

Steady state of periodically driven quantum systems

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Periodic driving is used to steer physical systems to unique stationary states or nonequilibrium steady states (NESS), producing enhanced properties inaccessible to nondriven systems. For open quantum systems, characterizing the NESS is challenging and existing results are generally limited to specific types of driving and the Born-Markov approximation. Here, we go beyond these limits by studying a generic periodically driven N -level quantum system interacting with a low-density thermal gas. Exploiting the framework of Floquet scattering theory, we establish general Floquet thermalization conditions constraining the nature of the NESS and the transition rates. Moreover, we examine theoretically the structure of the NESS in the high-temperature limit and find out that the NESS complies, rather surprisingly, with a uniform probability distribution (predicted by the Boltzmann law) for any driving. Numerical calculations illustrate our theoretical elaborations for a simple toy model.

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Introduction. Periodic driving is used in a wide range of fields to modify quantum systems in an *ad hoc* manner and to enhance specific properties such as tunneling suppression in Bose-Einstein condensate (BEC) [1], inducing topological state in semiconductor quantum wells [2] and creating topological and a quantum spin Hall insulator in lattices [3], reducing decoherence of a spin- $\frac{1}{2}$ particle [4], creating quantum heat machines [5]. This approach is known as Floquet engineering and has been studied for both closed [6,7] and open systems [8].

Periodically driven systems reach a periodic steady state or limiting cycle [9]. For systems interacting with a thermal bath, the periodic steady state can be mapped to a nonequilibrium steady state (NESS) [10]. The NESS ultimately determines the steady physical properties of the system. In general, determining the NESS is a complex problem, but some progress has been made under some restrictions on the driving. For example, in Refs. [11–13], the NESS of a high-frequency periodically driven system was found. To avoid the complexity of finding the NESS, an alternative approach can be taken: studying the conditions under which the NESS does not deviate from the thermal state or a thermal state with an effective Hamiltonian, also called the Floquet-Gibbs state (FGS). In these cases, all the properties or phases inaccessible in thermal equilibrium are lost, and therefore, these conditions should be avoided if any enhanced properties are desired. In

particular, under the Born-Markov approximation or weak coupling limit [14], detailed balance of the transition rates has been used to find conditions for a FGS [15–17]. These conditions are limited to specific driving Hamiltonians or frequencies, and, therefore, cannot be universally applied.

In this work, we study the NESS of a periodically driven N -level system interacting with a single thermal bath represented by a dilute gas. This allows us to employ a Lindblad equation beyond the weak coupling limit [18], to study the NESS beyond the Born-Markov approximation, and to find conditions for it to be a FGS. Exploiting the framework of Floquet scattering theory [19,20], we establish general Floquet thermalization conditions constraining the nature of the NESS and the transition rates. These conditions generalize the usual detailed balance-like restriction commonly found for rates of periodically driven system under the Born-Markov approximation [21]. Moreover, we examine theoretically the structure of NESS at high temperatures and find out that the high-temperature NESS approaches, rather surprisingly, to the uniformly distributed FGS, independently of the driving strength, frequency, or Hamiltonian. This happens despite the lack of compliance with detailed balance. As we show, the lack of deviation from a Boltzmann state in the high-temperature regime is a consequence of the unitarity of the scattering operator, which at high temperatures precludes the rate asymmetries required for a nonthermal steady state. In contrast, for lower temperatures, the NESS deviates from the Boltzmann equilibrium state, requiring the injection of energy in order to produce it. Our theoretical elaborations are illustrated by numerical calculations for a simple toy model.

Theoretical outline. Periodically driven N -level system coupled to dilute thermal gas. We consider an N -level quantum system that is subjected to a periodic driving with frequency ω . Moreover, the quantum system interacts with a single thermal bath at an inverse temperature $\beta = (k_B T)^{-1}$. We

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furthermore assume that the bath is a d -dimensional low-density gas composed of free particles that are inelastically scattered (without being created or annihilated) by the driven N -level system. This assumption allows us to write the low-density Lindblad master equation for the system evolution [18] (see S.7 in the Supplemental Material [22]). Such Lindblad master equation is valid for any bath-system coupling strength even beyond the Born-Markov approximation.

