{-\frac{1}{2}\overline{E_{\text{in}}}^2 L / \sqrt{2\pi\tilde{\Delta}_{\mathcal{P}}^2}}$ where $\overline{E_{\text{in}}} := (\overline{E_{\text{in}}}/L)/(\tilde{\Delta}_{\mathcal{P}}/\sqrt{L})$; here $\tilde{\Delta}_{\mathcal{P}}^2 = \Delta_{\mathcal{P}}^2 + \Delta_{\text{in}}^2$. Correspondingly, the average fraction $N_{\text{sh}}/N_{\text{FS}}$ is expected to be suppressed exponentially with increasing L .

Within the Gaussian model, the exponential suppression starts once L exceeds the crossover scale $L_W := 2/\overline{E_{\text{in}}}^2$. In this model, we can give qualitative estimates, e.g., $\overline{E_{\text{in}}}^2 = (2t_{\text{hop}}/V)^2 + 1 + 4/3(W/V)^2$. For the present situation $V = t_{\text{hop}}$, we are led to conclude that $L_W \approx (10 + (8/3)W^2)$, which is roughly consistent with what is seen in Fig. 2. Similarly, at $W \approx 3$, the disorder dominates the scale L_W , making it large and essentially inaccessible for us, which is also consistent with Fig. 2.

(3) At system sizes $L \lesssim L_W$, the satellite peak seen in Fig. 2 dominates the behavior of typical samples. The preceding argument suggests that this peak gradually disappears in the limit of large L . However, the satellite dominates the

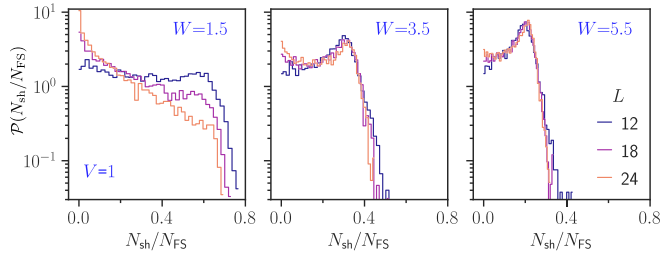


FIG. 2. The distribution of the normalized mass of the potential-energy shell, $N_{\text{sh}}/N_{\text{FS}}$, for different W , L .

transient dynamics, e.g., the sample-to-sample fluctuations, making them appear *smaller* in systems with *larger* disorder. More importantly, the satellite peak is a remnant of the non-interacting $V = 0$ limit, and the role of the interaction is to facilitate the flow of the weight from the peak to $N_{\text{sh}}/N_{\text{FS}} \rightarrow 0$ (see also Ref. [47]).

VI. CONNECTIVITY FLUCTUATIONS—FRAGMENTATION

Figure 2 implies that within a wide window of sizes $L \lesssim L_W$ a substantial fraction of the system remains part of the percolation cluster, with typical samples exhibiting a mass fraction exceeding $N_{\text{sh}}/N_{\text{FS}} \gtrsim 20\%$ at disorder values $W \gtrsim 3$. As is illustrated in Fig. 3, such rather massive potential-energy shells are not necessarily ergodic. The reason is that the shell consists of different parts that may or may not be connected; we refer to them as “fragments”. Clearly, for relating the

