

Quasiparticle properties of stationary light polaritons in atoms

U-Shin Kim^{*} and Yoon-Ho Kim[†]

Department of Physics, Pohang University of Science and Technology (POSTECH), Pohang 37673, Korea



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We theoretically analyze the quasiparticle properties of stationary light polariton (SLP) modes in atoms via perturbative approaches of effective atom-light interaction Hamiltonian. First, we explicitly show the five SLP eigenmodes of the Hilbert space of the SLP system. Then, the quasiparticle properties, such as group velocities and effective masses of SLPs, are investigated under various parametric conditions, which include the emergence of negative effective mass and effective mass singularity. Finally, we derive the intrinsic lifetimes of each SLP mode, which features a complementary relation among the lifetimes of SLP modes. Our results offer quantitative analysis of the quasiparticle properties in Hilbert space spanned by the SLP modes, and it is expected to contribute to SLP-based atom-light interaction applications, such as photonic quantum gates, optical simulation of many-body physics, and exotic dispersion engineering.

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I. INTRODUCTION

Coherent interactions between light and matter enable particlelike excitations consisting of hybrid light-matter components called polaritons. Specifically, stationary light polaritons (SLPs), the excitations from an atomic ensemble dressed by two counterpropagating electromagnetic fields, offer a unique property that retains their electromagnetic field nature even at a zero group velocity inside the atomic ensemble [1–5]. Due to the cavity-free trapping of electromagnetic field inside an atomic ensemble, SLP has brought forth novel quantum optical applications, such as cavity-free nonlinear optics [6,7], all-optical switchings [3], and photonic quantum gates [8,9]. Further, the SLP approach offers a new possibility for the optical simulation of quantum many-body physics, such as the crystallization of interacting photons [10], interacting relativistic theories [11], and high-temperature Bose-Einstein condensates (BEC) [12].

In this regard, the quantitative understanding of the quasiparticle properties of SLP modes is one of the prerequisites for realizing those SLP-based applications. So far, several studies have reported the dispersion properties of the SLP system, including conditional reflectivity [8,13], quadratic dispersion relation [12,14], and its effective mass [10,15]. However, the studies on the properties of SLPs have still been limited to a specific SLP mode and lack of explicit quantum state description of polariton mode, which has been one of the bottlenecks to describing the critical properties in the entire Hilbert space spanned by the SLP modes.

Here, we theoretically analyze the quasiparticle properties of SLP modes via perturbative approaches of effective atom-light interaction Hamiltonian. We first explicitly derive five eigenmodes of SLP and then present the group velocities and effective masses of each SLP mode, including negative effective masses and effective mass singularities. Finally, we investigate the intrinsic lifetimes of each SLP mode, which features a complementary relation of the lifetimes among the SLP eigenmodes. As our results offer explicit dispersion and decay properties for all SLP eigenmodes, our work is expected to contribute to SLP-based atom-light interaction applications, such as photonic quantum gates [8,9], optical simulations of many-body physics [11,16], exotic dispersion engineering [17,18], and dispersive quantum shock waves [19–21].

II. EIGENMODES OF SLPs

Let us first consider a one-dimensional atomic ensemble with a Λ -type level structure as shown in Fig. 1(a). When a slowly-varying probe field $\hat{\mathcal{E}}^+$ is injected into the atomic ensemble dressed by a coupling laser Ω_+ , i.e., electromagnetically induced transparency (EIT) scheme, the effective atom-light interaction Hamiltonian is described under the rotating-wave approximation (RWA) as [22–24]

$$\hat{\mathcal{H}} \simeq - \sum_j (\hbar g \sqrt{N} \hat{\mathcal{E}}^+ |e_j\rangle \langle g_j| + \hbar \Omega_+ |e_j\rangle \langle s_j|) + \text{H.c.}, \quad (1)$$

where $|g_j\rangle$, $|s_j\rangle$, and $|e_j\rangle$ represent the ground state, spin state, and excited state of the j -th atom as shown in Fig. 1(a), respectively. N corresponds to the number of atoms, and g corresponds to the atom-photon coupling constant. Under the ground-state approximation, the temporal evolutions of the collective atomic operators $\hat{\sigma}_{gs}$ and $\hat{\sigma}_{ge}$, defined as $\hat{\sigma}_{ge} = \frac{1}{\sqrt{N}} \sum_j |g_j\rangle \langle e_j|$ and $\hat{\sigma}_{gs} = \frac{1}{\sqrt{N}} \sum_j |g_j\rangle \langle s_j|$, are governed by Heisenberg equations as [15,25]

$$\dot{\hat{\sigma}}_{gs} = -\gamma_{gs} \hat{\sigma}_{gs} + i\Omega_+^* \hat{\sigma}_{ge}, \quad (2)$$

$$\dot{\hat{\sigma}}_{ge} = -\gamma_{ge} \hat{\sigma}_{ge} + ig\sqrt{N} \hat{\mathcal{E}}^+ + i\Omega_+ \hat{\sigma}_{gs}, \quad (3)$$

^{*}Contact author: usinkimphysics@gmail.com

[†]Contact author: yoonho72@gmail.com

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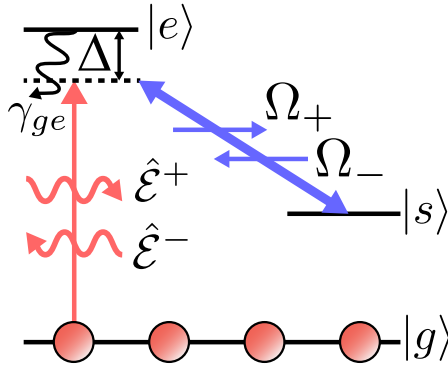


FIG. 1. The Λ -scheme atomic level structure for the generation of stationary light polaritons (SLPs). $\mathcal{E}^\pm$ correspond to two counterpropagating electromagnetic field components of probe while $\Omega_\pm$ correspond to the Rabi frequencies of two counterpropagating coupling lasers. All optical fields are red-detuned by Δ from $|e\rangle$, which has a dephasing rate of γ_{ge} .

where γ_{ge} and γ_{gs} are the dephasing rates for $|g\rangle \leftrightarrow |e\rangle$ transition and $|g\rangle \leftrightarrow |s\rangle$ transition, respectively. Also, the spatiotemporal propagation of probe field can be described in the slowly-varying envelop approximation as [22]

$$(\partial_t + c_0 \partial_z) \hat{\mathcal{E}}^+ = ig\sqrt{N} \hat{\sigma}_{ge}^+, \quad (4)$$

where c_0 is the speed of light in vacuum.