Periodic driving is described by exploiting the language of Floquet theory [19,20] (see S.1 in the Supplemental Material [22]). Due to driving, bare energy levels $\{E_j, 1 \leq j \leq N\}$ of the N -level system are converted into quasienergies $\{E_{j\nu}^{\text{QE}} = E_{j0}^{\text{QE}} + \nu\hbar\omega, 1 \leq j \leq N, \nu \in \mathbb{Z}\}$, where E_{j0}^{QE} reduces to E_j for zero driving, and the Floquet index ν labels here different Brillouin copies of the same physical Floquet state j .

Hereafter, we shall assume nondegenerate setup, $E_{j\nu}^{\text{QE}} \neq E_{j'\nu'}^{\text{QE}}$ for $(j\nu) \neq (j'\nu')$. In such nondegenerate cases, the physical populations $\wp_j$ of the driven N -level system are decoupled from coherences, and the latter decay to zero. From now on, we focus solely on the populations $\wp_j$ of the physical Floquet states j , which evolve according to the Pauli rate equation [23]:

$$\dot{\wp}_j = \sum_{j'=1}^N a_{jj'} \wp_{j'} - \wp_j \sum_{j'=1}^N a_{j'j}, \quad (1)$$

with $a_{j'j} = \sum_{\nu=-\infty}^{+\infty} a_{jj'}^{\nu}$. Here, $a_{jj'}^{\nu}$ refers to the rate of transition $j \rightarrow j'$ and the absorption of ν Floquet quanta by the system from the driving field. These transition rates are obtained using the framework of the low-density limit Lindblad equation. They are based on scattering theory and are constructed from the Floquet $\hat{T}$ -matrix elements through the following formula:

$$\begin{aligned} a_{jj'}^{\nu} = & \mathcal{N} Z^{-1} \int_{\mathbb{R}^d} d^d p \int_{\mathbb{R}^d} d^d p' e^{-\beta \frac{p^2}{2m}} \\ & \times \delta\left(\frac{p'^2}{2m} + E_{j'\nu}^{\text{QE}} - \frac{p^2}{2m} - E_{j0}^{\text{QE}}\right) \\ & \times |\langle \mathbf{p}'(j'\nu) | \hat{T} | \mathbf{p}(j0) \rangle|^2; \end{aligned} \quad (2)$$

see (S3) and S.7 in the Supplemental Material [22]. Here, the prefactor $\mathcal{N}$ is proportional to the gas particle density, $Z = \int_{\mathbb{R}^d} d^d p e^{-\beta \frac{p^2}{2m}}$, and $\langle \mathbf{p}'(j'\nu) | \hat{T} | \mathbf{p}(j0) \rangle$ are the on-shell Floquet $\hat{T}$ -matrix elements, encoding all information about scattering of a single gas particle on the driven N -level system (see S.1 in the Supplemental Material [22]). Vector $|\mathbf{p}(j0)\rangle$ represents an asymptotic in-state corresponding to a gas particle of momentum $\mathbf{p}$, which approaches the driven system residing in its in-Floquet state ($j0$). Similarly $\langle \mathbf{p}'(j'\nu) |$ refers to an asymptotic out-state corresponding to a gas particle of momentum $\mathbf{p}'$, which departs from the driven system residing in its out-Floquet state ($j'\nu$).

For $\dot{\wp}_j = 0$, we obtain from Eq. (1) the NESS, $\wp_j^{\text{NESS}}$, of the driven N -level system. The corresponding stationarity

condition,

$$\sum_{j'=1}^N a_{jj'} \wp_j^{\text{NESS}} - \wp_j^{\text{NESS}} \sum_{j'=1}^N a_{j'j} = 0, \quad (3)$$

together with the normalization requirement $\sum_j \wp_j^{\text{NESS}} = 1$, determines the NESS $\{\wp_j^{\text{NESS}}\}_{j=1}^N$ uniquely in terms of the total rates $\{a_{jj'}\}$.

