

Electromagnetic backaction in a resonant Bose gas: Controlled spectral shifts and limits of local response

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Resonant atoms modify the electromagnetic environment that in turn determines their optical response, giving rise to a self-consistent light-matter problem at finite density. We investigate this feedback for the hydrogen Lyman- α transition using a local effective-medium description combined with an electromagnetic Green's function treatment of the atomic self-energy. Within the controlled regime, the self-consistent dressing produces a systematic density-dependent modification of the spectrum, including a shift of the spectral centroid and an asymmetric redistribution of integrated spectral weight. Comparison with a density expansion through fourth order provides an internal validity test: The local description remains quantitatively controlled up to approximately $2.0 \times 10^{21} \text{ m}^{-3}$ and becomes marginal above this density. Formal continuation to higher densities produces much stronger spectral restructuring, but this occurs together with the loss of perturbative and local-response control and is therefore not interpreted as a quantitative prediction of resolved high-density modes. To assess experimental relevance without assuming a microscopic hydrogen collision model, we additionally introduce a parametric collisional broadening $\Gamma_{\text{coll}} = n\beta$. Collisional broadening becomes comparable to the controlled electromagnetic centroid shift for a characteristic coefficient of order $10^{-14} \text{ m}^3 \text{ s}^{-1}$. The robust prediction of the local theory is thus a density-dependent electromagnetic dressing of the resonant response, whereas its high-density breakdown identifies the regime in which a spatially resolved, nonlocal microscopic description of collective electromagnetic coupling becomes necessary.

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I. INTRODUCTION

At sufficiently low densities, the optical response of an atomic gas can usually be understood in terms of weakly perturbed single-particle resonances. Interactions with the surrounding medium then appear mainly as small radiative shifts, collisional broadening, or effective mean-field corrections. In this regime, the atom's resonant structure remains essentially unchanged.

This separation between the emitter and the environment becomes less clear once the gas itself forms a dense, resonant medium. The electromagnetic field generated by an atom is then scattered collectively by the surrounding atoms and acts back on the emitter. Because the surrounding medium consists of the same resonant transitions whose spectrum is being measured, the optical response becomes intrinsically self-consistent.

Recent experiments and theoretical studies have shown that resonant electromagnetic interactions can remain observable even in remarkably dilute and disordered atomic systems, where independent-emitter descriptions would normally be

expected to apply. Delocalized excitonic signals and multiparticle coherences have been observed in thermal vapors at mean interatomic separations far exceeding the optical wavelength [1–3]. Related behavior has also been explored in cooperative scattering experiments in dense cold atomic ensembles [4–8], in multidimensional spectroscopy of ultracold gases [9], and in recent studies of retarded dipole-dipole interactions in thermal atomic clouds [10]. Together, these developments indicate that resonant electromagnetic interactions can reorganize the optical response on mesoscopic scales even under conditions where perturbative descriptions would normally be expected to remain valid.

Ultracold Bose gases provide a particularly clean system for studying such behavior. In addition to long coherence times and controllable densities, Bose-Einstein condensates (BECs) exhibit strongly modified density correlations compared with thermal ensembles [11–13]. Hydrogen condensates are especially attractive in this context because their optical transitions and dipole matrix elements are accurately known and because experimentally realized densities approach the regime in which the interatomic spacing becomes comparable to the resonant electromagnetic wavelength [14,15].

Despite extensive work on dense Bose gases, most theoretical descriptions remain focused on collisional physics, contact interactions, or perturbative radiative corrections [16,17]. Comparatively little attention has been paid to the possibility that the electromagnetic environment generated collectively by the gas may itself become dynamically reconstructed through the resonant atomic response.

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In the present work, we investigate this regime for a resonant hydrogen Bose gas from the perspective of self-energy and electromagnetic backaction. In recent work, we have argued that electromagnetic environments can modify not only interaction energies but also the internal response properties of atoms and molecules through self-consistent feedback mechanisms [18,19]. Here, we extend this viewpoint to a resonant Bose gas, where the surrounding medium is itself generated collectively by the same optical transitions whose spectrum is being reconstructed.

The present treatment focuses specifically on the local self-consistent component of the electromagnetic backaction within a homogeneous effective-medium description. In this picture, the surrounding gas modifies the atomic response through a dynamically reconstructed local electromagnetic environment described by the coincidence-limit Green's tensor. Explicit spatially resolved dipole-dipole networks and nonlocal many-body correlation functions are not included at this level of approximation. This distinction becomes particularly important at increasing density, where a local description must itself be tested for quantitative consistency rather than extrapolated without an internal validity criterion.

Building on a Green's function formulation of medium-induced self-energies, we therefore develop a self-consistent effective-medium framework in which the dressed atomic response, dielectric environment, and electromagnetic feedback are determined simultaneously, and we accompany the nonlinear formulation by a systematic density expansion. Applying the theory to the hydrogen Lyman- α transition, we find that electromagnetic feedback produces a smooth density-dependent redistribution of spectral weight already in the regime where the local description remains controlled. In particular, the spectral centroid shifts toward lower frequencies while the distribution of spectral weight becomes increasingly asymmetric.

A central aim of the present revision is to distinguish such controlled electromagnetic dressing from the behavior obtained by formally continuing the nonlinear local closure to higher densities. By comparing successive orders of the density expansion and monitoring the strength of the effective-medium response, we identify a finite validity envelope for the homogeneous local theory. The pronounced spectral restructuring found upon continuation to larger densities occurs only after this expansion has lost quantitative control and is therefore not interpreted here as a prediction of resolved additional resonances. Instead, it signals the regime in which a spatially resolved, nonlocal treatment of the resonant atomic ensemble becomes necessary.

The observability of the controlled electromagnetic signatures also depends on competing broadening mechanisms. In particular, neutral-hydrogen collisions can produce density-dependent line broadening, while detailed line-shape calculations show that such effects generally depend on more than the asymptotic long-range interaction alone [20,21]. Rather than assigning a specific ultracold collision coefficient that is not presently available from the literature, we therefore treat collisional broadening parametrically and determine the range in which it becomes comparable to the electromagnetic shift predicted by the local theory.

The resulting picture is thus more restricted, but also more controlled, than a formal extrapolation of the self-consistent equations to arbitrary density. The local effective-medium model predicts controlled electromagnetic dressing in the dilute and intermediate-density regime, while its own breakdown provides a quantitative indication of where microscopic nonlocal electrodynamics must replace the homogeneous description.

II. SELF-CONSISTENT LOCAL ELECTROMAGNETIC RESPONSE

A. Atomic response and electromagnetic self-energy

We consider an ensemble of atomic hydrogen at number density n and ask how the optical response of an individual atom is modified by the electromagnetic environment generated by the surrounding resonant gas. The starting point is the dynamical dipole polarizability of an isolated atom,

$$\alpha_{ij}^{(0)}(\omega) = \frac{1}{\hbar} \sum_m \left[\frac{\langle 0|\hat{d}_i|m\rangle\langle m|\hat{d}_j|0\rangle}{\omega_{m0} - \omega - i0^+} + \frac{\langle 0|\hat{d}_j|m\rangle\langle m|\hat{d}_i|0\rangle}{\omega_{m0} + \omega + i0^+} \right], \quad (1)$$

with $\omega_{m0} = (E_m - E_0)/\hbar$. For an isotropic gas, we use the corresponding scalar response

$$\alpha^{(0)}(\omega) = \frac{1}{3} \sum_i \alpha_{ii}^{(0)}(\omega). \quad (2)$$

The electromagnetic environment is described by the dyadic Green's tensor, which we separate into its vacuum- and medium-induced parts,

$$\mathbf{G}(\mathbf{r}, \mathbf{r}', \omega) = \mathbf{G}^{(0)}(\mathbf{r}, \mathbf{r}', \omega) + \mathbf{G}^{(\text{med})}(\mathbf{r}, \mathbf{r}', \omega). \quad (3)$$

Only the medium-induced contribution is included in the environmental dressing discussed below. The vacuum radiative contribution is already contained in the free-atom response and must not be counted a second time.