Let us now introduce two counterpropagating probe fields ($\hat{\mathcal{E}}^+$ and $\hat{\mathcal{E}}^-$) in the atomic ensemble dressed by two counterpropagating coupling lasers (Ω_+ and Ω_-) as shown in Fig. 1, i.e., SLP scheme. The dynamics of SLP system can be described by the substitution of relevant parameters as [26–28]

$$\hat{\mathcal{E}}^+ \rightarrow \hat{\mathcal{E}}^+ e^{ik_0 z} + \hat{\mathcal{E}}^- e^{-ik_0 z}, \quad (5)$$

$$\Omega_+ \rightarrow \Omega_+ e^{ik_0 z} + \Omega_- e^{-ik_0 z}, \quad (6)$$

$$\hat{\sigma}_{ge} \rightarrow \hat{\sigma}_{ge}^+ e^{ik_0 z} + \hat{\sigma}_{ge}^- e^{-ik_0 z}, \quad (7)$$

$$\hat{\sigma}_{gs} \rightarrow \hat{\sigma}_{gs}^0 + \hat{\sigma}_{gs}^{+-} e^{2ik_0 z} + \hat{\sigma}_{gs}^{++} e^{-2ik_0 z}, \quad (8)$$

where k_0 is the carrier wave vector of probe fields and coupling lasers, which are assumed to be approximately same in the current Λ -level structure. $\hat{\mathcal{E}}^\pm$ correspond to the forward and backward propagating probe fields, respectively, and $\Omega_\pm$ correspond to the forward coupling (FWC) and backward coupling (BWC) lasers, respectively. The collective atomic excitation $\hat{\sigma}_{gs}$ in Eq. (8) is now spanned into three components with wave vectors of 0, $+2k_0$, and $-2k_0$, where $\hat{\sigma}_{gs}^0$ corresponds to the two-photon excitation by co-propagating probe fields and coupling lasers, whereas $\hat{\sigma}_{gs}^{+-}$ and $\hat{\sigma}_{gs}^{++}$ correspond to the two-photon excitations by counterpropagating probe fields and coupling lasers. The excitations with a wave

vector higher than $2k_0$ exhibit faster decays than the above terms by atomic motions due to their fast spatial oscillation, thus are neglected here [29]. Then, the temporal evolution of the collective atomic operators in Eqs. (2) and (3) should be modified as

$$\dot{\hat{\sigma}}_{ge}^+ = -\gamma_{ge}^+ \hat{\sigma}_{ge}^+ + ig\sqrt{N} \hat{\mathcal{E}}^+ + i\Omega_+ \hat{\sigma}_{gs}^0 + i\Omega_- \hat{\sigma}_{gs}^{+-}, \quad (9)$$

$$\dot{\hat{\sigma}}_{ge}^- = -\gamma_{ge}^- \hat{\sigma}_{ge}^- + ig\sqrt{N} \hat{\mathcal{E}}^- + i\Omega_- \hat{\sigma}_{gs}^0 + i\Omega_+ \hat{\sigma}_{gs}^{+-}, \quad (10)$$

$$\dot{\hat{\sigma}}_{gs}^0 = i\Omega_+^* \hat{\sigma}_{ge}^+ + i\Omega_-^* \hat{\sigma}_{ge}^- - \gamma_{gs}^0 \hat{\sigma}_{gs}^0, \quad (11)$$

$$\dot{\hat{\sigma}}_{gs}^{+-} = i\Omega_+^* \hat{\sigma}_{ge}^- - \gamma_{gs}^{+-} \hat{\sigma}_{gs}^{+-}, \quad (12)$$

$$\dot{\hat{\sigma}}_{gs}^{++} = i\Omega_-^* \hat{\sigma}_{ge}^+ - \gamma_{gs}^{++} \hat{\sigma}_{gs}^{++}, \quad (13)$$

where $\gamma_{ge}^\pm$, γ_{gs}^0 , γ_{gs}^{+-} , and γ_{gs}^{++} correspond to the dephasing rates of $\hat{\sigma}_{ge}^\pm$, $\hat{\sigma}_{gs}^0$, $\hat{\sigma}_{gs}^{+-}$, and $\hat{\sigma}_{gs}^{++}$, respectively.

Here, $\hat{\sigma}_{gs}^{+-}$ and $\hat{\sigma}_{gs}^{++}$, called the higher-order Raman coherence, are known to hinder the formation of SLPs, and thus are needed to be suppressed to study the quasiparticle properties of SLP modes [13,27,30]. To suppress the emergence of higher-order Raman coherence in a Λ -scheme, one can employ a detuning δ between two coupling lasers over the EIT bandwidth $\Delta\omega_{\text{EIT}}$ ($\Delta\omega_{\text{EIT}} \ll \delta$) [31–33]. Note that in a high optical depth (OD) limit given as $\frac{1}{4}(\Omega_+/\gamma_{ge})^4 \ll \text{OD}$, $\Delta\omega_{\text{EIT}} \ll \delta \ll \gamma_{ge}$ is achievable [34], and thus, we can assume $\gamma_{ge}^+ \approx \gamma_{ge}^-$ (unless, γ_{ge}^- should be described as $\gamma_{ge}^- = \gamma_{ge}^+ - i\delta$). For instance, the relevant OD regime is given as $2.3 \ll \text{OD}$ under $\Omega_+ = 2\pi \times 5$ MHz and $\gamma_{ge} = 2\pi \times 2.88$ MHz, which corresponds to the dephasing rate for D1 line of ^{87}Rb atom. Accordingly, we assume the higher-order Raman coherences are fully suppressed in this work, i.e., $\hat{\sigma}_{gs}^{+-} = \hat{\sigma}_{gs}^{++} = 0$. Note that the suppression of higher-order Raman coherence also justifies the truncation of $\hat{\sigma}_{gs}$ components with wave vector higher than $2k_0$.

The spatiotemporal propagation of probe fields in SLP scheme should also be modified from Eq. (4) into

$$(\partial_t + c_0 \partial_z) \hat{\mathcal{E}}^+ = ig\sqrt{N} \hat{\sigma}_{ge}^+, \quad (14)$$

$$(\partial_t - c_0 \partial_z) \hat{\mathcal{E}}^- = ig\sqrt{N} \hat{\sigma}_{ge}^-. \quad (15)$$

Equations (14) and (15) can be modified further into temporal evolutions by introducing the wave number k of the slowly-varying envelop of probe fields ($\partial_z \hat{\mathcal{E}}^\pm = \pm ik \hat{\mathcal{E}}^\pm$) as

$$\dot{\hat{\mathcal{E}}}^+ = -i(c_0 k \hat{\mathcal{E}}^+ - g\sqrt{N} \hat{\sigma}_{ge}^+), \quad (16)$$

$$\dot{\hat{\mathcal{E}}}^- = -i(-c_0 k \hat{\mathcal{E}}^- - g\sqrt{N} \hat{\sigma}_{ge}^-). \quad (17)$$