Floquet thermalization conditions. We recall that the Floquet $\hat{S}$ -matrix elements take the form

$$\begin{aligned} & \langle \mathbf{p}'(j'\nu) | \hat{S} | \mathbf{p}(j0) \rangle \\ & = \delta^d(\mathbf{p}' - \mathbf{p}) \delta_{j'j} \delta_{\nu 0} - 2\pi i \delta\left(\frac{p'^2}{2m} + E_{j'\nu}^{\text{QE}} - \frac{p^2}{2m} - E_{j0}^{\text{QE}}\right) \\ & \quad \times \langle \mathbf{p}'(j'\nu) | \hat{T} | \mathbf{p}(j0) \rangle; \end{aligned} \quad (4)$$

see S.2 in the Supplemental Material [22] and Refs. [19,20]. All scattering processes preserve unitarity, due to the unitary character of the quantum dynamical evolution. The resulting unitarity of $\hat{S}$ imposes in turn strong constraints on the associated $\hat{T}$ -matrix elements. Such constraints are also known as the optical theorem [see Eq. (36) in Ref. [24] and Eqs. (S5)–(S16) in the Supplemental Material [22]]. The just-mentioned optical theorem can be used to deduce constraints on the Floquet transition rates $a_{jj'}^{\nu}$. Namely, one finds that

$$\sum_{j'=1}^N \sum_{\nu'=-\infty}^{+\infty} a_{jj'}^{-\nu'} e^{-\beta E_{j'\nu'}^{\text{QE}}} = e^{-\beta E_{j0}^{\text{QE}}} \sum_{j'=1}^N \sum_{\nu=-\infty}^{+\infty} a_{j'j}^{+\nu}, \quad (5)$$

valid for all $1 \leq j \leq N$ and $\nu \in \mathbb{Z}$ (see Fig. 1 and S.2 in the Supplemental Material [22]).

The constraints (5) have important thermodynamic consequences for the nature of the NESS, as shown below. Conditions (5) are less restrictive than the standard detailed balancelike relations generally used in periodically driven

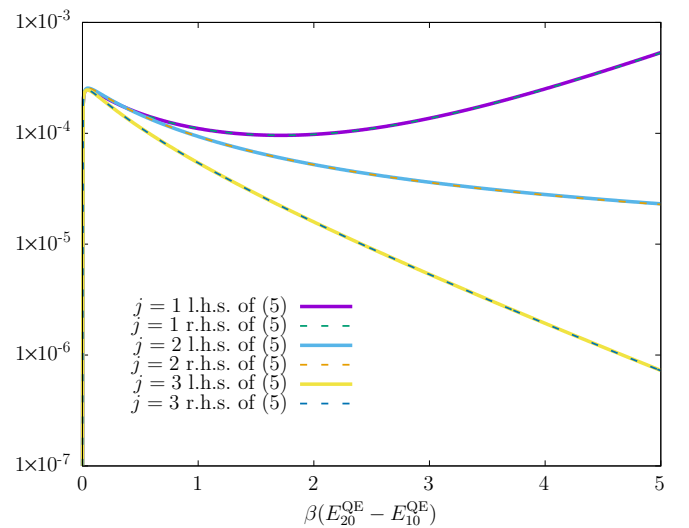


FIG. 1. Numerical check of the Floquet thermalization conditions (5) for a driven three-level system. Details of the used model can be found in Sec. III of the main text and in S.6 of the Supplemental Material [22].

open quantum systems: $a_{jj'}^{-\nu} e^{-\beta E_{j'v}^{\text{QE}}} = e^{-\beta E_{j0}^{\text{QE}}} a_{j'j}^{+\nu}$ [21]. The just-mentioned detailed balancelike relations are valid only in the Born-Markov approximation or for systems that do not break microreversibility [25] (that is, in the absence of a magnetic field or any other velocity-dependent forces). Because our dynamical evolution Eq. (1) reaches beyond the Born-Markov approximation, we will use the more general condition (5). Nevertheless, our results are also valid for regimes where the detailed balancelike relations apply.

