

Quasi-two-dimensional spin helix and magnon-induced singularity in twisted bilayer graphene

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Twisted bilayer graphene exhibits prominent correlated phenomena in two distinct regimes: a Kondo lattice near the magic angle, resembling heavy fermion systems, and a triangular correlated domain wall network under interlayer bias, akin to sliding Luttinger liquids previously introduced for cuprates. Combining these characteristics, here we investigate a system where interacting electrons in the domain wall network couple to localized spins. Owing to inter-domain-wall correlations, a quasi-two-dimensional spin helix phase within the localized spins emerges as a result of spatial phase coherence across parallel domain walls. Within the spin helix phase, magnons can induce a singularity, reflected in the scaling exponents of various correlation functions, accessible through electrical means and by adjusting the twist angle. We predict observable features in magnetic resonance and anisotropic paramagnetic spin susceptibility for the spin helix and the magnon-induced singularity, serving as experimental indicators of the interplay between the Kondo lattice and sliding Luttinger liquids. Integrating critical aspects of Luttinger liquid physics, magnetism, and Kondo physics in twisted bilayer graphene, our findings offer insights into similar correlated phenomena across a broad range of twisted van der Waals structures.

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Introduction. Twisted bilayer graphene (TBG) has emerged as a promising platform for exploring correlated phenomena. When the angle between the two layers are close to the magic angle, the Fermi velocity is dramatically suppressed [1–5], resulting in the formation of quasiflat energy bands and a significant enhancement in the density of states. Consequently, interactions between electrons become significant as compared to the bandwidth, leading to various correlated quantum states [6–9], such as superconductivity [10], correlated insulating states [11,12], strange metals [13,14], orbital ferromagnetism [15–18], and nematic order [19,20]. The many-body correlations are believed to originate from the strongly localized wave functions at AA-stacking regions [4,21–29]. Furthermore, recent investigations revealed that magic angle TBG shows characteristics akin to those of heavy fermion compounds, where localized moments develop near AA-stacking regions and couple to conduction electrons through spin-exchange coupling [29–36], thereby providing an additional perspective on Kondo lattice systems.

Remarkably, correlated phenomena in TBG can be achieved without relying on specific twisted structures. In particular, upon applying perpendicular electric fields, domain walls separating the AB- and BA-stacking areas are known to host gapless modes [37–52], which form a two-dimensional (2D) triangular quantum network. This network exhibits correlated phenomena and high tunability [53], with electrically adjustable parameters such as Fermi velocity, bandwidth, and

interaction strengths of the domain wall modes, which can further control the instability of the network toward various orders. This distinct regime highlights that TBG can host correlated phenomena across a broad range of configurations, independent of precise stacking or magic-angle conditions.

Motivated by these discoveries, here we explore a system that combines two distinct characteristics in TBG, including correlated network and the coupling between itinerant carriers and localized magnetic moments [Figs. 1(a)–1(c)]. The coupling between the interacting electrons and localized spins in such quasi-2D correlated networks has remained insufficiently explored. Specifically, we examine a system comprising interacting electrons that traverse the domain walls, coupling to localized moments distributed on the graphene layers through Kondo-type interaction [see Figs. 1(a) and 1(c)]. The first entity forms a triangular domain wall network [54,55], extending coupled wire models from earlier studies on cuprates [56–59]. The localized moments could be introduced through magnetic adatoms [60,61] or nuclear spins via isotope engineering [62,63]. The electrons mediate a spatially oscillating indirect coupling between these moments within and across parallel domain walls, leading to helical magnetic ordering of the localized moments at sufficiently low temperatures. In contrast to isolated one-dimensional systems [64–70], the spatial phase coherence developed in parallel domain walls leads to the formation of a quasi-2D spin helix [see Fig. 1(d)]. Within the spin helix phase, we identify a magnon-induced singularity, reflected in the scaling exponents of various correlation functions and the carrier velocity. This singularity is accessible not only by electrical means but also by varying the twist angle. We further predict observable features of the spin helix and magnon-induced singularity in magnetic resonance and paramagnetic susceptibility, providing an electrically

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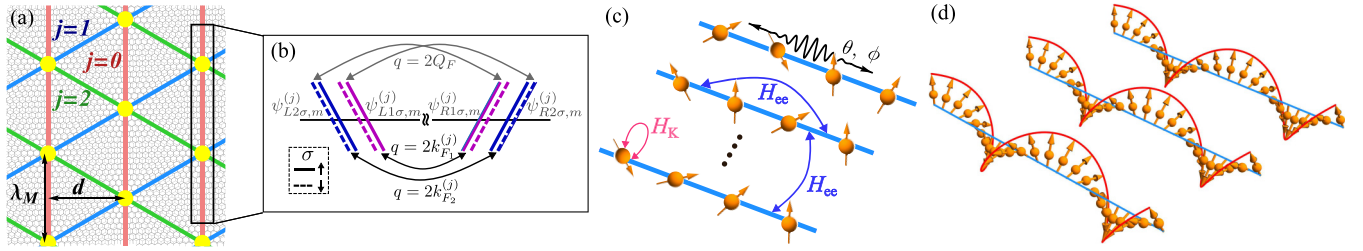


FIG. 1. (a) TBG domain wall network formed by three arrays (labeled by $j = \{0, 1, 2\}$) of parallel domain walls (red, blue, and green lines), each rotated by 120° relative to the others. The domain walls are indexed by $m \in [1, N_{\text{dw}}]$ with the number N_{dw} of domain walls per array. Here, λ_M represents the moiré wavelength and d is the distance between two adjacent domain walls. (b) Within a single domain wall, there are eight low-energy modes with Fermi wave vectors $k_{F_\delta, m}^{(j)}$ and represented by fermion fields $\psi_{\ell\delta\sigma, m}^{(j)}$, with the spin $\sigma \in \{\uparrow (\text{solid}), \downarrow (\text{dashed})\}$, propagation directions $\ell \in \{R, L\}$, and branches $\delta \in \{1 (\text{purple}), 2 (\text{blue})\}$. The arrows indicate backscattering processes with momentum transfers $q \in \{2k_{F_1, m}^{(j)}, 2k_{F_2, m}^{(j)}, 2Q_F \equiv k_{F_1, m}^{(j)} + k_{F_2, m}^{(j)}\}$. (c) Schematic of the electron-electron (e-e) interactions in parallel domain walls (blue lines) and spin-exchange interaction H_K with localized moments (represented as an orange arrow and sphere). The former includes contributions within and between parallel domain walls, leading to sliding Luttinger liquids characterized by bosonic fields (wavy curve) corresponding to the fermion fields in panel (b). (d) Sketch of spatially phase-coherent spin helices on three adjacent domain walls.

tunable platform for the interplay between correlated electrons and localized spins.

Model. Our Hamiltonian $H = H_{\text{ee}} + H_K$ includes the electronic subsystem, H_{ee} , and their coupling, H_K , to localized moments in TBG with a small but finite twist angle. Incorporating an interlayer bias in the continuum model [2,41], one can compute the density profile of the conduction electrons confined in the domain walls in order to construct the network model [53]. Using the standard bosonization [71], the electron subsystem, including both the kinetic energy and electron-electron interactions, is described by

$$H_{\text{ee}} = \frac{1}{N_{\text{dw}}} \sum_{j, q_\perp} \sum_{v, P} \int \frac{\hbar dr}{2\pi} \left[\frac{v_{vP}(q_\perp)}{\tilde{K}_{vP}(q_\perp)} |\partial_r \phi_{vP, q_\perp}^j(r)|^2 + v_{vP}(q_\perp) \tilde{K}_{vP}(q_\perp) |\partial_r \theta_{vP, q_\perp}^j(r)|^2 \right], \quad (1)$$

with the momentum component $\hbar q_\perp$ perpendicular to the domain walls. The bosonic fields ϕ and θ in Eq. (1) are labeled as the charge/spin $v \in \{c, s\}$ and symmetric/antisymmetric $P \in \{S, A\}$ sectors, respectively, corresponding to the linear combination of the eight gapless modes per domain wall in Fig. 1(b), and satisfy the commutation relation

$$\begin{aligned} & [\phi_{vP, q_\perp}^j(r), \theta_{v'P', q'_\perp}^{j'}(r')] \\ &= \frac{i\pi}{2} \delta_{jj'} \delta_{vv'} \delta_{PP'} \delta_{q_\perp, -q'_\perp} \text{sgn}(r' - r). \end{aligned} \quad (2)$$

