

Phenomenology of orbital torque, pumping, and mixing conductance in metallic bilayers

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The conversion between spin and orbital currents is at the origin of the orbital torque and its Onsager reciprocal, the orbital pumping. Here, we propose a phenomenological model to describe the orbital torque in magnetic bilayers composed of an orbital source (i.e., a light metal such as Ti, Ru, CuO_x, etc.) and a spin-orbit coupled magnet (i.e., typically Ni, (Co/Pt)_n, etc.). This approach accounts for spin-to-orbit and orbit-to-spin conversion in the ferromagnet and at the interface. We show that the orbital torque arises from a compromise between orbital current injection from the orbital source to the ferromagnet and spin current backflow from the ferromagnet back to the orbital source. We also discuss the concept of orbital-mixing conductance and introduce the “orbit-spin-mixing” and “spin-orbit-mixing” conductances that govern the orbital torque and orbital pumping, respectively.

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I. INTRODUCTION

Spin-orbit torque (SOT) induced by an electrical current has garnered significant attention for effectively manipulating the magnetization in spintronic heterostructures [1,2], standing out as a promising candidate for next-generation memory and logic devices [3,4]. In a bilayer composed of a nonmagnetic metal (N) and a ferromagnet (F), it encompasses a two-step process: the generation of a spin current via spin Hall effect (SHE) [5,6] or spin Rashba-Edelstein effect (SREE) [7,8] in layer N, followed by its absorption by the adjacent layer F. Because both SHE and SREE arise from spin-orbit coupling (SOC), efficient SOT devices require the use of heavy metals (Pt, W, etc.). An alternative to SOT can be achieved by using currents of orbital angular momentum rather than spin currents. Such orbital currents can be efficiently generated in light metals (Ti, Al, Zr, oxidized Cu) upon charge-orbit interconversion mechanisms such as the orbital Hall effect (OHE) [9,10] and the orbital Rashba-Edelstein effect [11,12]. In this case, an orbital current is generated in N in the absence of SOC, and is subsequently absorbed in F through orbit-to-spin conversion mediated by SOC [13]. This orbital torque and its parent effect, the orbital (Hall or Rashba) magnetoresistance, have been reported recently [14–18]. A

hallmark of the orbital torque is its dependence as a function of F's thickness. Since the orbital current is more likely to perform long-range transport than the spin current in F [19], the strength of the torque exhibits a steady enhancement with increasing F's thickness, while the conventional SOT quickly reaches saturation [17,18,20–23].

A practical way to model SOT in an N/F bilayer is to combine a drift-diffusion model that describes the generation of the spin current in N and the spin-mixing conductance boundary condition that describes the spin current absorption in F [24–27]. The concept of spin-mixing conductance is only valid in the absence of SOC [28]. To better account for such an effect, various strategies have been adopted, either using an effective interfacial layer [29], using more general Green's function techniques [30], or generalizing the magnetoelectronic circuit theory [31–33]. For orbital torque, the charge-to-orbital conversion has been computed from first principles [34–38], and the resulting torque in N/F bilayers has been obtained using, e.g., tight-binding modeling [13]. Such an approach provides an instructive microscopic description but lacks transparency. Therefore, a phenomenological approach similar to the one used to model SOT is highly desired. An attempt has been made to extend the spin drift-diffusion theory to orbital transport [17,39] by introducing a coupling length representing the conversion between spin and orbital moments in N, while the absorption of the spin and orbital currents is modeled by an effective mixing conductance at the N/F interface. This approach provides reasonable scaling but assumes that the orbital current is fully absorbed at the interface, which is incorrect, as pointed out by Ref. [19]: Since the orbital moment does not directly couple to the magnetization, it does not experience dephasing, in contrast with the spin moment. Recent microscopic theories

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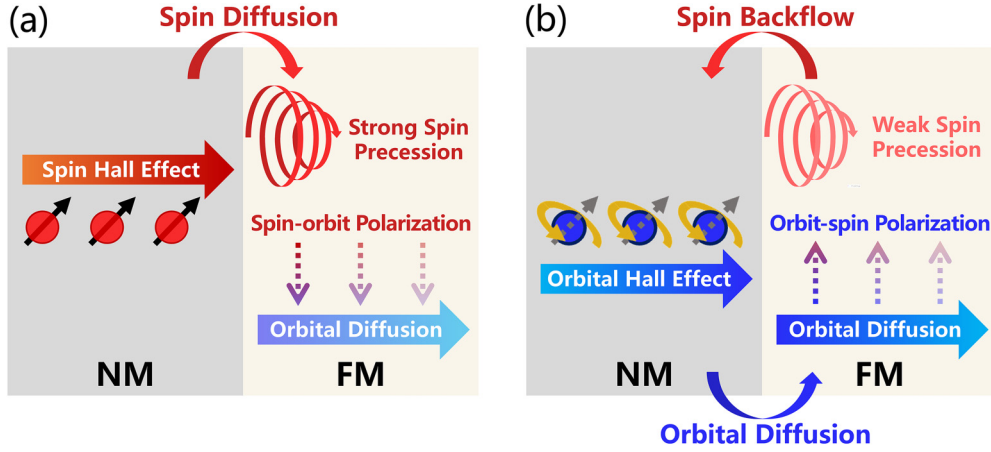


FIG. 1. Current intensities and diffusion directions for both spins and orbits when (a) SHE or (b) OHE is dominant in N.

[40,41] have pointed out additional features that enrich the physics of orbital transport. First, since the orbital momentum operator does not commute with the Hamiltonian in general, it is not a conserved quantity (even in the absence of SOC). This has two important consequences: (1) First, the orbital moment exhibits a much smaller diffusion coefficient than the spin or the charge [41]. In addition, an orbital moment penetrating a normal metal oscillates around the local crystal field [40]. (2) Second, in the presence of SOC, spin and orbital currents are coupled through a so-called spin-to-orbit and orbit-to-spin polarization [41]. This coefficient is different from the spin-orbit length introduced in Ref. [39], as further discussed in the present article. Although additional microscopic investigations are necessary to clarify the nature of these effects at interfaces and the impact of disorder and defects, these features are remarkable and must be accounted for to properly model orbital transport.