Then, the temporal evolution of the relevant operators describing SLP system can be represented in a single matrix dynamics, which is equivalent to the time-dependent Schrödinger equation (TDSE) with $\hbar = 1$, as

$$i\partial_t \begin{bmatrix} \hat{\mathcal{E}}^+ \\ \hat{\mathcal{E}}^- \\ \hat{\sigma}_{gs}^0 \\ \hat{\sigma}_{ge}^+ \\ \hat{\sigma}_{ge}^- \end{bmatrix} = \begin{bmatrix} kc_0 & 0 & 0 & -g\sqrt{N} & 0 \\ 0 & -kc_0 & 0 & 0 & -g\sqrt{N} \\ 0 & 0 & 0 & -\Omega_+^* & -\Omega_-^* \\ -g\sqrt{N} & 0 & -\Omega_+ & -i\gamma_{ge} & 0 \\ 0 & -g\sqrt{N} & -\Omega_- & 0 & -i\gamma_{ge} \end{bmatrix} \begin{bmatrix} \hat{\mathcal{E}}^+ \\ \hat{\mathcal{E}}^- \\ \hat{\sigma}_{gs}^0 \\ \hat{\sigma}_{ge}^+ \\ \hat{\sigma}_{ge}^- \end{bmatrix}, \quad (18)$$

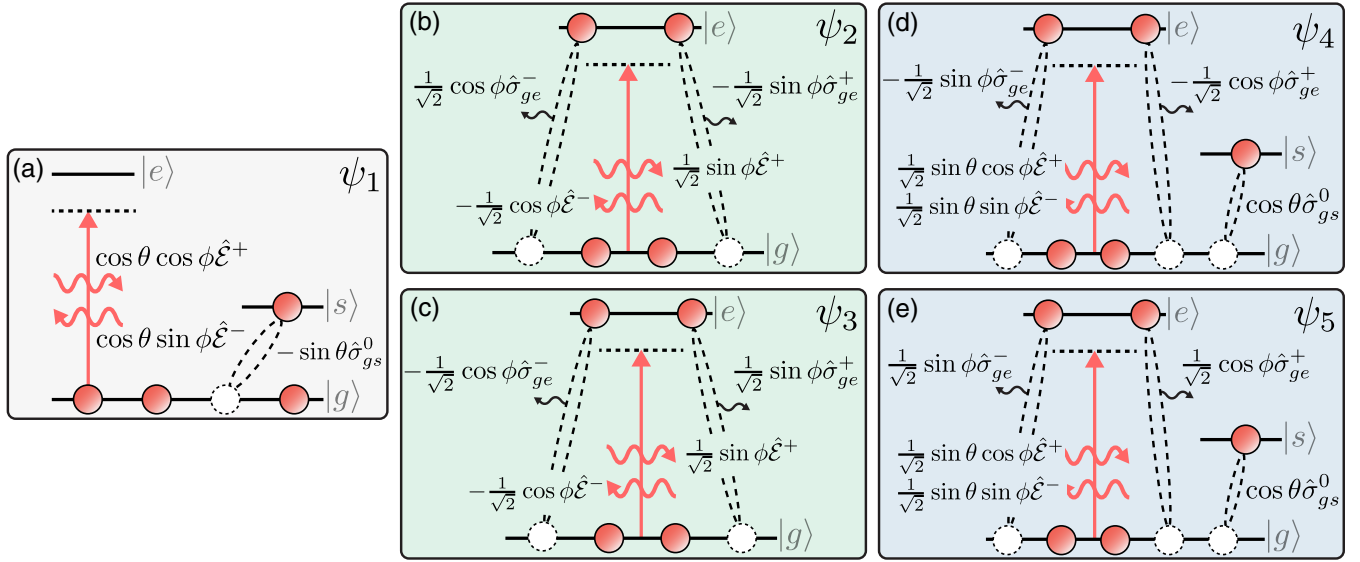


FIG. 2. The pictorial representation of the five eigenmodes in the SLP system, corresponding to Eqs. (20)–(24). (a) The stationary light dark-state polariton (SL-DSP; ψ_1), which exhibits a decay-free long lifetime. (b)–(e) The stationary light bright-state polaritons (SL-BSPs; ψ_2 to ψ_5), which exhibit limited $1/\epsilon$ lifetimes to $1/\gamma$.

where γ_{gs}^0 is assumed to be zero, i.e., the dipole-forbidden transition, and $\gamma_{ge}^+ = \gamma_{ge}^- = \gamma_{ge}$. Here, SLP system can be described as polariton modes under the adiabatic conditions of $\Omega_{\text{eff}} T_p \gg 1$ and $L_p \gg c_0/(g\sqrt{N})$, where $\Omega_{\text{eff}} = \sqrt{\Omega^2 + g^2 N}$, and T_p and L_p correspond to the temporal duration of probe pulse and characteristic length scale of polariton in the atomic ensemble, respectively [23,28]. Further, under $\Omega \gg \Delta$ and $\Omega \gg kc_0$ regimes, the effective Hamiltonian can be decomposed into three matrices: the unperturbed Hamiltonian describing the SLP eigenmodes, the perturbation matrix describing the dispersion of the polaritons ($\pm kc_0$), and the perturbation matrix describing the decay of the polaritons (γ_{ge}).

First, the unperturbed effective Hamiltonian of SLP system can be presented from the TDSE shown in Eq. (18) as

$$\hat{H}_0 = \begin{bmatrix} 0 & 0 & 0 & -g\sqrt{N} & 0 \\ 0 & 0 & 0 & 0 & -g\sqrt{N} \\ 0 & 0 & 0 & -\Omega_+^* & -\Omega_-^* \\ -g\sqrt{N} & 0 & -\Omega_+ & 0 & 0 \\ 0 & -g\sqrt{N} & -\Omega_- & 0 & 0 \end{bmatrix}. \quad (19)$$

The unperturbed effective Hamiltonian $\hat{H}_0$ now describes the SLP eigenmodes, which constitute the Hilbert space of SLP system. By introducing two mixing angles defined as $\tan \theta = g\sqrt{N}/\Omega$ and $\tan \phi = \Omega_-/\Omega_+$, where $\Omega^2 = \Omega_+^2 + \Omega_-^2$, the quantum states describing the five orthonormal quasiparticle modes of SLP system can be explicitly denoted in the basis of $[\hat{\mathcal{E}}^+, \hat{\mathcal{E}}^-, \hat{\sigma}_{gs}^0, \hat{\sigma}_{ge}^+, \hat{\sigma}_{ge}^-]^\top$ as

$$\psi_1 = [\cos \theta \cos \phi, \cos \theta \sin \phi, -\sin \theta, 0, 0]^\top, \quad (20)$$

$$\psi_2 = \frac{1}{\sqrt{2}}[\sin \phi, -\cos \phi, 0, -\sin \phi, \cos \phi]^\top, \quad (21)$$

$$\psi_3 = \frac{1}{\sqrt{2}}[\sin \phi, -\cos \phi, 0, \sin \phi, -\cos \phi]^\top, \quad (22)$$

$$\psi_4 = \frac{1}{\sqrt{2}}[\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta, -\cos \phi, -\sin \phi]^\top, \quad (23)$$

$$\psi_5 = \frac{1}{\sqrt{2}}[\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta, \cos \phi, \sin \phi]^\top, \quad (24)$$

which satisfies the Bosonic commutation relations under the weak-probe approximation [22]. The eigenmode of the SLP system ψ_1 , illustrated in Fig. 2(a), corresponds to the dark-state solution of the system, which can be called the stationary light dark-state polariton (SL-DSP) [12,28,31–33]. While the quantum state of SL-DSP is not coupled to either $\hat{\sigma}_{ge}^\pm$, which results into a decay-free long lifetime, the other four quasiparticle modes, the stationary light bright-state polaritons (SL-BSPs), illustrated in Figs. 2(b)–2(e), have the quantum states where the half of their population is directly coupled to $\hat{\sigma}_{ge}^\pm$, leading to finite lifetimes of the states.