Equation (1) is governed by $q_\perp$ -dependent functions $\tilde{K}_{vP}$ and v_{vP} serving as the effective interaction strength and carrier velocity generalized for the network. Since screened Coulomb interactions only enter the charge-symmetric sector ($vP = cS$) [53], for $vP \neq cS$ we take $\tilde{K}_{vP} = 1$ with v_{vP} given by the domain wall velocity v_{dw} . We further assume periodic boundary condition perpendicular to the domain walls within each array [54,56–59] and express $\tilde{K}_{cS}$ as a periodic function of $q_\perp$,

$$\tilde{K}_{cS}(q_\perp) = K_{cS} [1 + \lambda_1 \cos(q_\perp d) + \lambda_2 \cos(2q_\perp d)], \quad (3)$$

which we keep the first three Fourier components for simplicity. Here, we introduce the dimensionless parameters

$K_{cS} = (1 + \frac{U_{\text{ee}}}{\pi \hbar v_{\text{dw}}})^{-1/2}$ [71] and $\lambda_{1,2}$ to characterize the interaction strength, with U_{ee} estimated from the screened Coulomb interactions. The values of $\lambda_{1,2}$ are bounded by the condition $\tilde{K}_{cS}(q_\perp) > 0$ for $-\pi \leq q_\perp d \leq \pi$. The detailed derivation is shown in the Supplemental Material [72].

Interestingly, Eq. (1) generalizes the coupled-wire or sliding Luttinger liquid (SLL) Hamiltonian, as previously proposed for cuprates [56–59]. Alternative bosonized models for TBG have also been proposed [73–75], although they do not adopt the SLL description used here. Importantly, since the interaction-to-bandwidth ratio $U_{\text{ee}}/(\hbar v_{\text{dw}})$ can be estimated from the continuum model [53], the parameter K_{cS} can also be evaluated accordingly and exhibits electrical tunability—an advantage in two-dimensional moiré materials.

Before proceeding, we note that our analysis focuses on the low-temperature regime within the energy window set by the biased-induced local spectral gap in the AB- and BA-stacking regions [53]. In this regime, only domain wall modes participate in scattering processes, as the bulk modes in the moiré bands are gapped out [37,41–52]. Additionally, although scattering involving crossing domain walls may occur at their intersections, such processes are generally less relevant than those involving single or multiple parallel domain walls, since the corresponding operators enter the effective action without a spatial integral [54,55]. We therefore focus on scattering within a single domain wall or between parallel domain walls and omit the array index in the following.

The Kondo-type interaction H_K between conduction electrons and localized moments is given by

$$H_K = \sum_{k, m} \sum_{\mu, \sigma, \sigma'} \frac{J_K^\mu}{N_\perp} [\psi_{\sigma, m}^\dagger(r_k) \sigma_{\sigma\sigma'}^\mu \psi_{\sigma', m}(r_k)] S_m^\mu(r_k), \quad (4)$$

with the coupling J_K^μ with $\mu \in \{x, y, z\}$ (taking into account reduced $J_K^{x,y}$ for domain wall modes [72]), the number $N_\perp$ of localized moments within the transverse extent of the domain wall modes $\psi_{\sigma, m} \equiv \sum_{\ell, \delta} \psi_{\ell\delta\sigma, m}$, and the spin $S_m(r_k)$ (modeled as classical spins) near the domain wall with length S at the position labeled by r_k . Note that the interaction of this type in Eq. (4) is not unique to the Kondo interaction

in magnetic alloys or heavy fermions [76], as the hyperfine interaction also takes the same form. We also remark that the dipole-dipole interaction between the localized moments is significantly weaker than that of H_{ee} and H_K [77], and therefore not included in our analysis.

To proceed, we focus on the weak- J_K regime [64–69] and perform the Schrieffer-Wolff transformation on H_K by integrating out the electron degree of freedoms. Retaining terms up to second order in J_K , we obtain an indirect Ruderman-Kittel-Kasuya-Yosida (RKKY) interaction between localized moments [72],

$$H_R = \sum_{m,n} \sum_{\mu,\nu} \int \frac{dr dr'}{N_{\perp}^2} J_n^{\mu}(|r - r'|) S_{m+n}^{\mu}(r) S_m^{\nu}(r'), \quad (5)$$

with the spin operator $S_n^{\mu}(r_k)/a \rightarrow S_n^{\mu}(r)$ in continuum limit, short-distance cutoff a , and the spatially oscillating coupling strength J_n^{μ} proportional to the spin susceptibility of electrons between the n th-nearest-neighbor parallel domain walls. It is justified to focus on the weak- J_K regime here, since, for typical parameters, the Kondo temperature is well below all other relevant scales [64,65,69], so the localized spins are governed solely by the indirect RKKY interaction H_R . In momentum space, J_n^{μ} develops dips at momenta corresponding to scattering processes involving single or multiple parallel domain walls. As shown in Fig. 1(b), these backscattering processes include the intrabranch (Ω) processes with momentum transfer (projected onto the domain wall) $q = \pm 2k_{F_{1,2}}$ and the interbranch ($\Leftarrow$) ones with $q = \pm 2Q_F \equiv \pm(k_{F_1} + k_{F_2})$. The $\pm 2Q_F$ processes develop a global maximum (in absolute value) for $q_{\perp} = 0$, owing to more available states for scatterings [72]. To minimize energy, the localized moments tend to align with the Fourier component corresponding to the dip position $q = \pm 2Q_F$. Below, we discuss the ordering of these localized spins.

Spin helix formation and magnon spectrum. We now demonstrate that the localized moments tend to form a helical pattern with spatial period π/Q_F along the domain walls. Given the inherent C_3 rotational symmetry of our model, there is no preferred direction for the formation of the quasi-2D spin helix, making it equally probable to develop in any of the three arrays. Formally, we take the ansatz incorporating an offset phase ϑ_m depending on the domain wall index,

$$\langle S_m(r) \rangle = m_{2Q_F}(T) S N_{\perp} / a [\hat{x} \cos(2Q_F r + \vartheta_m) + \hat{y} \sin(2Q_F r + \vartheta_m)], \quad (6)$$

with the order parameter m_{2Q_F} satisfying $m_{2Q_F}(0) = 1$ and $m_{2Q_F}(T_{hx}) = 0$, and the ordering temperature T_{hx} . Before proceeding, we discuss the spatial rotational symmetry breaking in the ansatz. Namely, simultaneously establishing a helix in all three arrays would require the helix period π/Q_F to be commensurate with λ_M [see Fig. 1(a)], as the triangular network would otherwise lead to geometrical frustration. However, this commensurate condition necessitates precise tuning of the chemical potential. Under typical conditions, a quasi-2D spin helix will form within a single array as shown in Fig. 1(d), thereby breaking the C_3 rotational symmetry—a scenario we explore throughout this Letter.