The medium-induced self-energy can be written as [19,22–24]

$$\Sigma_{ab}(\omega) = -\frac{\omega^2}{\epsilon_0 c^2} \sum_{\mu} \sum_{ij} \langle a|\hat{d}_i|\mu\rangle G_{ij}^{(\text{med})}(\mathbf{r}_A, \mathbf{r}_A, \omega) \langle \mu|\hat{d}_j|b\rangle. \quad (4)$$

The dressed dynamics follow from the resolvent

$$\mathbf{R}(\omega) = [\hbar\omega\mathbf{I} - \mathbf{H}_0 - \boldsymbol{\Sigma}(\omega)]^{-1}, \quad (5)$$

which determines the renormalized dipole response,

$$\alpha_{ij}(\omega) = \frac{1}{\hbar} \sum_{ab} \langle 0|\hat{d}_i|a\rangle R_{ab}(\omega) \langle b|\hat{d}_j|0\rangle. \quad (6)$$

For a single dominant resonance, the feedback can be expressed in the scalar form

$$\alpha(\omega) = \frac{\alpha^{(0)}(\omega)}{1 - \alpha^{(0)}(\omega)\mathcal{F}(\omega)}, \quad (7)$$

where

$$\mathcal{F}(\omega) = \frac{\omega^2}{3\epsilon_0 c^2} \text{Tr} \mathbf{G}^{(\text{med})}(\mathbf{r}_A, \mathbf{r}_A, \omega) \quad (8)$$

is the medium-induced electromagnetic feedback kernel.

Equation (7) is not merely a perturbative line-shift formula. In the present problem, the medium response entering $\mathcal{F}(\omega)$ is itself constructed from the dressed polarizability. Consequently, α , the dielectric environment, and the local electromagnetic feedback have to be determined self-consistently.

B. Self-consistent resonant effective medium

The surrounding hydrogen gas is described as a homogeneous isotropic resonant medium. At the level of local linear response, we describe the dielectric function using a Clausius-Mossotti closure, augmented here by the phenomenological factor $g^{(2)}(0)$ introduced below [25–27],

$$\frac{\varepsilon(\omega) - 1}{\varepsilon(\omega) + 2} = \frac{ng^{(2)}(0)\alpha(\omega)}{3\varepsilon_0} \equiv x(\omega). \quad (9)$$

For the BEC-like reference calculation used throughout the main analysis, $g^{(2)}(0) = 1$. We retain the factor explicitly only to permit the phenomenological sensitivity test discussed in Sec. VID and Appendix B 2; its inclusion should not be interpreted as a microscopic incorporation of quantum statistics into the Clausius-Mossotti relation. It is useful to introduce the dimensionless local-density parameter

$$x(\omega) = \frac{ng^{(2)}(0)\alpha(\omega)}{3\varepsilon_0}, \quad (10)$$

such that

$$\varepsilon(\omega) = \frac{1 + 2x(\omega)}{1 - x(\omega)}. \quad (11)$$

The resulting feedback loop is therefore

$$\alpha(\omega) \longrightarrow \varepsilon(\omega) \longrightarrow \mathbf{G}^{(\text{med})}(\omega) \longrightarrow \mathcal{F}(\omega) \longrightarrow \alpha(\omega). \quad (12)$$

The nonlinearity arises because the same resonant polarizability that is being dressed determines the effective dielectric environment responsible for the dressing.

To describe the local field experienced by an atom embedded in the medium, we employ an Onsager real-cavity construction. Introducing

$$k_0 = \frac{\omega}{c}, \quad k_m = \sqrt{\varepsilon(\omega)}k_0, \quad T(\omega) = \frac{3\varepsilon(\omega)}{2\varepsilon(\omega) + 1}, \quad (13)$$

the radiative contribution to the medium-induced local feedback takes the vacuum-subtracted form

$$\mathcal{F}_{\text{rad}}(\omega) = \frac{ik_0^2}{6\pi\varepsilon_0}[T^2(\omega)k_m(\omega) - k_0]. \quad (14)$$

The subtraction of the vacuum term is essential: For $\varepsilon \rightarrow 1$, one has $T \rightarrow 1$ and $k_m \rightarrow k_0$, so that

$$\mathcal{F}_{\text{rad}} \longrightarrow 0. \quad (15)$$

The complete local feedback kernel used in the numerical calculations contains a quasistatic density contribution and the vacuum-subtracted radiative Onsager

contribution,

$$\mathcal{F}(\omega) = \mathcal{F}_{\text{stat}} + \mathcal{F}_{\text{rad}}(\omega), \quad (16)$$

where

$$\mathcal{F}_{\text{stat}} = \frac{ng^{(2)}(0)}{3\varepsilon_0} \quad (17)$$

and

$$\mathcal{F}_{\text{rad}}(\omega) = \frac{ik_0^2}{6\pi\varepsilon_0}[T^2(\omega)k_m(\omega) - k_0]. \quad (18)$$

Here, $g^{(2)}(0)$ accounts phenomenologically for the local pair correlation entering the density-dependent feedback. Unless stated otherwise, we use $g^{(2)}(0) = 1$, corresponding to the BEC-like case considered throughout the main analysis. The subtraction of the vacuum radiative contribution ensures that the feedback kernel vanishes continuously in the dilute limit. Indeed, as $n \rightarrow 0$, one has $\varepsilon \rightarrow 1$, $T \rightarrow 1$, and $k_m \rightarrow k_0$, while $\mathcal{F}_{\text{stat}} \rightarrow 0$. Consequently,

$$n \rightarrow 0: \quad \mathcal{F}(\omega) \rightarrow 0, \quad \alpha(\omega) \rightarrow \alpha^{(0)}(\omega). \quad (19)$$

The corrected kernel also restores the appropriate high-frequency behavior of the dressed atomic response. In the asymptotic limit,

$$|\omega| \rightarrow \infty: \quad \frac{\alpha(\omega)}{\alpha^{(0)}(\omega)} \rightarrow 1, \quad \alpha(\omega) \propto \omega^{-2}. \quad (20)$$

The dilute-density and high-frequency limits provide two independent consistency checks on the self-consistent construction and are satisfied by the numerical implementation used below.

C. Local-response approximation

The term “local” refers here to two related approximations. First, the surrounding atomic ensemble is replaced by a homogeneous dielectric function $\varepsilon(\omega)$ rather than represented by its discrete atomic positions. Second, the electromagnetic feedback acting on a selected atom is obtained from the co-incidence limit of the effective-medium Green’s tensor, rather than from an explicit sum over spatially resolved propagators $\mathbf{G}(\mathbf{r}_i, \mathbf{r}_j, \omega)$ between individual atoms.

This approximation retains repeated electromagnetic dressing through the self-consistent loop (12), but it does not retain the microscopic geometry of resonant dipole-dipole networks, finite-size correlations, or a nonlocal dielectric response. In particular, it should not be assumed that formal solutions of the nonlinear local closure remain quantitatively meaningful at arbitrary density.

We therefore treat the domain of validity of the local theory as a quantity to be determined rather than as an assumption. The numerical analysis below monitors both the strength of the effective-medium response and the convergence of a systematic density expansion. This allows us to distinguish a controlled local regime from a marginal regime and from the density range in which the homogeneous closure is no longer quantitatively reliable.

D. Density expansion and internal consistency

A convenient internal control is obtained by expanding the self-consistent response about the dilute-gas limit. We define

$$x_0(\omega) = \frac{ng^{(2)}(0)\alpha^{(0)}(\omega)}{3\varepsilon_0} \quad (21)$$

and write the dressed response as

$$\frac{\alpha(\omega)}{\alpha^{(0)}(\omega)} = \sum_{j=0}^{\infty} c_j(\omega)x_0^j, \quad c_0 = 1. \quad (22)$$

In the numerical calculations, we retain the expansion successively through first, second, third, and fourth order, which we denote by P_1 , P_2 , P_3 , and P_4 , respectively,

$$P_N(\omega) = \alpha^{(0)}(\omega) \sum_{j=0}^N c_j(\omega)x_0^j. \quad (23)$$

The coefficients through fourth order are obtained analytically from the same Onsager effective-medium closure used in the full nonlinear calculation. Their explicit expressions and the corresponding perturbative construction are given in Appendix A 3.

Agreement of successive truncations provides a direct measure of perturbative control. Conversely, a loss of hierarchy between $P_1, \dots, P_4$ indicates that a formal continuation of the nonlinear local solution can no longer be interpreted as a quantitatively controlled prediction. This criterion will be used below to establish the validity envelope of the local description.

E. Spectral observables

For comparison with the calculated spectra, we use the dimensionless linear susceptibility

$$\chi(\omega) = \frac{n\alpha(\omega)}{\varepsilon_0}. \quad (24)$$

The quantity $\text{Im } \chi$ characterizes the dissipative linear response of the medium. It should not be interpreted as an experimentally calibrated transmitted intensity; an explicit optical geometry would be required for such a conversion.