To quantitatively analyze the physical properties of each quasiparticle mode, we now introduce the total energy ω_i^{tot} of ψ_i as $\omega_i^{\text{tot}} = \omega_i^{(0)} + \omega_i^{(1)} + \omega_i^{(2)}$, where $\omega_i^{(0)}$, $\omega_i^{(1)}$, and $\omega_i^{(2)}$ correspond to the zeroth-, first-, and second-order energy correction for kc_0 given in $\Omega \gg kc_0$ regime, respectively. First, the intrinsic eigenenergy and lifetime of each quasiparticle mode can be analyzed by $\omega_i^{(0)}$ given by the perturbation matrix given from Eq. (18) as

$$\hat{D} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -i\gamma_{ge} & 0 \\ 0 & 0 & 0 & 0 & -i\gamma_{ge} \end{bmatrix}, \quad (25)$$

where $\gamma_{ge} = \gamma - i\Delta$, γ is the dephasing rate of $|e\rangle$, and Δ is the one-photon red-detuning of $\hat{\mathcal{E}}^\pm$ while the two-photon detuning is kept to 0 as shown in Fig. 1(a). Then, the corresponding energy of quasiparticle mode ψ_i up to the first-order of $\hat{\mathcal{D}}$ (and zeroth-order for kc_0), given as $\omega_i^{(0)} = \langle \psi_i | (\hat{\mathcal{H}}_0 + \hat{\mathcal{D}}) | \psi_i \rangle$, can be deduced into ($\hbar = 1$)

$$\omega_1^{(0)} = 0, \quad (26)$$

$$\omega_2^{(0)} = \left(\Omega \tan \theta - \frac{1}{2} \Delta \right) - \frac{i}{2} \gamma, \quad (27)$$

$$\omega_3^{(0)} = -\left(\Omega \tan \theta + \frac{1}{2} \Delta \right) - \frac{i}{2} \gamma, \quad (28)$$

$$\omega_4^{(0)} = \left(\Omega \sec \theta - \frac{1}{2} \Delta \right) - \frac{i}{2} \gamma, \quad (29)$$

$$\omega_5^{(0)} = -\left(\Omega \sec \theta + \frac{1}{2} \Delta \right) - \frac{i}{2} \gamma, \quad (30)$$

where $\omega_i^{(0)}$ is the corresponding complex energy of the quasiparticle mode ψ_i . One can find that the energy spectrums of SL-BSPs have finite bandwidths of $\gamma/2$ from Eqs. (27)–(30), which limit the $1/e$ lifetime of the states by $1/\gamma$. In contrast, SL-DSP does not have the energy bandwidth for the state as shown in Eq. (26), and thus has a decay-free long lifetime.

Then, in the same manner with $\hat{\mathcal{D}}$, the perturbation matrix describing the dispersion of each quasiparticle mode is given from Eq. (18) as

$$\hat{\mathcal{V}} = \begin{bmatrix} kc_0 & 0 & 0 & 0 & 0 \\ 0 & -kc_0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \quad (31)$$

The dispersion relations of the quasiparticle modes given by $\hat{\mathcal{V}}$ offer crucial physical quantities of SLP system, including the group velocities and effective masses. Let us first discuss the group velocities of each quasiparticle mode from the first-order correction for $\hat{\mathcal{V}}$, which is given as $\omega_i^{(1)} = \langle \psi_i | \hat{\mathcal{V}} | \psi_i \rangle$ for ψ_i . Then, the first-order energy corrections for each mode are given as

$$\omega_1^{(1)} = kc_0 \cos^2 \theta \cos 2\phi, \quad (32)$$

$$\omega_2^{(1)} = -\frac{1}{2} kc_0 \cos 2\phi, \quad (33)$$

$$\omega_3^{(1)} = -\frac{1}{2} kc_0 \cos 2\phi, \quad (34)$$

$$\omega_4^{(1)} = \frac{1}{2} kc_0 \sin^2 \theta \cos 2\phi, \quad (35)$$

$$\omega_5^{(1)} = \frac{1}{2} kc_0 \sin^2 \theta \cos 2\phi, \quad (36)$$

where $\omega_i^{(1)}$ is the corresponding first-order correction of the quasiparticle mode ψ_i for $\hat{\mathcal{V}}$. The group velocities of each

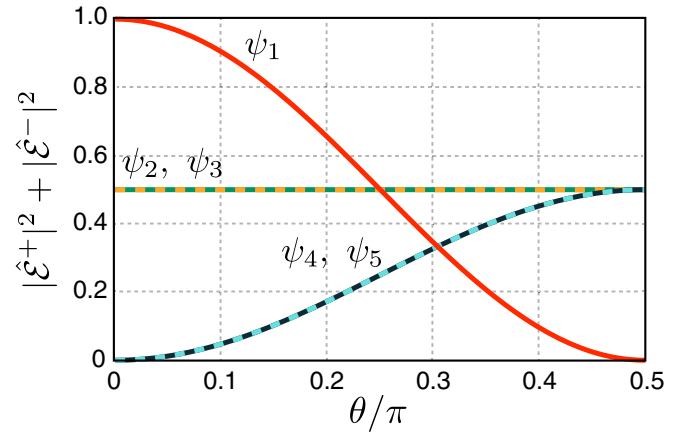


FIG. 3. The electric field components of SLP quasiparticles when $\tan \phi = 1$, i.e., zero-group-velocities for all quasiparticle modes. The red solid line corresponds to stationary light dark-state polariton (SL-DSP) ψ_1 , and the four dotted lines correspond to four stationary bright-state polaritons (SL-BSPs).

mode are then given from the relation $v_g = \partial \omega / \partial k$ as

$$v_{g,1} = c_0 \cos^2 \theta \cos 2\phi, \quad (37)$$

$$v_{g,2} = -\frac{1}{2} c_0 \cos 2\phi, \quad (38)$$

$$v_{g,3} = -\frac{1}{2} c_0 \cos 2\phi, \quad (39)$$

$$v_{g,4} = \frac{1}{2} c_0 \sin^2 \theta \cos 2\phi, \quad (40)$$

$$v_{g,5} = \frac{1}{2} c_0 \sin^2 \theta \cos 2\phi, \quad (41)$$

where $v_{g,i}$ is the corresponding group velocity of the quasiparticle mode ψ_i . One can find that in the physical parameter space of $0 \leq \{\theta, \phi\} \leq \pi/2$, ψ_2 and ψ_3 always propagates along the opposite direction to ψ_1 , ψ_4 , and ψ_5 . Also, when FWC and BWC are balanced ($\Omega_+ = \Omega_-$), i.e., $\phi = \pi/4$, the group velocities of the all quasiparticle modes become zero.

