

Large-size magnon cat states through intrinsic three-magnon nonlinearity

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The cat state, as an important tool for exploring the classical-quantum boundary, not only provides important tests of macroscopic quantum effects, but also has important applications in quantum metrology and quantum computation. The three-magnon process, which originates from the magnetic dipole-dipole interaction, has been less explored in the preparation of nonclassical magnon state. Here, we develop the magnon parametric coupling process that arises from the three-magnon scattering. The magnon parametric coupling strength is analytically derived and can reach a strong coupling regime within a discretized magnon mode system. Based on this, we propose a scheme to prepare the cat state without involving external nonlinearity in a magnetic system. Through the magnon parametric coupling, the extra magnon dissipation channel provided by the magnon-photon interaction induces an effective two-magnon loss channel, thereby steering the magnon into the cat state manifold. Notably, the size of the magnon cat state is inversely proportional to the parametric coupling strength, which facilitates the preparation of large-size cat states, even under environmental decoherence. Our work opens a path for the preparation of nonclassical states in magnon spintronics systems without extra nonlinear materials and also promotes the study of fundamental nonlinear physics on the magnetic platform.

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I. INTRODUCTION

Cat state [1], which is formed by coherent superposition of multiple coherent states [2–6], has unique properties different from classical states, i.e., the Wigner negativity and interference effect. It not only plays an important role in exploring classical-quantum boundaries [7,8], but also has significant applications in fault-tolerant quantum computation [9–12] and quantum metrology [13,14]. To date, cat states have been observed in numerous physical systems, such as electronics [15–17], photonics [7,18], atomic and molecular systems [8,19], superconducting quantum circuits [20,21], and trapped ions [8], ranging from microscopic to macroscopic devices. Cat state preparation protocols are also emerging. By selectively projecting measurements, the Rabi [22] and the radiative pressure-type [23] interactions can produce transient cat states. Adding or subtracting a single quantum operation in the squeezed vacuum state will result in a cat state situated on the side where the operation is implemented [24–26]. However, inevitably affected by environmental decoherence, the preparation and manipulation of high-fidelity cat states are challenging. To address this problem, the protocol of using engineered nonlinear loss [20,27–33] to prepare cat states has been theoretically proposed and experimentally explored. It shows that it can effectively suppress the decoherence of the

environment and protect the fragile cat states as long as the nonlinear loss rate is large enough.

Magnons, spin wave quantum, are collective spin excitations in magnetic materials [e.g., yttrium iron garnet (YIG)], which occupy a special place in the exploration of physical effects due to their long coherence time, low dissipation, high spin density, and wide frequency tunability [34–39]. Remarkable studies have been reported on the magnetic platform, ranging from magnon frequency combs [40–43], Bose-Einstein condensates [44], and magnon lasers [45–47] to magnon blockade [48], entanglement [49], and squeezing [50–52]. Of particular attention is the Kittel mode, a magnetized uniform precession mode, which is widely used to interact with other systems, such as microwave photons [53–56], optical photons [57–60], superconducting qubits [37–39], and phonons [49,60]. Recently, the increasing interest has been focused on the multimagnon nonlinear interaction [61–64], which originates from the intrinsic nonlinearity of magnetic materials including the three-magnon scattering process, etc. The Walker mode, a high-order magnon mode with distinct spatial profiles, plays an indispensable role in the multimagnon scattering process. For example, the simplest realized three-magnon confluence process, where the initial two Walker magnons with oppositely directed wave vectors result a near-zero wave vector Kittel magnon, has been explored theoretically and experimentally [61,63,65–70].

However, as the leading nonlinear interaction, the three-magnon coupling has rarely been explored for generating nonclassical states in magnetic systems. Meanwhile, it is also an intrinsic nonlinearity in magnetic materials, with the unique advantage of eliminating the need for external nonlinear materials [71]. The magnon cat state, a typical nonclassical state, has attracted increasing attention in modern

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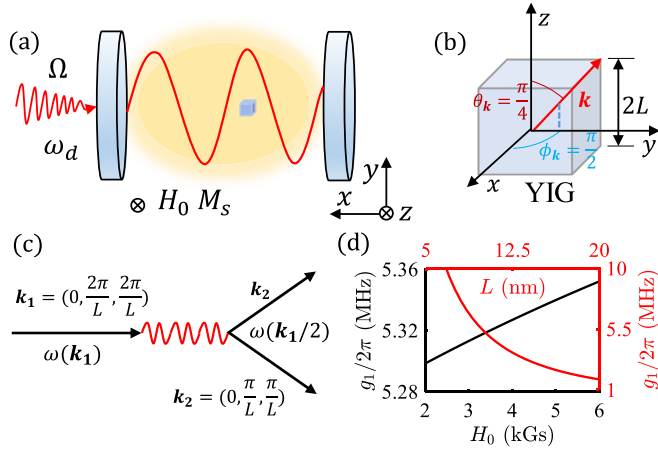


FIG. 1. The schematic diagram of magnon parametric coupling induced magnon cat state assisted by photon-magnon interaction. (a) The cubic YIG placed in the cavity interacts with microwave photons, and the external magnetic field H_0 is applied in the z direction. (b) Close-up view of the geometric structure of the cubic YIG with a standing wave magnon mode; L is the size parameter of YIG, ~ 10 nm. (c) A homogeneous spin wave on the x axis with wave number $\mathbf{k}_1 = (0, 2\pi/L, 2\pi/L)$ splits into a pair of magnons with wave number $\mathbf{k}_2 = (0, \pi/L, \pi/L)$. (d) The magnon parametric coupling strength g_1 vs magnetic field strength H_0 and the YIG size parameter L . The parameters are chosen as $4\pi M_s = 1.4$ kGs, $\mathbf{k}_1 = (0, 2\pi/L, 2\pi/L)$, and $\mathbf{k}_2 = (0, \pi/L, \pi/L)$; (1) black line: $L = 10$ nm and (2) red line: $H_0 = 6$ kGs.

quantum technology [26,72,73]. Here, we propose a scheme to prepare the large-size magnon cat state using the intrinsic nonlinearity of magnetism, inspired by recent experimental advances [71,74]. Significantly, we develop a magnon parametric coupling process, from which we analytically derive the parametric coupling strength and predict a strong coupling regime within a discretized magnon system. The magnon parametric coupling induces an effective magnon-photon parametric interaction through a second-order process to construct an additional two-magnon loss channel and steers the magnon into the cat state manifold. This scheme not only fundamentally differs from the previous work [33] but also significantly improves the two-magnon loss rate. The inverse proportionality between the size of magnon cat states and the parametric coupling strength facilitates the preparation of large-size cat states, even under the decoherence of the environment. Our work opens up a path to generate nonclassical magnon states without additional nonlinear materials and further promotes the study of fundamental nonlinear magnetism.

II. THE MAGNON PARAMETRIC COUPLING

Let us consider a cubic YIG with magnetization vector $\mathbf{M}(\mathbf{r}, t)$, as shown in Fig. 1, whose energy is given by $U_m = -\int \mathbf{M}^T(\mathbf{r}, t) \cdot \mathbf{H}_e(\mathbf{r}, t) d^3r - \frac{1}{2} \int \mathbf{M}^T(\mathbf{r}, t) \cdot \mathbf{H}_M(\mathbf{r}, t) d^3r$. The first term presents the Zeeman energy with an external magnetic field $\mathbf{H}_e(\mathbf{r}, t) = [H_x(\mathbf{r}, t), H_y(\mathbf{r}, t), H_z(\mathbf{r}, t)]^T$. The second term presents the interaction energy of the magnetization with an internal magnetic self-field $\mathbf{H}_M$, which can be written as $\mathbf{H}_M(\mathbf{r}, t) = -4\pi \int \mathbf{N}(\mathbf{r}, \mathbf{r}') \cdot \mathbf{M}^T(\mathbf{r}', t) d^3r'$. Here,

$\mathbf{N}(\mathbf{r}, \mathbf{r}')$ is a real 3×3 tensor Green's function operator with components $N_{ij}(\mathbf{r}, \mathbf{r}')$, i , and $j = x, y, z$. Three different energies, including dipole-dipole interaction energy, exchange energy, and anisotropic energy, will contribute to the self-field $\mathbf{H}_M(\mathbf{r}, t)$ [75]. In particular, the Green's tensor function of the dipole-dipole interaction is written as $N_{\text{dip}}(\mathbf{r}, \mathbf{r}') = \frac{1}{4\pi} \nabla(\nabla' \frac{1}{|\mathbf{r}-\mathbf{r}'|})$. The magnetization will interact with the self-Maxwell field induced by the dipole, thereby contributing to the three-magnon process. By defining the magnetization vector direction cosines $\alpha_{x,y,z}(\mathbf{r}, t) = \mathbf{M}_{x,y,z}(\mathbf{r}, t)/M_s$, and a scalar variable $\alpha_+(\mathbf{r}, t) = i\alpha_x(\mathbf{r}, t) + \alpha_y(\mathbf{r}, t)$, where M_s is the saturation magnetization, the linearized Landau-Lifshitz equation can be converted to coupled equations of motion for the quantum annihilation and creation operators $m(\mathbf{r}, t)$ and $m^\dagger(\mathbf{r}, t)$ of the magnon [75],

$$i \frac{dm(\mathbf{r}, t)}{dt} = \frac{\delta U}{\delta m^\dagger(\mathbf{r}, t)}, \quad -i \frac{dm^\dagger(\mathbf{r}, t)}{dt} = \frac{\delta U}{\delta m(\mathbf{r}, t)}, \quad (1)$$

where $U = \frac{|\gamma|U_m}{M_s V_{\text{eff}}}$ is the unit of frequency with effective mode volume V_{eff} . Here, $\gamma/2\pi = 28$ GHz/T is the gyromagnetic ratio. The annihilation operator $m(\mathbf{r}, t)$ is associated with the direction cosine, $\alpha_+(\mathbf{r}, t) = m(\mathbf{r}, t)\sqrt{2 - m(\mathbf{r}, t)m^\dagger(\mathbf{r}, t)}$, and $\alpha_z(\mathbf{r}, t) = 1 - m(\mathbf{r}, t)m^\dagger(\mathbf{r}, t)$. We can rewrite $\mathbf{M}(\mathbf{r}, t)$ as $\mathbf{M}(\mathbf{r}, t) = M_s \mathbf{T} \cdot [\alpha_+(\mathbf{r}, t); \alpha_+^\dagger(\mathbf{r}, t); \alpha_z(\mathbf{r}, t)]$, where $\mathbf{T}$ is a matrix given by

$$\mathbf{T} = \frac{1}{2} \begin{pmatrix} -i & i & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}. \quad (2)$$