In the present work, we develop a drift-diffusion model that describes the spin and orbital transport induced by SHE and OHE in N/F bilayers. This approach accounts for spin and orbital diffusion inside F, as well as spin-to-orbit conversion in F and at the N/F interface. It enables us to clarify the parameters that control the orbital torque, demonstrating the interplay between orbital current injection and spin current backflow. Finally, we discuss the concept of orbital-mixing conductance and its connection to orbital pumping.

II. THEORY

We intend to describe the phenomenology of orbital torque in an N/F bilayer, as depicted in Fig. 1. The theory of spin transfer torque has been studied in great detail over the past two decades, pointing out the central role of spin precession and dephasing in F [42–45]. Since the spin dephasing length is usually short in most transition metal ferromagnets (typically, ≈ 1 nm), the physics governing the absorption of an incoming spin current in a ferromagnet is often modeled by the so-called interfacial spin-mixing conductance [28,46,47], i.e., a boundary condition that connects the interfacial spin current with the drop in spin chemical potential across the interface. It is tempting to develop a similar concept for orbital torque, the “orbital-mixing conductance,” that connects the impinging

orbital current with the drop in orbital chemical potential. Nonetheless, since orbital currents diffuse over much longer distances than spin currents in ferromagnets [17,18,20–23], a comprehensive model needs to properly account for the actual spin precession and dephasing inside F.

A. Two-channel spin-orbit transport

We assume that N accommodates both OHE and SHE. The spin diffusion equations in the presence of both SOC and magnetic exchange have been studied intensively over the past two decades, resulting in rather complex intermingling of intrinsic and extrinsic SHE, spin swapping, spin precession, dephasing, and relaxation [44,48,49]. To keep our theory tractable, we chose to retain only the terms that are the most relevant to the phenomena we are interested in. Assuming that in a nonmagnetic metal, the spin and orbital transport are governed by diffusion, relaxation, SHE, and OHE, the diffusion equations that connect the spin (orbital) chemical potential $\mu_{s(o)}^N$ to the spin (orbital) current tensor $\mathbf{J}_{s(o)}^N$ read [48]

$$\mathbf{J}_s^N = -\sigma_s^N \partial_z \mu_s^N + \sigma_{\text{SHE}} \hat{\sigma} \times \mathbf{E}, \quad \partial_z^2 \mu_s^N = \frac{\mu_s^N}{\lambda_s^{N2}}, \quad (1)$$

$$\mathbf{J}_o^N = -\sigma_o^N \partial_z \mu_o^N + \sigma_{\text{OHE}} \hat{l} \times \mathbf{E}, \quad \partial_z^2 \mu_o^N = \frac{\mu_o^N}{\lambda_o^{N2}}. \quad (2)$$

Here, $\hat{\sigma}$ ($\hat{l}$) is the unit vector of the spin (orbital) polarization, $\lambda_{s(o)}^N$ is the spin (orbital) relaxation length, $\sigma_{s(o)}^N$ is the spin (orbital) longitudinal conductivity, and $\sigma_{\text{SHE(OHE)}}$ is the spin (orbital) Hall conductivity. In other words, we consider that the spin and orbital degrees of freedom propagate along distinct channels. Recent theories suggest that the orbital relaxation is primarily governed by the crystal field interaction, in contrast with spin relaxation, which is mostly governed by spin-orbit coupling [50,51]. Without loss of generality, the spin and orbital currents are expressed in units of a charge current to avoid unnecessarily cumbersome notation.

When SOC is present, the two channels are coupled, and thus, an incoming spin current is accompanied by an orbital current and vice versa [41]. We disregard this effect in Eqs. (1) and (2) because it does not add new physics and only leads to renormalized transport coefficients. Nonetheless, in F, this

effect is crucial to convert the incoming orbital current into a spin current and drive the orbital torque. Therefore, the drift-diffusion equations can be modified as

$$\mathbf{J}_s^F = -\sigma_s^F \partial_z \mu_s^F - P_{os} \sigma_o^F \partial_z \mu_o^F, \quad (3)$$

$$\mathbf{J}_o^F = -\sigma_o^F \partial_z \mu_o^F - P_{so} \sigma_s^F \partial_z \mu_s^F, \quad (4)$$

where P_{os} and P_{so} stand for the orbit-to-spin and spin-to-orbit polarization, respectively. In Ref. [41], we have computed these coefficients in realistic materials and demonstrated that the spin-to-orbit and orbit-to-spin polarizations can be as large as spin polarization in transition metals (i.e., several tens of percent). In addition, the itinerant spin density directly couples to the magnetization via the exchange coupling, so that the spin chemical potential obeys [44,45,49,52,53]

$$\frac{1}{\sigma_s^F} \partial_z \mathbf{J}_s^F = -\frac{1}{\lambda_{sf}^2} \mu_s^F - \frac{1}{\lambda_J^2} \mu_s^F \times \mathbf{m} - \frac{1}{\lambda_\phi^2} \mathbf{m} \times (\mu_s^F \times \mathbf{m}), \quad (5)$$

where λ_{sf} is the spin diffusion length, λ_J is the spin precession length, and λ_ϕ is the spin dephasing length. Hence, the component of the spin density that is aligned on the magnetization $\mathbf{m}$ simply diffuses into F, whereas the transverse components experience a damped precession [44,45,49,52,53]. In contrast, the orbital density only couples to the crystal field potential, which induces its relaxation [50]. The drift-diffusion equation is therefore

$$\frac{1}{\sigma_o^F} \partial_z \mathbf{J}_o^F = -\frac{\mu_o^F}{\lambda_{F2}^2}. \quad (6)$$

This expression is the simplest phenomenological form of the orbital diffusion and neglects the precession of the orbital momentum around the crystal field predicted in Refs. [40,41], and that remained to be thoroughly discussed. In addition, we point out that the direct coupling between spin and orbital chemical potential introduced phenomenologically in Ref. [39] is absent from our equations, as, to the best of our knowledge, this term still requires firm microscopic grounds. In contrast, the coupling between orbital and spin degrees of freedom ($\sim P_{so}, P_{os}$), which enters the definition of the spin and orbital currents in Eqs. (3) and (4), mixes spin and orbital moments in Eqs. (5) and (6), thereby leading to intertwined spin and orbital profiles, as discussed in Sec. III.