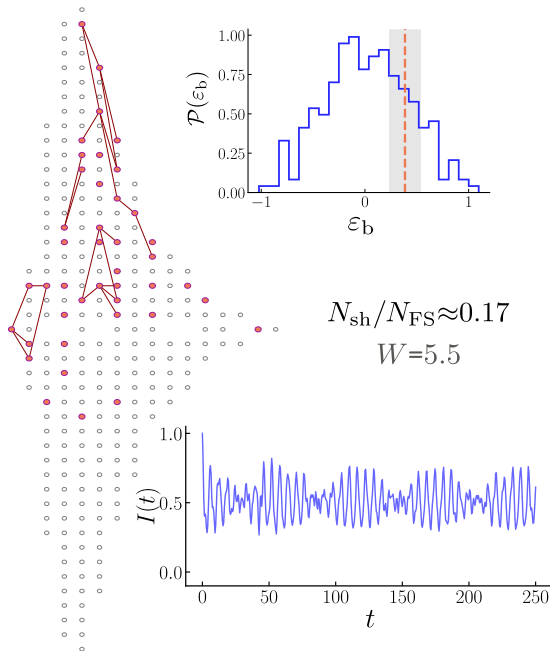


FIG. 3. Similar data as in Fig. 1 for disorder strength $W = 5.5$ for $L = 10$. With W growing beyond $W \approx 3.5$, the energy shell is seen to become increasingly fragmented. Correspondingly, $I(t)$ does not decay. The resulting oscillations in $I(t)$, which is a hallmark of fragmentation, are analyzed in detail in the Supplemental Material [47] (see Fig. S1 therein).

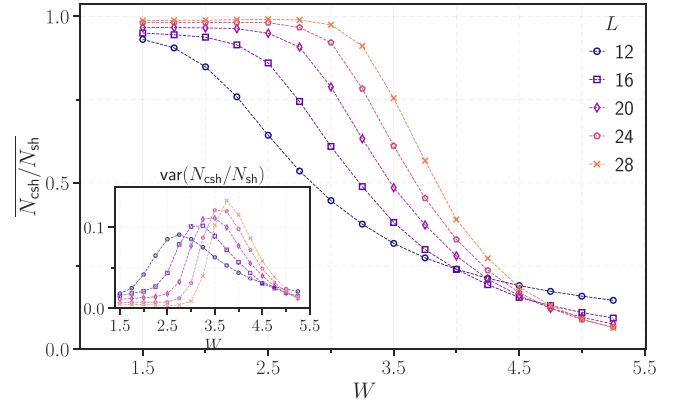


FIG. 4. The size of the cluster that contains the initial state, N_{csh} , normalized to the size of the corresponding potential-energy shell, N_{sh} , as a function of the disorder strength W for system sizes $L = 12, \dots, 28$. Inset: The corresponding fluctuations of the ratio $N_{\text{csh}}/N_{\text{sh}}$.

potential-energy shell to the relaxation dynamics, the shell geometry is to be carefully characterized [52].

In order to detect fragmentation, we study in Fig. 4 the statistics of the mass of the fragment, N_{csh} , that contains the initial state [Néel]. At moderate to weak disorder, in nearly all samples, the potential-energy shell is singly connected, $N_{\text{csh}}/N_{\text{sh}} \approx 1$, corresponding to the connected fixed point. In particular, as seen in the inset, the variance of $N_{\text{csh}}/N_{\text{sh}}$ is very close to zero at $W \lesssim 3$ once the system size exceeds a value of $L \simeq 24$. Fragmentation implies that the initial state is not connected with most of the potential-energy shell, i.e., $N_{\text{csh}}/N_{\text{sh}} \ll 1$. As seen in Fig. 4, this is the dominating behavior at strong disorder, $W \gtrsim 4.5$.

Upon inspecting the evolution of the traces seen in Fig. 4, the question naturally arises whether there is a critical disorder, W_{frag} , at which phase transition takes place, similar to percolation, which separates a phase dominated by fragmentation from a singly connected regime. The range of system sizes at our disposal is not suitable to provide a definite answer by extrapolating $L \rightarrow \infty$. We notice, however, that for a sharp transition to occur, all traces will have to intersect in a single point in the limit of large L , which defines the critical disorder W_{frag} . Since the intersection point of two neighboring traces in Fig. 4 is seen to move rapidly to larger disorder, a conservative lower bound would be $W_{\text{frag}} \gtrsim 4$ with the distinct possibility that there is no bound at all.

Even for a critical point floating toward infinite disorder, a meaningful definition of a strong-disorder phase can be given within a familiar scenario of turnaround flow [53–55]. This point has also been emphasized in recent work on the MBL transition [48]. The basic idea is readily identified in Fig. 4: In the range $4 \lesssim W \lesssim 4.5$, the volume fraction $N_{\text{csh}}/N_{\text{sh}}$ is seen to first decrease before it increases again at larger L . At even larger W , the L window at which $N_{\text{csh}}/N_{\text{sh}}$ decreases can become big enough so as to be interpreted as the Renormalization Group (RG) flow approaching a fully fragmented fixed point. The flow reversal at the largest system sizes is then interpreted as instability of the fragmented fixed point.