Through Taylor expansion, the Hamiltonian U can be expanded in a power series $U = U^{(0)} + U^{(1)} + U^{(2)} + U^{(3)} + \dots$. We just expand $\alpha_+(\mathbf{r}, t)$ to the first order as $\alpha_+(\mathbf{r}, t) \approx \sqrt{2}m(\mathbf{r}, t)$ and use intrinsic mode expansion $m(\mathbf{r}, t) = \sum_{\mathbf{k}} m_{\mathbf{k}}(t) \cos(k_x x) \cos(k_y y) \cos(k_z z)$, where $\mathbf{k} = (k_x, k_y, k_z) = (n\pi/L_x, m\pi/L_y, p\pi/L_z)$ is wave number with n, m, p integers and $L_i (i = x, y, z) \sim 10$ nm is the size parameter of the YIG material [74,76–81]. Then, one can obtain the effective Hamiltonian of three-magnon interaction ($\hbar = 1$) $H_m = \sum_{\mathbf{k}} \omega_{\mathbf{k}} m_{\mathbf{k}}^\dagger m_{\mathbf{k}} - \sum_{\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3} g_{\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3} (m_{\mathbf{k}_1}^\dagger m_{\mathbf{k}_2} m_{\mathbf{k}_3} + m_{\mathbf{k}_1} m_{\mathbf{k}_2}^\dagger m_{\mathbf{k}_3}^\dagger)$. We focus on the magnon parametric process, i.e., $m_{\mathbf{k}_1} m_{\mathbf{k}_2}^{\dagger 2} (\mathbf{k}_2 = \mathbf{k}_3)$, and the Hamiltonian can be reduced to

$$H_m = \omega_1 m_1^\dagger m_1 + \omega_2 m_2^\dagger m_2 - g_1 (m_2^{\dagger 2} m_1 + m_2^2 m_1^\dagger), \quad (3)$$

where ω_1, ω_2 are the resonance frequencies of the magnon with $\mathbf{k}_1$ and $\mathbf{k}_2$ wave numbers. $g_1 \propto \frac{\omega_M}{\sqrt{2}} (iN_{xz, \mathbf{k}} + N_{yz, \mathbf{k}})$ shows parametric interaction strength with $\omega_M = \gamma 4\pi M_s$. $N_{xz, \mathbf{k}}$ and $N_{yz, \mathbf{k}}$ are the Fourier coefficients of the tensor factor $N_{\text{dip}}(\mathbf{r}, \mathbf{r}')$, which can be obtained by using the Poisson equation of the free Green's function. For a spin-wave number $\mathbf{k}$, with azimuthal and polar angles $\phi_{\mathbf{k}}$ and $\theta_{\mathbf{k}}$, respectively, the components are given by $N_{xz, \mathbf{k}} = \sin\theta_{\mathbf{k}} \cos\theta_{\mathbf{k}} \cos\phi_{\mathbf{k}}$ and $N_{yz, \mathbf{k}} = \sin\theta_{\mathbf{k}} \cos\theta_{\mathbf{k}} \sin\phi_{\mathbf{k}}$. We consider a homogeneous spin wave on the x axis with wave number $\mathbf{k}_1 = (0, 2\pi/L, 2\pi/L)$ splitting into a pair of magnons with wave number $\mathbf{k}_2 = (0, \pi/L, \pi/L)$, which indicates that $\theta_{\mathbf{k}} = \pi/4$ and $\phi_{\mathbf{k}} = \pi/2$. In Fig. 1(d), we plot the magnon parametric coupling strength versus magnetic field strength H_0 , which shows a positive correlation (black line). For $L = 10$ nm YIG

materials with saturation magnetization $4\pi M_s = 1.4$ kGs and $H_0 = 6$ kGs, the magnon parametric coupling strength can reach theoretically $g_1/2\pi = 5.36$ MHz. For the currently achievable magnon dissipation rates $\kappa_{m_1}/2\pi \sim 0.4$ and $\kappa_{m_2}/2\pi \sim 0.2$ MHz [38,82], the magnon parametric coupling strength is in a strong coupling regime. Here, the strong coupling regime is defined as the condition where the coupling strength exceeds the dissipation rates ($g_1 > \kappa_{m_1}, \kappa_{m_2}$) [38].

III. THE MAGNON CAT STATE PREPARATION PROTOCOL

Since the magnon mode is discrete, we can selectively couple the photon mode to the magnon mode m_1 . Meanwhile, energy conservation will also be challenging to be satisfied in the discrete magnon mode, resulting in the parametric processes being nonresonant and typically negligible. However, nonresonant magnon parametric processes can contribute to the resonant photon-magnon parametric processes, mediated by the virtual magnon. The Hamiltonian describing this process is written as

$$\begin{aligned} H = & \omega_a a^\dagger a + \omega_1 m_1^\dagger m_1 + \omega_2 m_2^\dagger m_2 \\ & - g_1 (m_2^\dagger m_1 + m_2 m_1^\dagger) + g (a m_1^\dagger + a^\dagger m_1) \\ & + \Omega (a e^{i\omega_d t} + a^\dagger e^{-i\omega_d t}), \end{aligned} \quad (4)$$

where a is the annihilation operator of photon with frequency ω_a . g is the photon-magnon coupling strength. The photon mode is driven by a coherent laser with driving strength Ω and frequency ω_d . We rewrite the Hamiltonian in the interaction picture by assuming that $\omega_d \approx \omega_a \approx 2\omega_2$,

$$\begin{aligned} H = & g(a m_1^\dagger e^{i\Delta t} + a^\dagger m_1 e^{-i\Delta t}) - g_1 (m_1^\dagger m_2^\dagger e^{i\Delta t} + m_1 m_2^\dagger e^{-i\Delta t}) \\ & + \Omega (a + a^\dagger), \end{aligned} \quad (5)$$

where $\Delta = \omega_1 - \omega_a$ is the detuning of the magnon m_1 to the photon mode a . There is an effective second-order coupling between a and m_2 through time averaging [83,84] when $\Delta \gg \{g_1, g\}$, and the effective second-order Hamiltonian is written as

$$H_{\text{avg}}^{(2)} = -\frac{g_1^2}{\Delta} m_2^\dagger m_2^\dagger + \frac{g_1 g}{\Delta} (a^\dagger m_2^\dagger + a m_2^\dagger) + \Omega (a + a^\dagger), \quad (6)$$

where the second-order term $-\frac{g_1^2}{\Delta} a^\dagger a$ can be eliminated by shifting the eigenfrequency of the mode a . The dissipative dynamic evolution of the system is determined by the master equation

$$\dot{\rho} = -i[H_{\text{avg}}^{(2)}, \rho] + (\kappa_a/2)\mathcal{L}[a]\rho + (\kappa_m/2)\mathcal{L}[m_2]\rho, \quad (7)$$

where κ_a and κ_m are the single-photon loss and single-magnon loss rates, respectively, and $\mathcal{L}[o]\rho = 2o\rho o^\dagger - o^\dagger o\rho - \rho o^\dagger o$. When $\kappa_m \ll \kappa_a$, the loss channel of photon mode will induce the two-magnon loss process, which can be captured by the master equation

$$\dot{\rho} = -i[H_{\text{eff}}, \rho] + (\kappa_{\text{eff}}/2)\mathcal{L}[m_2^2]\rho. \quad (8)$$

Here, $H_{\text{eff}} = -\frac{g_1^2}{\Delta} m_2^\dagger m_2^\dagger + J(m_2^\dagger - m_2^2)$ and $J = \frac{i2\Omega g_1 g}{\kappa_a \Delta}$. $\kappa_{\text{eff}} = \frac{4g_1^2 g^2}{\kappa_a \Delta^2}$ is the effective two-magnon loss rate. Compared with the publication [33], a significant advantage of current

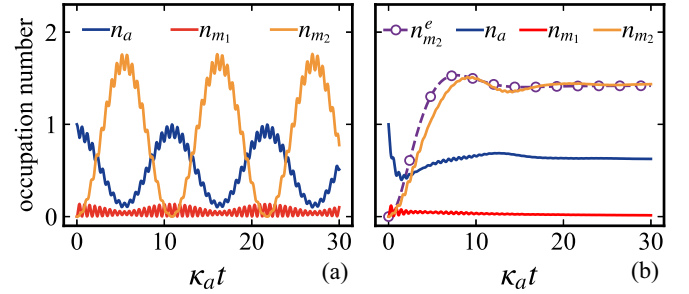


FIG. 2. (a) The evolution of the occupancy numbers of different modes controlled by the original Hamiltonian in the absence of dissipation and driving. (b) The time evolution of different modes using both the original Hamiltonian (solid line) and the effective Hamiltonian (dashed line) with driving and dissipation. The parameters are chosen as (a) $\Delta = 10\kappa_a$, $g_1 = \kappa_a$, and $g = 2\kappa_a$, (b) $\Omega = 0.5\kappa_a$ and $\kappa_m = 0.001\kappa_a$.

work is that the two-magnon loss rate induced by the second-order process is improved by an order of magnitude when $\Delta \gg \{g_1, g\}$. For example, the two-magnon loss rate is 25 times greater than the one in the previous work for $\Delta = 10\kappa_a$, $g_1 = 0.5\kappa_a$, and $g = 2\kappa_a$, enabling the preparation of the magnon cat state with higher speed and fidelity (see Sec. V for a detailed comparison). The equation (8) includes all the terms necessary to generate the magnon cat state. The first term describes the evolution of the magnon according to H_{eff} , including the coherent two-magnon squeezing term with strength J and the self-Kerr nonlinear term with strength $-\frac{g_1^2}{\Delta}$ induced by the second-order process. The second term is two-magnon loss with rate $\frac{\kappa_{\text{eff}}}{2}$. Under the action of these two processes, the magnon will naturally evolve from the initial state to the even or odd-cat states.