For zero driving, $a_{j'j}^v = \delta_{v0} a_{j'j}$. Accordingly, relations (5) boil down to

$$\sum_{j' \neq j} a_{jj'} e^{-\beta E_{j'}} = e^{-\beta E_j} \sum_{j' \neq j} a_{j'j}, \quad (6)$$

for all $1 \leq j \leq N$. Properties (6) represent nothing else than the thermalization conditions encountered in the theory of nondriven open quantum systems [25,26]. We have thus shown that unitarity of the total (system + bath) dynamics implies the thermalization of a nondriven system that undergoes a nonunitary reduced dynamics. This represents a mechanism of emergence of thermalization of a reduced system under a total unitary dynamics. Its connection with other mechanisms such as the eigenstate thermalization hypothesis [27,28] should be further explored. An analogy between Eqs. (5) and (6) motivates us to hereafter call our more general relations (5) as the Floquet thermalization conditions. We note that constraints (5) and (6) are fundamental preconditions that should be fulfilled by the rates of the Pauli dynamics of any physical N -level system coupled to a dilute thermal gas.

Structure of the NESS. The Floquet thermalization conditions (5) can also be redisplayed as follows:

$$\sum_{j'} a_{jj'} e^{-\beta E_{j0}^{\text{QE}}} \langle e^{\beta v \hbar \omega} \rangle_{jj'} = e^{-\beta E_{j0}^{\text{QE}}} \sum_{j'} a_{j'j}, \quad (7)$$

where

$$\langle e^{\beta v \hbar \omega} \rangle_{jj'} = a_{jj'}^{-1} \sum_v a_{jj'}^v e^{\beta v \hbar \omega} = \frac{\sum_v a_{jj'}^v e^{\beta v \hbar \omega}}{\sum_v a_{jj'}^v}. \quad (8)$$

The term $\langle e^{\beta v \hbar \omega} \rangle_{jj'}$ represents a measure of how much the NESS deviates from the thermal state due to driving. This becomes clearer after using Eq. (7) and rewriting Eq. (3) as

$$\sum_{j'} a_{jj'} \left(\frac{\phi_{jj'}^{\text{NESS}}}{\phi_j^{\text{NESS}}} - e^{-\beta(E_{j0}^{\text{QE}} - E_{j0}^{\text{QE}})} \langle e^{\beta v \hbar \omega} \rangle_{jj'} \right) = 0. \quad (9)$$

If $\langle e^{\beta v \hbar \omega} \rangle_{jj'} = 1$ for all j and j' , Eq. (9) implies that the steady state is thermal. This is the case for the nondriven case, and also for the high-temperature limit, where $\langle e^{\beta v \hbar \omega} \rangle_{jj'} \rightarrow 1$. Therefore, at infinite temperature, the steady state is thermal and has thus uniform population distribution, $\phi_j^{\text{NESS}} = 1/N$. This uniformity can also be deduced from the Floquet thermalization conditions. Namely, for $\beta = 0$, Eq. (7) implies that the transition rates are symmetric in the sense that the sum of transition rates of the processes flowing out of the state j is balanced by the sum of transition rates of processes going into state j . Such symmetry results in a steady state with a homogeneous distribution.

For a thermal steady state, on average no energy is exchanged with either the bath or the driving. The energy

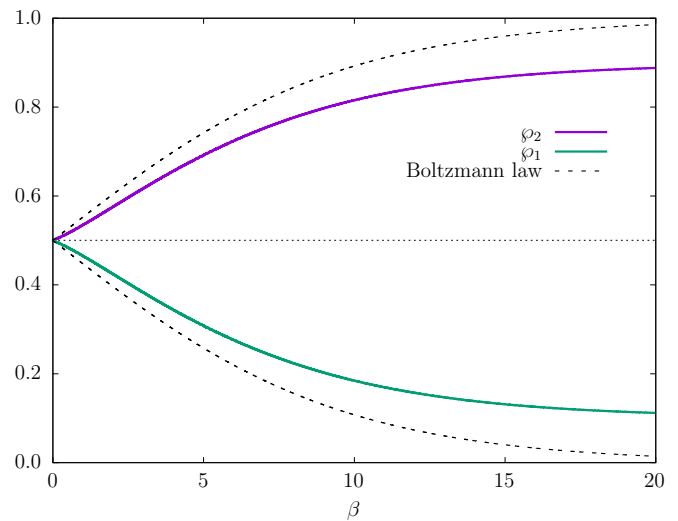


FIG. 2. The structure of NESS for a simple toy model of a driven two-level system. The uniform population distribution emerges at $\beta = 0$. Details of the used model can be found in Sec. III of the main text and in S.6 of the Supplemental Material [22].