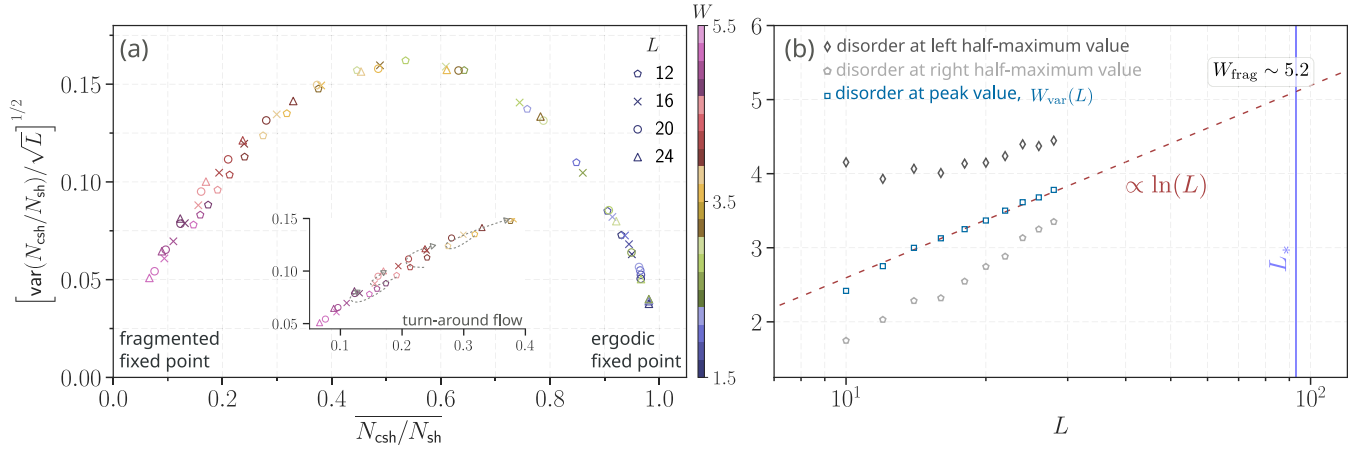


FIG. 5. (a) Direct comparison of variance and mean values, reploting the data Fig. 4. The colors indicate different disorder values W as given in the legend. An approximate collapse is achieved after normalizing the ordinate with the heuristic factor of $\sqrt{L}$. Inset: Enlarged left part of the main plot, showing the turnaround flow. (b) Flow of the maximum position W_{var} of $\text{var}(N_{\text{csh}}/N_{\text{sh}})$ with L , as extracted from data such as shown in inset of Fig. 4 (blue squares). The dashed line indicates a logarithmic extrapolation; the vertical line represents the system size L_* at which the turnaround of the RG flow from strong to weak coupling is cut off. The other two traces indicate the evolution of the disorder values at which $\text{var}(N_{\text{csh}}/N_{\text{sh}})$ takes half the peak value (left, diamonds and right, pentagons). All traces taken together incorporate the evolution of the peak's shape with system size.

VII. EVOLUTION OF FRAGMENTATION WITH SYSTEM SIZE

In an attempt to settle the question whether W_{frag} is finite upon $L \rightarrow \infty$, we compare in Fig. 5(a) the average $N_{\text{csh}}/N_{\text{sh}}$ to the variance for the entire spread of disorder values and system sizes. In the regime of weak to moderate disorder, $N_{\text{csh}}/N_{\text{sh}} \lesssim 1$, we observe a collapse of the data consistent with $\text{var}(N_{\text{csh}}/N_{\text{sh}}) = \sqrt{L}f(N_{\text{csh}}/N_{\text{sh}})$, where $f(x)$ denotes a parameterless function of its argument. The $\sqrt{L}$ rescaling is empirical and is introduced to organize the finite-size data; we do not interpret it as a controlled scaling theory. The master curve hosts families of data points that share the same disorder, W . With L increasing, the data points per family flow along the master curve toward the weak-disorder fixed point $N_{\text{csh}}/N_{\text{sh}} = 1$.