In order to test the effectiveness of the second-order Hamiltonian, we plot the time evolution of different modes under the original Hamiltonian in Eq. (4) without dissipation and driving, as shown in Fig. 2(a). Here, we choose the initial state as $|100\rangle$, where the ket $|n_a n_{m_1} n_{m_2}\rangle$ refers to the product state with n_a photons a , n_{m_1} magnons m_1 , and n_{m_2} magnons m_2 . Under the condition of large detuning, there is a Rabi oscillation between the photon mode a and the magnon mode m_2 , where the creation and annihilation of a single photon will cause the annihilation and creation of a pair of magnons. However, the magnon mode m_1 is rarely excited and does not have energy exchange with other modes, which proves the effectiveness of the second-order Hamiltonian [Fig. 2(a)]. Furthermore, the time evolution of the occupation number of different modes, considering the driving and dissipation, is plotted in Fig. 2(b), where the solid and dashed lines exhibit the evolution of different modes according to the original and effective Hamiltonian, respectively. The results show the evolution of magnon mode m_2 under two Hamiltonians agree well on long timescales, thus further validating our second-order and adiabatically eliminated process.

In the absence of single-magnon loss, the squeezing, self-Kerr, and the loss terms restrict the magnon to two-magnon operations and conserve the magnon excitation-number parity; thus, stable magnon cat states can be generated. From Eq. (8), the steady-state solution of magnon mode m_2 can be

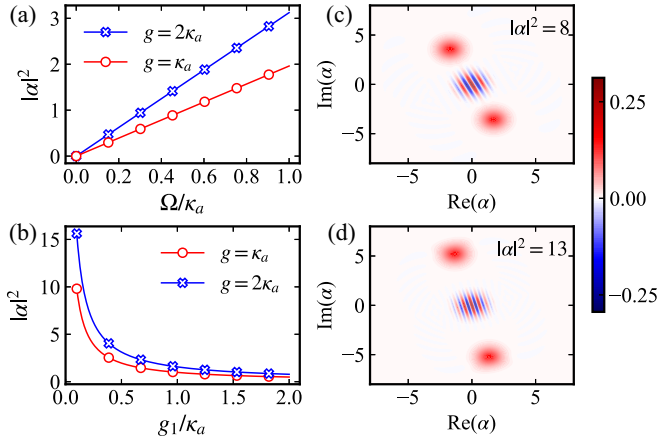


FIG. 3. (a), (b) The size of the generated cat state vs driving strength and parametric coupling strength under different photon-magnon couplings. (c), (d) The Wigner function of the stable magnon cat state using the adiabatically eliminated master equation (8) with different sizes. The parameters are chosen as (a) $g_1 = \kappa_a$, (b) $\Omega = 0.5\kappa_a$, (c) $g_1 = 0.2\kappa_a$, $g = 2\kappa_a$, and $\Omega = 0.5\kappa_a$, and (d) $g_1 = 0.1\kappa_a$, $g = 3\kappa_a$, and $\Omega = 0.5\kappa_a$. Other parameters are the same as in Fig. 2.

derived analytically. Not unexpectedly, it is a cat state with an amplitude $\alpha = \sqrt{\frac{iJ}{\kappa_{\text{eff}}/2 - ig_1^2/\Delta}}$. In particular, steady state of the magnon will be an even-cat state with an even magnon number distribution if initialized in a vacuum state, or be an odd-cat state with an odd magnon number distribution if initialized in single excited state. Due to the introduction of the self-Kerr term induced by the second-order process, the amplitude α is no longer a pure real number, but is instead accompanied by the rotation of the cat state in the phase space. To see this more clearly, we numerically plot the Wigner function $W(\alpha, \alpha^*)$ of the steady-state density matrix ρ in Eq. (8) and present in Figs. 3(c) and 3(d). Here, the Wigner function is defined as

$$W(\alpha, \alpha^*) = \frac{1}{\pi^2} \int e^{\eta^* \alpha - \eta \alpha^*} \chi(\eta) d^2\eta,$$

where $\chi(\eta) = \text{Tr}(\rho e^{\eta m^\dagger - \eta^* m})$ is the characteristic function of the density matrix ρ . Starting from initial state, the Wigner distribution of m_2 mode undergoes a rotation in phase space. At the same time, the Wigner negativity and interference fringes also gradually appear and m_2 mode eventually stabilizes on the cat state manifold [Figs. 3(c) and 3(d)].

Our scheme can produce large-size cat states. The size of a magnon cat state is characterized by $|\alpha|^2$, where α is the amplitude of the coherent state. When the size of a cat state exceeds 1, the two coherent states are approximately orthogonal, i.e., $\langle -\alpha | \alpha \rangle \approx 0$. Therefore, a cat state with a size greater than 1 is regarded as a large-size cat state [23,26]. By inserting $J = \frac{i2\Omega g_1 g}{\kappa_a \Delta}$ and $\kappa_{\text{eff}} = \frac{4g_1^2 g^2}{\kappa_a \Delta^2}$ into the amplitude α , we can get $|\alpha|^2 = \sqrt{\frac{4g^2 \Delta^2}{(\kappa_a^2 \Delta^2 + 4g^4) g_1}} \frac{\Omega}{g_1}$, and it is clear that $|\alpha|^2$ is proportional to the driving strength Ω and inversely proportional to the magnon parametric coupling strength g_1 . Therefore, on the one hand, the size of the cat state can be enhanced by increasing the driving strength Ω . Under the fixed interaction

strength $g = \kappa_a$ and $g_1 = \kappa_a$, the size of the magnon cat state versus the driving strength Ω is depicted in Fig. 3(a), which exhibits a linear dependence with Ω . The stronger the driving, the larger size of the cat state. Meanwhile, the size of the cat state will increase with the enhancement of the photon-magnon coupling strength g [blue line in Fig. 3(a)]. On the other hand, by reducing the parametric coupling strength g_1 , the size of the cat state can be increased more significantly. When $\Omega = 0.5\kappa_a$ is fixed, the size of the magnon cat state versus g_1 under different photon-magnon coupling strengths is plotted in Fig. 3(b), where it shows an inversely proportional dependence. On the left side of the plot where g_1/κ_a is close to 0, the size of the magnon cat state will be very large. Specifically, when $g = 2\kappa_a$, the cat state with size $|\alpha|^2 = 15$ can be generated near $g_1/\kappa_a = 0.1$. Compared with small-size cat states, the two superposed coherent-state components of large-size cat state have almost no overlap in phase space [Figs. 3(c) and 3(d)] and are distinguishable by appropriate quadrature measurements [23,85,86], which can not only be better utilized to explore the classical-quantum boundary, but also holds greater value for noise-tolerant quantum computing and other quantum applications [9,87].

IV. EFFECTS OF DECOHERENCE

So far, we have discussed the preparation of stable large-size cat states without single-magnon loss. However, single-magnon loss is inevitable in real physical systems. The single-magnon loss can be captured by the master equation

$$\dot{\rho} = -i[H_{\text{eff}}, \rho] + (\kappa_{\text{eff}}/2)\mathcal{L}[m^2]\rho + (\kappa_m/2)\mathcal{L}[m]\rho. \quad (9)$$

Unlike the squeezing term and the two-magnon loss term, the single-magnon loss will change the parity of the state, e.g., from even parity to odd parity and vice versa, resulting in decoherence of the coherent superposition state. Therefore, the magnon will eventually evolve from the cat state to the statistical mixed state of two coherent states. However, the single-magnon loss process can be strongly suppressed in our scheme as long as $\kappa_{\text{eff}} \gg \kappa_m$. Figure 4 shows the time evolution of the fidelity F of the magnon state under the influence of single-magnon loss. Here, F is defined as $F = \text{Tr}(\rho_0 \rho)$, where ρ_0 is the ideal Schrödinger cat state and ρ is the actual density matrix of the magnon coming from Eq. (9). From Fig. 4(a), it clearly shows that as the two-magnon loss rate increases, the cat state is generated more quickly and with higher fidelity, which indicates that a higher two-magnon loss rate more significantly suppresses single-magnon loss. Figure 4(b) shows the evolution of the magnon state at $\kappa_{\text{eff}}/\kappa_m = 10$. The Wigner negativity and interference fringes quickly reach the peak, and then are gradually destroyed under the influence of single-magnon loss, transitioning into a statistical mixed state of coherent states. In addition, following the analysis of the last section, we investigate the effectiveness of generating large-size cat state considering the single-magnon loss, as shown in Figs. 4(c) and 4(d). For an ultralarge cat state with size $|\alpha|^2 = 14$, a fidelity of up to $F_{\text{max}} = 80\%$ can be guaranteed [Fig. 4(c)]. By calculating the effective two-magnon loss rate of the system at this time, we can obtain $\kappa_{\text{eff}}/\kappa_m = 3$, showing that the two-magnon loss rate suppresses the single-magnon loss to induce a large-size magnon cat state generation. The