exchange with the driving is comprised of multiple processes, each involving a specific number of Floquet excitations, ν . The probability of each such process is the product between the state population, ϕ_j^{NESS} and the transition rate $a_{jj'}^v$. Using the fact that at infinite temperature the steady state is proportional to the identity; i.e., it is a thermal state, a zero energy exchange with the driving imposes a further constraint on the transition rates at $\beta = 0$. Namely,

$$\hbar \omega \sum_j \sum_{j'} \sum_{\nu=-\infty}^{\nu=+\infty} \nu a_{jj'}^v \big|_{\beta=0} = 0; \quad (10)$$

or, equivalently,

$$\hbar \omega \sum_j \sum_{\nu=-\infty}^{\nu=+\infty} \nu a_{jj}^v \big|_{\beta=0} + \hbar \omega \sum_{j>j'} \sum_{\nu=-\infty}^{\nu=+\infty} \nu (a_{jj'}^v + a_{j'j}^v) \big|_{\beta=0} = 0. \quad (11)$$

(see Fig. 3) our just given thermodynamic argument makes the result (10) independent of the driving strength, frequency, and the Hamiltonian, and therefore it can be universally applied to any driving. Equation (10) or (11) can also be derived in an alternative manner not relying on thermodynamics, as a mere consequence of the Floquet thermalization conditions (see S.3 in the Supplemental Material [22]).

Based on Eq. (11), we conjecture that

$$(a_{jj'}^{+\nu} + a_{j'j}^{+\nu}) \big|_{\beta=0} = (a_{jj'}^{-\nu} + a_{j'j}^{-\nu}) \big|_{\beta=0}; \quad (12)$$

otherwise fine tuning will be required to comply with Eq. (11). The just presented conjecture can actually be proven to hold for a relatively broad class of situations. Namely, assuming that the system-particle coupling $\hat{V}$ provides the matrix element $\langle \mathbf{p}'(j'\nu) | \hat{V} | \mathbf{p}(j0) \rangle$, which does not fall off to zero too quickly with energy (see S.3 in the Supplemental Material for a quantitative criterion [22]), one encounters an extra

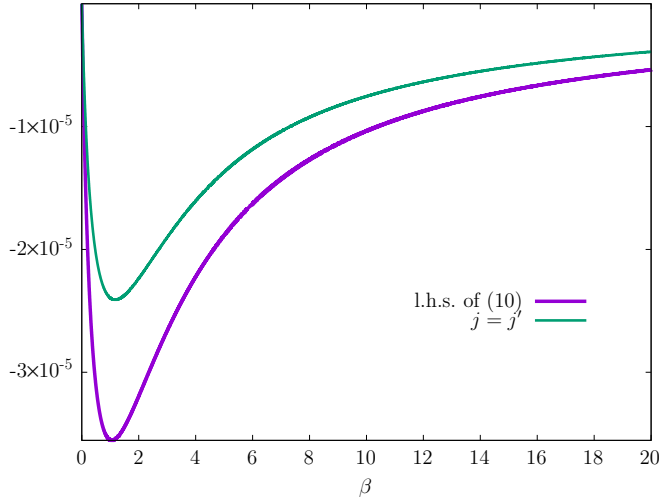


FIG. 3. Quantity $\hbar\omega \sum_j \sum_{j'} \sum_{v=-\infty}^{v=+\infty} v a_{jj'}^v$ plotted against β for a simple toy model of a driven two-level system. One can see that the property (10) holds at $\beta = 0$. Interestingly, the green line shows that even $\hbar\omega \sum_j \sum_{v=-\infty}^{v=+\infty} v a_{jj}^v$ vanishes at $\beta = 0$, rationalization of such behavior is left as an open question for possible further research. Details of the used model can be found in Sec. III of the main text and in S.6 of the Supplemental Material [22].

symmetry property

$$a_{jj'}^{+v} \big|_{\beta=0} = a_{j'j}^{-v} \big|_{\beta=0}, \quad (13)$$

which is even much stronger than Eq. (12). Relation (13) is implied by incorporating into Eq. (2), the Born approximation $\langle \langle \mathbf{p}'(j'v) | \hat{T} | \mathbf{p}(j0) \rangle \rangle \mapsto \langle \langle \mathbf{p}'(j'v) | \hat{V} | \mathbf{p}(j0) \rangle \rangle$, such a step becomes exact for the above stated kinds of $\hat{V}$ (see S.3 in the Supplemental Material for details [22]).