Figure 3 shows the electromagnetic field populations of each quasiparticle mode plotted against θ/π at a zero-group-velocity, i.e., $\phi = \pi/4$. Note that θ is defined as $\tan \theta = g\sqrt{N}/\Omega$ and thus can be manipulated by the optical power of FWC and BWC. One can verify that at an arbitrary θ , all the quasiparticle modes still contain finite electromagnetic field populations, where the population is decreasing for ψ_1 , increasing for ψ_4 and ψ_5 , and kept constant for ψ_2 and ψ_3 through θ . The unique property of SLP system that retains finite electromagnetic field populations at a zero group velocity gives a stark contrast to EIT scheme, where the EIT DSP is completely mapped into the atomic coherence without electromagnetic field population at the zero group velocity [22]. The cavity-free trapping of the electromagnetic field inside an atomic ensemble in the SLP system offers a physical analogy to cavity quantum electrodynamics with enhanced scalability and controllability, opening novel approaches to the atom-light interactions, such as cavity-free quantum optics [6,7], all-optical switchings [3], photon blockades [13], and photonic quantum gates [8,9].

III. EFFECTIVE MASSES OF SLPS

Let us now discuss the effective masses of each polariton mode given from the second-order energy correction for $\hat{\mathcal{V}}$, which is given as $\sum_{i \neq j} |\langle \psi_j | \hat{\mathcal{V}} | \psi_i \rangle|^2 / (\omega_i^{(0)} - \omega_j^{(0)})$ for ψ_i . The total energies of each polariton mode spanned up to $(kc_0)^2$ are given as

$$\omega_1^{\text{tot}}(k)/\Omega = -\frac{\cos^2 \theta}{2 \tan^2 \theta} \left(\frac{\Delta}{\Omega} \right) \left(\frac{kc_0}{\Omega} \right)^2, \quad (42)$$

$$\omega_2^{\text{tot}}(k)/\Omega = \tan \theta - \frac{1}{2} \left(\frac{\Delta}{\Omega} \right) + \frac{1}{2} \left(\frac{kc_0}{\Omega} \right)^2 \left[\frac{\cos^2 \theta}{2 \tan^2 \theta} \left(\frac{\Delta}{\Omega} \right) + \frac{2}{\tan 2\theta} \right], \quad (43)$$

$$\omega_3^{\text{tot}}(k)/\Omega = -\tan \theta - \frac{1}{2} \left(\frac{\Delta}{\Omega} \right) + \frac{1}{2} \left(\frac{kc_0}{\Omega} \right)^2 \left[\frac{\cos^2 \theta}{2 \tan^2 \theta} \left(\frac{\Delta}{\Omega} \right) - \frac{2}{\tan 2\theta} \right], \quad (44)$$

$$\omega_4^{\text{tot}}(k)/\Omega = \sec \theta - \frac{1}{2} \left(\frac{\Delta}{\Omega} \right) + \frac{1}{2} \left(\frac{kc_0}{\Omega} \right)^2 \sin \theta \tan \theta, \quad (45)$$

$$\omega_5^{\text{tot}}(k)/\Omega = -\sec \theta - \frac{1}{2} \left(\frac{\Delta}{\Omega} \right) - \frac{1}{2} \left(\frac{kc_0}{\Omega} \right)^2 \sin \theta \tan \theta, \quad (46)$$

where $\omega_i^{\text{tot}}(k)$ corresponds to the total energy of the quasiparticle mode ψ_i up to $(kc_0)^2$ under $\tan \phi = 1$ and $\gamma = 0$. One can find that the total energies of the quasiparticle modes, including SL-DSP and the other four SL-BSPs, exhibit quadratic dispersions for kc_0 near $k = 0$, leading to nonzero effective masses. Note that the SLPs not only have a positive effective mass, but also a negative mass under a certain choice of Δ/Ω and θ , which can be controlled in an experimental scheme. The emergence of negative effective mass in the SLP system can be understood as the dispersion engineering via the atom-light interaction, which achieves the negative group velocity dispersion regime (anomalous dispersion), and it opens a possibility to explore the soliton physics in an atomic system with controllable dispersion properties [17,35]. Further, The nonzero effective mass of SLPs gives a stark contrast to the EIT polaritons, which have linear dispersions to kc_0 , and thus a zero effective masses near $k = 0$. The quadratic and linear dispersion can be switched back and forth by turning on and off one of the coupling lasers, and this tunability allows the dispersion engineering of the polariton modes, leading to the possibility for the coherent manipulations of photons with a suppressed photonic loss [9].

The parameter dependences of the effective mass can be studied further in details by deducing them from the given relation $m^{-1} = \partial^2 \omega^{\text{tot}}(k) / \partial k^2$. For given Δ/Ω and θ , the dimensionless effective masses of each quasiparticle mode are given as

$$(m_1 c_0^2 / \Omega)^{-1} = -\left(\frac{\Delta}{\Omega} \right) \frac{\cos^2 \theta}{\tan^2 \theta}, \quad (47)$$

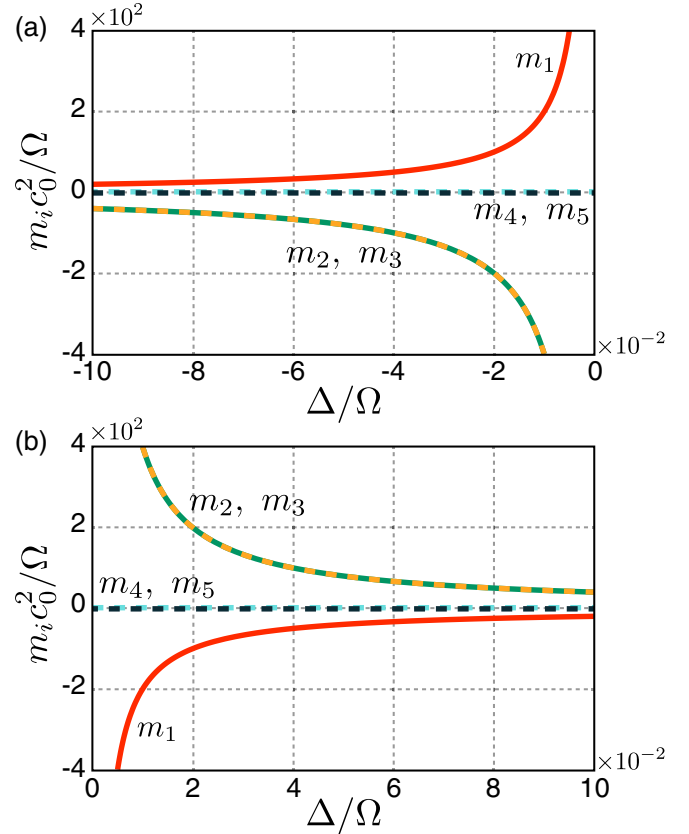


FIG. 4. The effective mass of SLP eigenmode near $k = 0$ when $\tan \theta = \tan \phi = 1$ and $\gamma = 0$. (a) The effective masses from $\Delta/\Omega = -10^{-1}$ to 0, (b) The effective masses from $\Delta/\Omega = 0$ to 10^{-1} . The red solid line corresponds to SL-DSP ψ_1 , and the dotted lines correspond to other four BSPs, ψ_2 , ψ_3 , ψ_4 , and ψ_5 .