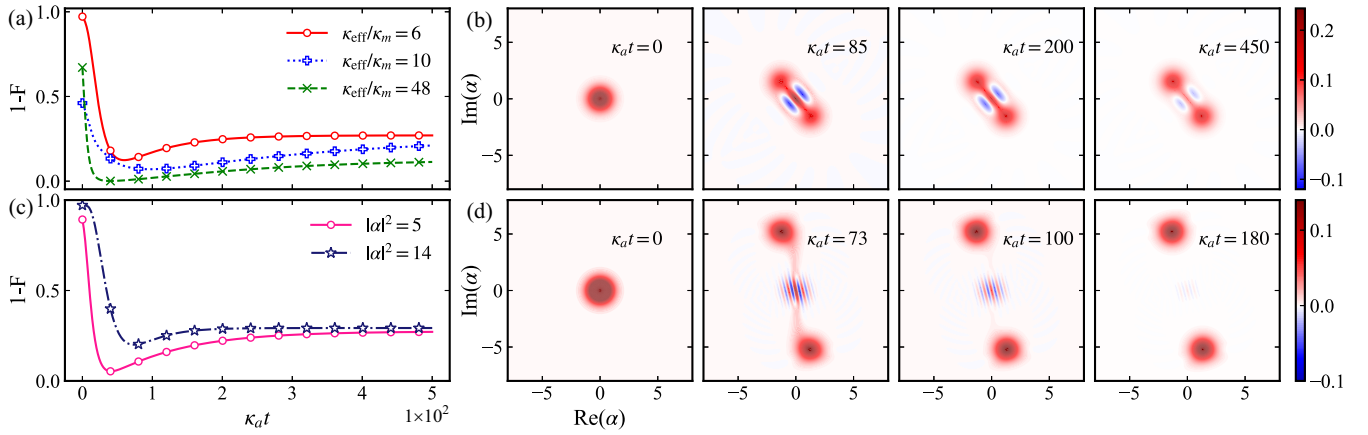


FIG. 4. (a) The time evolution of the fidelity F under different two-magnon losses. (b) The time evolution of Wigner function of the magnon cat state under a fixed parameter ratio $\kappa_{\text{eff}}/\kappa_m = 10$. (c) The time evolution of the fidelity of the large-size cat state. (d) The time evolution of Wigner function of the magnon cat state with a large size $|\alpha|^2 = 14$. The parameters are chosen as $\Delta = 10\kappa_a$, $\kappa_m = 0.001\kappa_a$, and $\Omega = 0.5\kappa_a$, (b) $g_1 = 0.5\kappa_a$ and $g = \kappa_a$, and (d) $g_1 = 0.1\kappa_a$ and $g = 3\kappa_a$. Other parameters are the same as in Fig. 2.

corresponding Wigner negativity of the magnon state and the interference fringes of the coherent superposition are also shown in Fig. 4(d), representing that the large-size magnon cat state can be effectively generated.

V. DISCUSSION

Here, we discuss the differences and advantages between the current work and the previous work [33]. First, in current work, the nonlinearity for preparing the magnon cat state originates from the nonlinear coupling process between magnons, such as the magnon parametric coupling here, rather than the external photon parametric nonlinearity. This magnon parametric coupling process belongs to the intrinsic nonlinearity of the magnetic materials, so to prepare the magnon cat state through this intrinsic nonlinearity does not require additional nonlinearity, such as the photon $\chi^{(2)}$ nonlinearity in previous work. Second, in our current scheme, two distinct magnon modes and only one photon mode are considered. In contrast, in the previous scheme, two photon modes and only one magnon mode were considered. Therefore, in our current scheme, only one photon cavity is required instead of the two cavities in the previous work, thus saving resources. Third, due to the conversion of nonlinearity from photon parametric coupling to magnon parametric coupling, the construction of the two-magnon loss channel of the target magnon is reduced from the third-order process in the previous work to the second-order process, such as from $H(t) \int H(t_1)H(t_2)dt_1dt_2$ to $H(t) \int H(t_1)dt_1$ [83,84]. On the one hand, the second-order effective process will induce a Kerr nonlinearity of the target magnon, such as $m_2^\dagger m_2^\dagger m_2 m_2$, which is unavailable in the third-order process and leads to a rotation of the magnon cat state in phase space. On the other hand, the two-magnon loss rate of the target magnon derived from the second-order effective process has different dependences on the parameters of the system compared to the third-order process, e.g., $\kappa_{\text{eff}}^{(2)} = \frac{4g_1^2 g^2}{\kappa_a \Delta^2}$ for the second-order process and $\kappa_{\text{eff}}^{(3)} = \frac{4g_1^2 g^4}{\kappa_a \Delta^4}$ for the third-order process. The two-magnon loss rate in the second-order process is inversely proportional to the square of the detuning Δ , while the two-magnon loss rate in the

third-order process is inversely proportional to the fourth power of the detuning Δ . Therefore, under large detuning conditions, i.e., $\Delta \gg \{g_1, g\}$, the two-magnon loss rate achieved by the second-order process can be significantly higher than the two-magnon loss rate induced by the third-order process. Thus, the fidelity and the preparation time of the magnon cat state can be improved. In Fig. 5, we plot the fidelity of the prepared magnon cat state versus time for different coupling strengths g and g_1 under the two processes. The left (right) figure shows the fidelity for the current (previous) work. The black dotted line marks the instant when the fidelity reaches its maximum during the evolution of the magnon cat state. On the one hand, under the same parameter conditions, the maximum fidelity in our current work reaches $F_{\text{max}} = 0.97$ and $F_{\text{max}} = 0.99$ [Figs. 5(a) and 5(c)], which are higher than the corresponding values of $F_{\text{max}} = 0.79$ and $F_{\text{max}} = 0.72$ in

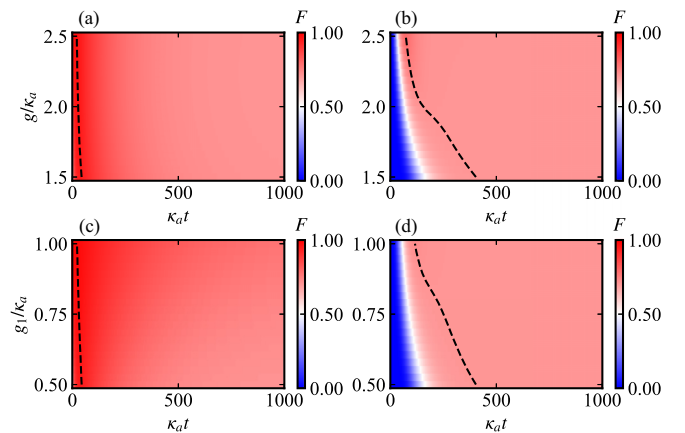


FIG. 5. (a), (c) The time evolution of the fidelity F for current work under different coupling strengths. (b), (d) The time evolution of the fidelity F for previous work under different coupling strengths. The black line is the maximum fidelity under different coupling strengths. The parameters are chosen as $\Omega = 0.5\kappa_a$, $\Delta = 10\kappa_a$ where (a), (b) $g_1 = 0.5\kappa_a$, and (c), (d) $g = 1.5\kappa_a$. Other parameters are the same as in Fig. 2.

the previous work [Figs. 5(b) and 5(d)]. On the other hand, the instant (black dotted line) to reach the maximum fidelity in our current work is shorter than that in the previous work, indicating a faster generation of the magnon cat state. Therefore, our current work offers significant improvements in both fidelity and operation time over the previous work.

VI. CONCLUSION

Here, we propose a protocol to generate the large-size magnon cat states using the magnon parametric interaction. We analytically derive the expression of the magnon parametric interaction and predict the strong coupling regime in a nanoscale YIG. Through the nonresonant magnon parametric coupling, an extra magnon dissipative channel is constructed via the magnon-photon interaction. This channel induces an effective two-magnon loss process, and then steers the magnon into the cat state manifold. This protocol allows for straightforward preparation of large-size magnon cat states with high fidelity, and is effective against environmental decoherence. Our work has extensive applications in quantum metrology and quantum computing. Meanwhile, the magnon parametric coupling process developed here can also find potential value in preparation of other nonclassical magnon states, e.g., the squeezed state, and single-magnon state, and advances the study of nonlinear physics in magnon spintronics.