Moreover, if extra symmetries on the driving are considered (see S.3 in the Supplemental Material [22]), it is possible to get yet another additional strong symmetry property of the rates, namely,

$$a_{jj'}^{+v} \big|_{\beta=0} = a_{j'j}^{-v} \big|_{\beta=0}. \quad (14)$$

In this case, Eq. (11) has important physical consequences. Namely, it implies that the NESS is thermal also for small values of β , that is, for high but finite temperatures up to $\mathcal{O}(\beta)$. To demonstrate this, we carry out a Taylor expansion of Eq. (9) around $\beta = 0$ (see S.5 in the Supplemental Material [22]). Notice that the first-order expansion term of $\langle e^{\beta v \hbar \omega} \rangle_{jj'}$ is proportional to the left-hand side of Eq. (11). Defining $\wp_{jj'}^{\text{corr}}$ as the first-order correction to the steady state population rates, that is, $\frac{\wp_{jj'}^{\text{NESS}}}{\wp_{jj'}^{\text{NESS}}} = 1 + \beta \wp_{jj'}^{\text{corr}} + \mathcal{O}(\beta^2)$, we get

$$\begin{aligned} \wp_{jj'}^{\text{corr}} &= -(E_{j'0}^{\text{QE}} - E_{j0}^{\text{QE}}) + \frac{\sum_v v a_{j'j}^v \big|_{\beta=0}}{\sum_v a_{j'j}^v \big|_{\beta=0}} \\ &= -(E_{j'0}^{\text{QE}} - E_{j0}^{\text{QE}}), \end{aligned} \quad (15)$$

where we used the fact that no energy is exchanged with the driving field at $\beta = 0$, i.e., Eq. (11), to obtain the second equality. The right-hand side of Eq. (15) is nothing but the first-order correction to the thermal state. Therefore, the NESS equals to the thermal state up to $\mathcal{O}(\beta)$ (see Fig. 4).

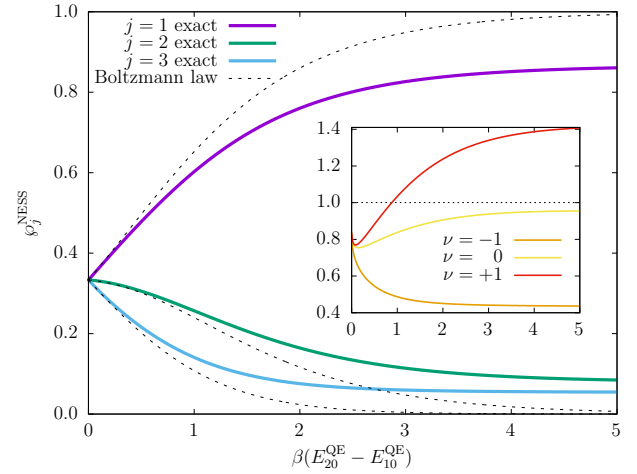


FIG. 4. The structure of NESS plotted as a function of normalized inverse temperature $\beta(E_{20}^{\text{QE}} - E_{10}^{\text{QE}})$. Note the special behavior of the populations $\wp_{jj'}^{\text{NESS}}$ in the $\beta \rightarrow 0$ and $\beta \rightarrow \infty$ limits. Inset: $(a_{jj'}^{-v} e^{-\beta E_{j'v}^{\text{QE}}}) / (e^{-\beta E_{j0}^{\text{QE}}} a_{j'j}^v)$ plotted here against $\beta(E_{20}^{\text{QE}} - E_{10}^{\text{QE}})$ for the case of $j = 1$ and $j' = 2$. Showing explicitly that the detailed balance condition does not hold. Specifications of the used model can be found in Sec. III of the main text and in S.6 of the Supplemental Material [22]. Note that the property (14) does apply for the present case, ensuring compliance with the Boltzmann law up to $\mathcal{O}(\beta)$.