For line-shape comparisons, we introduce the detuning

$$\Delta = \frac{\omega - \omega_0}{2\pi} \quad (25)$$

and evaluate all integrated observables over the same fixed frequency window. The spectral centroid is defined by

$$\bar{\Delta} = \frac{\int_{\Delta_{\min}}^{\Delta_{\max}} d\Delta \Delta \text{Im } \chi(\Delta)}{\int_{\Delta_{\min}}^{\Delta_{\max}} d\Delta \text{Im } \chi(\Delta)}. \quad (26)$$

To characterize the asymmetric redistribution of spectral weight, we also define the integrated contributions on the

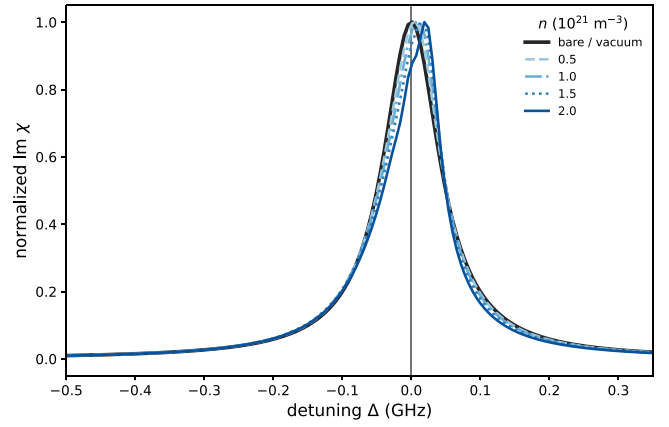


FIG. 1. Normalized dissipative response of the hydrogen Lyman- α transition within the controlled local-response regime. The curves show the isolated-atom response and the self-consistently dressed spectra for $n = 0.5, 1.0, 1.5$, and $2.0 \times 10^{21} \text{ m}^{-3}$. Each spectrum is normalized independently to its maximum to emphasize changes in line shape; the corresponding unnormalized susceptibility is shown in Appendix A 1. The vertical line marks zero detuning from the bare Lyman- α resonance.

red- and blue-detuned sides,

$$W_r = \int_{\Delta_{\min}}^0 d\Delta \text{Im } \chi(\Delta), \quad (27)$$

$$W_b = \int_0^{\Delta_{\max}} d\Delta \text{Im } \chi(\Delta), \quad (28)$$

and the corresponding ratio

$$R_{rb} = \frac{W_r}{W_b}. \quad (29)$$

The position of the largest sampled spectral value can additionally be used as a dominant-mode diagnostic. Unlike the centroid and integrated weights, however, this quantity is intrinsically sensitive to the frequency grid when the spectral maximum moves by less than one grid interval. We therefore use centroid- and integral-based quantities as the primary observables in the main text and retain the dominant-mode analysis only as a numerical diagnostic in Appendix A 2.

III. CONTROLLED ELECTROMAGNETIC DRESSING OF LYMAN α

A. Density evolution of the spectrum

We now apply the self-consistent local-response model to the hydrogen Lyman- α transition. Rather than formally continuing the nonlinear solution to arbitrarily large densities, we first restrict the analysis to the density range in which the local effective-medium description remains controlled according to the consistency tests discussed in Sec. IV.

Figure 1 shows the resulting dissipative response for densities between $n = 0.5 \times 10^{21}$ and $2.0 \times 10^{21} \text{ m}^{-3}$, together with the isolated-atom reference. To emphasize changes of the spectral line shape, each curve is normalized to its own maximum. The corresponding absolute susceptibility is shown in Appendix A 1.

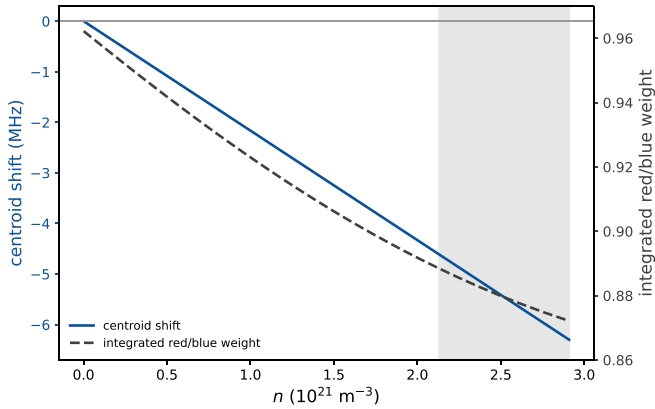


FIG. 2. Robust spectral observables as functions of atomic density. The spectral centroid shift (left axis) and integrated red-to-blue spectral-weight ratio (right axis) are evaluated from the self-consistently dressed Lyman- α response. The shaded regions indicate the controlled and marginal regimes of the local effective-medium description established independently in Sec. IV.

At the lowest densities, the dressed spectrum remains close to the isolated-atom line shape. Increasing density progressively redistributes spectral weight. The response becomes increasingly asymmetric, with the red- and blue-detuned parts of the line evolving differently, while the central resonant feature remains clearly identifiable. The deformation develops continuously with density and does not require the appearance of additional resolved resonances.

This behavior is the direct spectral signature of the feedback loop in Eq. (12). The resonant atomic polarizability modifies the effective dielectric response of the gas, which in turn changes the local electromagnetic Green's tensor and hence the polarizability itself. Even before this feedback becomes nonperturbative, the resulting frequency dependence distorts the two sides of the resonance differently and produces a frequency-dependent redistribution of spectral weight across the resonance.

The magnitude of the effect remains modest in the controlled regime. This matters for interpreting Fig. 1: The local theory robustly predicts smooth, density-dependent dressing and asymmetric redistribution of spectral weight, rather than the dramatic multipeak reconstruction obtained by formally extending the nonlinear closure to substantially higher densities. The latter regime is examined separately in Sec. IV.

B. Robust spectral observables

The normalized spectra make line-shape deformation visible, but they do not provide the most robust quantitative measure of electromagnetic dressing. In particular, the position of the largest sampled spectral value can become sensitive to the frequency grid when the maximum's displacement is comparable to the numerical frequency spacing. We therefore characterize the controlled response primarily through the spectral centroid and the integrated red-to-blue spectral-weight ratio defined in Sec. II E.

Figure 2 shows that both observables evolve smoothly with density. The centroid develops an increasingly negative shift, reaching approximately -4.4 MHz at the conservative bound-

ary of the controlled regime and approximately -6.3 MHz at the outer boundary of the marginal regime. Relative to the natural Lyman- α linewidth of approximately 99.8 MHz, these shifts correspond to only a few percent of the linewidth.

At the same time, the integrated red-to-blue spectral-weight ratio decreases continuously with increasing density. This behavior is not inconsistent with the negative centroid shift, since the two observables probe different properties of the line shape: R_{rb} compares the total spectral weight on either side of zero detuning, whereas the centroid additionally weights that spectral response by its detuning. Their simultaneous evolution therefore demonstrates that the self-consistent dressing cannot be described as a rigid displacement of an otherwise unchanged line; both the distribution and the frequency moment of the spectral weight change with density.

The centroid and integrated spectral weights are particularly useful because they average over the full line shape rather than relying on the position of a single frequency-grid point. They remain smooth under the numerical resolution tests and under changes for which the sampled dominant-mode position exhibits discrete steps. We therefore use these integral quantities as the primary measures of electromagnetic dressing in the remainder of the paper. The grid sensitivity of the dominant spectral mode is analyzed in Appendix A 2.

The results of Figs. 1 and 2 establish the central prediction that can be made without extrapolating the local theory beyond its controlled domain: Self-consistent electromagnetic backaction produces a continuous, density-dependent modification of the resonant hydrogen response, manifested by a spectral shift and an asymmetric redistribution of spectral weight. The next section determines quantitatively how far in density this local description can be trusted.

IV. VALIDITY OF THE LOCAL SELF-CONSISTENT DESCRIPTION

A. Perturbative convergence

The self-consistent local-response equations can formally be solved beyond the dilute-density regime. The existence of such a solution, however, does not by itself establish that the homogeneous local approximation remains quantitatively controlled. We therefore use the density expansion introduced in Sec. II D as an internal convergence test.

Figure 3 compares the successive approximations P_1 – P_4 over representative densities. All approximants are generated from the same Onsager effective-medium closure and differ only in the highest retained power of the dilute-density parameter x_0 . Their explicit coefficients are given in Appendix A 3.

At low density, successive orders of the expansion rapidly approach one another. The electromagnetic dressing is therefore perturbative in this regime, and the response is continuously connected to the isolated-atom limit. With increasing density, the differences between successive orders grow, but an ordered hierarchy remains visible over an intermediate range. This is the regime in which higher-order electromagnetic dressing is important while the expansion remains quantitatively useful.