At the strongest disorder, $W > 5$, the family flow seen in Fig. 5(a) is directed opposite, namely, toward the fragmented fixed point at which N_{csh} becomes a set of measure zero within the potential-energy shell. Crucially, as witnessed in the inset of Fig. 5(a), at slightly weaker disorder, a turnaround is visible: The initial flow is directed toward the fragmented fixpoint before it redirects once a system size has been crossed that depends on the disorder strength $L^*(W)$. Moreover, it is only at $L > L^*$ that the flow returns to the master curve $f(x)$; correspondingly, the data for the regime of strong disorder, $0 < N_{\text{csh}}/N_{\text{sh}} \lesssim 0.4$, collapse only partially onto the master curve, namely, after the turnaround.

Being limited to system sizes in our numerics, we can observe the turnaround proper only for disorder values so that $L^*(W) \lesssim 24$, which implies the upper limit $W \approx 4.6$. The clustering of larger- W data at the largest accessible sizes suggests that the turnaround persists beyond this range. The collapse in Fig. 5(a) gives $[\text{var}(N_{\text{csh}}/N_{\text{sh}})]^{1/2} \lesssim 0.161$. Since $0 \leq N_{\text{csh}}/N_{\text{sh}} \leq 1$, the variance is bounded by $1/4$. If the peak reaches this maximal value, one obtains $[\frac{1/4}{\sqrt{L_*}}]^{1/2} \approx 0.161$,

and hence $L_* \approx 93$. At system sizes exceeding L_* , the RG flow observed in Fig. 5(a) is confined to regions either left or right of the maximum, without the possibility to cross. This suggests the following scenario that at $L \gtrsim L_*$ all points with $N_{\text{csh}}/N_{\text{sh}} < 1/2$ fall into the domain of attraction of the fragmented fixed point, while the others flow toward ergodicity.

Since L_* is larger than the system sizes reliably accessible in current computational studies, $L \lesssim 28$, the extrapolation necessarily remains heuristic. Nevertheless, the result is of interest because it permits an extrapolated estimate of W_{frag} . In Fig. 5(b), we follow the evolution of the maximum position of the variance, $W_{\text{var}}(L)$, displayed in Fig. 4, inset, with system size L . It is of interest since one might identify $W_{\text{frag}} \approx W_{\text{var}}(L_*)$. Within the limited set of system sizes available, we have $W_{\text{var}}(L) \sim \ln(L)$ and the extrapolation to L_* yields $W_{\text{frag}} \approx 5$.

VIII. CONCLUSION

We analyze quenches in the one-dimensional disordered t - V model (1) within a Fock-space representation. At the core of our analysis, we introduce the concept of a potential-energy shell, which essentially contains all Fock-space basis states that are energetically accessible within the relaxation dynamics. Crucially, the shell is a property of the quench protocol, which is defined by the Hamiltonian *together* with the initial state, taken to be the Néel state in this work and also other initial states [47].

As one would expect, we observe a close relationship between dynamical observables, such as the sublattice imbalance, and the geometric properties of the shell, i.e., its size (“mass”), connectivity, and spatial extent (measured in units of one spin-flip distance).

(1) The potential-energy-shell concept allows us to explain the peculiar fact that sample-to-sample fluctuations of imbalance relaxations in small samples, $L \lesssim 20$, are very large at

moderate disorder [17]: They are understood as a manifestation of fluctuations of the mass of the shell.

(2) The potential-energy concept implies shell fragmentation above a finite-size scale $W_{\text{frag}}(L)$. This provides a geometric mechanism for the pronounced slowing down of relaxation and the strong sample-to-sample fluctuations observed in the disorder regime associated with MBL studies.

(3) The crossover scale $W_{\text{frag}}(L)$ shifts with increasing L to larger values, which implies the existence of a parameter region that exhibits a turnaround flow driven by the increasing system size. The physical picture behind the flow turning around is the increasing growth (“mass”) of the fragment that contains the initial state. In other words, the number of many-body resonances proliferates with increasing L , providing more and more relaxation channels.