ACKNOWLEDGMENTS

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DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

APPENDIX A: DERIVATION OF PARAMETRIC COUPLED HAMILTONIAN

The Hamiltonian of the magnetic material is written as [75,88]

$$H_1 = - \int \mathbf{M}^T(\mathbf{r}, t) \cdot \mathbf{H}_e(\mathbf{r}, t) d^3r - \frac{1}{2} \int \mathbf{M}^T(\mathbf{r}, t) \cdot \mathbf{H}_M(\mathbf{r}, t) d^3r, \quad (\text{A1})$$

where the first term is the Zeeman energy with the external magnetic field $\mathbf{H}_e(\mathbf{r}, t)$, and $\mathbf{M}(\mathbf{r}, t)$ is the magnetization vector. The second term presents the interaction energy between the internal magnetic self-field $\mathbf{H}_M(\mathbf{r}, t)$ and the magnetization vector. The internal magnetic self-field $\mathbf{H}_M(\mathbf{r}, t)$ can be written as

$$\mathbf{H}_M(\mathbf{r}, t) = -4\pi \int \mathbf{N}(\mathbf{r}, \mathbf{r}') \cdot \mathbf{M}(\mathbf{r}', t) d^3r'. \quad (\text{A2})$$

Here, $\mathbf{N}(\mathbf{r}, t)$ is a 3×3 tensor Green's function operator with components $N_{ij}(\mathbf{r}, \mathbf{r}')$, $i, j = x, y, z$. The self-field $\mathbf{H}_M(\mathbf{r}, t)$ consists of three parts, including Maxwellian dipole field, non-Maxwellian exchange field, and uniaxial anisotropic field. Specifically, the dipole field can be expressed through a Green's function of the form

$$\mathbf{N}_{\text{dip}}(\mathbf{r}, \mathbf{r}') = \frac{1}{4\pi} \nabla \left(\nabla' \frac{1}{|\mathbf{r} - \mathbf{r}'|} \right). \quad (\text{A3})$$

The exchange field can be expressed as

$$\mathbf{N}_{\text{ex}}(\mathbf{r}, \mathbf{r}') = -\alpha_{\text{ex}} \delta(\mathbf{r} - \mathbf{r}') \mathbf{I} \nabla \cdot \nabla'. \quad (\text{A4})$$

$\alpha_{\text{ex}} = 2A/4\pi M_s^2$ is an exchange parameter, where A is an exchange constant and M_s is the saturation magnetization of the material, and $\mathbf{I}$ is an identity matrix. For uniaxial anisotropic fields, we can write $\mathbf{N}$ as

$$\mathbf{N}_{\text{uniax}}(\mathbf{r}, \mathbf{r}') = -\frac{H_u}{4\pi M_s} \delta(\mathbf{r} - \mathbf{r}') \mathbf{e} \otimes \mathbf{e}, \quad (\text{A5})$$

where $H_u = 2K/M_s$ is an effective uniaxial anisotropic field parameter, where K is a uniaxial anisotropy energy density, and $\mathbf{e}$ represents the direction of the anisotropic axis. For simplicity, we will omit the uniaxial anisotropy energy in subsequent calculations.

For the dynamic evolution of the magnetization vector $\mathbf{M}_s$, one can find a suitable canonical transformation by defining

$$\alpha_+(\mathbf{r}, t) = m(\mathbf{r}, t) \sqrt{2 - m(\mathbf{r}, t) m^\dagger(\mathbf{r}, t)}, \quad \alpha_z(\mathbf{r}, t) = 1 - m(\mathbf{r}, t) m^\dagger(\mathbf{r}, t). \quad (\text{A6})$$

Here, $\alpha_+(\mathbf{r}, t) = i\alpha_x(\mathbf{r}, t) + \alpha_y(\mathbf{r}, t)$ is a scalar variable and $m(\mathbf{r}, t)$ and $m^\dagger(\mathbf{r}, t)$ are the canonical variables corresponding to quantum annihilation and creation operators.

Through the above definition, the Landau-Lifshitz torque equation

$$\frac{d\mathbf{M}(\mathbf{r}, t)}{dt} = -\gamma \mathbf{M}(\mathbf{r}, t) \times \mathbf{H}_{\text{eff}}(\mathbf{r}, t), \quad (\text{A7})$$

where $\gamma/2\pi = 28$ GHz/T is the gyromagnetic ratio and $\mathbf{H}_{\text{eff}} = -\frac{1}{V_{\text{eff}}} \frac{\delta H_1}{\delta \mathbf{M}}$ is a net effective field, is transformed into a coupled dynamic equation of $m(\mathbf{r}, t)$, and $m^\dagger(\mathbf{r}, t)$ in standard Hamiltonian form

$$i \frac{dm(\mathbf{r}, t)}{dt} = \frac{\delta U}{\delta m^\dagger(\mathbf{r}, t)}, \quad -i \frac{dm^\dagger(\mathbf{r}, t)}{dt} = \frac{\delta U}{\delta m(\mathbf{r}, t)}. \quad (\text{A8})$$

Here, $U = \frac{\gamma H_1}{M_s V_{\text{eff}}}$ is a Hamiltonian function with units of frequency. V_{eff} is the normalized coefficient of the spatial distribution of the magnon modes. We can rewrite the magnetization vector as $\mathbf{M}(\mathbf{r}, t) = M_s \mathbf{T} \cdot \boldsymbol{\alpha}(\mathbf{r}, t)$, where $\mathbf{T}$ is a 3×3 matrix given by

$$\mathbf{T} = \frac{1}{2} \begin{pmatrix} -i & i & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}. \quad (\text{A9})$$

The vector $\alpha(\mathbf{r}, t)$ can be directly linked to the canonical variables $m(\mathbf{r}, t)$ and $m^\dagger(\mathbf{r}, t)$,

$$\begin{aligned}\alpha(\mathbf{r}, t) &= \begin{pmatrix} \alpha_+(\mathbf{r}, t) \\ \alpha_+^\dagger(\mathbf{r}, t) \\ \alpha_z(\mathbf{r}, t) \end{pmatrix} \\ &= \begin{pmatrix} m(\mathbf{r}, t)\sqrt{2 - m(\mathbf{r}, t)m^\dagger(\mathbf{r}, t)} \\ \sqrt{2 - m(\mathbf{r}, t)m^\dagger(\mathbf{r}, t)}m^\dagger(\mathbf{r}, t) \\ 1 - m(\mathbf{r}, t)m^\dagger(\mathbf{r}, t) \end{pmatrix}. \quad (\text{A10})\end{aligned}$$

To obtain the third-order magnon interaction, we need to expand $\alpha_+(\mathbf{r}, t)$ and retain only the first term

$$\alpha_+(\mathbf{r}, t) = \sqrt{2}m(\mathbf{r}, t). \quad (\text{A11})$$

For $m(\mathbf{r}, t)$, we assume the free boundary condition $\frac{\partial m(\mathbf{r}, t)}{\partial n}|_{n=-L, L} = 0$, ($n = x, y, z$), so its spatial distribution can be written as the sum of a series of different eigenmodes $m(\mathbf{r}, t) = \sum_{\mathbf{k}} m_{\mathbf{k}}(t)\cos(k_x x)\cos(k_y y)\cos(k_z z)$, where $\mathbf{k} = (k_x, k_y, k_z) = (n\pi/L, m\pi/L, p\pi/L)$ is the wave number with size parameter L and n, m, p are the integers. Therefore, for the finite-size YIG material of cubic type, the magnon mode will be similar to the optical mode in the cavity. Only some specific wavelengths and frequencies can exist [76–81]. For the tensor Green's function $N(\mathbf{r}, \mathbf{r}')$, it is expanded into Fourier series by doing periodic continuation as follows:

$$\begin{aligned}N_{\mathbf{k}} &= \int N(\mathbf{r}, \mathbf{r}')\cos(\mathbf{k} \cdot (\mathbf{r} - \mathbf{r}'))d^3(\mathbf{r} - \mathbf{r}') \\ &= \begin{pmatrix} N_{xx, \mathbf{k}} & N_{xy, \mathbf{k}} & N_{xz, \mathbf{k}} \\ N_{yx, \mathbf{k}} & N_{yy, \mathbf{k}} & N_{yz, \mathbf{k}} \\ N_{zx, \mathbf{k}} & N_{zy, \mathbf{k}} & N_{zz, \mathbf{k}} \end{pmatrix}. \quad (\text{A12})\end{aligned}$$

So $N(\mathbf{r}, \mathbf{r}') = \frac{1}{4L^3} \sum_{\mathbf{k}} N_{\mathbf{k}} \cos(\mathbf{k} \cdot (\mathbf{r} - \mathbf{r}'))$ and finally the form of the self-field $\mathbf{H}_M(\mathbf{r}, t)$ is given by

$$\mathbf{H}_M(\mathbf{r}, t) = -4\pi \sum_{\mathbf{k}} N_{\mathbf{k}} \cdot \mathbf{M}_{\mathbf{k}}(t)\cos(k_x x)\cos(k_y y)\cos(k_z z), \quad (\text{A13})$$

where $\mathbf{M}_{\mathbf{k}}(t)$ corresponds to the intrinsic mode expansion of the vector $\alpha(\mathbf{r}, t)$. Here, we use the orthogonality of functions, such as $\int_{-L}^L \cos(\frac{m\pi x}{L})\cos(\frac{n\pi x}{L})\sin(\frac{p\pi x}{L})dx = 0$. Bringing these results into the Hamiltonian function U , the Hamiltonian function U that is truncated to the third-order process is written as [under the rotating-wave approximation]

$$U^{(2)} = \sum_{\mathbf{k}} \omega_{\mathbf{k}} b_{\mathbf{k}}^\dagger b_{\mathbf{k}} - \sum_{k_1, k_2, k_3} g_{k_1, k_2, k_3} (b_{k_1}^\dagger b_{k_2} b_{k_3} + b_{k_1} b_{k_2}^\dagger b_{k_3}^\dagger), \quad (\text{A14})$$