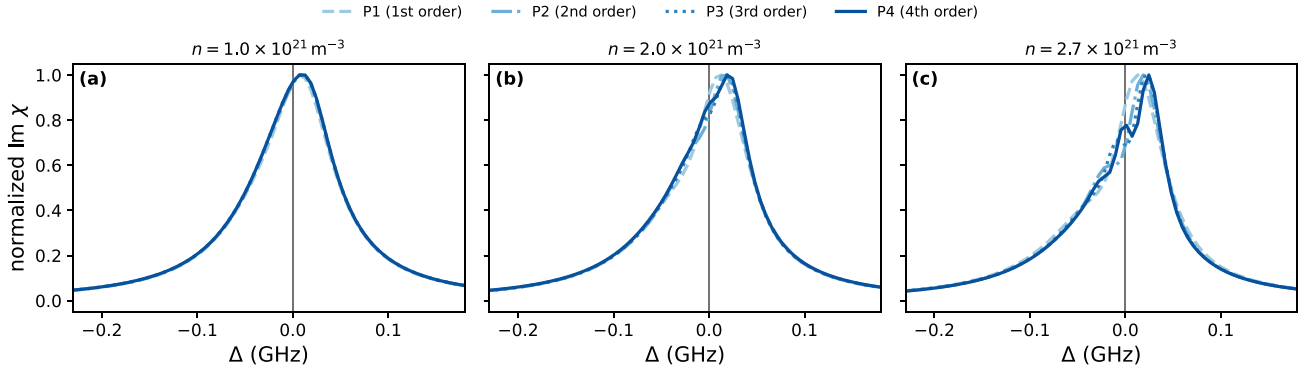


FIG. 3. Perturbative convergence of the normalized dissipative response near the Lyman- α resonance at (a) $n = 1.0 \times 10^{21} \text{ m}^{-3}$, (b) $n = 2.0 \times 10^{21} \text{ m}^{-3}$, and (c) $n = 2.7 \times 10^{21} \text{ m}^{-3}$. The curves show the successive perturbative approximations P_1 – P_4 defined in Eq. (23). At low density, the four orders are nearly indistinguishable. Higher-order corrections become visible on approaching the boundary of the controlled local-response regime, while at $n = 2.7 \times 10^{21} \text{ m}^{-3}$ the increasing separation between successive orders signals the deterioration of perturbative convergence. The vertical line marks zero detuning from the bare Lyman- α resonance.

At still larger densities, this hierarchy deteriorates. To quantify this behavior, we define the pointwise relative difference between the third- and fourth-order approximants as

$$E_{34}(\omega, n) = \frac{|P_4(\omega, n) - P_3(\omega, n)|}{\max[|P_4(\omega, n)|, |P_3(\omega, n)|, \alpha_{\text{floor}}]}. \quad (30)$$

Here, α_{floor} is a small numerical scale introduced only to avoid an ill-defined relative error at zeros of the response. We monitor both the magnitude of E_{34} across the spectral window and whether the successive differences decrease with perturbative order.

The loss of convergence is not identified from a particular visual feature of a spectrum. It follows instead from the behavior of successive approximations to the same self-consistent theory. This provides a theory-based criterion for determining where the local closure ceases to be a controlled continuation of its dilute-density limit.

B. Local validity envelope

Perturbative convergence is complemented by two measures of the strength of the local electromagnetic feedback. We monitor

$$\eta_K(n) = \max_{\omega} |\alpha^{(0)}(\omega) \mathcal{F}(\omega, n)| \quad (31)$$

and

$$\eta_{\varepsilon}(n) = \max_{\omega} |\varepsilon(\omega, n) - 1|. \quad (32)$$

The first measures the magnitude of the electromagnetic correction entering the dressed polarizability, while the second measures the departure of the effective medium from the dilute dielectric limit.

These thresholds are not intended to define a mathematically sharp boundary of the local theory. Rather, they provide conservative numerical criteria for separating a regime in which several independent diagnostics simultaneously indicate perturbative control from one in which that control progressively deteriorates. The conclusions below do not rely on interpreting either boundary as a singular physical density.

We classify the response as *controlled* when

$$\eta_K < 0.3, \quad \eta_{\varepsilon} < 0.3, \quad (33)$$

at least 95% of the spectral window satisfies

$$E_{34} < 0.01, \quad (34)$$

and both P_3 and P_4 retain the passive sign of the dissipative response, $\text{Im } \chi(\omega) \geq 0$, throughout the spectral window. With these conservative criteria, the local model remains controlled up to approximately

$$n_{\text{ctrl}} \simeq 2.0 \times 10^{21} \text{ m}^{-3}. \quad (35)$$

We additionally define a *marginal* regime in which the corresponding bounds are relaxed to

$$\eta_K < 0.5, \quad \eta_{\varepsilon} < 0.5, \quad (36)$$

with at least 95% of the spectral window satisfying $E_{34} < 0.05$, together with an improving perturbative hierarchy over at least 90% of the frequency window and no passivity violation in P_3 or P_4 . The outer boundary of this regime lies at approximately

$$n_{\text{marg}} \simeq 2.9 \times 10^{21} \text{ m}^{-3}. \quad (37)$$

Above this range, we classify the homogeneous local solution as *uncontrolled*.

Figure 4 summarizes these diagnostics and shows how the controlled, marginal, and uncontrolled regimes emerge from their combined behavior. The conservative controlled boundary at $n \simeq 2.0 \times 10^{21} \text{ m}^{-3}$ is reached while the individual diagnostics still evolve smoothly; it therefore represents an operational validity criterion rather than a singular physical transition. Beyond this density, the perturbative hierarchy and the effective-medium diagnostics progressively deteriorate, leading to the outer marginal boundary near $n \simeq 2.9 \times 10^{21} \text{ m}^{-3}$.

The density boundaries can also be expressed in terms of the mean interparticle spacing

$$a = n^{-1/3}. \quad (38)$$

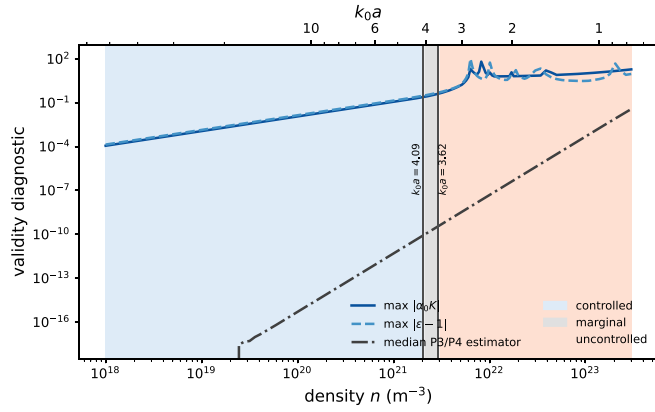


FIG. 4. Validity envelope of the homogeneous local effective-medium description. The diagnostics quantify the strength of the electromagnetic feedback and the convergence of the perturbative density expansion. The conservative controlled regime extends to approximately $2.0 \times 10^{21} \text{ m}^{-3}$, followed by a marginal interval up to approximately $2.9 \times 10^{21} \text{ m}^{-3}$. The upper axis expresses the density in terms of the corresponding mean interparticle spacing through $k_0 a$, with $a = n^{-1/3}$.

At the conservative boundary, $n \simeq 2.0 \times 10^{21} \text{ m}^{-3}$, we obtain

$$a \simeq 79.1 \text{ nm}, \quad k_0 a \simeq 4.09, \quad (39)$$

whereas at the outer marginal boundary, $n \simeq 2.9 \times 10^{21} \text{ m}^{-3}$,

$$a \simeq 70.1 \text{ nm}, \quad k_0 a \simeq 3.62. \quad (40)$$

An important consequence is that the loss of quantitative control occurs well before the simple geometric condition $k_0 a \sim 1$ is reached. Consequently, $k_0 a \sim 1$ cannot by itself be used as the criterion for the breakdown of the local theory. Instead, the limitation results from the combined growth of the resonant electromagnetic feedback and the deterioration of the density expansion.

Formal continuation of the nonlinear local equations beyond this envelope produces substantially stronger spectral reconstruction. In particular, sharp structures and strongly deformed spectra emerge at densities for which the perturbative hierarchy has already ceased to provide quantitative control. Representative formally continued solutions are shown in Appendix B 1.

We therefore do not interpret these high-density structures as quantitative predictions of additional resolved resonances. Rather, they demonstrate that the self-consistent electromagnetic feedback becomes strongly nonlinear in the same density range in which the homogeneous local approximation loses its internal control. The present theory robustly predicts smooth electromagnetic dressing below the validity boundary; the formal high-density reconstruction identifies a regime in which a more microscopic description is required.

In particular, once the homogeneous local closure becomes uncontrolled, the replacement of the discrete atomic ensemble by a local dielectric function can no longer be assumed to capture the relevant electromagnetic propagation. A natural extension is therefore a spatially resolved, nonlocal formulation in which the Green's tensor connects individual atomic positions and the response depends explicitly on interparticle

separation and spatial correlations. We return to this point in Sec. VIC.