$$(m_2 c_0^2 / \Omega)^{-1} = \frac{\cos^2 \theta}{2 \tan^2 \theta} \left(\frac{\Delta}{\Omega} \right) + \frac{2}{\tan 2\theta}, \quad (48)$$

$$(m_3 c_0^2 / \Omega)^{-1} = \frac{\cos^2 \theta}{2 \tan^2 \theta} \left(\frac{\Delta}{\Omega} \right) - \frac{2}{\tan 2\theta}, \quad (49)$$

$$(m_4 c_0^2 / \Omega)^{-1} = \sin \theta \tan \theta, \quad (50)$$

$$(m_5 c_0^2 / \Omega)^{-1} = -\sin \theta \tan \theta, \quad (51)$$

where m_i corresponds to the effective mass of ψ_i near $k = 0$ under $\tan \phi = 1$ and $\gamma = 0$. Let us first investigate the dependence of the effective masses to the one-photon detuning Δ at a given θ . Figure 4 shows $m_i c_0^2 / \Omega$ plotted against Δ/Ω under $\tan \theta = \tan \phi = 1$ and $\gamma = 0$. Figure 4(a) shows the effective masses in $\Delta < 0$ regime, and Fig. 4(b) shows those in $0 < \Delta$ regime. One can find that the effective masses of ψ_1 , ψ_2 , and ψ_3 can be tuned over the positive and negative mass regimes by adjusting Δ/Ω , while the effective masses of ψ_4 and ψ_5 are kept constant through Δ/Ω . For the blue-detuned regime ($\Delta < 0$), ψ_1 exhibits a positive effective mass while ψ_2 and ψ_3 exhibit negative masses, and vice versa for red-detuned regime ($\Delta > 0$). Specifically, the effective mass of SL-DSP, ψ_1 , is inversely proportional to Δ/Ω , and it offers a stationary and decay-free quasiparticle mode with a tunable effective mass in a wide range in both positive and negative

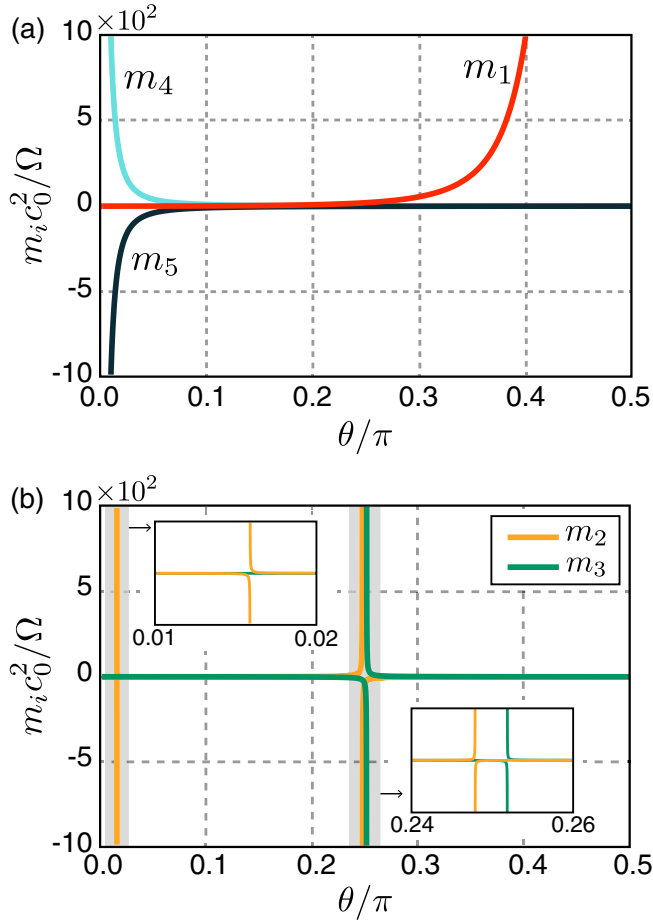


FIG. 5. The effective masses of SLPs near $k = 0$ plotted against θ/π when $\tan \phi = 1$, $\gamma = 0$, and $\Delta/\Omega = -0.1$. (a) The effective masses for ψ_1 , ψ_4 , and ψ_5 . (b) The effective masses for ψ_2 and ψ_3 .

mass regimes. The properties of SL-DSP, where the effective mass can be controlled in the both regimes, opens the possibilities of engineering exotic photonic quantum states, such as the crystallization of interacting photons [10], photonic simulation of relativistic field theory [11], high-temperature BEC [12], and the self-trapping of photonic gas in an atomic ensemble [17].

Let us now investigate the dependence of the effective masses to θ at a given Δ/Ω , where $\tan \theta = g\sqrt{N}/\Omega$ and thus can be controlled by the optical power of FWC and BWC. Figure 5 shows $m_i c_0^2 / \Omega$ plotted against θ/π under $\tan \phi = 1$, $\gamma = 0$, and $\Delta/\Omega = -0.1$. As shown in Fig. 5(a), the effective mass of ψ_1 monotonically increases for θ/π in $0 \leq \theta \leq \pi/2$ and diverges at $\theta = \pi/2$, while the effective mass of ψ_4 monotonically decreases in the same parameter regime and diverges at $\theta = 0$. Note that $m_5 = -m_4$ always holds for ψ_4 and ψ_5 . Here again, the effective mass of ψ_1 offers a tunable effective mass in a wide range, which can be optically controlled by adjusting θ , which can be achieved by adjusting the optical power of FWC and BWC. Note that the negative effective mass of ψ_1 can be achieved by choosing $\Delta > 0$, i.e., a red-detuned Δ . Figure 5(b) shows the effective masses of ψ_2 and ψ_3 , and unlike the other quasiparticle modes, m_2 and m_3 exhibit singularities near $\theta = 0$ and $\theta = \pi/4$. The

insets show the magnified plots near the singularities. The analytic condition of singularities can be calculated from from Eqs. (48) and (49) as

$$\tan^5 \theta - \tan \theta \mp \frac{\Delta}{2\Omega} = 0, \quad (52)$$

where ψ_2 and ψ_3 exhibit the effective mass singularities when Eq. (52) has real solutions for θ in $0 \leq \theta \leq \pi/2$. Here, $\mp \Delta/(2\Omega)$ correspond to the singularities for m_2 and m_3 , respectively. In $\Delta/\Omega \ll 1$ regime, the positions of singularities can be approximated up to the first-order of Δ near $\theta = 0$ and $\theta = \pi/4$ from Eq. (52). For $\theta \approx 0$, the singularities arise at