To proceed, we derive the magnon spectrum from Eqs. (5) and (6) using spin-wave analysis, retaining only the

leading-order terms in the small parameter $1/(N_{\perp} S) \ll 1$. This procedure leads to a 2×2 matrix, whose twice-positive eigenvalue gives the magnon dispersion [72],

$$\begin{aligned} \hbar\omega(q, q_{\perp}) &= \frac{S}{2^{3/2} N_{\perp}} \sqrt{J_{q_{\perp}=0}^x(2Q_F) - J_{q_{\perp}}^z(q)} \\ &\times \sqrt{2J_{q_{\perp}=0}^x(2Q_F) - J_{q_{\perp}}^x(2Q_F + q) - J_{q_{\perp}}^x(2Q_F - q)}, \quad (7) \end{aligned}$$

where $J_{q_{\perp}}^{\mu}(q)$ is the Fourier transform of $J_n^{\mu}(r)$. The resulting magnon spectrum for representative parameters is shown in Fig. S4 of the Supplemental Material [72]. For $q_{\perp} = 0$, Goldstone zero modes are present at $q = 0, \pm 2Q_F$, corresponding to the breaking of spin rotational symmetry in the helix phase [67,69]; for $q_{\perp} \neq 0$, the Goldstone modes acquire finite energy. In realistic systems, however, these zero modes can be gapped due to finite-size effects [68,78], circumventing the Mermin-Wagner theorem for the thermodynamic limit. Namely, in a domain wall of length L , the momentum is quantized in unit of $q = \pi/L$ and the magnon energy is approximately constant (see Fig. S4 in the Supplemental Material [72]), $\hbar\omega(q = \frac{\pi}{L}, q_{\perp}) \approx \hbar\omega_0 \equiv S|J_{q_{\perp}=0}^x(2Q_F)|/2N_{\perp}$, set by the RKKY energy scale $J_{q_{\perp}=0}^x(2Q_F)$.

Next, we obtain the magnetic energy gain from Eqs. (5) and (6), highlighting a key difference compared to isolated channels [68,78]. Specifically, owing to the nonlocal contributions $J_{n \neq 0}^{\mu}$ in Eq. (5), the energy is minimized when the offset ϑ_m is uniform across domain walls [72]. Combined with the fact that the global maximum of $|J_{n \neq 0}^{\mu}|$ occurs at $q_{\perp} = 0$, this leads to the development of spatial phase coherence among spin helices in distinct domain walls in parallel to each other [see Fig. 1(d)]. With the numerically computed magnon energy $\hbar\omega_0$, we estimate the ordering temperature T_{hx} while self-consistently incorporating effects of the spatially rotating magnetic field induced by spin ordering [72]. Notably, reflecting the 2D nature of the system, the contributions from $J_{n \neq 0}^{\mu}$ result in an increase in T_{hx} more than an order of magnitude. Additionally, the helix-induced field couples to the electron spins, opening a partial gap in the domain wall spectrum. This gap leads to a Peierls energy gain, further stabilizing the spin helix. Consequently, we establish that the system forms a C_3 -breaking spatially phase-coherent quasi-2D spin helix at sufficiently low temperatures.

Magnon-induced singularity. In the spin helix phase, the magnons can lead to spin flips of electrons, which is known to influence electrical transport through backscattering [78–80]. Here, we instead look into the magnon-induced forward scattering and explore its effects on the scaling dimensions of various operators. To this end, we express Eq. (4) as $H_K \approx \langle H_K \rangle_{hx} + H_{em}$, with the expectation value $\langle \cdots \rangle_{hx}$ with respect to the spin helix phase. The coupling between the electron spin density and the magnon-induced spin flip can be formulated as

$$H_{em} = g_{em} \int \frac{dr}{2\pi} \sum_m [\partial_r \phi_{s,m}(r)] \Pi_m(r), \quad (8)$$

with the electron-magnon coupling strength $g_{em} \equiv -2J_K \sqrt{a S m_{2Q_F}} / (\hbar\omega_0 N_{\perp})$, $\Pi_m(r) \sim [a_m(r) - a_m^{\dagger}(r)]$, and magnon field a_m . In the bosonic language, magnons couple

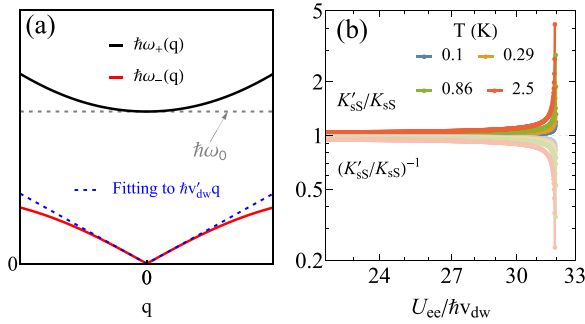


FIG. 2. (a) Excitation spectrum $\hbar\omega_{\pm}(q)$ (black/red curves) in electron-magnon-coupled systems. For small q , the lower band $\omega_{-}(q)$ follows linear dispersion with velocity v'_{dw} (blue dashed line). (b) Modified scaling exponent (K'_{ss}) as a function of the interaction strength (U_{ee}) for various temperatures and $L = 0.5 \mu\text{m}$, $N_{\perp} = 80$, $J_K = 1 \text{ meV}$, $\lambda_1 = \lambda_2 = 0.2$, and $N_{dw} = 20$.

to the spin-symmetric boson ($\phi_{ss,m}$) within each domain wall, which can therefore influence the electron subsystem. We present the corresponding excitation spectrum in Fig. 2(a), which is characterized by a gapped ω_{+} branch with band bottom ω_0 and a gapless branch ω_{-} with modified velocity v'_{dw} .

Since the electron-magnon coupling enters the Hamiltonian in quadratic form, we diagonalize the full system and compute the scaling dimension K'_{ss} ($1/K'_{ss}$) of the operator $e^{i\phi_{ss}}$ ($e^{i\theta_{ss}}$). This allows us to compare them to the one without magnons,

$$\frac{K'_{ss}}{K_{ss}} = \left(1 - \frac{J_K^2 a S K_{ss} m_{2Q_F}}{\hbar^2 v_{dw} \omega_0}\right)^{-\frac{1}{2}}. \quad (9)$$

As illustrated in Fig. 2(b), a singularity arises when the quantity in the parentheses of Eq. (9) vanishes. This is reflected in the divergence (vanishing) of the modified scaling dimension of the correlation function of ϕ_{ss} (θ_{ss}). The magnon-induced singularity can be observed through physical quantities such as paramagnetic susceptibility or spin relaxation rate, which depends on the renormalized parameter K'_{ss} and carrier velocity $v'_{dw} \propto 1/K'_{ss}$, and will be discussed later. The singularity can be experimentally accessed by adjusting the temperature and the interaction strength, U_{ee} , the latter of which is tunable through twist angle and interlayer bias [53]. More precisely, increasing the distance from a metallic gate, decreasing the twist angle, and/or increasing the bias voltage enhances the ratio $U_{ee}/(\hbar v_{dw})$ (see the Supplemental Material [72] for details). In contrast, we found that the exponents are robust against J_K (as $\omega_0 \propto J_K^2$) and the interaction parameters $\lambda_{1,2}$. A similar divergence driven by phonons has been discussed in (quasi-)one-dimensional systems [53,81–85].

Realization and transport features. Having demonstrated the general picture of the spin helix formation, we now discuss two scenarios for its realizations. The first one is TBG fabricated using ^{13}C isotopes, in which conduction electrons and nuclear spins couple through the hyperfine interaction. While detailed investigations of hyperfine coupling, similar to studies on semiconductors [86], remain absent for moiré systems, it is noteworthy that an experimental hyperfine coupling strength of $O(100 \mu\text{eV})$ has been reported

in nanotubes [63], exceeding the theoretical value [62]. We estimate $T_{hx} = O(10 \text{ mK})$ for $J_K = O(\mu\text{eV})$ and typical parameters for the electron subsystem (i.e., those adopted for Fig. S3 in the Supplemental Material [72]). The second scenario involves magnetic adatoms deposited on graphene layers, interacting with domain wall modes through exchange coupling. This has some parallels with previous studies on monolayer graphene [60,61,87–89], where an exchange coupling of 5 meV has been observed in samples with fluorine adatoms [60,61]. In this scenario, the exchange coupling generally exceeds the hyperfine coupling considered in the first scenario, and it can be further enhanced due to the increased electron density within the domain walls. We find that T_{hx} can reach up to $O(\text{K})$ for $J_K = O(\text{meV})$.