(4) We provide a caveat: While fragmentation and many-body localization are related phenomena, they remain fundamentally different. The full many-body quantum dynamics also includes tunneling between fragments and quantum interference. Thus, the dynamics is not confined to a single fragment. From the perspective of the potential-energy shell, the MBL problem involves competition between localization on a fragmented backbone and delocalization due to tunneling, which makes the MBL problem so interesting and challenging.

(5) Since shell fragmentation is a geometric property of the Fock-space graph, its characteristic scale W_{frag} should not be identified with the MBL critical disorder W_{MBL} , which additionally depends on quantum tunneling and interference effects. It is nevertheless useful to contrast the present Fock-space problem with localization on conventional random regular graphs (RRGs). In the latter, the critical disorder W_{RRG} depends on a connectivity K that is kept fixed in the thermodynamic limit, and the on-site disorder is uncorrelated [56]. The Fock-space graph generated by the local many-body Hamiltonian considered here differs in both respects: Its connectivity

grows with system size, while the Fock-space energies are strongly correlated. These structural differences suggest that the stability and finite-size flow of localization on the present Fock-space graph can differ substantially from those of conventional RRG models [41]. Determining how this distinction ultimately affects the MBL transition requires information beyond that provided by the geometric fragmentation analysis presented here.

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DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

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- [1] M. Schreiber, S. S. Hodgman, P. Bordia, H. P. Lüschen, M. H. Fischer, R. Vosk, E. Altman, U. Schneider, and I. Bloch, Observation of many-body localization of interacting fermions in a quasirandom optical lattice, *Science* **349**, 842 (2015).
 - [2] D. A. Abanin, E. Altman, I. Bloch, and M. Serbyn, *Colloquium: Many-body localization, thermalization, and entanglement*, *Rev. Mod. Phys.* **91**, 021001 (2019).
 - [3] F. Alet and N. Laflorencie, Many-body localization: An introduction and selected topics, *C. R. Phys.* **19**, 498 (2018).
 - [4] P. Sierant, M. Lewenstein, A. Scardicchio, L. Vidmar, and J. Zakrzewski, Many-body localization in the age of classical computing, *Rep. Prog. Phys.* **88**, 026502 (2025).
 - [5] I. V. Gornyi, A. D. Mirlin, and D. G. Polyakov, Interacting electrons in disordered wires: Anderson localization and low- T transport, *Phys. Rev. Lett.* **95**, 206603 (2005).
 - [6] D. M. Basko, I. L. Aleiner, and B. L. Altshuler, Metal insulator transition in a weakly interacting many electron system with localized single particle states, *Ann. Phys.* **321**, 1126 (2006).
 - [7] S. Bera, G. De Tomasi, F. Weiner, and F. Evers, Density propagator for many-body localization: Finite-size effects, transient subdiffusion, and exponential decay, *Phys. Rev. Lett.* **118**, 196801 (2017).
 - [8] F. Weiner, F. Evers, and S. Bera, Slow dynamics and strong finite-size effects in many-body localization with random and quasiperiodic potentials, *Phys. Rev. B* **100**, 104204 (2019).
 - [9] R. K. Panda, A. Scardicchio, M. Schulz, S. R. Taylor, and M. Žnidarič, Can we study the many-body localisation transition? *Europhys. Lett.* **128**, 67003 (2020).
 - [10] M. Kiefer-Emmanouilidis, R. Unanyan, M. Fleischhauer, and J. Sirker, Evidence for unbounded growth of the number entropy in many-body localized phases, *Phys. Rev. Lett.* **124**, 243601 (2020).
 - [11] J. Šuntajs, J. Bonča, T. Prosen, and L. Vidmar, Ergodicity breaking transition in finite disordered spin chains, *Phys. Rev. B* **102**, 064207 (2020).
 - [12] D. Sels and A. Polkovnikov, Dynamical obstruction to localization in a disordered spin chain, *Phys. Rev. E* **104**, 054105 (2021).
 - [13] D.A. Abanin, J.H. Bardarson, G. De Tomasi, S. Gopalakrishnan, V. Khemani, S.A. Parameswaran, F. Pollmann,