B. Boundary conditions

As mentioned in the introduction, at the N/F interface, spin and orbital moments are expected to experience enhanced relaxation [29–33]. In the absence of a microscopic theory of interfacial orbital relaxation, we choose to model this effect by inserting an effective interfacial layer “Int,” assuming that the spin and orbital currents are continuous at both FM/Int and Int/NM interfaces. In other words, (spin and orbital) currents and chemical potentials are connected by a set of conductances, akin to Ohm’s law,

$$\mathbf{J}_s^i = \mathbf{J}_s^{i+1} = G_s(\mu_s^i - \mu_s^{i+1}) + G_{os}(\mu_o^i - \mu_o^{i+1}), \quad (7)$$

$$\mathbf{J}_o^i = \mathbf{J}_o^{i+1} = G_o(\mu_o^i - \mu_o^{i+1}) + G_{so}(\mu_s^i - \mu_s^{i+1}), \quad (8)$$

where i is the i th layer (F, N, Int), $G_{s(o)}$ is the interfacial spin (orbital) conductance, and $G_{os(so)}$ models the transfer between spin and orbital momenta that may occur in the presence of interfacial SOC. These interfacial spin-orbit conductances play the same role as the spin-orbit polarizations introduced above: They unlock the conversion of an incoming orbital current into a spin current and enable orbital torque. The coefficient G_{os} is of particular interest to us as it connects the orbital chemical potential to the spin current and acts like an orbital-spin mixing conductance, as discussed further below. Notice that, in principle, these interfacial conductances are 3×3 tensors as spin and orbital precession might take place at the interface. Understanding these effects would require a systematic *ab initio* treatment that goes beyond the scope of the present work.

As mentioned above, the spin precession and dephasing taking place in F usually occur over a short distance, typically ~ 1 nm. Therefore, it is often convenient to assume that the spin density injected inside it is immediately absorbed close to the interface, i.e., $\mu_s^F \rightarrow 0$. In this case, in the limit of FM thickness larger than the typical spin decoherence length and neglecting the interfacial spin and orbital memory loss, the boundary conditions are reduced to

$$\mathbf{J}_s^N = G_{s,r}^{\uparrow\downarrow} \mathbf{m} \times (\mu_s^N \times \mathbf{m}) + G_{s,i}^{\uparrow\downarrow} \mathbf{m} \times \mu_s^N + G_{os}(\mu_o^N - \mu_o^F), \quad (9)$$

$$\mathbf{J}_o^F = \mathbf{J}_o^N = G_o(\mu_o^N - \mu_o^F) + G_{so} \mu_s^N, \quad (10)$$

where $G_s^{\uparrow\downarrow} = G_{s,r}^{\uparrow\downarrow} + iG_{s,i}^{\uparrow\downarrow}$ is the spin-mixing conductance [46] (the factor “2” appearing in the original definition is absorbed in $G_s^{\uparrow\downarrow}$). In the next section, we solve this set of equations to compute the orbital torque in N/F bilayers.

III. ANATOMY OF ORBITAL TORQUE

A. Definition of the SOT

In the following, we assume that the torque originates from the transfer of spin angular momentum from the itinerant electrons to the magnetization. In other words, we assume that the orbital current does not directly participate in the SOT [54]. As a result, the torque $\boldsymbol{\tau}$ reads

$$\boldsymbol{\tau} = - \int_0^{d_F} \mathbf{m} \times (\nabla \cdot \mathbf{J}_s \times \mathbf{m}) = \mathbf{m} \times (\mathbf{J}_s^N(0) \times \mathbf{m}), \quad (11)$$

where $\mathbf{J}_s^N(0)$ is the spin current at the N/F interface. In other words, the SOT strength is solely characterized by the interfacial spin current, regardless of its origin (SHE or OHE). In the configuration we adopt, the bilayer is extended along the z direction, and the electric field is applied along $\mathbf{x}$. As a result, SHE and OHE generate a current of angular momentum polarized along $\mathbf{y}$. According to Eq. (5), the penetration of the spin into F is accompanied by a precession, and consequently, the SOT possesses two components, $\boldsymbol{\tau} = \tau_{\parallel} \mathbf{m} \times (\mathbf{y} \times \mathbf{m}) + \tau_{\perp} \mathbf{m} \times \mathbf{y}$, usually referred to as the in-plane and perpendicular components. The former is responsible for the dampinglike torque and the latter for the fieldlike torque [2]. In the following, we express the torque as a complex quantity,

defined $\tau = \tau_{\parallel} + i\tau_{\perp}$. With this formulation, the SOT takes the general form

$$\tau_{\text{SHE}} = \eta_s \sigma_{\text{SHE}} E \left(1 - \cosh^{-1} \frac{d_N}{\lambda_s^N} \right), \quad (12)$$

$$\tau_{\text{OHE}} = \eta_o \sigma_{\text{OHE}} E \left(1 - \cosh^{-1} \frac{d_N}{\lambda_o^N} \right), \quad (13)$$