$$\theta_0 \approx \frac{|\Delta|}{2\Omega}, \quad (53)$$

where θ_0 corresponds to the singularity near $\theta \approx 0$, and it arises for m_2 when $\Delta < 0$ and for m_3 when $\Delta > 0$. For $\theta \approx \pi/4$, the singularities arise at

$$\theta_{\pi/4} \approx \frac{\pi}{4} \pm \frac{1}{16} \frac{\Delta}{\Omega}, \quad (54)$$

where $\theta_{\pi/4}$ is the singularities near $\pi/4$, and $\pm \Delta/(16\Omega)$ correspond to m_2 and m_3 , respectively. Thus, the singularities near $\theta = \pi/4$ are degenerate when $\Delta = 0$, and the degeneracy is broken when $\Delta \neq 0$ with a separation of $\Delta\theta_{\pi/4} = |\Delta|/(8\Omega)$. The singularities of effective mass shown in ψ_2 and ψ_3 open possibilities for exotic dispersion engineering such as an optical simulation of the spin-orbit-coupled BEC dispersion by spatial modulation of atomic populations [17,18], the negative kinetic temperature [36], and dispersive quantum shockwave at the mass singularities [19–21].

IV. LIFETIMES OF SLPS

Let us finally investigate the lifetimes of each quasiparticle mode under $\gamma \neq 0$ assumption. The lifetimes are given from the imaginary parts of the total energies, which are spanned up to $(kc_0)^2$ as

$$\text{Im} [\omega_1^{\text{tot}}(k)] = -(kc_0)^2 \frac{\gamma}{4\Omega^2 + \gamma^2}, \quad (55)$$

$$\text{Im} [\omega_2^{\text{tot}}(k)] = -\frac{1}{2}\gamma + \frac{1}{2}(kc_0)^2 \frac{\gamma}{4\Omega^2 + \gamma^2}, \quad (56)$$

$$\text{Im} [\omega_3^{\text{tot}}(k)] = -\frac{1}{2}\gamma + \frac{1}{2}(kc_0)^2 \frac{\gamma}{4\Omega^2 + \gamma^2}, \quad (57)$$

$$\text{Im} [\omega_4^{\text{tot}}(k)] = -\frac{1}{2}\gamma, \quad (58)$$

$$\text{Im} [\omega_5^{\text{tot}}(k)] = -\frac{1}{2}\gamma, \quad (59)$$

where $\omega_i^{\text{tot}}(k)$ corresponds to the total energy of ψ_i under $\tan \theta = \tan \phi = 1$ and $\Delta = 0$. Then, the $1/e$ lifetimes of ψ_i are given as $\tau_i = -1/(2\text{Im} [\omega_i^{\text{tot}}(k)])$, which can be deduced as

$$\tau_1/\tau_e = \frac{(\gamma/\Omega)^2 + 4}{(kc_0/\Omega)^2}, \quad (60)$$

$$\tau_2/\tau_e = 2 \frac{(\gamma/\Omega)^2 + 4}{(\gamma/\Omega)^2 + 4 - (kc_0/\Omega)^2}, \quad (61)$$

$$\tau_3/\tau_e = 2 \frac{(\gamma/\Omega)^2 + 4}{(\gamma/\Omega)^2 + 4 - (kc_0/\Omega)^2}, \quad (62)$$

$$\tau_4/\tau_e = 2, \quad (63)$$

$$\tau_5/\tau_e = 2, \quad (64)$$

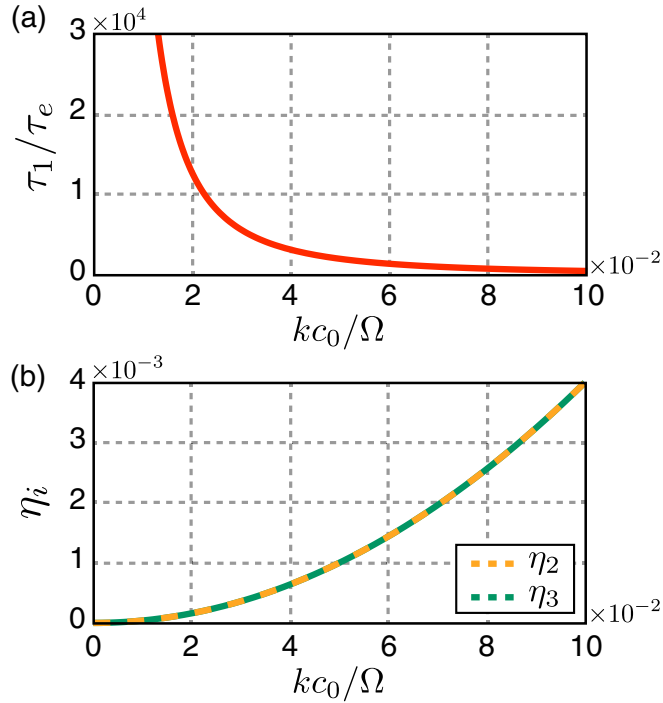


FIG. 6. (a) The lifetime of SL-DSP at a given kc_0/Ω when $\tan \theta = \tan \phi = 1$ and $\gamma/\Omega = 1$. (b) The fractional enhancement for the lifetime of ψ_2 and ψ_3 at a given kc_0/Ω when $\tan \theta = \tan \phi = 1$ and $\gamma/\Omega = 1$, where the fractional enhancement is given as $\eta_i = (\tau_i(k) - \tau_i(0))/\tau_e$. Note that while the lifetime of ψ_1 significantly decreases in a scale of $10^4\tau_e$, the lifetimes of ψ_2 and ψ_3 only increase in the scale of $10^{-3}\tau_e$.

where τ_i is the $1/e$ lifetime of ψ_i , and $\tau_e = (2\gamma)^{-1}$, which corresponds to the lifetime of the excited state $|e\rangle$.

Figure 6(a) shows τ_1/τ_e plotted against kc_0/Ω for ψ_1 when $\gamma/\Omega = 1$. Note that the lifetimes of ψ_4 and ψ_5 are kept constant as $2\tau_e$. As shown in Fig. 6(a), the decay-free mode ψ_1 starts to exhibit a finite lifetime decreasing with a rate of $1/(kc_0)^2$ at a nonzero k . Note that the decreased lifetime of ψ_1 can be compensated by increasing Ω from the relation of $\tau_1/\tau_e = (\gamma^2 + 4\Omega^2)/(k^2c_0^2)$, and thus, one can find that the lifetime of SL-DSP can be enhanced by employing FWC and BWC with stronger optical powers.