- A.C. Potter, M. Serbyn, and R. Vasseur, Distinguishing localization from chaos: Challenges in finite-size systems, *Ann. Phys.* **427**, 168415 (2021).
- [14] P. Sierant and J. Zakrzewski, Challenges to observation of many-body localization, *Phys. Rev. B* **105**, 224203 (2022).
- [15] A. Morningstar, L. Colmenarez, V. Khemani, D. J. Luitz, and D. A. Huse, Avalanches and many-body resonances in many-body localized systems, *Phys. Rev. B* **105**, 174205 (2022).
- [16] D. M. Long, P. J. D. Crowley, V. Khemani, and A. Chandran, Phenomenology of the prethermal many-body localized regime, *Phys. Rev. Lett.* **131**, 106301 (2023).
- [17] F. Evers, I. Modak, and S. Bera, Internal clock of many-body delocalization, *Phys. Rev. B* **108**, 134204 (2023).
- [18] D. Aceituno Chávez, C. Artiago, T. Klein Kvorning, L. Herviou, and J. H. Bardarson, Ultraslow growth of number entropy in an ℓ -bit model of many-body localization, *Phys. Rev. Lett.* **133**, 126502 (2024).
- [19] J. Colbois, F. Alet, and N. Laflorencie, Statistics of systemwide correlations in the random-field XXZ chain: Importance of rare events in the many-body localized phase, *Phys. Rev. B* **110**, 214210 (2024).
- [20] O. S. Barišić, J. Kokalj, I. Balog, and P. Prelovšek, Dynamical conductivity and its fluctuations along the crossover to many-body localization, *Phys. Rev. B* **94**, 045126 (2016).
- [21] V. Khemani, S. P. Lim, D. N. Sheng, and D. A. Huse, Critical properties of the many-body localization transition, *Phys. Rev. X* **7**, 021013 (2017).
- [22] E. V. H. Doggen, F. Schindler, K. S. Tikhonov, A. D. Mirlin, T. Neupert, D. G. Polyakov, and I. V. Gornyi, Many-body localization and delocalization in large quantum chains, *Phys. Rev. B* **98**, 174202 (2018).
- [23] S. Nandy, F. Evers, and S. Bera, Dephasing in strongly disordered interacting quantum wires, *Phys. Rev. B* **103**, 085105 (2021).
- [24] The Fock-space perspective has been successfully adopted before, for example, in understanding electron lifetimes in interacting quantum dots by mapping to a single-particle localization problem in Fock space [25]. More recently, significant progress has been made by approximating Fock space as a Bethe lattice (a tree without loops) [26–29] or by the random regular graph [30–35]. Also, the MBL transition has been investigated from the Fock-space perspective [36–42], including certain aspects of dynamics [43–46].
- [25] B. L. Altshuler, Y. Gefen, A. Kamenev, and L. S. Levitov, Quasiparticle lifetime in a finite system: A nonperturbative approach, *Phys. Rev. Lett.* **78**, 2803 (1997).
- [26] K. S. Tikhonov and A. D. Mirlin, Fractality of wave functions on a Cayley tree: Difference between tree and locally treelike graph without boundary, *Phys. Rev. B* **94**, 184203 (2016).
- [27] M. Sonner, K. S. Tikhonov, and A. D. Mirlin, Multifractality of wave functions on a Cayley tree: From root to leaves, *Phys. Rev. B* **96**, 214204 (2017).
- [28] G. Biroli, A. K. Hartmann, and M. Tarzia, Critical behavior of the Anderson model on the Bethe lattice via a large-deviation approach, *Phys. Rev. B* **105**, 094202 (2022).
- [29] T. Rizzo and M. Tarzia, The localized phase of the Anderson model on the Bethe lattice, *arXiv:2406.18748*.