While the mesoscopic length scales discussed here should enable spin-sensitive scanning probes [90–92] to image helix formation, the presence of metallic gates may render this approach inapplicable. We therefore search for additional observable features. Since the spin helix can generate a spatially rotating magnetic field, which gaps out half of the electron modes, we expect the quantized conductance, for instance, in setups in Ref. [93], to reduce with onset at T_{hx} , providing an indirect probe for the spin helix formation [64,65,69,70]. Similar conductance reduction has been observed in GaAs quantum wires [94]. Moreover, one can probe the spin helix and the magnon-induced singularity through magnetic properties such as magnetic resonance and paramagnetic susceptibility, which we discuss next.

Magnetic resonance. The transition into the quasi-2D spin helix state, along with the properties on both sides of the transition, can be detected by measuring the spin relaxation rate $1/T_1$ in magnetic resonance experiments [95]. This rate captures the local dynamics of the magnetic moments due to the exchange interaction H_K and displays two regimes as displayed in Fig. 3 and described by

$$\frac{1}{T_1} \propto \begin{cases} T^{2g_{\leftarrow}^x - 2}, & T > T_{hx}, \\ [1 - m_{2Q_F}(T)] T^{2\bar{K} - 2}, & T \ll T_{hx}, \end{cases} \quad (10a)$$

$$T_1 \propto \begin{cases} T^{2g_{\leftarrow}^x - 2}, & T > T_{hx}, \\ [1 - m_{2Q_F}(T)] T^{2\bar{K} - 2}, & T \ll T_{hx}, \end{cases} \quad (10b)$$

where $(1 - m_{2Q_F}) \propto T^{\alpha}$ with a numerical exponent α obtained from fitting. Here, $g_{\leftarrow}^x$ and $\bar{K}$ are the fractional power-law exponents of the spin susceptibility for temperatures above and below T_{hx} , respectively. Their explicit forms are determined by the interacting strength through K_{cs} (see the Supplemental Material [72]):

$$g_{\leftarrow}^x = \frac{1}{4}(\bar{\Delta}_{\phi_{cs},n=0} + 3), \quad (11a)$$

$$\bar{K} = \int_{\pi/N_{dw}}^{\pi} \frac{d(q_{\perp}d)}{\pi} \sqrt{\frac{\tilde{K}_{cs}^2(q_{\perp}) + 3\tilde{K}_{cs}(q_{\perp})}{3\tilde{K}_{cs}(q_{\perp}) + 1}}, \quad (11b)$$

where $\bar{\Delta}_{\phi_{cs},n} \equiv (2\pi)^{-1} \int_{-\pi}^{\pi} d(q_{\perp}d) \tilde{K}_{cs}(q_{\perp}) \cos(nq_{\perp}d)$. In Eq. (11), the condition $K_{vp \neq cs} = 1$ has been used. We note that the analysis here can also be generalized for isolated channels. Above T_{hx} , the SLL phase exhibits a power-law temperature dependence in Eq. (10a), characterized by a fractional exponent $g_{\leftarrow}^x$, generalizing the conventional Korringa law for Fermi liquids [97,98], similarly to Rashba nanowires [99].

Sufficiently below T_{hx} , the decay takes a distinct form shown in Eq. (10b), where the factor $(1 - m_{2Q_F})$ weights

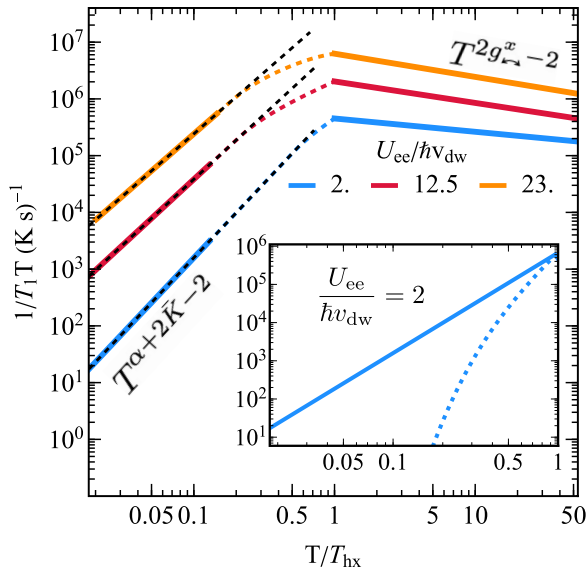


FIG. 3. Temperature (T) dependence of spin relaxation rate $1/(TT_1)$ for various interaction strengths (U_{ee}). The ordering temperature (T_{hx}) separates two regimes described by Eqs. (10a) and (10b). The black dashed lines represent the power-law fit for the low- T regime, while the colored dashed curves serve as visual guides for $T \lesssim T_{hx}$. The inset shows the contributions from gapped (dashed) and remaining gapless (solid) modes. The other parameter values are given in the caption of Fig. 2.

the disordered part of the localized spins. As mentioned, the formation of a helix induces a partial gap in the domain wall spectrum, which suppresses the relaxation channel from the gapped modes, giving rise to a rate of exponential form. As shown in the inset of Fig. 3, the relaxation is thus dominated by the remaining gapless modes, characterized by the effective parameter $\tilde{K}$. Consequently, we obtain a generalized power-law decay distinct from the conventional exponential suppression in fully gapped systems [100,101]. Additionally, upon approaching the magnon-induced singularity, the excitation velocity vanishes and the density of states becomes singular, leading to a divergence in the low-temperature regime of Fig. 3.

For temperatures slightly below T_{hx} , the crossover behavior in this regime (dashed curves in Fig. 3) may require further analysis beyond our approach [80,102]. To access this additional crossover scale, one should consider the dynamics of the local moments and the increased thermal population of magnons, which may influence the polarization of the local spins and thereby affect the spin relaxation rate. Nevertheless, the primary prediction here is the distinct power laws in Eqs. (10a) and (10b). Notably, our numerical results

for typical parameters indicate that the power law in the helix phase is generally steeper (specifically, $\alpha + 2\tilde{K} - 2 > 2 - 2g_{\leftarrow}^x$). Given the nanoscale nature of our target systems, the predicted features can be detected through resistively detected spin resonance experiments [103–109].

Anisotropic paramagnetic susceptibility. We examine the paramagnetic susceptibility $\chi_u = \mu_0(\partial M/\partial B)$ with the permeability μ_0 , the total magnetization M of the electron subsystem, and the in-plane external magnetic field $\mathbf{B} = B\hat{n}$ forming an angle φ with the domain wall direction [where $\hat{n} = (\sin \varphi, 0, \cos \varphi)$]. Taking into account the helix-induced effective field and electron-magnon coupling, we derive the contribution to the paramagnetic susceptibility, as summarized in Table I and detailed in Sec. S.VI of the Supplemental Material [72]. In particular, for $T < T_{hx}$, we get a $\cos^2 \varphi$ dependence of the anisotropic susceptibility, indicating the spontaneous breaking of rotational symmetry. The maximum χ_u thus identifies the specific domain wall direction where the spin helix forms, with magnitude depending on the e-e interaction and the moiré pattern. Notably, as the magnon-induced singularity is approached [see Fig. 2(b)], the paramagnetic susceptibility peaks, serving as an experimental indicator of the magnon-induced singularity.

Conclusions. We demonstrate the formation of a spatially phase-coherent planar spin helix and a magnon-induced singularity in TBG networks with observable features. Owing to its 2D nature, direct probing of the helix formation or magnon spectrum with inelastic neutron scatterings [110–115], resonant inelastic x-ray scatterings [116–119], or Lorentz microscope [90] might not be practical. Alternatively, spin resonance and paramagnetic susceptibility provide viable options; since the former is extensive, fabricating sizable TBG samples will be advantageous [120]. Given that conventional 2D Kondo lattice models typically describe noninteracting electrons, observing the features predicted here could unveil an electrically tunable platform where interacting electrons and Kondo physics interplay. This approach is instrumental in revealing various quantum phases, including skyrmion lattices stabilized by external magnetic fields [121,122], strange metals [123–126], and topological heavy fermion superconductivity [127].