where $\omega_{\mathbf{k}} = \sqrt{A_{\mathbf{k}}^2 - |B_{\mathbf{k}}|^2}$ is the resonance frequency of the magnon, with $A_{\mathbf{k}} = \omega_H + \frac{\omega_M}{2}(N_{xx, \mathbf{k}} + N_{yy, \mathbf{k}})$ and $B_{\mathbf{k}} = \frac{\omega_M}{2}(-N_{xx, \mathbf{k}} + N_{yy, \mathbf{k}} + 2iN_{xy, \mathbf{k}})$. Here, $\omega_H = \gamma(H_0 - 4\pi M_s N_{zz, 0})$ and $\omega_M = \gamma 4\pi M_s$. $g_{k_1, k_2, k_3} = \frac{1}{2}[(D_{k_3} u_{k_3} + D_{k_3}^* v_{k_3})(u_{k_2} u_{k_1} + v_{k_2} v_{k_1}^*) + (D_{k_2} u_{k_2} + D_{k_2}^* v_{k_2})(u_{k_1} u_{k_3} + v_{k_3} v_{k_1}^*) + (D_{k_1} u_{k_1} + D_{k_1}^* v_{k_1})(u_{k_3} v_{k_2} + v_{k_3} u_{k_2})]$ is the three-magnon interaction strength with $D_{\mathbf{k}} = \frac{\omega_M F}{\sqrt{2}}(iN_{xz, \mathbf{k}} + N_{yz, \mathbf{k}})$, where F is the overlap factor of the three-magnon

spatial distribution. In the derivation of Eq. (A14), we omit the $U^{(0)}$ and $U^{(1)}$ processes. Since the $U^{(0)}$ process does not contain magnon operators, we throw it away. For the $U^{(1)}$ process, it can be written as $U^{(1)} = -\sum(\gamma H_{\perp, \mathbf{k}}^* - D_{\mathbf{k}}^* \Delta(\mathbf{k}))m_{\mathbf{k}}(t) + \text{H.c.}$, where $H_{\perp, \mathbf{k}} = \frac{H_{y, \mathbf{k}} + iH_{x, \mathbf{k}}}{\sqrt{2}}$ is the Fourier coefficient of the external magnetic field $H_e(\mathbf{r}, t)$ and $\Delta(\mathbf{k})$ is a Kronecker delta function. Since we only consider a static magnetic field H_0 , H_x, H_y in z, x, y directions, which refers to magnetic field that does not change with space or time, i.e., $\frac{dH_e(\mathbf{r}, t)}{dt} = 0$ and $\frac{\partial H_e(\mathbf{r}, t)}{\partial n}|_{n=x, y, z} = 0$, $H_{\perp, \mathbf{k}} = 0$ for $\mathbf{k} \neq 0$. The $U^{(1)}$ process simplifies to $U^{(1)} = -(H_L - D_0)m_0^\dagger - (H_L^* - D_0^*)m_0$, where $H_L = \frac{\gamma}{\sqrt{2}}(iH_x + H_y)$ and $D_0 = \frac{\gamma 4\pi M_s}{\sqrt{2}}(iN_{xz, 0} + N_{yz, 0})$. First of all, we can notice that the first-order process $U^{(1)}$ only contains uniform magnon mode operator m_0 , and does not contain high-order nonuniform magnon operator $m_{\mathbf{k}}$. Second, the condition $H_L - D_0 = 0$ defines static equilibrium, corresponding to the well-known condition for the stable static magnetization state in a uniformly magnetized sample [75,77]. Third, $N_{xz, 0}$ ($N_{yz, 0}$) denotes the mean value of the self-fields in the x (y) direction induced by the uniform magnon mode, which are equal to 0 [89]. Therefore, the $U^{(1)}$ process will disappear. At the same time, we use the Holstein-Primakoff (HP) transformation to diagonalize the Hamiltonian U [75]:

$$\begin{pmatrix} b_{\mathbf{k}}(t) \\ b_{\mathbf{k}}^\dagger(t) \end{pmatrix} = \begin{pmatrix} u_{\mathbf{k}} & v_{\mathbf{k}} \\ v_{\mathbf{k}}^* & u_{\mathbf{k}} \end{pmatrix}^{-1} \begin{pmatrix} m_{\mathbf{k}}(t) \\ m_{\mathbf{k}}^\dagger(t) \end{pmatrix}, \quad (\text{A15})$$

where $u_{\mathbf{k}} = \sqrt{\frac{A_{\mathbf{k}} + \omega_{\mathbf{k}}}{2\omega_{\mathbf{k}}}}$ and $v_{\mathbf{k}} = -\frac{B_{\mathbf{k}}}{|B_{\mathbf{k}}|} \sqrt{\frac{A_{\mathbf{k}} - \omega_{\mathbf{k}}}{2\omega_{\mathbf{k}}}}$ are the HP transform coefficients. Direct derivation of the spin wave dispersion relation reads

$$\begin{aligned}\omega_{\mathbf{k}} &= f(\mathbf{k}) \\ &= \sqrt{(\omega_H + \omega_M \alpha_{ex} \mathbf{k}^2)(\omega_H + \omega_M \alpha_{ex} \mathbf{k}^2 + \omega_M \sin^2 \theta_{\mathbf{k}})}. \quad (\text{A16})\end{aligned}$$

When the size of the YIG material is $\sim nm$, the exchange spin waves will be dominant and the frequency difference between two adjacent modes can reach several tens of gigahertz. Therefore, when considering the magnon-photon interaction, we can selectively couple the photon mode a with frequency ω_a to a specific magnon mode $b_{\mathbf{k}_1}$ with frequency $\omega_{\mathbf{k}_1}$. Here, $\Delta = \omega_{\mathbf{k}_1} - \omega_a$ is detuning. For the three-magnon coupling, the energy conservation and spatial overlap between the different magnon modes will limit the process, making the energy exchange between the three magnons typically weak because it is difficult for them to satisfy both conditions simultaneously in the discrete magnon modes, and thus can be neglected [74,90]. However, through magnon-photon coupling, these conditions can be relaxed, for example, loosening energy conservation. Specifically, when the photon mode a and magnon mode $b_{\mathbf{k}_2}$ are parametrically resonant, i.e., $\omega_a = 2\omega_{\mathbf{k}_2}$, the photon mode a will be indirectly coupled to the magnon mode $b_{\mathbf{k}_2}$ through the auxiliary mode $b_{\mathbf{k}_1}(\mathbf{k}_1 = 2\mathbf{k}_2)$. Here, we use the fact that the resonant three-magnon processes are not allowed (i.e., if the energy conservation laws for the three-magnon process are not satisfied), but the three-magnon

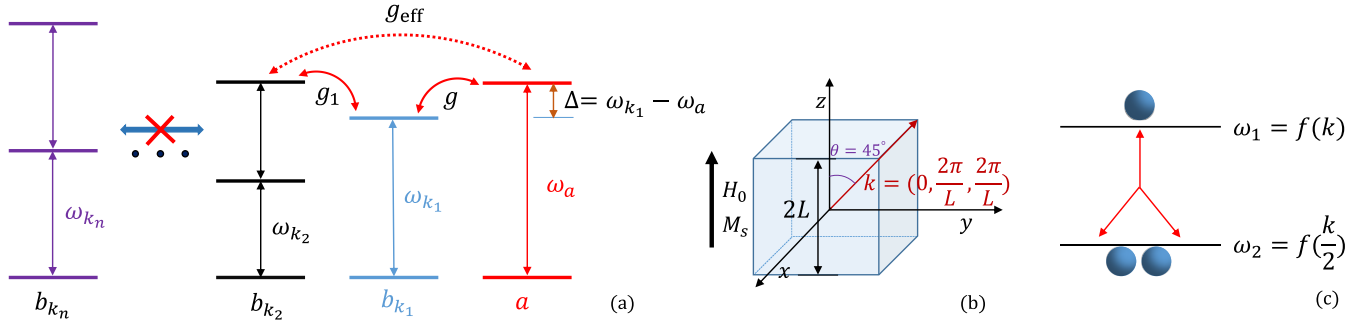


FIG. 6. (a) The schematic diagram of the nonresonant magnon parametric coupling contributing to the resonant photon-magnon parametric coupling scheme. (b) A schematic diagram of the structure of a cubic YIG with a length $2L$, where $\mathbf{k} = (0, 2\pi/L, 2\pi/L)$ marks the a set of eigenmode of magnons, representing uniformity along the x axis and polar angles θ_k along the z axis. The external magnetic field H_0 and saturation magnetization M_s are oriented along the z axis. (c) A magnon with wave number $\mathbf{k}$ and frequency $\omega_1 = f(\mathbf{k})$ splits into two magnons with wave number $\mathbf{k}/2$ and frequency $\omega_2 = f(\mathbf{k}/2)$.

interaction efficiency is nonzero. Additionally, we choose the two lowest-order magnon modes to avoid the other three-magnon processes in which the sum of the two split magnon frequencies resonates at the driving frequency ω_a ; thus, the summation can be released, as shown in Fig. 6(a). The Hamiltonian that captures this process is written as

$$H = \omega_a a^\dagger a + \omega_1 b_1^\dagger b_1 + \omega_2 b_2^\dagger b_2 - g_1(b_2^{\dagger 2} b_1 + b_2^2 b_1^\dagger) + g(ab_1^\dagger + a^\dagger b_1), \quad (\text{A17})$$

where a is the annihilation operator of photon with frequency ω_a . g is the photon-magnon coupling strength. The plasmonic microcavity is capable of confining electromagnetic waves within a subwavelength or extremely subwavelength spatial region, which can be used to enhance the light-matter interaction [91]. Through the plasmonic microcavity, the interaction between the nonuniform magnon and photon modes can be significantly enhanced. On the other hand, a planar superconducting lumped-element LC resonator can also strongly confine the microwave field in the vicinity of the spin to enhance the spin-resonator interaction [92], which can be utilized to enhance the coupling strength between nonuniform magnon and photon modes. g_1 is the parametric interaction strength. So we derive the Hamiltonian in the main text. Our scheme relaxes the conditions of the three-magnon process, which greatly enhances the experimental feasibility. The two-magnon study has also been experimentally reported [71,74].