- [30] I. García-Mata, O. Giraud, B. Georgeot, J. Martin, R. Dubertrand, and G. Lemarié, Scaling theory of the Anderson transition in random graphs: Ergodicity and universality, *Phys. Rev. Lett.* **118**, 166801 (2017).
- [31] S. Bera, G. De Tomasi, I. M. Khaymovich, and A. Scardicchio, Return probability for the Anderson model on the random regular graph, *Phys. Rev. B* **98**, 134205 (2018).
- [32] I. García-Mata, J. Martin, R. Dubertrand, O. Giraud, B. Georgeot, and G. Lemarié, Two critical localization lengths in the Anderson transition on random graphs, *Phys. Rev. Res.* **2**, 012020(R) (2020).
- [33] G. De Tomasi, S. Bera, A. Scardicchio, and I. M. Khaymovich, Subdiffusion in the Anderson model on the random regular graph, *Phys. Rev. B* **101**, 100201(R) (2020).
- [34] K. Tikhonov and A. Mirlin, From Anderson localization on random regular graphs to many-body localization, *Ann. Phys.* **435**, 168525 (2021).
- [35] P. Sierant, M. Lewenstein, and A. Scardicchio, Universality in Anderson localization on random graphs with varying connectivity, *SciPost Phys.* **15**, 045 (2023).
- [36] N. Macé, F. Alet, and N. Laflorencie, Multifractal scalings across the many-body localization transition, *Phys. Rev. Lett.* **123**, 180601 (2019).
- [37] S. Roy, D. E. Logan, and J. T. Chalker, Exact solution of a percolation analog for the many-body localization transition, *Phys. Rev. B* **99**, 220201(R) (2019).
- [38] F. Pietracaprina and N. Laflorencie, Hilbert-space fragmentation, multifractality, and many-body localization, *Ann. Phys.* **435**, 168502 (2021).
- [39] G. De Tomasi, I. M. Khaymovich, F. Pollmann, and S. Warzel, Rare thermal bubbles at the many-body localization transition from the Fock space point of view, *Phys. Rev. B* **104**, 024202 (2021).
- [40] J. Sutradhar, S. Ghosh, S. Roy, D. E. Logan, S. Mukerjee, and S. Banerjee, Scaling of the Fock-space propagator and multifractality across the many-body localization transition, *Phys. Rev. B* **106**, 054203 (2022).
- [41] T. Scoquart, I. V. Gornyi, and A. D. Mirlin, Role of Fock-space correlations in many-body localization, *Phys. Rev. B* **109**, 214203 (2024).
- [42] S. Roy and D. E. Logan, The Fock-space landscape of many-body localisation, *J. Phys.: Condens. Matter* **37**, 073003 (2025).
- [43] I. Creed, D. E. Logan, and S. Roy, Probability transport on the Fock space of a disordered quantum spin chain, *Phys. Rev. B* **107**, 094206 (2023).
- [44] G. Biroli, A. K. Hartmann, and M. Tarzia, Large-deviation analysis of rare resonances for the many-body localization transition, *Phys. Rev. B* **110**, 014205 (2024).
- [45] T. Scoquart, I. V. Gornyi, and A. D. Mirlin, Scaling of many-body localization transitions: Quantum dynamics in Fock space and real space, *Phys. Rev. B* **112**, 064203 (2025).
- [46] Z.-H. Sun, Y.-Y. Wang, J. Cui, H. Fan, and M. Heyl, Characterizing dynamical criticality of many-body localization transitions from a Fock-space perspective, *Phys. Rev. B* **111**, 094210 (2025).
- [47] See Supplemental Material at <https://link.aps.org/supplemental/10.1103/hvqr-6n3b> for the dynamical correlation in the fragmented regime, the noninteracting $V = 0$ data, the fragmented landscape for other initial states, and a discussion of fragmentation in a power-law interacting model, which includes Refs. [57–67] in addition to other reference already mentioned above.