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TABLE I. Paramagnetic spin susceptibility χ_u in terms of the overall scale, $\chi_u^0 \equiv \frac{\mu_0(g\mu_B)^2}{\pi\hbar v_{dw}}$. Here, we have $\tilde{K}_{cs}(q_{\perp} = 0) = K_{cs}(1 + \lambda_1 + \lambda_2)$ from Eq. (3). The detailed derivation is provided in Sec. S.VI of the Supplemental Material [72].

Temperature regime	$T \rightarrow 0$	$T \ll T_{hx}$	$T > T_{hx}$
Susceptibility χ_u/χ_u^0	$\frac{K_{ss} \cos^2 \varphi}{1 + \tilde{K}_{cs}(q_{\perp} = 0)K_{ss}}$	$\frac{K_{ss} \cos^2 \varphi}{(K_{ss}/K'_{ss})^2 + \tilde{K}_{cs}(q_{\perp} = 0)K_{ss}}$	K_{ss}

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Data availability. The data that support the findings of this Letter are openly available [128].

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- [1] J. M. Lopes Dos Santos, N. M. Peres, and A. H. Castro Neto, Graphene bilayer with a twist: Electronic structure, *Phys. Rev. Lett.* **99**, 256802 (2007).
 - [2] R. Bistritzer and A. H. MacDonald, Moiré bands in twisted double-layer graphene, *Proc. Natl. Acad. Sci. USA* **108**, 12233 (2011).
 - [3] G. Catarina, B. Amorim, E. V. Castro, J. M. Lopes, and N. Peres, *Handbook of Graphene* (John Wiley & Sons, New York, 2019), Vol. 8, pp. 177–231.
 - [4] G. Tarnopolsky, A. J. Kruchkov, and A. Vishwanath, Origin of magic angles in twisted bilayer graphene, *Phys. Rev. Lett.* **122**, 106405 (2019).
 - [5] W.-C. Wang, F.-W. Chen, K.-S. Lin, J. T. Hou, H.-C. Lin, and M.-Y. Chou, Role of the Fermi ring in determining the magic angles in twisted bilayer graphene, *Phys. Rev. B* **110**, 115154 (2024).
 - [6] E. Y. Andrei and A. H. MacDonald, Graphene bilayers with a twist, *Nat. Mater.* **19**, 1265 (2020).
 - [7] L. Balents, C. R. Dean, D. K. Efetov, and A. F. Young, Superconductivity and strong correlations in moiré flat bands, *Nat. Phys.* **16**, 725 (2020).
 - [8] K. P. Nuckolls and A. Yazdani, A microscopic perspective on moiré materials, *Nat. Rev. Mater.* **9**, 460 (2024).
 - [9] E. Y. Andrei, D. K. Efetov, P. Jarillo-Herrero, A. H. MacDonald, K. F. Mak, T. Senthil, E. Tutuc, A. Yazdani, and A. F. Young, The marvels of moiré materials, *Nat. Rev. Mater.* **6**, 201 (2021).
 - [10] Y. Cao, V. Fatemi, S. Fang, K. Watanabe, T. Taniguchi, E. Kaxiras, and P. Jarillo-Herrero, Unconventional superconductivity in magic-angle graphene superlattices, *Nature (London)* **556**, 43 (2018).
 - [11] Y. Cao, J. Y. Luo, V. Fatemi, S. Fang, J. D. Sanchez-Yamagishi, K. Watanabe, T. Taniguchi, E. Kaxiras, and P. Jarillo-Herrero, Superlattice-induced insulating states and valley-protected orbits in twisted bilayer graphene, *Phys. Rev. Lett.* **117**, 116804 (2016).
 - [12] Y. Cao, V. Fatemi, A. Demir, S. Fang, S. L. Tomarken, J. Y. Luo, J. D. Sanchez-Yamagishi, K. Watanabe, T. Taniguchi, E. Kaxiras, R. C. Ashoori, and P. Jarillo-Herrero, Correlated insulator behaviour at half-filling in magic-angle graphene superlattices, *Nature (London)* **556**, 80 (2018).
 - [13] H. Polshyn, M. Yankowitz, S. Chen, Y. Zhang, K. Watanabe, T. Taniguchi, C. R. Dean, and A. F. Young, Large linear-in-temperature resistivity in twisted bilayer graphene, *Nat. Phys.* **15**, 1011 (2019).
 - [14] Y. Cao, D. Chowdhury, D. Rodan-Legrain, O. Rubies-Bigorda, K. Watanabe, T. Taniguchi, T. Senthil, and P. Jarillo-Herrero, Strange metal in magic-angle graphene with near Planckian dissipation, *Phys. Rev. Lett.* **124**, 076801 (2020).
 - [15] H. Polshyn, J. Zhu, M. A. Kumar, Y. Zhang, F. Yang, C. L. Tschirhart, M. Serlin, K. Watanabe, T. Taniguchi, A. H. MacDonald, and A. F. Young, Electrical switching of magnetic order in an orbital Chern insulator, *Nature (London)* **588**, 66 (2020).
 - [16] A. L. Sharpe, E. J. Fox, A. W. Barnard, J. Finney, K. Watanabe, T. Taniguchi, M. A. Kastner, and D. Goldhaber-Gordon, Emergent ferromagnetism near three-quarters filling in twisted bilayer graphene, *Science* **365**, 605 (2019).
 - [17] M. Serlin, C. L. Tschirhart, H. Polshyn, Y. Zhang, J. Zhu, K. Watanabe, T. Taniguchi, L. Balents, and A. F. Young, Intrinsic quantized anomalous Hall effect in a moiré heterostructure, *Science* **367**, 900 (2020).
 - [18] A. L. Sharpe, E. J. Fox, A. W. Barnard, J. Finney, K. Watanabe, T. Taniguchi, M. A. Kastner, and D. Goldhaber-Gordon, Evidence of orbital ferromagnetism in twisted bilayer graphene aligned to hexagonal boron nitride, *Nano Lett.* **21**, 4299 (2021).
 - [19] Y. Cao, D. Rodan-Legrain, J. M. Park, N. F. Yuan, K. Watanabe, T. Taniguchi, R. M. Fernandes, L. Fu, and P. Jarillo-Herrero, Nematicity and competing orders in superconducting magic-angle graphene, *Science* **372**, 264 (2021).
 - [20] C. Rubio-Verdú, S. Turkel, Y. Song, L. Klebl, R. Samajdar, M. S. Scheurer, J. W. Venderbos, K. Watanabe, T. Taniguchi, H. Ochoa, L. Xian, D. M. Kennes, R. M. Fernandes, Á. Rubio, and A. N. Pasupathy, Moiré nematic phase in twisted double bilayer graphene, *Nat. Phys.* **18**, 196 (2022).
 - [21] M. Koshino, N. F. Q. Yuan, T. Koretsune, M. Ochi, K. Kuroki, and L. Fu, Maximally localized Wannier orbitals and the extended Hubbard model for twisted bilayer graphene, *Phys. Rev. X* **8**, 031087 (2018).
 - [22] H. C. Po, L. Zou, A. Vishwanath, and T. Senthil, Origin of Mott insulating behavior and superconductivity in twisted bilayer graphene, *Phys. Rev. X* **8**, 031089 (2018).
 - [23] Y. Xie, B. Lian, B. Jäck, X. Liu, C. L. Chiu, K. Watanabe, T. Taniguchi, B. A. Bernevig, and A. Yazdani, Spectroscopic signatures of many-body correlations in magic-angle twisted bilayer graphene, *Nature (London)* **572**, 101 (2019).
 - [24] J. Kang and O. Vafek, Symmetry, maximally localized Wannier states, and a low-energy model for twisted bilayer graphene narrow bands, *Phys. Rev. X* **8**, 031088 (2018).
 - [25] D. Wong, K. P. Nuckolls, M. Oh, B. Lian, Y. Xie, S. Jeon, K. Watanabe, T. Taniguchi, B. A. Bernevig, and A. Yazdani, Cascade of electronic transitions in magic-angle twisted bilayer graphene, *Nature (London)* **582**, 198 (2020).
 - [26] Y. Saito, F. Yang, J. Ge, X. Liu, T. Taniguchi, K. Watanabe, J. I. Li, E. Berg, and A. F. Young, Isospin Pomeranchuk effect in twisted bilayer graphene, *Nature (London)* **592**, 220 (2021).
 - [27] A. Rozen, J. M. Park, U. Zondiner, Y. Cao, D. Rodan-Legrain, T. Taniguchi, K. Watanabe, Y. Oreg, A. Stern, E. Berg, P. Jarillo-Herrero, and S. Ilani, Entropic evidence for a Pomeranchuk effect in magic-angle graphene, *Nature (London)* **592**, 214 (2021).