where η_s and η_o are the (complex) spin and orbital transmission efficiencies, and d_N is the thickness of layer N. The detailed expression of these efficiencies is rather complex and difficult to interpret transparently since they involve the interplay between spin and orbital diffusion, interfacial scattering, and spin-orbit interconversion. Therefore, before discussing the numerical results, it is worth inspecting the general form of these two transmission efficiencies in certain limits. To do so, we define the effective (spin and orbital) conductance $\tilde{G}_{s(o)}^{N(F)}$ that quantifies the ability for a given metal (N or F) to absorb the (spin or orbital) current,

$$\tilde{G}_{s(o)}^{N(F)} = \frac{\sigma_{s(o)}^{N(F)}}{\lambda_{s(o)}^{N(F)}}, \quad (14)$$

where the (possibly complex) spin (orbital) diffusion lengths read $\tilde{\lambda}_s^F = (\alpha - i\beta)^{-1}$ and $\tilde{\lambda}_{s(o)}^{N(F)} = \lambda_{s(o)}^{N(F)} \coth[d_{N(F)}/\lambda_{s(o)}^{N(F)}]$. The coefficients α , β are defined as

$$\alpha, \beta = \frac{1}{2} \left[\sqrt{\left(\frac{1}{\lambda_{sf}^2} + \frac{1}{\lambda_{\phi}^2} \right)^2 + \frac{1}{\lambda_J^4} \pm \frac{1}{\lambda_{sf}^2} \pm \frac{1}{\lambda_{\phi}^2}} \right]^{\frac{1}{2}}. \quad (15)$$

The larger the conductivity and the smaller the diffusion length, the larger the ability of the metal to absorb a (spin or orbital) current. In addition, we assume that the thickness of F is larger than the spin dephasing length, $e^{2d_F(\alpha-i\beta)} \gg 1$, and we account for SOC perturbatively so that the bulk and interfacial spin-to-orbit (orbit-to-spin) polarizations, $P_{os(so)}$ and $\gamma_{so(os)} = G_{os(so)}/G_{o(s)}$, are considered up to the first order only. These limits, physically reasonable, allow us to determine explicit expressions of the spin and orbital transmission efficiencies.

B. Spin Hall effect-driven SOT

To clarify the physics behind the orbital torque, let us first consider the standard case of the SHE-driven torque ($\sigma_{\text{SHE}} \neq 0$, $\sigma_{\text{OHE}} = 0$). Considering first the diffusion of the spin current into F [Eq. (5)], the spin transmission efficiency reads

$$\eta_s = \left(1 + \frac{\tilde{G}_s^N}{\tilde{G}_s^F} \right)^{-1}. \quad (16)$$

The SOT efficiency is determined by the relative strengths between the effective spin conductances of N and F. For example, if N is highly conductive, the itinerant spins are likely to stay in N instead of penetrating F, reducing the proportion of transmitted spin current and, thus, the SOT. In other words, to maximize the SHE-driven SOT, the bilayer should be designed such that $\tilde{G}_s^F \gg \tilde{G}_s^N$. If SOC is turned on in F, the spin current is accompanied by an orbital current [see Eqs. (3) and (4)]. The contribution of this orbital current to

the SOT is indirect as it simply renormalizes the efficiency by a factor $1 + \gamma_{so}\gamma_{os} \approx 1$. This negligible contribution of the orbital currents is rather obvious since the primary mechanism for SOT is the SHE. When the complete absorption of the spin current at the interface is assumed, Eq. (16) remains valid with $\tilde{G}_s^F \rightarrow G_s^{\uparrow\downarrow}$. The spin-mixing conductance $G_{\uparrow\downarrow}$ includes the noncollinear spin precession contributions, so the resultant η_s can have both real and imaginary parts, corresponding to dampinglike torque and fieldlike torque, respectively.

C. Orbital Hall effect-driven SOT

The orbital torque efficiency, η_o , presents a very different structure that manifests the alternative scenario governing orbital injection. Assuming that spin-to-orbit conversion takes place in F [Eqs. (3) and (4)], we obtain the efficiency

$$\eta_o = P_{os} \left(1 + \frac{\tilde{G}_s^F}{\tilde{G}_s^N} \right)^{-1} \left(1 + \frac{\tilde{G}_o^N}{\tilde{G}_o^F} \right)^{-1}. \quad (17)$$

The efficiency is the product of three terms. First, the term $\sim (1 + \frac{\tilde{G}_s^F}{\tilde{G}_s^N})^{-1}$ represents the injection of the orbital current from N into F. Once in F, this orbital current is converted into a spin current via the orbit-to-spin polarization factor P_{os} . Finally, the last term $\sim (1 + \frac{\tilde{G}_o^N}{\tilde{G}_o^F})^{-1}$ represents the injection of the spin current from F into N. In other words, since the SOT is due to the absorption of the spin current, it is directly related to the spin current backflow from F to N, as illustrated in Fig. 1(b). This is a distinctive feature compared to SHE-driven SOT that scales with the direct injection of the spin current from N to F [see Eq. (16)].

Equation (17) corresponds to a scenario where the orbit-to-spin conversion takes place in F, as achieved in Refs. [22,23,39]. Alternatively, the orbit-to-spin conversion can be boosted by inserting a thin layer with strong SOC between N and F, e.g., $\text{CuO}_x/\text{Pt/TmG}$ [14], $\text{Cr}/(\text{Gd}, \text{Pt})/\text{Co}$ [16], or $\text{Co}/\text{Pt}/\text{CuO}_x$ [59–61]. In this case, the conversion rather happens at the N/F interface [Eqs. (7) and (8)], and the expression of η_o changes to

$$\eta_o = \gamma_{os} \frac{\tilde{G}_s^F}{G_s} \left(1 + \frac{\tilde{G}_s^F}{\tilde{G}_s^N} \right)^{-1} \left[1 + \tilde{G}_o \left(\frac{1}{G_o} + \frac{1}{\tilde{G}_o^F} \right) \right]^{-1}. \quad (18)$$

As depicted in Fig. 1(b), it contains the orbital transmission, G_o , the orbit-to-spin conversion, G_{os} , the spin precession, $\tilde{G}_s^F$, and the spin backflow, G_s . If one assumes that the spin current is absorbed at the interface, we simply need to make the substitution $\tilde{G}_s^F, G_s \rightarrow G_s^{\uparrow\downarrow}$.