- [48] J. Niedda, G. B. Testasecca, G. Magnifico, F. Balducci, C. Vanoni, and A. Scardicchio, Renormalization group analysis of the many-body localization transition in the random-field XXZ chain, *Phys. Rev. B* **112**, 144201 (2025).
- [49] Y. Prasad and A. Garg, Initial state dependent dynamics across the many-body localization transition, *Phys. Rev. B* **105**, 214202 (2022).
- [50] The variance of the on-site energies does not incorporate a term resulting from the kinetic energy because it reflects the distribution of eigenvalues of $\hat{H}'$; indeed, the variance of the on-site energies, $\langle b | \hat{H}^2 - E_b^2 | b \rangle$, averaged over the Fock space is given by $\Delta_{\mathcal{P}}^2 + \frac{1}{2} \Delta_{\text{Néel}}^2$.
- [51] E. J. Torres-Herrera, G. De Tomasi, M. Schiulaz, F. Pérez-Bernal, and L. F. Santos, Self-averaging in many-body quantum systems out of equilibrium: Approach to the localized phase, *Phys. Rev. B* **102**, 094310 (2020).
- [52] Also in the opposite case, $N_{\text{sh}}/N_{\text{FS}} \ll 1$, there is not necessarily a direct connection between the mass of the shell and the relaxation behavior. Even if the shell has a mass of measure zero, the quench that it carries can be ergodic if the shell is space filling; i.e., some of its sites can be found in any region of Fock space.
- [53] J. Cardy, Effect of random impurities on fluctuation-driven first-order transitions, *J. Phys. A: Math. Gen.* **29**, 1897 (1996).
- [54] J. Cardy, Quenched randomness at first-order transitions, *Phys. A: Stat. Mech. Appl.* **263**, 215 (1999).
- [55] Q. Zhu, X. Wan, R. Narayanan, J. A. Hoyos, and T. Vojta, Emerging criticality in the disordered three-color Ashkin-Teller model, *Phys. Rev. B* **91**, 224201 (2015).
- [56] J.-N. Herre, J. F. Karcher, K. S. Tikhonov, and A. D. Mirlin, Ergodicity-to-localization transition on random regular graphs with large connectivity and in many-body quantum dots, *Phys. Rev. B* **108**, 014203 (2023).
- [57] A. D. Mirlin and F. Evers, Multifractality and critical fluctuations at the Anderson transition, *Phys. Rev. B* **62**, 7920 (2000).
- [58] I. A. Maceira and A. M. Läuchli, Thermalization dynamics in closed quantum many body systems a precision large scale exact diagonalization study, [arXiv:2409.18863](https://arxiv.org/abs/2409.18863).
- [59] S. Roy, J. T. Chalker, and D. E. Logan, Percolation in Fock space as a proxy for many-body localization, *Phys. Rev. B* **99**, 104206 (2019).
- [60] P. Prelovšek, M. Mierzejewski, J. Krsnik, and O. S. Barišić, Many-body localization as a percolation phenomenon, *Phys. Rev. B* **103**, 045139 (2021).
- [61] K. S. Tikhonov, A. D. Mirlin, and M. A. Skvortsov, Anderson localization and ergodicity on random regular graphs, *Phys. Rev. B* **94**, 220203 (2016).
- [62] C. Vanoni, B. L. Altshuler, V. E. Kravtsov, and A. Scardicchio, Renormalization group analysis of the Anderson model on random regular graphs, *Proc. Natl. Acad. Sci. USA* **121** (2024).
- [63] B. L. Altshuler, V. E. Kravtsov, A. Scardicchio, P. Sierant, and C. Vanoni, Renormalization group for anderson localization on high-dimensional lattices, *Proc. Natl. Acad. Sci. USA* **122** (2025).
- [64] G. De Tomasi, D. Hetterich, P. Sala, and F. Pollmann, Dynamics of strongly interacting systems: From Fock-space fragmentation to many-body localization, *Phys. Rev. B* **100**, 214313 (2019).
- [65] A. L. Burin, Many-body delocalization in a strongly disordered system with long-range interactions: Finite-size scaling, *Phys. Rev. B* **91**, 094202 (2015).
- [66] K. S. Tikhonov and A. D. Mirlin, Many-body localization transition with power-law interactions: Statistics of eigenstates, *Phys. Rev. B* **97**, 214205 (2018).
- [67] F. Evers and A. D. Mirlin, Anderson transitions, *Rev. Mod. Phys.* **80**, 1355 (2008).