V. COMPETITION WITH COLLISIONAL BROADENING

A. Parametric collision model

The electromagnetic dressing discussed above is not the only density-dependent contribution to the Lyman- α line shape. At finite density, binary collisions provide an additional source of dephasing and spectral broadening. To assess whether the electromagnetic shifts predicted within the controlled local-response regime can remain spectroscopically relevant in the presence of such processes, we introduce a phenomenological collisional decay rate

$$\Gamma_{\text{coll}}(n) = n\beta, \quad (41)$$

where β is an effective collisional broadening coefficient. With the convention used here, the corresponding contribution to the ordinary-frequency full width at half maximum is

$$\Delta\nu_{\text{coll}} = \frac{\Gamma_{\text{coll}}}{2\pi} = \frac{n\beta}{2\pi}. \quad (42)$$

We deliberately treat β as an external parameter rather than attempting to derive a quantitative hydrogen self-broadening coefficient from the long-range part of the interatomic interaction alone. Microscopic calculations of collision-induced broadening and line shapes require the relevant molecular interaction potentials and their associated collision dynamics, including contributions from intermediate and short interatomic separations [20,21]. A reduction of this physics to a single asymptotic interaction coefficient would therefore introduce an additional approximation whose quantitative accuracy cannot be established within the present local effective-medium description.

Equation (41) is consequently used as a sensitivity model: Rather than predicting β , we determine how large the collisional contribution would have to be to compete with the electromagnetic spectral modification obtained above. This formulation also separates the validity question of the electromagnetic model from the independent microscopic problem of hydrogen collisional broadening.

B. Collision sensitivity and observability

As a reference scale, the isolated Lyman- α transition in the present calculation has an ordinary-frequency natural linewidth of approximately 99.8 MHz (FWHM). Within the controlled local-response regime, the electromagnetic centroid displacement is considerably smaller: It reaches only a few percent of the natural linewidth near the upper end of the controlled density interval. Collisional broadening can therefore become relevant to the observability of the electromagnetic contribution even when it does not dominate the total linewidth.

To quantify this competition, we compare the electromagnetic centroid displacement with the additional linewidth generated by Eq. (42). For a given density, the characteristic coefficient for which both frequency scales become equal is

$$\beta_{\text{eq}}(n) = \frac{2\pi |\Delta\nu_{\text{cent}}(n)|}{n}, \quad (43)$$

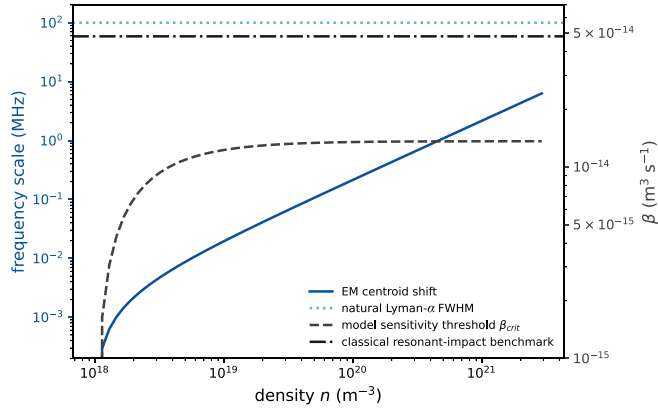


FIG. 5. Sensitivity of the controlled electromagnetic spectral modification to phenomenological collisional broadening. The electromagnetic centroid shift is compared with the collisional linewidth generated by $\Gamma_{\text{coll}} = n\beta$. The collision coefficient β is treated as an external parameter rather than as a calculated property of hydrogen. The indicated reference values provide scales for comparison with microscopic collision calculations; they are not inputs to the self-consistent electromagnetic model. The intersection identifies the value of β for which collisional broadening becomes comparable to the electromagnetic centroid shift.

where $\Delta\nu_{\text{cent}}$ denotes the electromagnetic centroid shift relative to the isolated-atom spectrum. Importantly, β_{eq} is a sensitivity threshold and not a prediction of the physical hydrogen collision coefficient.

Figure 5 shows that the competition between the two mechanisms occurs for coefficients of order $10^{-14} \text{ m}^3 \text{ s}^{-1}$ within the controlled density range. Using the conservative local-model boundary, equality between the collisional FWHM and the electromagnetic centroid displacement occurs at approximately

$$\beta_{\text{eq}} \simeq 1.36 \times 10^{-14} \text{ m}^3 \text{ s}^{-1} = 1.36 \times 10^{-8} \text{ cm}^3 \text{ s}^{-1}. \quad (44)$$

This number should again be interpreted as an observability threshold. This comparison concerns only the relative electromagnetic and collisional frequency scales and does not by itself establish experimental resolvability, which also depends on instrumental resolution, signal-to-noise ratio, and the measurement geometry. If it is comparable to or larger than this scale, collisional broadening becomes an increasingly important limitation on resolving the electromagnetic spectral modification.

The same phenomenological damping also affects the mathematical range over which the local self-consistent equations remain regular. Increasing collisional damping suppresses resonant feedback and thereby extends the density interval over which the local solution satisfies the adopted validity criteria. This regularizing effect, however, does not imply improved spectroscopic observability: The increased absolute electromagnetic shift is accompanied by an increased total linewidth. The corresponding continuation of the local validity boundary with collisional damping is documented in Appendix C 1.

A quantitative prediction for a particular hydrogen gas or condensate therefore requires microscopic collision

information beyond the scope of the present electromagnetic effective-medium model. Instead, this analysis establishes the relevant scale against which such a calculation must be compared. In this sense, Eq. (44) provides a direct interface between the controlled electromagnetic calculation and a future microscopic treatment of hydrogen collision dynamics.

VI. DISCUSSION

A. What is predicted robustly?

The analysis above separates two qualitatively different regimes of the self-consistent electromagnetic response. At sufficiently low density, the local effective-medium description is perturbatively controlled and provides a robust prediction of density-dependent electromagnetic dressing of the Lyman- α transition. The effect is not restricted to a rigid shift of the resonance. Instead, the spectral weight is redistributed asymmetrically, producing a density-dependent centroid displacement and a change of the integrated red-to-blue spectral-weight ratio. These effects are already present well inside the regime in which successive perturbative orders remain convergent.

Within this regime, spectral moments and integrated quantities provide the most reliable observables. In particular, the centroid shift and integrated spectral weights evolve smoothly with density and remain stable under the numerical tests described above. In contrast, observables based on the position of an individual sampled maximum are considerably more sensitive to frequency resolution. The controlled prediction of the present theory should therefore be understood primarily in terms of a smooth density-dependent redistribution of spectral weight, rather than the motion of a single sharply defined spectral peak.

A second robust conclusion concerns the evolution of the feedback itself. Increasing density progressively enhances higher-order terms in the electromagnetic self-consistency loop. Figure 3 shows this directly: The successive approximations P_1 – P_4 are nearly indistinguishable at low density, whereas their separation becomes appreciable on approaching the boundary of the controlled regime. Electromagnetic back-action therefore evolves from a weak perturbative correction toward a regime in which higher-order contributions become increasingly important. The local approximation loses quantitative control during this evolution, before the strongest spectral reconstruction produced by formal continuation of the self-consistent equations is reached.

B. What is not predicted by the present theory?

This validity boundary places an important restriction on the interpretation of the self-consistent solutions. In particular, the present calculation does not provide a controlled prediction of the number, positions, or widths of individual spectral modes obtained by formally continuing the local equations to high density. In that regime, successive perturbative approximations cease to form a controlled hierarchy and the implicit nonlinear response develops a pronounced sensitivity to branch selection. The strongly reconstructed spectra obtained from such a continuation are shown in Appendix B 1

for diagnostic purposes, but are not interpreted as quantitative spectral predictions.

Their appearance nevertheless contains useful physical information. The failure occurs because the electromagnetic feedback becomes strong enough that the assumptions underlying the homogeneous local closure can no longer be separated from the resulting spectral response. The formal high-density structures should therefore be interpreted as a diagnostic of the onset of strong collective electromagnetic feedback and, simultaneously, of the breakdown of the approximation used to describe that feedback. They motivate a more microscopic treatment, but they do not establish a particular high-density mode multiplicity.

The present theory also does not predict a microscopic collisional linewidth. As discussed in Sec. V, the coefficient β is deliberately retained as an external parameter because a quantitative description of hydrogen self-broadening requires collision physics beyond the electromagnetic effective-medium closure considered here. Likewise, the local dielectric response does not constitute a complete many-body theory of the atomic gas. By construction, we average out spatial correlations, recurrent scattering, and the discrete positions of individual emitters.