Although derived in the limit of small orbital-spin interconversion coefficients, these different expressions are instrumental in determining the scenario of OHE-driven SOT. Importantly, using different assumptions, such as bulk versus interfacial orbit-to-spin conversion and interfacial absorption of the incoming spin current (i.e., adopting the spin-mixing conductance approach), does not modify the overall picture. To optimize the OHE-driven SOT, one needs to maximize the injection of the orbital current from N into F and maximize the spin current backflow from F into N. So ideally, the materials composing the bilayer should be such that $\tilde{G}_s^F \gg \tilde{G}_s^N$ and $\tilde{G}_o^F \gg \tilde{G}_o^N$.

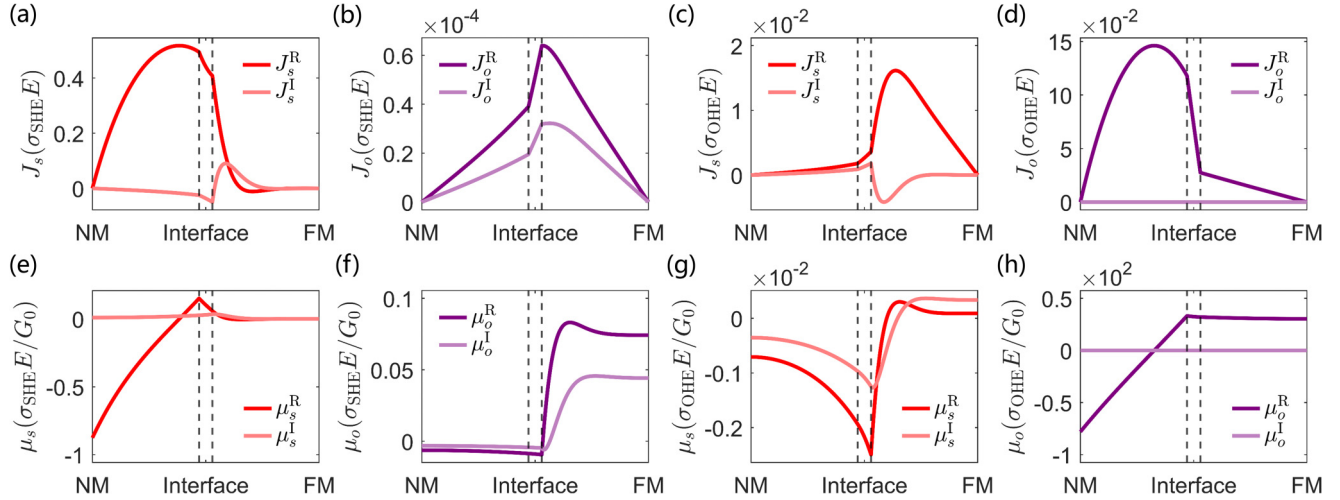


FIG. 2. (a), (c) Spin and (b), (d) orbital current profiles induced by SHE [panels (a), (b), (e), and (f)] and OHE [panels (c), (d), (g), and (h)]. The corresponding spin and orbital chemical potential profiles are shown in panels (e), (g) and (f), (h), respectively. In the definition of the chemical potentials, we introduce the scaling parameter $G_0 = 10^{15} \Omega^{-1} \text{m}^{-2}$. The superscripts R and I refer to the real $[\propto \mathbf{m} \times (\mathbf{y} \times \mathbf{m})]$ and imaginary parts $[\propto \mathbf{m} \times \mathbf{y}]$ of the spin and orbital vectors. The material parameters are set to $d_N = d_F = 5 \text{ nm}$, $\lambda_s^N = \lambda_{sf} = 3 \text{ nm}$ [55,56], $\lambda_o^N = 6 \text{ nm}$ [57], $\lambda_o^F = 15 \text{ nm}$ [58], $\lambda_\phi = 1 \text{ nm}$, $\lambda_J = 0.5 \text{ nm}$, $\sigma_s^N = \sigma_s^F = 3 \times 10^6 \Omega^{-1} \text{m}^{-1}$, $\sigma_o^N = \sigma_o^F = 4 \times 10^4 \Omega^{-1} \text{m}^{-1}$, $P_{so} = 0.02$, and $P_{os} = 0.8$ [41]. To account for interfacial spin and orbital memory loss, we inserted an effective interfacial layer of thickness $d_{\text{int}} = d_N/8$ and spin (orbital) relaxation length $\lambda_{s(o)}^{\text{int}} = \lambda_{s(o)}^N/2$.

IV. ORBITAL TORQUE MODELING

A. Current and chemical potential profiles

The expressions of the SOT discussed in the previous section were obtained assuming thick F and weak SOC. Let us now inspect the SOT efficiency by computing the orbital and spin currents across the N/F bilayer using the drift-diffusion model exposed in Sec. II A. For these calculations, we account for the spin precession inside F [Eq. (5)]. The parameters are indicated in the caption of Fig. 2. Notice that, according to Ref. [41], $\sigma_s \gg \sigma_o$, which results in $P_{os} \gg P_{so}$. This makes the spin-to-orbit and orbit-to-spin conversions highly asymmetric.

As a benchmark, we first consider the case of SHE-driven SOT, and we reproduce the (spin and orbital) current and density profiles in Figs. 2(a) and 2(b). The spin current and density profile shown in Figs. 2(a) and 2(e) are well known and have been discussed elsewhere (see, e.g., Ref. [26]). Since spin-to-orbit conversion takes place in F, the penetration of the spin current is accompanied by a tiny orbital current ($J_o/J_s \sim 10^{-4}$) [Fig. 2(b)]. Notice that the orbital density diffuses over much longer distances but only weakly affects the SOT, as discussed in more detail below.