- [28] N. Tilak, X. Lai, S. Wu, Z. Zhang, M. Xu, R. d. A. Ribeiro, P. C. Canfield, and E. Y. Andrei, Flat band carrier confinement in magic-angle twisted bilayer graphene, *Nat. Commun.* **12**, 4180 (2021).
- [29] H. Shi and X. Dai, Heavy-fermion representation for twisted bilayer graphene systems, *Phys. Rev. B* **106**, 245129 (2022).
- [30] Z. D. Song and B. A. Bernevig, Magic-angle twisted bilayer graphene as a topological heavy fermion problem, *Phys. Rev. Lett.* **129**, 047601 (2022).
- [31] Y.-Z. Chou and S. Das Sarma, Kondo lattice model in magic-angle twisted bilayer graphene, *Phys. Rev. Lett.* **131**, 026501 (2023).
- [32] H. Hu, B. A. Bernevig, and A. M. Tsvelik, Kondo lattice model of magic-angle twisted-bilayer graphene: Hund's rule, local-moment fluctuations, and low-energy effective theory, *Phys. Rev. Lett.* **131**, 026502 (2023).
- [33] H. Hu, G. Rai, L. Crippa, J. Herzog-Arbeitman, D. C  zlug  zru, T. Wehling, G. Sangiovanni, R. Valent  , A. M. Tsvelik, and B. A. Bernevig, Symmetric Kondo lattice states in doped strained twisted bilayer graphene, *Phys. Rev. Lett.* **131**, 166501 (2023).
- [34] D. C  lug  ru, H. Hu, R. L. Merino, N. Regnault, D. K. Efetov, and B. A. Bernevig, The thermoelectric effect and its natural heavy fermion explanation in twisted bilayer and trilayer graphene, *arXiv:2402.14057*.
- [35] R. L. Merino, D. C  lug  ru, H. Hu, J. D  ez-M  rida, A. D  ez-Carl  n, T. Taniguchi, K. Watanabe, P. Seifert, B. A. Bernevig, and D. K. Efetov, Interplay between light and heavy electron bands in magic-angle twisted bilayer graphene, *Nat. Phys.* **21**, 1078 (2025).
- [36] L. L. H. Lau and P. Coleman, Topological mixed valence model for twisted bilayer graphene, *Phys. Rev. X* **15**, 021028 (2025).
- [37] P. San-Jose and E. Prada, Helical networks in twisted bilayer graphene under interlayer bias, *Phys. Rev. B* **88**, 121408(R) (2013).
- [38] J. S. Alden, A. W. Tsen, P. Y. Huang, R. Hovden, L. Brown, J. Park, D. A. Muller, and P. L. McEuen, Strain solitons and topological defects in bilayer graphene, *Proc. Natl. Acad. Sci. USA* **110**, 11256 (2013).
- [39] L. Ju, Z. Shi, N. Nair, Y. Lv, C. Jin, J. Velasco, C. Ojeda-Aristizabal, H. A. Bechtel, M. C. Martin, A. Zettl, J. Analytis, and F. Wang, Topological valley transport at bilayer graphene domain walls, *Nature (London)* **520**, 650 (2015).
- [40] L.-J. Yin, H. Jiang, J.-B. Qiao, and L. He, Direct imaging of topological edge states at a bilayer graphene domain wall, *Nat. Commun.* **7**, 11760 (2016).
- [41] D. K. Efimkin and A. H. MacDonald, Helical network model for twisted bilayer graphene, *Phys. Rev. B* **98**, 035404 (2018).
- [42] B. Tsim, N. N. Nam, and M. Koshino, Perfect one-dimensional chiral states in biased twisted bilayer graphene, *Phys. Rev. B* **101**, 125409 (2020).
- [43] P. Rickhaus, J. Wallbank, S. Slizovskiy, R. Pisoni, H. Overweg, Y. Lee, M. Eich, M. H. Liu, K. Watanabe, T. Taniguchi, T. Ihn, and K. Ensslin, Transport through a network of topological channels in twisted bilayer graphene, *Nano Lett.* **18**, 6725 (2018).
- [44] S. Huang, K. Kim, D. K. Efimkin, T. Lovorn, T. Taniguchi, K. Watanabe, A. H. MacDonald, E. Tutuc, and B. J. Leroy, Topologically protected helical states in minimally twisted bilayer graphene, *Phys. Rev. Lett.* **121**, 037702 (2018).
- [45] H. Yoo, R. Engelke, S. Carr, S. Fang, K. Zhang, P. Cazeaux, S. H. Sung, R. Hovden, A. W. Tsen, T. Taniguchi, K. Watanabe, G. C. Yi, M. Kim, M. Luskin, E. B. Tadmor, E. Kaxiras, and P. Kim, Atomic and electronic reconstruction at the van der Waals interface in twisted bilayer graphene, *Nat. Mater.* **18**, 448 (2019).
- [46] S. G. Xu, A. I. Berdyugin, P. Kumaravadivel, F. Guinea, R. Krishna Kumar, D. A. Bandurin, S. V. Morozov, W. Kuang, B. Tsim, S. Liu, J. H. Edgar, I. V. Grigorieva, V. I. Fal'ko, M. Kim, and A. K. Geim, Giant oscillations in a triangular network of one-dimensional states in marginally twisted graphene, *Nat. Commun.* **10**, 4008 (2019).
- [47] S. Carr, D. Massatt, S. B. Torrisi, P. Cazeaux, M. Luskin, and E. Kaxiras, Relaxation and domain formation in incommensurate two-dimensional heterostructures, *Phys. Rev. B* **98**, 224102 (2018).
- [48] M. Andelkovi  , L. Covaci, and F. M. Peeters, DC conductivity of twisted bilayer graphene: Angle-dependent transport properties and effects of disorder, *Phys. Rev. Mater.* **2**, 034004 (2018).
- [49] M. Fleischmann, R. Gupta, F. Wulfschl  ger, S. Theil, D. Weckbecker, V. Meded, S. Sharma, B. Meyer, and S. Shallcross, Perfect and controllable nesting in minimally twisted bilayer graphene, *Nano Lett.* **20**, 971 (2020).
- [50] T. Hou, Y. Ren, Y. Quan, J. Jung, W. Ren, and Z. Qiao, Metallic network of topological domain walls, *Phys. Rev. B* **101**, 201403 (2020).
- [51] N. R. Walet and F. Guinea, The emergence of one-dimensional channels in marginal-angle twisted bilayer graphene, *2D Mater.* **7**, 015023 (2019).
- [52] J. D. Verbakel, Q. Yao, K. Sotthewes, and H. J. W. Zandvliet, Valley-protected one-dimensional states in small-angle twisted bilayer graphene, *Phys. Rev. B* **103**, 165134 (2021).
- [53] H.-C. Wang and C.-H. Hsu, Electrically tunable correlated domain wall network in twisted bilayer graphene, *2D Mater.* **11**, 035007 (2024).
- [54] C. Chen, A. H. Castro Neto, and V. M. Pereira, Correlated states of a triangular net of coupled quantum wires: Implications for the phase diagram of marginally twisted bilayer graphene, *Phys. Rev. B* **101**, 165431 (2020).
- [55] C.-H. Hsu, D. Loss, and J. Klinovaja, General scattering and electronic states in a quantum-wire network of moir   systems, *Phys. Rev. B* **108**, L121409 (2023).
- [56] V. J. Emery, E. Fradkin, S. A. Kivelson, and T. C. Lubensky, Quantum theory of the smectic metal state in stripe phases, *Phys. Rev. Lett.* **85**, 2160 (2000).
- [57] A. Vishwanath and D. Carpentier, Two-dimensional anisotropic non-Fermi-liquid phase of coupled Luttinger liquids, *Phys. Rev. Lett.* **86**, 676 (2001).
- [58] R. Mukhopadhyay, C. L. Kane, and T. C. Lubensky, Sliding Luttinger liquid phases, *Phys. Rev. B* **64**, 045120 (2001).
- [59] R. Mukhopadhyay, C. L. Kane, and T. C. Lubensky, Crossed sliding Luttinger liquid phase, *Phys. Rev. B* **63**, 081103 (2001).
- [60] X. Hong, K. Zou, B. Wang, S. H. Cheng, and J. Zhu, Evidence for spin-flip scattering and local moments in dilute fluorinated graphene, *Phys. Rev. Lett.* **108**, 226602 (2012).