C. Relation to microscopic collective optics

The loss of validity of the local description at increasing density has a natural interpretation in the context of microscopic collective optics. A homogeneous dielectric function replaces the discrete atomic configuration by a local constitutive relation and thereby removes the explicit spatial propagation and correlations between individual dipoles. Microscopic calculations have shown that the optical response of resonant atomic ensembles can depart from the conventional continuous-medium description as light-mediated dipole-dipole correlations and recurrent scattering become important [5–8].

Complementary spectroscopic experiments have directly revealed long-range dipole-dipole interactions and many-body correlations in dilute thermal and ultracold atomic ensembles [1–3,9,10]. In particular, spatially resolved retarded dipole-dipole coupling has recently been identified as an essential ingredient for reproducing collective spectroscopic signals in a thermal atomic cloud [10]. These results reinforce the conclusion suggested by our validity analysis: Once the microscopic spatial organization of the ensemble participates explicitly in the optical response, its description solely through a scalar local $\epsilon(\omega)$ is no longer generally sufficient.

The validity analysis therefore suggests a specific extension of the present framework. Instead of closing the electromagnetic problem through a homogeneous local permittivity, the field-mediated interaction may be retained explicitly through the spatial Green's tensor $\mathbf{G}(\mathbf{r}, \mathbf{r}', \omega)$ that couples dipoles at distinct positions $\mathbf{r}$ and $\mathbf{r}'$. The response then becomes intrinsically nonlocal, with the atomic distribution and its spatial correlations entering explicitly. At the statistical level, these correlations can be characterized by quantities such as

$$g^{(2)}(\mathbf{r}, \mathbf{r}'), \quad (45)$$

rather than by a local density alone. Equivalently, for a discrete configuration of atoms, the self-consistent problem becomes a coupled dipole network in which propagation between different atomic positions is mediated by $\mathbf{G}(\mathbf{r}_i, \mathbf{r}_j, \omega)$.

Such a construction would retain the central physical mechanism studied here, electromagnetic backaction of the environment on the atomic response, while removing the assumption that the surrounding ensemble can be represented by a homogeneous local medium. Importantly, the present results identify the regime in which this extension becomes necessary: The breakdown is encountered while the mean interparticle spacing remains a non-negligible electromagnetic length scale, rather than only in an extreme continuum limit. The transition from the controlled local regime to a spatially resolved nonlocal theory is therefore a consequence of the validity analysis itself, rather than merely a generic direction for future work.

D. Quantum-statistical correlations

Quantum statistics provide an additional source of spatial correlation that is absent from the homogeneous local closure. To estimate the possible sensitivity of the electromagnetic feedback to this effect, we have also considered a phenomenological modification in which the relevant local feedback contribution is weighted by the zero-separation pair-correlation factor $g^{(2)}(0)$. The cases $g^{(2)}(0) = 1$ and $g^{(2)}(0) = 2$ provide, respectively, BEC-like and thermal limiting values for this restricted test. The resulting comparison is shown in Appendix B 2.

This procedure should not be interpreted as a microscopic theory of Bose correlations. Multiplication of a local feedback term by $g^{(2)}(0)$ changes the strength of the prescribed local interaction but does not introduce the spatial dependence of the pair-correlation function, recurrent propagation between different atoms, or the correlation-induced modification of the electromagnetic Green's tensor. Accordingly, the comparison is used only as a sensitivity test and not as a quantitative prediction of the difference between condensed and thermal hydrogen.

A consistent treatment of quantum-statistical effects belongs instead to the nonlocal extension outlined above, where $g^{(2)}(\mathbf{r}, \mathbf{r}')$ can enter together with the spatial Green's tensor and the atomic response. Such a formulation would make it possible to distinguish electromagnetic collective effects arising from density alone from those associated specifically with the quantum statistics and spatial correlations of the gas.

VII. CONCLUSIONS

We have investigated the self-consistent electromagnetic dressing of the hydrogen Lyman- α transition within a homogeneous local effective-medium description. In the revised formulation, the atomic polarizability, dielectric response, and medium-induced electromagnetic self-energy are determined within the same Onsager local-field closure, with the vacuum contribution consistently separated from the medium-induced feedback.

The resulting theory predicts a density-dependent modification of the resonant response already within a regime in which the local description remains quantitatively controlled.

The most robust signatures are a smooth shift of the spectral centroid and a redistribution of integrated spectral weight. These quantities remain considerably more stable than the position of an individual sampled spectral maximum and therefore provide the appropriate observables for characterizing the electromagnetic dressing.

A central result of the present analysis is that the range of validity of the local self-consistent description can be established independently of any visually selected spectral crossover. Successive density-expansion orders, the magnitude of the electromagnetic feedback, the departure from the dilute dielectric limit, and passivity provide complementary internal consistency tests. With the conservative criteria adopted here, the local description is controlled up to approximately $2.0 \times 10^{21} \text{ m}^{-3}$ and becomes marginal above this density, with the outer marginal boundary near $2.9 \times 10^{21} \text{ m}^{-3}$.

Formal continuation of the non-linear equations to higher densities produces much stronger spectral restructuring, including sharp and multifeature line shapes. However, this restructuring develops only after the perturbative hierarchy and other internal validity diagnostics have begun to deteriorate. We therefore do not regard the detailed number, positions, or widths of these high-density spectral features as controlled predictions of the present theory. Instead, their emergence demonstrates that electromagnetic feedback is becoming nonperturbative in a regime in which the homogeneous local approximation itself is no longer sufficient. The robust conclusion is thus the onset and progressive growth of self-consistent electromagnetic backaction, rather than a particular high-density multippeak spectrum.

We have additionally quantified the competition with collisional broadening without assigning an unsupported microscopic collision coefficient to low-temperature hydrogen. Treating $\Gamma_{\text{coll}} = n\beta$ parametrically shows that collisional broadening becomes comparable to the controlled electromagnetic centroid shift for a characteristic coefficient of order $10^{-14} \text{ m}^3 \text{ s}^{-1}$, with $\beta_{\text{eq}} \simeq 1.36 \times 10^{-14} \text{ m}^3 \text{ s}^{-1}$ at the conservative validity boundary. This value is an observability threshold, not a prediction of the hydrogen collision coefficient. A quantitative experimental assessment therefore requires microscopic collision information appropriate to the relevant hydrogen state and temperature.

The validity analysis also identifies the natural theoretical extension of the present approach. At densities beyond the controlled local regime, the atomic ensemble should no longer be represented solely by a homogeneous $\epsilon(\omega)$. A spatially resolved formulation based on $\mathbf{G}(\mathbf{r}, \mathbf{r}', \omega)$ can retain electromagnetic propagation between distinct atoms and incorporate the spatial pair correlation function $g^{(2)}(\mathbf{r}, \mathbf{r}')$. Such a non-local description would provide the appropriate framework for determining how the controlled low-density electromagnetic dressing evolves into the collective many-body optical response at higher densities.

DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

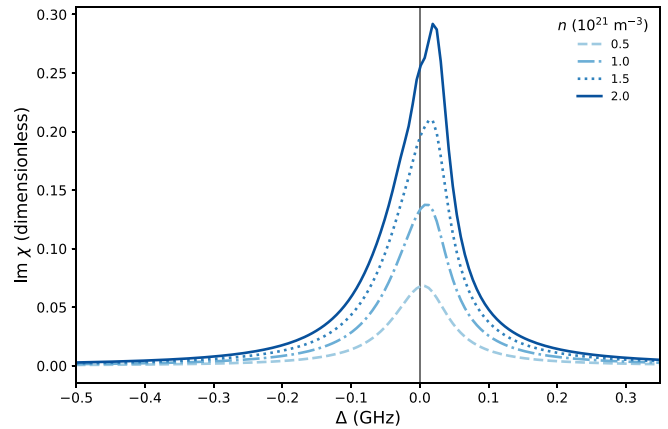


FIG. 6. Unnormalized dissipative susceptibility $\text{Im } \chi(\omega)$ in the controlled local-response regime. The curves correspond to the same densities used for the normalized spectra in Fig. 1. The vertical line marks zero detuning from the bare Lyman- α resonance.

APPENDIX A: ADDITIONAL SPECTRAL AND NUMERICAL DIAGNOSTICS

The main text focuses on spectral quantities that remain robust within the controlled local-response regime. In this Appendix, we provide complementary diagnostics concerning the absolute response amplitude and the numerical sensitivity of peak-based observables.

1. Absolute susceptibility

Figure 6 shows the unnormalized dissipative susceptibility corresponding to the normalized spectra presented in Fig. 1. As defined in Eq. (24), the dimensionless susceptibility is $\chi(\omega) = n\alpha(\omega)/\epsilon_0$. Normalizing each spectrum to its individual maximum, as in Fig. 1, isolates line-shape changes. The unnormalized quantity retains, in addition, the density dependence of the macroscopic response amplitude.