Figure 6(b) shows the fractional enhancement of the lifetime against kc_0/Ω for ψ_2 and ψ_3 when $\gamma/\Omega = 1$, i.e., $\eta_{2,3} = (\tau_{2,3}(k) - \tau_{2,3}(0))/\tau_e$. One can find that the lifetime of ψ_2 and ψ_3 exhibit a fractional enhancement of $\sim 10^{-3}\tau_e$ for $(kc_0/\Omega) \sim 10^{-1}$, which is extremely small in comparison to the decreased lifetime of ψ_1 . The enhancement of lifetime comes from the eigenmode mixing in $\{\psi_1, \psi_2, \psi_3\}$ space at a finite kc_0/Ω value, where the dispersion matrix $\hat{V}$ rearranges the imaginary parts of them as shown in Eqs. (55)–(57). Thus, a complementary relation for the lifetimes $\sum_{i=1}^3 \tau_i^{-1} = \tau_e^{-1}$ always holds even for a finite Δ , where τ_2 and τ_3 start to deviate for $\Delta \neq 0$. Note that the complementary relation can be rewritten as $\sum_{i=1}^5 \tau_i^{-1} = 2\tau_e^{-1}$ by including τ_4 and τ_5 , which are kept constant as $\tau_4 = \tau_5 = 2\tau_e$ for varying Δ and kc_0 in this regime.

V. CONCLUSION

In summary, we have investigated the dispersion and decay properties of SLP quasiparticle modes via perturbative approaches. We first derived the five SLP eigenmodes from the atom-light interaction Hamiltonian, and then the quasiparticle properties, such as group velocities and effective masses, are explicitly presented from the perturbation matrices describing their dispersion and decay properties. We then showed the tunability of the effective masses for experimental parameters, leading to the emergence of negative effective masses and effective mass singularities of SLP modes at certain conditions. Finally, the lifetimes of each quasiparticle mode are presented, which features a complementary relation of the lifetimes among the SLP modes. As our results report dispersion and decay properties of SLP quasiparticles in detail, including interesting effective mass and lifetime properties, our work is expected to contribute to rich applications of atom-light interactions based on SLP, such as photonic quantum gates [8,9], optical simulations of many-body physics [10–12], exotic dispersion engineering [17,18], and dispersive quantum shock waves [19–21].

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DATA AVAILABILITY

No data were created or analyzed in this study.

APPENDIX: DYNAMICS OF STATIONARY LIGHT POLARITONS

Here, we derive the dynamics of stationary light polaritons (SLPs) in detail, which is shown in Eqs. (9)–(13). By substituting the modified electric fields and atomic coherence given in Eqs. (9)–(13) to Eq. (2), the dynamics of SLPs is calculated to be

$$\begin{aligned} \partial_t (\hat{\sigma}_{gs}^0 + \hat{\sigma}_{gs}^{-+} e^{2ik_0z} + \hat{\sigma}_{gs}^{+-} e^{-2ik_0z}) \\ = i\Omega_+^* \hat{\sigma}_{ge}^+ + i\Omega_-^* \hat{\sigma}_{ge}^- + i\Omega_+^* \hat{\sigma}_{ge}^- e^{-2ik_0z} + i\Omega_-^* \hat{\sigma}_{ge}^+ e^{2ik_0z} \\ - \gamma_{gs} (\hat{\sigma}_{gs}^0 + \hat{\sigma}_{gs}^{-+} e^{2ik_0z} + \hat{\sigma}_{gs}^{+-} e^{-2ik_0z}), \end{aligned} \quad (A1)$$

which leads to Eqs. (11)–(13) by ordering in terms of the wave number of each term. Note that we introduce the different dephasing rate for $\hat{\sigma}_{gs}^0$, $\hat{\sigma}_{gs}^{-+}$, and $\hat{\sigma}_{gs}^{+-}$ as γ_{gs}^0 , γ_{gs}^{-+} , and γ_{gs}^{+-} , respectively, to keep the general description, which cannot be directly captured in Eq. (A1). First, when there is a detuning δ between forward coupling (FWC) and backward coupling (BWC), which is essential in the experimental implementations of the SLP system [3,27,31–33], γ_{gs}^{-+} and γ_{gs}^{+-} start to deviate from each other and should be described as $\gamma_{gs}^{-+} = \gamma_{gs}^{+-} + 2i\delta$. Thus, to keep the generality of dynamics, one should introduce the distinct γ_{gs}^{-+} and γ_{gs}^{+-} . Second, the separation between γ_{gs}^0 and γ_{gs}^{-+} (γ_{gs}^{+-}) is due to the fact that the higher-order Raman coherences ($\hat{\sigma}_{gs}^{-+}$ and

$\hat{\sigma}_{gs}^{+-}$) experience faster decay rate by atomic motions than zeroth-order Raman coherence ($\hat{\sigma}_{gs}^0$) due to their higher wave number [27].

In the same manner, the dynamics of SLPs can be deduced by substituting Eqs. (5)–(8) to Eq. (3) as

$$\begin{aligned} \partial_t(\hat{\sigma}_{ge}^+ e^{ik_0 z} + \hat{\sigma}_{ge}^- e^{-ik_0 z}) &= ig\sqrt{N}(\hat{\mathcal{E}}^+ e^{ik_0 z} + \hat{\mathcal{E}}^- e^{-ik_0 z}) \\ &+ i(\Omega_+ \hat{\sigma}_{gs}^0 + \Omega_- \hat{\sigma}_{gs}^{+-}) e^{ik_0 z} + i(\Omega_- \hat{\sigma}_{gs}^0 + \Omega_+ \hat{\sigma}_{gs}^{+-}) e^{-ik_0 z} \\ &+ i\Omega_+ \hat{\sigma}_{gs}^{+-} e^{3ik_0 z} + i\Omega_- \hat{\sigma}_{gs}^{+-} e^{-3ik_0 z} \\ &- \gamma_{ge}(\hat{\sigma}_{ge}^+ e^{ik_0 z} + \hat{\sigma}_{ge}^- e^{-ik_0 z}). \end{aligned} \quad (\text{A2})$$

Here, one can find that Eq. (A2) is not self-consistent due to the $\pm 3k_0$ wave number terms, i.e., $\Omega_+ \hat{\sigma}_{gs}^{+-} e^{3ik_0 z}$ and $\Omega_- \hat{\sigma}_{gs}^{+-} e^{-3ik_0 z}$. However, considering the regime that the higher-order Raman coherences ($\hat{\sigma}_{gs}^{+-}$ and $\hat{\sigma}_{gs}^{+-}$) are strongly suppressed by the atomic level structure [27] or the atomic motion [1], the $\pm 3k_0$ wave number terms should be suppressed to a greater extent due to its higher wave number. Therefore, we can truncate the wave number up to $\pm 2k_0$ and neglect $\Omega_+ \hat{\sigma}_{gs}^{+-} e^{3ik_0 z}$ and $\Omega_- \hat{\sigma}_{gs}^{+-} e^{-3ik_0 z}$ to get Eqs. (9) and (10) [27], which shows good agreement to experimental results [27,31,32]. Here again, note that we introduce the different dephasing rate for $\hat{\sigma}_{ge}^+$ and $\hat{\sigma}_{ge}^-$ as γ_{ge}^+ and γ_{ge}^- , respectively, to keep the general description ($\gamma_{ge}^+ = \gamma_{ge}^- + i\delta$ when detuning δ is introduced between FWC and BWC).

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