Let us now pay attention to the spin and orbital current profiles induced by OHE, displayed in Figs. 2(c) and 2(d). Since the transport is driven by OHE in N, the spin current flowing in N is small ($J_s/J_o \sim 10^{-2}$) [Figs. 2(c) and 2(g)], whereas the orbital current displays the conventional exponential profile reflecting the Hall effect [Figs. 2(d) and 2(h)] (compare with [Figs. 2(a) and 2(c)]). Once in F, where orbit-to-spin conversion occurs, the spin current is quickly absorbed close to the interface, experiencing a damped oscillation [Figs. 2(c) and 2(g)], whereas the orbital current slowly diffuses [Figs. 2(d) and 2(h)].

Finally, we stress that the nonequilibrium spin and orbital moments (current and density) possess two components,

in-plane $[\propto \mathbf{m} \times (\mathbf{y} \times \mathbf{m})]$ and perpendicular to the $(\mathbf{m}, \mathbf{y})$ plane $[\propto \mathbf{m} \times \mathbf{y}]$, and represented, respectively, by the real and imaginary parts of $J_{s(o)}$ and $\mu_{s(o)}$. Figure 2(d) shows that OHE first generates an orbital current polarized along $\mathbf{m} \times (\mathbf{y} \times \mathbf{m})$, which remains dominant during the full diffusion and orbit-to-spin conversion process. The orbital-to-spin conversion in F results in a spin current polarized along the same direction, $\propto \mathbf{m} \times (\mathbf{y} \times \mathbf{m})$. Due to exchange coupling, this newly generated spin current precesses around the magnetization, inducing a perpendicular component, $\propto \mathbf{m} \times \mathbf{y}$ [see Fig. 2(c)]. The perpendicular component of orbital current subsequently generated by spin-to-orbit conversion is too small to be detectable in Fig. 2(d).

B. Thickness dependence of orbital torque

A hallmark of the orbital contribution to dampinglike SOT is the dependence of the torque efficiency on the thickness of F [17,18,20–23]. Figures 3(a) and 3(d) correspond to SHE-driven SOT and Figs. 3(b) and 3(e) correspond to OHE-driven SOT. In Fig. 3(a), the SHE-driven SOT rapidly saturates with the thickness of F, and its magnitude is nearly independent of the spin diffusion length. This is because the spin current absorption $\sim \alpha - i\beta$ is dominantly determined by the spin precession λ_J and spin dephasing λ_ϕ lengths close to the interface [26]. On the contrary, the OHE-driven SOT [Fig. 3(b)] shows a clear dependence on the thickness of F and its orbital diffusion length, as the orbital moment does not directly couple to the magnetization. It is important to note that the orbital torque decreases when increasing λ_o^F , which seems counterintuitive. In fact, the weak orbital relaxation in F enables the orbital backflow into N, thereby reducing the amount of orbital current effectively injected into F. Conversely, the thickness dependence of the orbital torque is almost insensitive to the