APPENDIX B: DERIVATION OF THE MAGNON PARAMETRIC COUPLING STRENGTH

The three-magnon interaction originates from the interaction between magnetization and self-Maxwell field induced by magnetization. From the expression of the three-magnon interaction $g_{k_1, k_2, k_3} \propto D_{\mathbf{k}} = \frac{\omega_M}{\sqrt{2}}(iN_{xz, \mathbf{k}} + N_{yz, \mathbf{k}})$, we observe that only the dipole interaction gives rise to the three-magnon process. Consequently, we only need to calculate the Fourier components $N_{xz, \mathbf{k}}$ and $N_{yz, \mathbf{k}}$ of the dipole tensor Green's function $N_{\text{dip}}(\mathbf{r}, \mathbf{r}')$. We refer to the dipole tensor Green's function again,

$$N_{\text{dip}}(\mathbf{r}, \mathbf{r}') = \frac{1}{4\pi} \nabla \left(\nabla' \frac{1}{|\mathbf{r} - \mathbf{r}'|} \right). \quad (\text{B1})$$

From this expression, we note that $\frac{1}{|\mathbf{r} - \mathbf{r}'|}$ is a Green's function in free space, which satisfies the differential equation

$$\nabla^2 \frac{1}{|\mathbf{r} - \mathbf{r}'|} = -4\pi \delta(\mathbf{r} - \mathbf{r}'). \quad (\text{B2})$$

We first carry out the Fourier transform of the Green's function $\frac{1}{|\mathbf{r} - \mathbf{r}'|} = \frac{1}{V} \sum_{\mathbf{k}} G(\mathbf{k}) \cos(\mathbf{k} \cdot (\mathbf{r} - \mathbf{r}'))$, where the sin term is omitted due to orthogonality $\int_{-L}^L \cos(\frac{m\pi x}{L}) \sin(\frac{p\pi x}{L}) dx = 0$ and $\int_{-L}^L \cos(\frac{m\pi x}{L}) \cos(\frac{n\pi x}{L}) \sin(\frac{p\pi x}{L}) dx = 0$, and substitute it into the differential equation (B2), and the Fourier component $G(\mathbf{k})$ of the Green's function can be calculated as $G(\mathbf{k}) = \frac{4\pi}{k^2}$. The Green's function is then rewritten as

$$\frac{1}{|\mathbf{r} - \mathbf{r}'|} = \frac{1}{V} \sum_{\mathbf{k}} \frac{4\pi}{k^2} \cos(\mathbf{k} \cdot (\mathbf{r} - \mathbf{r}')).$$

We first calculate $N_{xz, \mathbf{k}}$. Substituting it into Eq. (A12), we can obtain

$$\begin{aligned} N_{xz, \mathbf{k}} &= \int N_{\text{dip}}(\mathbf{r}, \mathbf{r}') \cos(\mathbf{k} \cdot (\mathbf{r} - \mathbf{r}')) d^3(\mathbf{r} - \mathbf{r}') \\ &= \frac{1}{V} \int \frac{1}{4\pi} \nabla_z \left(\nabla'_x \sum_{\mathbf{k}'} \frac{4\pi}{k'^2} \cos(\mathbf{k}' \cdot (\mathbf{r} - \mathbf{r}')) \right) \\ &\quad \times \cos(\mathbf{k} \cdot (\mathbf{r} - \mathbf{r}')) d^3(\mathbf{r} - \mathbf{r}') \\ &= \frac{1}{V} \sum_{\mathbf{k}'} \int \frac{k'_x k'_z}{k'^2} \cos(\mathbf{k}' \cdot (\mathbf{r} - \mathbf{r}')) \\ &\quad \times \cos(\mathbf{k} \cdot (\mathbf{r} - \mathbf{r}')) d^3(\mathbf{r} - \mathbf{r}') \\ &= \frac{k_x k_z}{k^2}. \end{aligned} \quad (\text{B3})$$

So we get $N_{xz, \mathbf{k}} = \frac{k_x k_z}{k^2}$. Using the spherical coordinate $k_x = k \sin \theta_k \cos \phi_k$ and $k_z = k \cos \theta_k$, simplify to get $N_{xz, \mathbf{k}} = \sin \theta_k \cos \phi_k \cos \theta_k$. Similarly, we can obtain $N_{yz, \mathbf{k}} = \sin \theta_k \sin \phi_k \cos \theta_k$. We consider a homogeneous spin wave on the x axis with wave number $\mathbf{k}_1 = (0, 2\pi/L, 2\pi/L)$ splitting into a pair of magnons with wave number $\mathbf{k}_2 = (0, \pi/L, \pi/L)$, which indicates that $\theta_k = \pi/4$, $\phi_k = \pi/2$, and $D_{\mathbf{k}_1} = D_{\mathbf{k}_2} = D_{\mathbf{k}_3}$. In addition, the three-magnon spatial overlap coefficient is $F = \frac{1}{8}$ in this case. We numerically

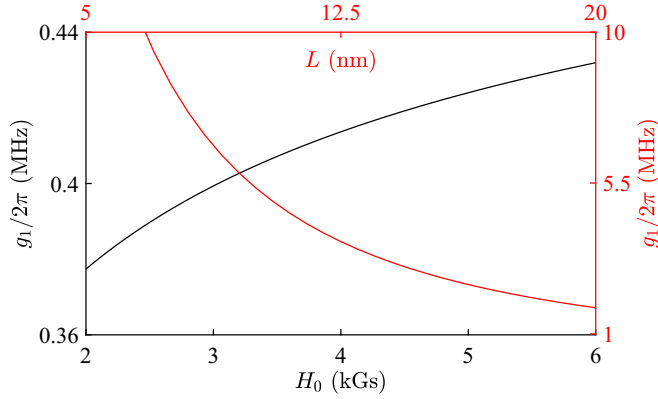


FIG. 7. The magnon parametric coupling strength g_1 vs magnetic field strength H_0 and the YIG size parameter L . The parameters are chosen as follows (a) black line: $L = 50$ nm, $4\pi M_s = 1.4$ kGs, $\alpha_{\text{ex}} = 3 \times 10^{-16}$ m², $\mathbf{k}_1 = (0, 2\pi/L, 2\pi/L)$, and $\mathbf{k}_2 = (0, \pi/L, \pi/L)$; (b) red line: $\alpha_{\text{ex}} = 3 \times 10^{-16}$ m², $\mathbf{k}_1 = (0, 2\pi/L, 2\pi/L)$, $\mathbf{k}_2 = (0, \pi/L, \pi/L)$, $H_0 = 5$ kGs, and $4\pi M_s = 1.4$ kGs.

plot the curves of magnon parametric coupling g_1 versus magnetic field strength H_0 and YIG size L , as shown in Fig. 7. When saturation magnetization $4\pi M_s = 1400$ Gs, the magnon parametric coupling strength can reach theoretically strong coupling regime. Therefore, we choose the magnon parametric coupling of $\sim \kappa_a$, which is experimentally feasible in our scheme.

APPENDIX C: DERIVATION OF SECOND-ORDER EFFECTIVE HAMILTONIAN

The total Hamiltonian of the system can be written as

$$H = \omega_a a^\dagger a + \omega_1 m_1^\dagger m_1 + \omega_2 m_2^\dagger m_2 - g_1 (m_2^{\dagger 2} m_1 + m_2^2 m_1^\dagger) + g(am_1^\dagger + a^\dagger m_1) + \Omega(ae^{i\omega_d t} + a^\dagger e^{-i\omega_d t}), \quad (\text{C1})$$

where the photon mode a is driven by a coherent laser with driving strength Ω and frequency ω_d . Transforming it into the interaction picture by assuming that $\omega_d \approx \omega_a \approx 2\omega_2$, one can obtain

$$H = g(am_1^\dagger e^{i\Delta t} + a^\dagger m_1 e^{-i\Delta t}) - g_1(m_1^\dagger m_2^2 e^{i\Delta t} + m_1 m_2^{\dagger 2} e^{-i\Delta t}) + \Omega(a + a^\dagger), \quad (\text{C2})$$

where $\Delta = \omega_1 - \omega_a$ is the detuning of the magnon m_1 to the photon mode a . Following the formula of time average given in Refs. [83,84],

$$H_{\text{eff}} = H_0 + \sum_{n,m} \frac{1}{\omega_{nm}^+} [h_m^\dagger, h_n] e^{i(\omega_m - \omega_n)t}, \quad (\text{C3})$$

where H_0 is the time-independent part and $\omega_{nm}^+ = \frac{1}{2}(\frac{1}{\omega_n} + \frac{1}{\omega_m})$, we can get the effective second-order process Hamiltonian. For $h_1 = ga^\dagger m_1$ and $h_2 = g_1 m_1 m_2^{\dagger 2}$, $\omega_1 = \omega_2 = \Delta$, we can

calculate

$$\begin{aligned} [h_1^\dagger, h_1] &= -\frac{g^2}{\Delta} a^\dagger a, \\ [h_2^\dagger, h_2] &= -\frac{g_1^2}{\Delta} m_2^{\dagger 2} m_2^2, \\ [h_1^\dagger, h_2] &= \frac{gg_1}{\Delta} a m_2^{\dagger 2}, \\ [h_2^\dagger, h_1] &= \frac{gg_1}{\Delta} a^\dagger m_2^2, \end{aligned} \quad (\text{C4})$$

where we have used $m_1^\dagger m_1 |0\rangle = 0$. The first term $-\frac{g^2}{\Delta} a^\dagger a$ can be canceled by offsetting the eigenfrequency of the photon mode a . Therefore, we obtain the effective second-order process Hamiltonian appearing in the main text:

$$H_{\text{avg}}^{(2)} = -\frac{g_1^2}{\Delta} m_2^{\dagger 2} m_2^2 + \frac{g_1 g}{\Delta} (a^\dagger m_2^2 + a m_2^{\dagger 2}) + \Omega(a + a^\dagger). \quad (\text{C5})$$