The fact that the steady state approaches the thermal state at high temperatures has important implications for any practical realization of a periodically driven system, because the strength of the driving is always limited. Once the maximal driving strength is determined, one should be careful to avoid working at too high temperatures, otherwise the steady state will be thermal.

The precise meaning of “too high temperatures” depends upon whether the driving complies with the extra symmetry property (14) or not. If it does not, deviations from the thermal state start at first order of β and high temperatures are defined as

$$\beta \ll \frac{1}{-(E_{j'0}^{\text{QE}} - E_{j0}^{\text{QE}}) + \frac{d\langle e^{\beta v \hbar \omega} \rangle_{jj'}}{d\beta} \big|_{\beta=0}}. \quad (16)$$

On the other hand, if the condition (14) is matched, the thermal state deviations start only at the second order of β , and the creation of nonthermal states could be more challenging. In this case, the notion of high temperature can be estimated by calculating the second-order β expansion of Eq. (9) and comparing it to the first-order correction. If

$$\beta \ll \frac{2(E_{j'0}^{\text{QE}} - E_{j0}^{\text{QE}})}{\frac{d^2\langle e^{\beta v \hbar \omega} \rangle_{jj'}}{d\beta^2} \big|_{\beta=0} + (E_{j'0}^{\text{QE}} - E_{j0}^{\text{QE}})^2}, \quad (17)$$

then the NESS is well approximated by a thermal state (see Fig. 4). The effects of driving on determining the high-temperature regime are captured here by $\frac{d^2\langle e^{\beta v \hbar \omega} \rangle_{jj'}}{d\beta^2} \big|_{\beta=0}$.

In contrast to the high-temperature behavior, at low temperature ($\beta \rightarrow \infty$), the NESS asymptotes always deviate from the thermal state (see Figs. 2 and 4). This is analytically proven in S.4 of the Supplemental Material [22].

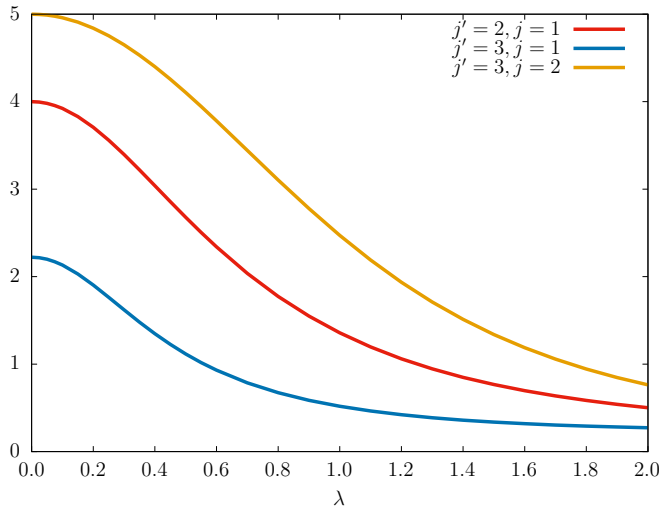


FIG. 5. High-temperature limit [right-hand side of Eq. (17)]. Here, the λ -dependence saturates for large values of λ . Therefore, for the specific system and type of driving used in our calculation, the pertinent NESS is basically thermal for $\beta \ll 0.2$, regardless upon the strength of driving. This saturation phenomenon deserves to be explored in more detail in the future. Specifications of the used three-level model complying with Eq. (14) can be found in Sec. III of the main text and S.6 of the Supplemental Material [22].

Numerical test. We have tested our theoretical findings using a toy model of a periodically driven two- and three-level system interacting with a dilute thermal gas. The Hamiltonian of our driven two-level system takes the form

$$\hat{H}_S(t) = E_2 |2\rangle\langle 2| + E_1 |1\rangle\langle 1| + \lambda \cos(\omega t) |1\rangle\langle 2| + \lambda \cos(\omega t) |2\rangle\langle 1|; \quad (18)$$

this corresponds to, e.g., an atomic system driven by laser. The Hamiltonian of our driven three-level system takes the form

$$\hat{H}_S(t) = (E_3 + \lambda \cos(\omega t)) |3\rangle\langle 3| + E_2 |2\rangle\langle 2| + (E_1 - \lambda \cos(\omega t)) |1\rangle\langle 1|. \quad (19)$$