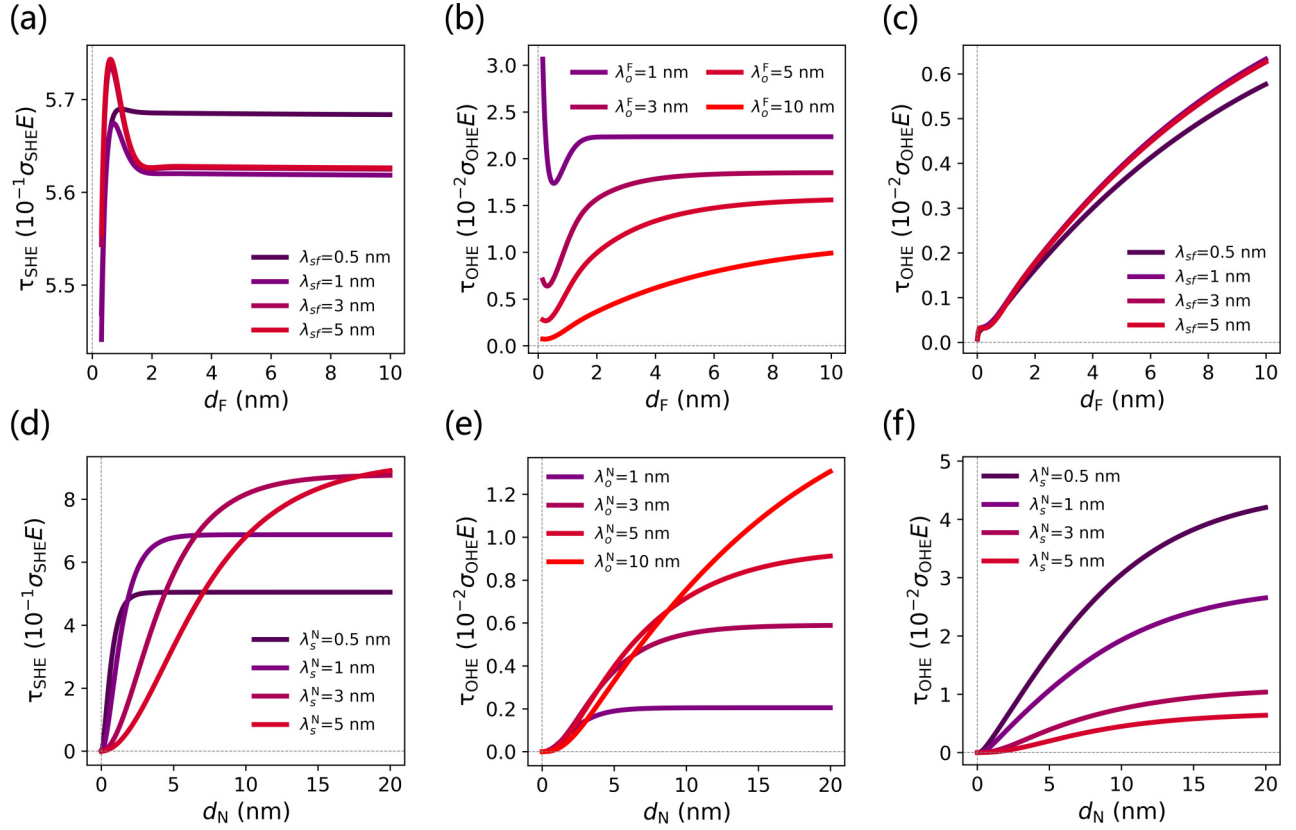


FIG. 3. Dampinglike SOT induced by SHE as a function of (a) d_F and (d) d_N for various values of the spin relaxation length (a) λ_{sf} and (d) λ_s^N . Dampinglike SOT induced by OHE as a function of d_F for various values of (b) the orbital relaxation length λ_o^F and (c) the spin relaxation length λ_{sf} . Dampinglike SOT induced by OHE as a function of d_N for various values of (e) the orbital relaxation length λ_o^N and (f) the spin relaxation length λ_s^N . Unless otherwise specified, the parameters are the same as in Fig. 2.

spin diffusion length in F, λ_{sf} , as long as it remains smaller than the spin dephasing length λ_ϕ [see Fig. 3(c)].

The dependence of SOT on the thickness of layer N is reported in Figs. 3(d) and 3(e). Here, as we consider the total SOT, the accumulation term $(1 - \cosh^{-1} d_N/\lambda_{s(o)}^N)$ and the efficiency term $\eta_{s(o)}$ are in competition with each other, leading to different correlations between the SOT and $\lambda_{s(o)}^N$ depending on the thickness. The accumulation term increases with d_N and saturates for $d_N \approx 5\lambda_{s(o)}^N$, whereas the efficiency $\eta_{s(o)}$ increases with $d_N/\lambda_{s(o)}^N$. In other words, the orbital torque can be optimized by choosing a short λ_o^F , a long λ_o^N , and $d_N \approx 5\lambda_o^N$. Interestingly, the thickness dependence of the orbital torque is very sensitive to the spin diffusion length in N, λ_s^N : Reducing λ_s^N enhances the spin current backflow and leads to a larger torque [see Fig. 3(f)].

C. Orbital-mixing conductance and orbital pumping

We conclude this work by commenting on the nature of the orbital-mixing conductance and connecting it to orbital pumping. As mentioned in the introduction, given the tremendous success of the concept of spin-mixing conductance, it is appealing to define an orbital version of this interfacial boundary condition. The spin-mixing conductance, which connects the interfacial spin chemical potential to the interfacial spin current, has been originally defined using the scattering-matrix theory, $G_s^{\uparrow\downarrow} = (2e^2/hA) \sum_{nm} (\delta_{nm} - r_{nn}^\uparrow r_{nn}^{\downarrow*})$, where r_{nn}^σ is the

spin-dependent reflection coefficient for the n, m modes at the N/F interface [28,46,47]. In the framework of the drift-diffusion theory developed here, we obtain the alternative definition

$$G_s^{\uparrow\downarrow} = \left[\frac{1}{G_s} + \frac{1}{\sigma_s^F(\alpha - i\beta)} \right]^{-1}, \quad (19)$$

which expresses the spin-mixing conductance as a function of the spin precession, dephasing, and relaxation lengths in F.

One can conceive different definitions for the orbital-mixing conductance. The definition that is the closest to the spin-mixing conductance connects the interfacial orbital chemical potential to the interfacial *orbital* current, G_o [see Eq. (8)]. However, because the orbital current is absorbed far away from the interface and does not directly produce a torque on the magnetization, this definition is not particularly useful for modeling SOT. A more practical definition connects the interfacial orbital chemical potential to the interfacial *spin* current that is quickly absorbed close to the interface, as discussed in the previous section. The definition of this “orbit-spin-mixing conductance” differs depending on the adopted model. If the orbit-to-spin conversion is accomplished inside F, we obtain

$$G_{os}^m = G_o P_{os}. \quad (20)$$

The orbit-spin-mixing conductance arises from the transmission of orbital currents at the N/F interface followed by the

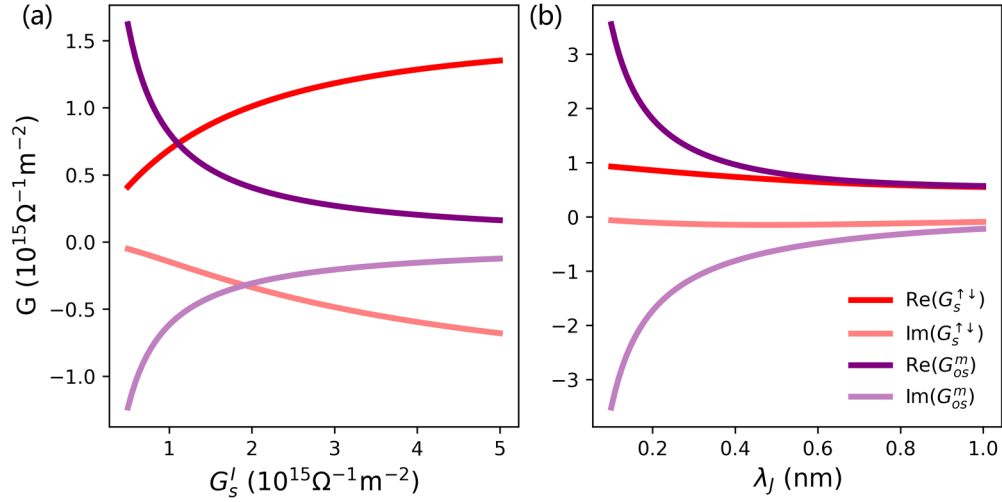


FIG. 4. Spin-mixing conductance (red), $G_s^{\uparrow\downarrow}$, and orbit-spin-mixing conductance (purple), G_{os}^m , as a function of (a) the interfacial spin conductance and (b) the spin precession length. Without loss of generality, we fix $\lambda_J = 0.5 \text{ nm}$ in panel (a) and $G_s = 1 \times 10^{15} \Omega^{-1} \text{m}^{-2}$ in panel (b). The other parameters are set to $G_{os} = 5 \times 10^{14} \Omega^{-1} \text{m}^{-2}$, $\sigma_s^F = 1 \times 10^6 \Omega^{-1} \text{m}^{-1}$, $\lambda_{sf} = 3 \text{ nm}$, and $\lambda_\phi = 1 \text{ nm}$.