- [61] L. Fritz and M. Vojta, The physics of Kondo impurities in graphene, *Rep. Prog. Phys.* **76**, 032501 (2013).
- [62] J. Fischer, B. Trauzettel, and D. Loss, Hyperfine interaction and electron-spin decoherence in graphene and carbon nanotube quantum dots, *Phys. Rev. B* **80**, 155401 (2009).
- [63] H. O. Churchill, A. J. Bestwick, J. W. Harlow, F. Kuemmeth, D. Marcos, C. H. Stwertka, S. K. Watson, and C. M. Marcus, Electron-nuclear interaction in ^{13}C nanotube double quantum dots, *Nat. Phys.* **5**, 321 (2009).
- [64] B. Braunecker, P. Simon, and D. Loss, Nuclear magnetism and electron order in interacting one-dimensional conductors, *Phys. Rev. B* **80**, 165119 (2009).
- [65] B. Braunecker, P. Simon, and D. Loss, Nuclear magnetism and electronic order in ^{13}C nanotubes, *Phys. Rev. Lett.* **102**, 116403 (2009).
- [66] T. Meng and D. Loss, Helical nuclear spin order in two-subband quantum wires, *Phys. Rev. B* **87**, 235427 (2013).
- [67] J. Klinovaja, P. Stano, A. Yazdani, and D. Loss, Topological superconductivity and Majorana fermions in RKKY systems, *Phys. Rev. Lett.* **111**, 186805 (2013).
- [68] T. Meng, P. Stano, J. Klinovaja, and D. Loss, Helical nuclear spin order in a strip of stripes in the quantum Hall regime, *Eur. Phys. J. B* **87**, 203 (2014).
- [69] C.-H. Hsu, P. Stano, J. Klinovaja, and D. Loss, Antiferromagnetic nuclear spin helix and topological superconductivity in ^{13}C nanotubes, *Phys. Rev. B* **92**, 235435 (2015).
- [70] C.-H. Hsu, F. Ronetti, P. Stano, J. Klinovaja, and D. Loss, Universal conductance dips and fractional excitations in a two-subband quantum wire, *Phys. Rev. Res.* **2**, 043208 (2020).
- [71] T. Giamarchi, *Quantum Physics in One Dimension* (Oxford University Press, New York, 2003).
- [72] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/h5sd-v51h> for details on the sliding Luttinger liquid description of the domain-wall network in TBG, the RKKY interaction and spin susceptibility, the proof of the formation of a quasi-2D spin helix, and derivations of the magnon spectra, magnon-induced singularity, spin relaxation rate, and paramagnetic spin susceptibility.
- [73] X. C. Wu, C. M. Jian, and C. Xu, Coupled-wire description of the correlated physics in twisted bilayer graphene, *Phys. Rev. B* **99**, 161405(R) (2019).
- [74] Y. Z. Chou, Y. P. Lin, S. Das Sarma, and R. M. Nandkishore, Superconductor versus insulator in twisted bilayer graphene, *Phys. Rev. B* **100**, 115128 (2019).
- [75] J. M. Lee, M. Oshikawa, and G. Y. Cho, Non-Fermi liquids in conducting two-dimensional networks, *Phys. Rev. Lett.* **126**, 186601 (2021).
- [76] A. C. Hewson, *The Kondo Problem to Heavy Fermions, Cambridge Studies in Magnetism* (Cambridge University Press, Cambridge, England, 1993).
- [77] D. Paget, G. Lampel, B. Sapoval, and V. I. Safarov, Low field electron-nuclear spin coupling in gallium arsenide under optical pumping conditions, *Phys. Rev. B* **15**, 5780 (1977).
- [78] C.-H. Hsu, P. Stano, J. Klinovaja, and D. Loss, Effects of nuclear spins on the transport properties of the edge of two-dimensional topological insulators, *Phys. Rev. B* **97**, 125432 (2018).
- [79] C.-H. Hsu, P. Stano, J. Klinovaja, and D. Loss, Nuclear-spin-induced localization of edge states in two-dimensional topological insulators, *Phys. Rev. B* **96**, 081405(R) (2017).
- [80] C.-H. Hsu, P. Stano, J. Klinovaja, and D. Loss, Helical liquids in semiconductors, *Semicond. Sci. Technol.* **36**, 123003 (2021).
- [81] J. Bardeen, Electron-vibration interactions and superconductivity, *Rev. Mod. Phys.* **23**, 261 (1951).
- [82] G. Wentzel, The interaction of lattice vibrations with electrons in a metal, *Phys. Rev.* **83**, 168 (1951).
- [83] T. Martin and D. Loss, Phase diagram for a Luttinger liquid coupled to phonons in one dimension, *Int. J. Mod. Phys. B* **09**, 495 (1995).
- [84] D. Loss and T. Martin, Wentzel-Bardeen singularity and phase diagram for interacting electrons coupled to acoustic phonons in one dimension, *Phys. Rev. B* **50**, 12160 (1994).
- [85] C.-H. Hsu, Interaction- and phonon-induced topological phase transitions in double helical liquids, *Nanoscale Horiz.* **9**, 1725 (2024).
- [86] J. Schliemann, A. Khaetskii, and D. Loss, Electron spin dynamics in quantum dots and related nanostructures due to hyperfine interaction with nuclei, *J. Phys.: Condens. Matter* **15**, R1809 (2003).
- [87] B. Uchoa, T. G. Rappoport, and A. H. Castro Neto, Kondo quantum criticality of magnetic adatoms in graphene, *Phys. Rev. Lett.* **106**, 016801 (2011).
- [88] T. O. Wehling, A. V. Balatsky, M. I. Katsnelson, A. I. Lichtenstein, and A. Rosch, Orbitaly controlled Kondo effect of Co adatoms on graphene, *Phys. Rev. B* **81**, 115427 (2010).
- [89] J. Ren, H. Guo, J. Pan, Y. Y. Zhang, X. Wu, H.-G. Luo, S. Du, S. T. Pantelides, and H.-J. Gao, Kondo effect of cobalt adatoms on a graphene monolayer controlled by substrate-induced ripples, *Nano Lett.* **14**, 4011 (2014).
- [90] Y. Togawa, T. Koyama, K. Takayanagi, S. Mori, Y. Kousaka, J. Akimitsu, S. Nishihara, K. Inoue, A. S. Ovchinnikov, and J. Kishine, Chiral magnetic soliton lattice on a chiral helimagnet, *Phys. Rev. Lett.* **108**, 107202 (2012).
- [91] A. Dussaux, P. Schönherr, K. Koumpouras, J. Chico, K. Chang, L. Lorenzelli, N. Kanazawa, Y. Tokura, M. Garst, A. Bergman, C. L. Degen, and D. Meier, Local dynamics of topological magnetic defects in the itinerant helimagnet FeGe, *Nat. Commun.* **7**, 12430 (2016).
- [92] X. Z. Yu, Y. Onose, N. Kanazawa, J. H. Park, J. H. Han, Y. Matsui, N. Nagaosa, and Y. Tokura, Real-space observation of a two-dimensional skyrmion crystal, *Nature (London)* **465**, 901 (2010).
- [93] Z. Hou, K. Yuan, and H. Jiang, Arrays of one-dimensional conducting channels in minimally twisted bilayer graphene, *Phys. Rev. B* **110**, L161406 (2024).
- [94] C. P. Scheller, T. M. Liu, G. Barak, A. Yacoby, L. N. Pfeiffer, K. W. West, and D. M. Zumbühl, Possible evidence for helical nuclear spin order in GaAs quantum wires, *Phys. Rev. Lett.* **112**, 066801 (2014).
- [95] The spin relaxation rate investigated here also supplements the predicted absorption frequency in Ref. [96], which is governed by the RKKY energy scale with external magnetic fields producing side peaks.
- [96] P. Stano and D. Loss, NMR response of nuclear-spin helix in quantum wires with hyperfine and spin-orbit interaction, *Phys. Rev. B* **90**, 195312 (2014).
- [97] J. Korringa, Nuclear magnetic relaxation and resonance line shift in metals, *Physica* **16**, 601 (1950).