The absolute response increases with density while retaining the gradual line-shape deformation discussed in Sec. III A. Consequently, the asymmetric spectral redistribution seen in the normalized spectra is not generated by the normalization procedure. The normalization only removes the overall density-dependent amplitude to make changes in spectral shape more readily visible.

We emphasize that $\chi(\omega)$ is the linear susceptibility of the homogeneous effective medium. The present calculation does not specify a particular experimental transmission or absorption geometry. Converting to an absolute detected optical signal would also require the sample geometry, propagation length, and the corresponding electromagnetic propagation problem.

2. Grid sensitivity of the dominant spectral mode

In addition to the integral observables employed in the main text, one may define the dominant spectral mode as

$$\Delta_{\text{max}} = \arg \max_{\Delta} \text{Im } \chi(\Delta). \quad (\text{A1})$$

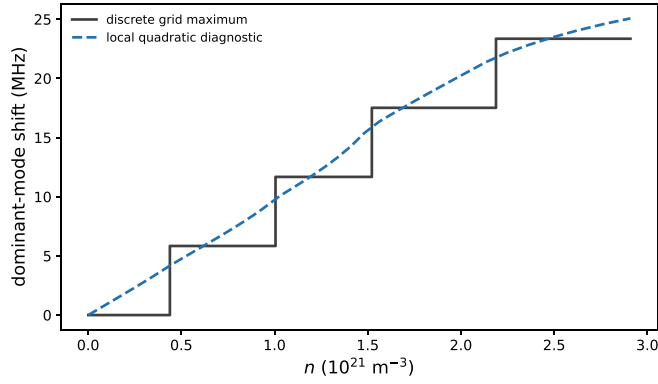


FIG. 7. Numerical sensitivity of the dominant-mode position to the frequency discretization. Direct selection of the largest sampled spectral value produces discrete changes of the inferred peak position as the maximum moves between neighboring frequency points. A local estimate of the maximum substantially reduces this discretization effect. The behavior motivates the use of moment- and integral-based observables in the main analysis.

When $\Delta_{\max}$ is estimated directly from the largest value on a discrete frequency grid, its apparent position can change only in integer multiples of the grid spacing. This becomes relevant when the physical shift of the maximum is comparable to the numerical frequency resolution. Figure 7 illustrates this discretization effect and compares the directly sampled peak position with a local estimate of the spectral maximum.

As seen in Fig. 7, the steplike behavior of the sampled maximum is therefore a property of the estimator rather than a discontinuity of the underlying spectrum. In contrast, the centroid and integrated spectral weights defined in Sec. II E use information from the complete spectral profile and evolve smoothly with density.

For this reason, the dominant-mode position is retained only as a secondary diagnostic. Neither the boundary of the controlled local-response regime nor the principal conclusions of the present work are inferred from discrete changes of $\Delta_{\max}$.

3. Density-expansion coefficients

The perturbative solutions P_N introduced in Eq. (23) are obtained by expanding the same self-consistent Onsager effective-medium closure used for the full nonlinear response. For completeness, we give here the explicit expansion employed in the numerical calculations.

We define the dilute-density expansion parameter as

$$x_0(\omega) = \frac{ng^{(2)}(0)\alpha^{(0)}(\omega)}{3\epsilon_0} \quad (\text{A2})$$

and introduce the dimensionless radiative quantity

$$g(\omega) = \alpha^{(0)}(\omega) \frac{i}{6\pi\epsilon_0} \left(\frac{\omega}{c}\right)^3. \quad (\text{A3})$$

The dressed polarizability is then expanded as

$$\frac{\alpha(\omega)}{\alpha^{(0)}(\omega)} = \sum_{j=0}^{\infty} c_j[g(\omega)]x_0^j(\omega), \quad (\text{A4})$$

with

$$c_0(g) = 1, \quad (\text{A5})$$

$$c_1(g) = \frac{7g+2}{2}, \quad (\text{A6})$$

$$c_2(g) = \frac{196g^2 + 103g + 8}{8}, \quad (\text{A7})$$

$$c_3(g) = \frac{3430g^3 + 2625g^2 + 491g + 16}{16}, \quad (\text{A8})$$

$$c_4(g) = \frac{2\,68\,912g^4 + 2\,70\,284g^3 + 81\,630g^2 + 7963g + 128}{128}. \quad (\text{A9})$$

Accordingly, the order- N approximation used throughout the convergence analysis is

$$P_N(\omega) = \alpha^{(0)}(\omega) \sum_{j=0}^N c_j[g(\omega)]x_0^j(\omega), \quad N = 0, \dots, 4. \quad (\text{A10})$$

Thus, $P_0 = \alpha^{(0)}$ is the isolated-atom response, while P_1, P_2, P_3 , and P_4 successively retain the first four density-dependent corrections generated by the same local self-consistent closure. They are therefore successive truncations of a single density expansion rather than independent response models.

The expansion provides an internal consistency test of the nonlinear solution. In a perturbatively controlled regime, increasing the truncation order should reduce the difference between successive approximations. Conversely, the loss of such an improving hierarchy indicates that the formal nonlinear solution is no longer supported by a controlled expansion about the dilute-density limit. This criterion is used in Sec. IV A as one of the diagnostics defining the validity envelope of the local effective-medium description.

APPENDIX B: FORMAL CONTINUATION AND PHENOMENOLOGICAL CORRELATION TESTS

The local self-consistent equations can formally be continued beyond the density interval in which the perturbative hierarchy remains controlled. This Appendix documents the resulting behavior and separates it from the quantitative predictions made in the main text. We additionally report a phenomenological test of the sensitivity of the local feedback to the equal-position pair-correlation factor $g^{(2)}(0)$.

1. Formal continuation beyond the controlled regime

Figure 8 compares representative spectra inside the controlled local-response regime with spectra obtained by formally continuing the same nonlinear closure to substantially higher densities.

Within the controlled regime, the electromagnetic dressing evolves smoothly, as discussed in Sec. III. Upon formal continuation to higher density, the nonlinear equations generate much sharper spectral features and a substantially stronger reconstruction of the line shape. However, the appearance of these features coincides with the deterioration of the

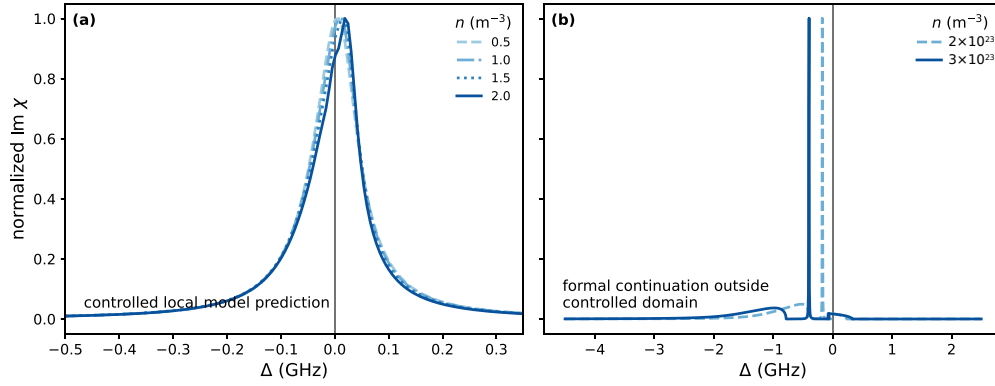


FIG. 8. Comparison of spectra obtained within the controlled local-response regime with spectra obtained by formal continuation of the nonlinear local closure beyond its validity envelope. The pronounced structures in the formally continued solutions occur in the same density range in which the perturbative hierarchy and other internal consistency diagnostics cease to provide quantitative control. These curves are therefore shown as diagnostics of the nonlinear local equations and are not interpreted as predictions of resolved physical resonances.

perturbative hierarchy discussed in Sec. IV A and with the loss of the local validity criteria established in Sec. IV B.

The existence of a numerical solution of the implicit nonlinear equations is therefore not, by itself, sufficient to establish a physical high-density spectrum. In this regime, root selection and the continuation of the self-consistent solution become nontrivial, and the detailed high-density structures are not robust enough to be identified with additional resolved resonances.

The formal solutions nevertheless contain useful information: They indicate that the local electromagnetic feedback becomes strongly nonlinear before the homogeneous description can be extrapolated to arbitrarily large density. Determining the optical response beyond this point requires a spatially resolved theory in which propagation between individual atoms and nonlocal dipole-dipole correlations are retained explicitly.