orbit-to-spin conversion in F. It is worth noticing that no spin precession terms appear in Eq. (20) since the spin current is simply dragged by the orbital current in F, as reported in Fig. 2(b). If the orbit-to-spin conversion occurs at the interface, we obtain

$$G_{os}^m = \frac{G_{os}}{G_s} \sigma_s^F (\alpha - i\beta). \quad (21)$$

Likewise, we can attribute the orbit-spin-mixing conductance to three substeps: the orbit-to-spin conversion by G_{os} , the spin precession by $\sigma_s^F (\alpha - i\beta)$, and the spin backflow by G_s . It is distinct from the bulk orbit-to-spin conversion scenario, as the spin current is not dragged by the orbital current but arises from interfacial “filtering”; as a consequence, the spin current backflow controls the magnitude of the orbit-spin-mixing conductance.

Figure 4 displays the dependence of the (complex) spin-mixing conductance (red lines) and the orbit-spin-mixing conductance (purple) as a function of (a) the interfacial spin conductance and (b) the spin precession length. Compared to the spin-mixing conductance, the orbit-spin-mixing conductance is generally more sensitive to the changes in both interfacial and ferromagnetic parameters. It is also worth noticing that the spin-mixing conductance and the orbit-spin-mixing conductance show different signs of correlation with the interfacial spin conductance, reflecting the diffusion and backflow of spin as illustrated in Fig. 1.

This concept enables us to propose a phenomenological expression for the orbital pumping, recently investigated using microscopic models [19,62,63] and studied experimentally [64]. Based on Eqs. (9) and (10), we obtain the response tensor that connects generalized currents with generalized forces [65]

$$\begin{pmatrix} \mathbf{I}_s \\ \mathbf{I}_o \\ \partial_t \mathbf{m} \end{pmatrix} = \begin{pmatrix} \frac{A}{e^2} G_s^{\uparrow\downarrow} & \frac{A}{e^2} G_{os}^m & \mathcal{L}_{ms} \\ \frac{A}{e^2} G_{so}^m & \frac{A}{e^2} G_o & \mathcal{L}_{mo} \\ \mathcal{L}_{sm} & \mathcal{L}_{om} & \frac{\gamma}{M_s \Omega} \mathbf{m} \times \end{pmatrix} \begin{pmatrix} e \Delta \boldsymbol{\mu}_s \\ e \Delta \boldsymbol{\mu}_o \\ \mathbf{F}_m \end{pmatrix}. \quad (22)$$

Here, $\mathbf{I}_{s,o} = (A/e) \mathbf{J}_{s,o}$ is particle current associated with the spin and orbital current density, A is the contact area between N and F, and G_{os}^m and G_{so}^m are the effective orbit-spin-mixing and spin-orbit-mixing conductances introduced above. $\mathbf{F}_m = \partial_t W \approx -(M_s \Omega / \gamma) \mathbf{m} \times \partial_t \mathbf{m}$ is the magnetic driving force, where M_s is the saturation magnetization (in A/m) and Ω is the volume of the magnetic layer. The matrix elements are 3×3 tensors in angular momentum space. The torque is defined $\mathbf{T} = (\mu_B / e d M_s) \mathbf{m} \times (\mathbf{J}_s \times \mathbf{m})$ and is readily connected to $e \Delta \boldsymbol{\mu}_{s,o}$ via Eq. (22), giving

$$\mathcal{L}_{sm} = \frac{\mu_B}{e^2 d M_s} G_s^{\uparrow\downarrow}, \quad \mathcal{L}_{om} = \frac{\mu_B}{e^2 d M_s} G_{os}^m. \quad (23)$$

Applying Onsager’s reciprocity [65], we obtain $\mathcal{L}_{ms} = \mathcal{L}_{sm}$ and $\mathcal{L}_{mo} = (\mu_B / e^2 d M_s) G_{so}^m$. The first term produces the well-known spin pumping, $\mathbf{J}_s = (\hbar/e) G_s^{\uparrow\downarrow} \mathbf{m} \times \partial_t \mathbf{m}$, whereas the second one quantifies the orbital pumping, i.e., $\mathbf{J}_o = (\hbar/e) G_{so}^m \mathbf{m} \times \partial_t \mathbf{m}$.

V. CONCLUSION

In this work, we have developed a phenomenological model to describe the orbital torque in magnetic bilayers accounting for spin-orbit interconversion in F and at the N/F interface. It enables us to clarify the parameters that control the orbital torque, demonstrating the interplay between orbital current injection and spin current backflow. The current and potential profiles indicate that, despite its short dephasing length, the spin current exhibits a long propagation in F, driven by the orbit-spin polarization of the smoothly diffusing orbital current. Our calculations also reveal that a long orbital relaxation length in F results in a smoother thickness dependence but also in a larger saturation. This suggests that the orbital torque scales with the orbital relaxation length. Finally, we discuss the concept of orbital-mixing conductance and introduce the orbit-spin-mixing and spin-orbit-mixing conductances that govern the orbital torque and orbital pumping, respectively.

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DATA AVAILABILITY

The datasheets for the figures are available on the Zenodo server [66].

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