In Eqs. (18) and (19), ω represents the driving frequency and λ controls the driving strength. The interaction with the low-density thermal gas is described by a repulsive localized potential that acts on the gas particle and a system operator that is not diagonal in the eigenbasis of $\hat{H}_S(t)$ (for details, see S.6 in the Supplemental Material [22]). The corresponding $\hat{T}$ matrix elements can be numerically calculated as detailed in S.6 of the Supplemental Material [22], where also all values of the parameters of the two models are included. Using Eq. (2), the transition rates can finally be obtained. Hereafter, we shall adopt the following system parameters: $E_1 = -0.25$, $E_2 = +0.25$, $\omega = 0.7$, $\lambda = 0.5$ (two-level system), and $E_1 = -0.5$, $E_2 = 0.0$, $E_3 = 0.4$, $\omega = 1.35$, $\lambda = 0.5$ (three-level system, in Fig. 5, the parameter λ varies continuously).

Our above-presented analytic formulas also offer potentially important criteria for testing numerical calculations. In particular, the Floquet thermalization conditions (5) and the constraints (11) on the rates at $\beta = 0$ can be used to test the

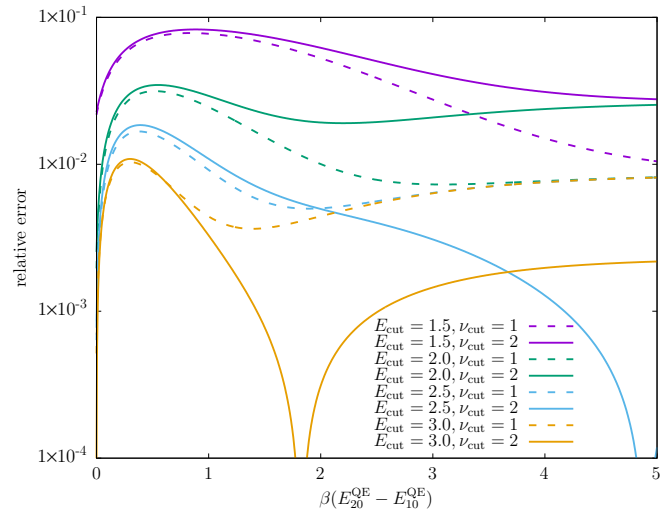


FIG. 6. Floquet thermalization conditions (5) used as a criterion of accuracy of the calculated transition rates, $a_{jj'}^v$. Vertical axis corresponds here to the relative error, i.e., to $2|(\text{left-hand side of (5)} - (\text{right-hand side of (5)}))|/((\text{left-hand side of (5)} + (\text{right-hand side of (5)})))$. E_{cut} = cutoff energy of the incoming gas particles, ν_{cut} = cutoff value of magnitude of the Floquet index ν . The relative error gradually decreases when E_{cut} and ν_{cut} are lifted up. Specifications of the used three-level model can be found in Sec. III of the main text and in S.6 of the Supplemental Material [22].

accuracy of numerical calculations of these transition rates. The former point is illustrated in Fig. 6.

Concluding remarks. In summary, we have studied a generic periodically driven N -level quantum system interacting with a low-density thermal gas. We have found that unitarity of the scattering matrix (4) gives rise to the Floquet thermalization conditions (5), constraining the transition rates $a_{jj'}^v$ of Eq. (2). This forces the NESS to be close to a uniformly distributed (thermal) state at high temperatures, and suppresses the steady energy exchange with external driving, and the steady heat flow with the bath. Therefore, we conclude that high temperatures reduce the possibility of driving a system away from uniformly distributed thermal equilibrium. The just-mentioned finding holds universally for *any driving and strength of the system-bath coupling* (although the notion of high enough temperatures depends on the driving type).

Moreover, we note that our quantum mechanical derivation of the standard thermalization conditions (6), valid for non-driven systems, represents a insightful contribution of its own right, with potential implications in the field of emergence of thermalization. Finally, conditions (5) also represent a tool to test the accuracy of numerical calculations.

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Data availability. Available from the corresponding author upon request.

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