- [98] P. Coleman, *Introduction to Many-Body Physics* (Cambridge University Press, Cambridge, England, 2015).
- [99] A. A. Zyuzin, T. Meng, V. Kornich, and D. Loss, Nuclear spin relaxation in Rashba nanowires, *Phys. Rev. B* **90**, 195125 (2014).
- [100] P. M. Singer, P. Wzietek, H. Alloul, F. Simon, and H. Kuzmany, NMR evidence for gapped spin excitations in metallic carbon nanotubes, *Phys. Rev. Lett.* **95**, 236403 (2005).
- [101] B. Dóra, M. Gulácsi, F. Simon, and H. Kuzmany, Spin gap and Luttinger liquid description of the NMR relaxation in carbon nanotubes, *Phys. Rev. Lett.* **99**, 166402 (2007).
- [102] J. I. Väyrynen, F. Geissler, and L. I. Glazman, Magnetic moments in a helical edge can make weak correlations seem strong, *Phys. Rev. B* **93**, 241301 (2016).
- [103] M. Dobers, K. v. Klitzing, J. Schneider, G. Weimann, and K. Ploog, Electrical detection of nuclear magnetic resonance in GaAs–Al_xGa_{1–x}As heterostructures, *Phys. Rev. Lett.* **61**, 1650 (1988).
- [104] M. Fanciulli, *Electron Spin Resonance and Related Phenomena in Low-Dimensional Structures* (Springer, Berlin, Heidelberg, 2009), Vol. 115.
- [105] N. Kumada, K. Muraki, and Y. Hirayama, Low-frequency spin dynamics in a canted antiferromagnet, *Science* **313**, 329 (2006).
- [106] N. Kumada, K. Muraki, and Y. Hirayama, NMR evidence for spin canting in a bilayer $\nu = 2$ quantum Hall system, *Phys. Rev. Lett.* **99**, 076805 (2007).
- [107] L. Tiemann, G. Gamez, N. Kumada, and K. Muraki, Unraveling the spin polarization of the $\nu = 5/2$ fractional quantum Hall state, *Science* **335**, 828 (2012).
- [108] L. Tiemann, T. D. Rhone, N. Shibata, and K. Muraki, NMR profiling of quantum electron solids in high magnetic fields, *Nat. Phys.* **10**, 648 (2014).
- [109] E. Morissette, J. X. Lin, D. Sun, L. Zhang, S. Liu, D. Rhodes, K. Watanabe, T. Taniguchi, J. Hone, J. Pollanen, M. S. Scheurer, M. Lilly, A. Mounce, and J. I. Li, Dirac revivals drive a resonance response in twisted bilayer graphene, *Nat. Phys.* **19**, 1156 (2023).
- [110] S. Lovesey, *Theory of Neutron Scattering from Condensed Matter: Nuclear Scattering, International Series of Monographs on Physics* (Clarendon Press, Oxford, 1984).
- [111] S. V. Maleyev, Investigation of spin chirality by polarized neutrons, *Phys. Rev. Lett.* **75**, 4682 (1995).
- [112] S. V. Grigoriev, Y. O. Chetverikov, D. Lott, and A. Schreyer, Field induced chirality in the helix structure of Dy/Y multilayer films and experimental evidence for Dzyaloshinskii-Moriya interaction on the interfaces, *Phys. Rev. Lett.* **100**, 197203 (2008).
- [113] G. L. Squires, *Introduction to the Theory of Thermal Neutron Scattering*, 3rd ed. (Cambridge University Press, Cambridge, England, 2012).
- [114] B. Willis and C. Carlile, *Experimental Neutron Scattering* (Oxford University Press, New York, 2017).
- [115] A. Furrer, J. Mesot, and T. Strässle, *Neutron Scattering in Condensed Matter Physics* (World Scientific, Singapore, 2009).
- [116] L. J. Ament, M. Van Veenendaal, T. P. Devereaux, J. P. Hill, and J. Van Den Brink, Resonant inelastic x-ray scattering studies of elementary excitations, *Rev. Mod. Phys.* **83**, 705 (2011).
- [117] Faris Gel'mukhanov, M. Odelius, S. P. Polyutov, A. Föhlisch, and V. Kimberg, Dynamics of resonant x-ray and Auger scattering, *Rev. Mod. Phys.* **93**, 035001 (2021).
- [118] J. van den Brink, in *Quantum Materials: Experiments and Theory*, edited by E. Pavarini, E. Koch, J. van den Brink, and G. Sawatzky (Forschungszentrum Jülich GmbH Institute for Advanced Simulation, Jülich, 2016), Chap. 12, pp. 316–346.
- [119] S. Fatale, S. Moser, and M. Grioni, Magnetic excitations in soft x-ray RIXS: Recent developments, *J. Electron Spectrosc. Relat. Phenom.* **200**, 274 (2015).
- [120] Y.-C. Chen, W.-H. Lin, W.-S. Tseng, C.-C. Chen, G. Rossman, C.-D. Chen, Y.-S. Wu, and N.-C. Yeh, Direct growth of mm-size twisted bilayer graphene by plasma-enhanced chemical vapor deposition, *Carbon* **156**, 212 (2020).
- [121] T. Okubo, S. Chung, and H. Kawamura, Multiple- q states and the skyrmion lattice of the triangular-lattice Heisenberg antiferromagnet under magnetic fields, *Phys. Rev. Lett.* **108**, 017206 (2012).
- [122] Y. Togawa, Y. Kousaka, K. Inoue, and J.-I. Kishine, Symmetry, structure, and dynamics of monoaxial chiral magnets, *J. Phys. Soc. Jpn.* **85**, 112001 (2016).
- [123] Y. Komijani and P. Coleman, Model for a ferromagnetic quantum critical point in a 1D Kondo lattice, *Phys. Rev. Lett.* **120**, 157206 (2018).
- [124] Y. Komijani and P. Coleman, Emergent critical charge fluctuations at the Kondo breakdown of heavy fermions, *Phys. Rev. Lett.* **122**, 217001 (2019).
- [125] J. Wang, Y.-Y. Chang, C.-Y. Mou, S. Kirchner, and C.-H. Chung, Quantum phase transition in a two-dimensional Kondo-Heisenberg model: A dynamical Schwinger-boson large- n approach, *Phys. Rev. B* **102**, 115133 (2020).
- [126] J. Wang, Y.-Y. Chang, and C.-H. Chung, A mechanism for the strange metal phase in rare-earth intermetallic compounds, *Proc. Natl. Acad. Sci. USA* **119**, e2116980119 (2022).
- [127] T. Hazra and P. Coleman, Triplet pairing mechanisms from Hund's-Kondo models: Applications to UTe₂ and CeRh₂As₂, *Phys. Rev. Lett.* **130**, 136002 (2023).
- [128] Y.-Y. Chang and K. Saito, Codes for research on helix in twisted bilayer graphene (version 2), Zenodo (2025), <https://doi.org/10.5281/zenodo.15572017>.