2. Phenomenological sensitivity to pair correlations

The local model contains the equal-position second-order correlation factor $g^{(2)}(0)$ through the effective-medium density parameter and the quasistatic feedback contribution. In the main analysis, we use

$$g^{(2)}(0) = 1, \quad (\text{B1})$$

corresponding to the BEC-like reference case. To assess the sensitivity of the local closure to this parameter, we additionally compare it with the thermal-gas value

$$g^{(2)}(0) = 2. \quad (\text{B2})$$

The comparison demonstrates that increasing $g^{(2)}(0)$ strengthens the effective local feedback and correspondingly moves the breakdown of the local approximation to lower physical densities. This is expected from the appearance of $ng^{(2)}(0)$ in the local effective-medium parameter.

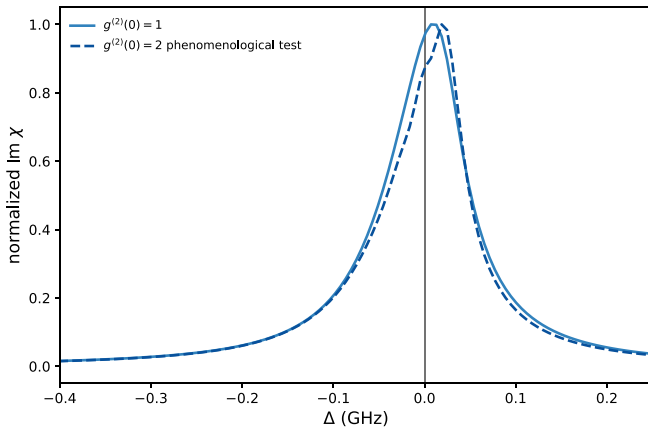


FIG. 9. Phenomenological sensitivity of the local self-consistent response to the equal-position pair-correlation factor $g^{(2)}(0)$. The BEC-like case $g^{(2)}(0) = 1$ is compared with the thermal value $g^{(2)}(0) = 2$. This comparison tests the sensitivity of the local closure to the strength of local pair correlations; it is not a microscopic calculation of quantum-statistical many-body correlations.

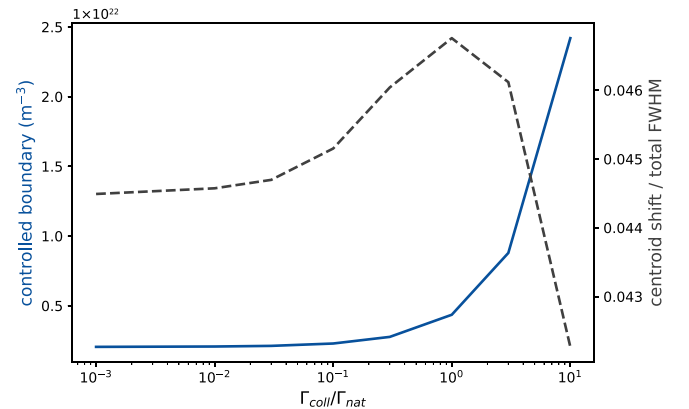


FIG. 10. Effect of phenomenological collisional damping on the validity boundary of the local self-consistent model and on the relative electromagnetic spectral signal evaluated at that boundary. Increasing damping weakens the resonant nonlinear feedback and allows the local equations to remain numerically controlled to larger densities. The resulting increase of the absolute accessible density does not imply a corresponding increase of the electromagnetic signal relative to the total linewidth.

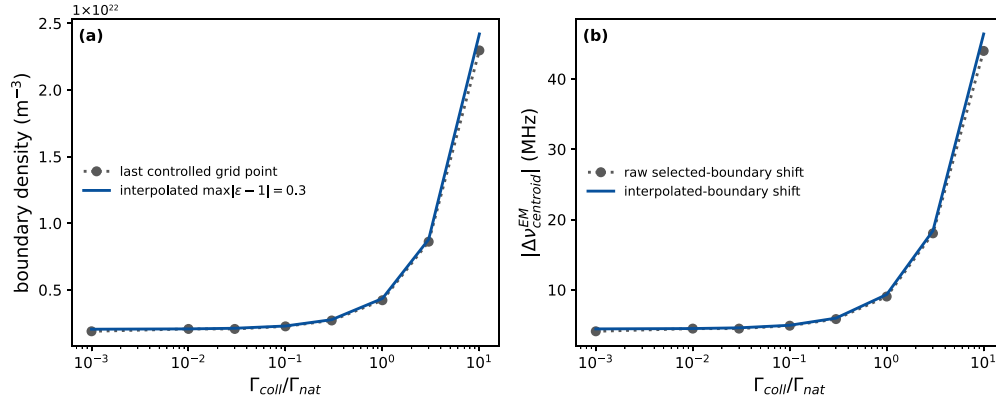


FIG. 11. Numerical determination of the collision-dependent validity boundary. Direct selection of the last sampled density satisfying the local validity criterion produces a small grid-dependent discretization. Interpolation between the neighboring density points that bracket the threshold yields a smooth estimate of the boundary and of the spectral centroid evaluated there. No interpolation or smoothing of the spectra themselves is involved.

The result should, however, be interpreted only as a phenomenological sensitivity test. Replacing $g^{(2)}(0)$ by its thermal value does not introduce the spatial correlation function $g^{(2)}(\mathbf{r}, \mathbf{r}')$ or the corresponding nonlocal electromagnetic propagation between correlated atomic positions. Consequently, Fig. 9 does not constitute a microscopic prediction of the difference between BEC and thermal optical spectra. Such a calculation would require the nonlocal extension discussed in Sec. VIC.

APPENDIX C: COLLISIONAL BROADENING AND VALIDITY DIAGNOSTICS

The collision analysis in Sec. V introduces a phenomenological homogeneous broadening rate

$$\Gamma_{\text{coll}} = n\beta, \quad (\text{C1})$$

where β is treated as an external collision coefficient. The purpose of the analysis is not to assign a microscopic low-temperature hydrogen collision rate, but to determine how an additional density-dependent linewidth affects both the visibility of the electromagnetic dressing and the numerical validity of the local self-consistent solution.

1. Effect of collisional damping on the local validity boundary

Additional homogeneous damping modifies the resonant polarizability and therefore also changes the strength of the nonlinear electromagnetic feedback. Figure 10 shows how the upper density boundary of the local description changes as the collisional linewidth is increased.

The extension of the controlled density range with increasing damping can be understood from the reduced resonant atomic response. Broadening the bare resonance lowers the maximum polarizability and thereby reduces the strength of the self-consistent effective-medium feedback. Collisions can therefore regularize the nonlinear local equations in a mathematical sense.

This regularization should not be confused with improved experimental observability. Although the density at which the local model reaches its validity boundary increases with

collisional damping, the total linewidth increases at the same time. The electromagnetic shift therefore remains a small fraction of the broadened spectral width. Additional damping can stabilize the local closure while simultaneously making its electromagnetic spectral signature more difficult to resolve.

2. Numerical determination of the collision-shifted boundary

The validity boundary used in the collision analysis is determined from the same local-response criterion employed in the zero-collision calculation. On a finite density grid, selecting the last density satisfying the criterion introduces a small discretization of the inferred boundary. This effect is illustrated in Fig. 11.

The apparent steplike structure obtained from direct grid selection is not a physical discontinuity. It arises because the boundary moves continuously with damping while the available density values are discrete. To remove this purely numerical quantization, we interpolate in $\ln n$ between the two neighboring densities that bracket the chosen validity threshold and evaluate the corresponding centroid by interpolation over the same interval.

This procedure is used only to locate the density at which the continuous validity criterion is crossed. The calculated spectra themselves are neither smoothed nor interpolated. The resulting correction to the centroid is small compared with the spectral linewidth and does not affect the physical conclusions of the collision-sensitivity analysis.

3. Interpretation of the collision coefficient

The parameter β in Eq. (C1) is deliberately left as an external quantity. In ordinary-frequency units, the corresponding collisional contribution to the full width at half maximum is

$$\Delta \nu_{\text{coll, FWHM}} = \frac{n\beta}{2\pi}. \quad (\text{C2})$$

The sensitivity analysis can therefore be inverted to determine the value of β for which collisional broadening becomes comparable to a specified electromagnetic spectral shift. This construction provides an observability criterion without requiring a microscopic collision model.

A quantitative prediction of β for ultracold resonant hydrogen would require additional collision physics that is outside the present effective-medium treatment. Detailed calculations of hydrogen self-broadening show that linewidths can depend on the full molecular interaction potentials and on intermediate- and short-range encounters, in addition to the asymptotic resonant interaction [20,21]. Moreover, published calculations generally refer to specific transitions and

temperature regimes that need not coincide with the ultracold Lyman- α system considered here.

For these reasons, the collision analysis in the present work should be read as a sensitivity study: It determines how large a collision coefficient would have to be to compete with the controlled electromagnetic signal, but does not claim that the corresponding value is the microscopic low-temperature H-H collision coefficient.

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