APPENDIX D: THE STABLE MAGNON CAT STATE INDUCED BY TWO-MAGNON LOSS CONTAINS KERR NONLINEARITY

(1) The evolution of the magnon state is determined by the master equation

$$\dot{\rho} = -i[H_{\text{eff}}, \rho] + (\kappa_{\text{eff}}/2)\mathcal{L}[m_2^2]\rho, \quad (\text{D1})$$

where $H_{\text{eff}} = -\frac{g_1^2}{\Delta} m_2^{\dagger 2} m_2^2 + J(m_2^2 - m_2^{\dagger 2})$ is the effective Hamiltonian, $J = \frac{i2\Omega g_1 g}{\kappa_a \Delta}$ is the squeezing term, $\mathcal{L}[o]\rho = 2o\rho o^\dagger - o^\dagger o\rho - \rho o^\dagger o$ is the Lindblad operator, and $\kappa_{\text{eff}} = \frac{4g_1^2 g^2}{\kappa_a \Delta^2}$ is the two-magnon loss rate. The steady-state solution of Eq. (D1) is a cat state, and its parity is determined by the initial state. We assume that the steady state is a superposition of two coherent states ($|\alpha\rangle + |-\alpha\rangle$), where α is the amplitude of the coherent state. By setting $\dot{\rho} = 0$ and inserting $\rho = (|\alpha\rangle + |-\alpha\rangle)(\langle\alpha| + \langle-\alpha|)$ into Eq. (D1), and then multiplying from the left by coherent state $\langle\beta|$ and from the right by coherent state $|\beta\rangle$, we can obtain

$$\begin{aligned} & -i(K\beta^{*2}\alpha^2\rho_s + J(\alpha^2 - \beta^{*2}))\rho_s \\ & + i((K\alpha^{*2}l^2\rho_s + J(l^2 - \alpha^{*2})\rho_s)) \\ & + \frac{\kappa_{\text{eff}}}{2}(2\alpha^2\alpha^{*2}\rho_s - \beta^{*2}\alpha^2\rho_s - \alpha^{*2}l^2\rho_s) = 0, \end{aligned} \quad (\text{D2})$$

where $K = -\frac{g_1^2}{\Delta}$ and $\rho_s = (\langle\beta|\alpha\rangle + \langle\beta|-\alpha\rangle)(\langle\alpha|l\rangle + \langle-\alpha|l\rangle)$. Here, we use that $m_2|\alpha\rangle = \alpha|\alpha\rangle$. Equation (D2) holds for any β and l , so

$$\begin{aligned} & -i(K\beta^{*2}\alpha^2 + J(\alpha^2 - \beta^{*2})) + i((K\alpha^{*2}l^2 + J(l^2 - \alpha^{*2}))) \\ & + \frac{\kappa_{\text{eff}}}{2}(2\alpha^2\alpha^{*2} - \beta^{*2}\alpha^2 - \alpha^{*2}l^2) = 0 \end{aligned} \quad (\text{D3})$$

is required. Taking the complex conjugate of Eq. (D3), and noting that $J^* = -J$, we get

$$\begin{aligned} & i(K\beta^2\alpha^{*2} - J(\alpha^{*2} - \beta^2)) \\ & - i((K\alpha^2l^{*2} - J(l^{*2} - \alpha^2))) \\ & + \frac{\kappa_{\text{eff}}}{2}(2\alpha^2\alpha^{*2} - \beta^2\alpha^{*2} - \alpha^2l^{*2}) = 0. \end{aligned} \quad (\text{D4})$$

Subtracting Eq. (D3) from Eq. (D4), we can obtain

$$\begin{aligned} & i(K\beta^2\alpha^{*2} + K\beta^{*2}\alpha^2 + J\beta^2 - J\beta^{*2}) \\ & + i(-K\alpha^2 l^{*2} - K\alpha^{*2} l^2 + J l^{*2} - J l^2) \\ & + \frac{\kappa_{\text{eff}}}{2}(-\beta^2\alpha^{*2} - \alpha^2 l^{*2} + \beta^{*2}\alpha^2 + \alpha^{*2} l^2) = 0. \end{aligned} \quad (\text{D5})$$

Through some algebras, we can get

$$\begin{aligned} & \left(i(K\alpha^2 - J) + \frac{\kappa_{\text{eff}}}{2}\alpha^2\right)\beta^{*2} \\ & + \left(i(K\alpha^{*2} + J) + \frac{\kappa_{\text{eff}}}{2}(-\alpha^{*2})\right)\beta^2 \\ & + \left(i(-K\alpha^2 + J) + \frac{\kappa_{\text{eff}}}{2}(-\alpha^2)\right)l^{*2} \\ & + \left(i(-K\alpha^{*2} - J) + \frac{\kappa_{\text{eff}}}{2}\alpha^{*2}\right)l^2 = 0. \end{aligned} \quad (\text{D6})$$

Due to the arbitrariness of β and l , this requires that the coefficients of $\beta^2(\beta^{*2})$ and $l^2(l^{*2})$ are equal to 0. Fortunately, when $i(K\alpha^2 - J) + \frac{\kappa_{\text{eff}}}{2}\alpha^2 = 0$, all the coefficients will be 0. Therefore, we get the equation about α . Simplifying the equation $i(K\alpha^2 - J) + \frac{\kappa_{\text{eff}}}{2}\alpha^2 = 0$, one can obtain

$$\alpha^2 = \frac{iJ}{\frac{\kappa_{\text{eff}}}{2} + iK}, \quad (\text{D7})$$

which is the amplitude of the stable magnon cat state. Therefore, we prove that the stable solution of Eq. (D1) is a cat state. We can carry out limit analysis. By inserting $J = \frac{i2\Omega g_1 g}{\kappa_a \Delta}$, $\kappa_{\text{eff}} = \frac{4g_1^2 g^2}{\kappa_a \Delta^2}$, and $K = -\frac{g_1^2}{\Delta}$ into the amplitude $|\alpha|^2$, we can get $|\alpha|^2 = \sqrt{\frac{4g_1^2 \Delta^2}{(\kappa_a^2 \Delta^2 + 4g_1^4)} \frac{g_1 \Omega}{g_1^2}}$. The two-magnon loss channel cannot be constructed when the magnon parametric coupling interaction $g_1 = 0$, so the magnon cat state cannot be prepared, and this expression is meaningless. When $g_1 \rightarrow 0^+$, the denominator is the higher-order infinitesimal of the molecule, indicating that the amplitude of the magnon cat state will be very large at this time.

(2) In addition, we give another proof of the stable solution of Eq. (D1). By setting $\dot{\rho} = 0$, one can obtain

$$-i[H_{\text{eff}}, \rho] + (\kappa_{\text{eff}}/2)\mathcal{L}[m_2^2]\rho = 0. \quad (\text{D8})$$

Substituting the effective Hamiltonian H_{eff} and the two-magnon loss rate κ_{eff} , the steady-state equation reads

$$\hat{D}|\psi\rangle\langle\psi|m^{\dagger 2} - m^{\dagger 2}\hat{D}|\psi\rangle\langle\psi| + \text{H.c} = 0, \quad (\text{D9})$$

where $\hat{D} = ((\frac{\kappa_{\text{eff}}}{2} + iK)m^2 - iJ)$ and we have set $\rho = |\psi\rangle\langle\psi|$. This equation shows that the steady state $|\psi\rangle$ satisfies

$$\left(\left(\frac{\kappa_{\text{eff}}}{2} + iK\right)m^2 - iJ\right)|\psi\rangle = 0. \quad (\text{D10})$$

Setting $\alpha^2 = \frac{iJ}{\frac{\kappa_{\text{eff}}}{2} + iK}$, we can rewrite Eq. (D10) as

$$(m^2 - \alpha^2)|\psi\rangle = 0. \quad (\text{D11})$$

Expressing $|\psi\rangle$ in terms of the Fock state $|n\rangle$ as

$$|\psi\rangle = \sum_n c_n |n\rangle,$$

where $n \geq 0$ refers to the number of excited magnons and c_n is the probability amplitude. The condition in Eq. (D11) gives a recursion relation as follows:

$$c_{n+2}\sqrt{n+2}\sqrt{n+1} = \alpha^2 c_n. \quad (\text{D12})$$

The recursion relation in Eq. (D12) shows that, when the magnon is initially in an even Fock state $|n\rangle$, i.e., in the ground state $|0\rangle$, the steady state $|\psi\rangle$ can be expressed as

$$|\psi\rangle = \sqrt{\frac{2}{e^{|\alpha|^2} + e^{-|\alpha|^2}}} \sum_{n=\text{even}} \frac{\alpha^n}{\sqrt{n!}} |n\rangle. \quad (\text{D13})$$

Similarly, when the magnon is initially in the odd Fock state $|n\rangle$, i.e., in the single-excitation state $|1\rangle$, the steady state $|\psi\rangle$ becomes

$$|\psi\rangle = \sqrt{\frac{2}{e^{|\alpha|^2} - e^{-|\alpha|^2}}} \sum_{n=\text{odd}} \frac{\alpha^n}{\sqrt{n!}} |n\rangle. \quad (\text{D14})$$

From Eqs. (D13) and (D14), we know that the state $|\psi\rangle$ is the cat state. Therefore, the steady-state solution of Eq. (D1) is the cat state.

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